Including Quantification in Defeasible Reasoning for the Description Logic $\mathcal{EL}_\bot$

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Abstract. Defeasible Description Logics (DDLs) can state defeasible concept inclusions and often use rational closure according to the KLM postulates for reasoning. If in DDLs with quantification a defeasible subsumption relationship holds between concepts, it can also hold if these concepts appear nested in existential restrictions. Earlier reasoning algorithms did not detect this kind of relationships. We devise a new form of canonical models that extend classical ones for $\mathcal{EL}_\bot$ by elements that satisfy increasing amounts of defeasible knowledge and show that reasoning w.r.t. these models yields the missing rational entailments.

1 Introduction

Description Logics (DLs) *concepts* describe groups of objects by means of other concepts (unary FOL predicates) and roles (binary relations). Such concepts can be related to other concepts in the TBox. Technically, the TBox is a theory constraining the interpretation of concepts. The lightweight DL $\mathcal{EL}$ allows as complex concepts: conjunction and existential restriction which is a form of existential quantification. A prominent DL reasoning problem is to compute subsumption relationships between two concepts. Such a relationship holds, if all instances of one concept must be necessarily instances of the other (w.r.t. the TBox). In $\mathcal{EL}$ subsumption can be computed in polynomial time [1] which made it the choice as the basis for OWL 2 EL, standardised by the W3C. In $\mathcal{EL}$ satisfiability is trivial, since no negation can be expressed. $\mathcal{EL}_\bot$ can express disjointness of concepts and is thus more interesting for non-monotonic reasoning.

Defeasible DLs (DDLs) are non-monotonic extensions of DLs and were intensively investigated [4,5,2,3,6]. Most DDLs allow to state additional relationships between concepts that characterize typical instances and that can get defeated if they cause an inconsistency. The notions of defeasibility and typicality are closely related: the more defeasible information is used for reasoning about a concept, the more typical its instances are regarded. Reasoning problems for DDLs are defined w.r.t. different entailment relations using different forms of closure. We consider rational closure, which is the basis for stronger forms of entailment and based on the KLM postulates [7]. Casini and Straccia lifted these postulates for

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propositional logic to DLs [4]. In [3] Casini et al. devise a reduction for computing rational closure by materialisation, where the idea is to encode a consistent subset of the defeasible statements as a concept and then use this in classical subsumption queries as additional constraint for the (potential) sub-concept in the query. Their translation of the KLM postulates to DLs yields rational entailments of propositional nature, but does not regard existential restrictions. To be precise, an existential restriction, e.g. $\exists r.B$ requires existence of an element related via role r to an element belonging to concept B—which cannot be expressed in propositional logic. By neglecting quantification in the translation of the postulates, a defeasible implication between concepts need not hold, if these occur nested in existential restrictions. This is at odds with the basic principle of defeasible reasoning: defeasible information *should* be used for reasoning if no more specific knowledge contradicts it. Consequently, the materialisation-based approach [3] misses entailments, since it treats all role successors uniformly as non-typical concept members. Quantified concepts are disregarded in several algorithms for defeasible reasoning in DLs that employ materialisation, such as lexicographic [5] and relevant [3] closure. This even holds for preferential model semantics, by the result of Britz et al. in [2], that entailments of preferential model semantics coincide with those of rational closure in [3].

In this paper we devise a reasoning algorithm for rational entailment for $\mathcal{EL}_\bot$ that derives defeasible knowledge for concepts in existential restrictions by a redution to classical reasoning. To this end we extend the well-known canonical models for $\mathcal{EL}$ to *typicality models* that use domain elements to represent differing levels of typicality of a concept. For a simple form of these typicality models we show that it entails the same consequences as the materialisation-based approach [3]. We devise an extension of these models that capture the maximal typicality of each role successor element individually and thereby facilitates defeasible reasoning also for concepts nested in existential restrictions. These maximal typicality models yield (potentially) more rational entailments. Due to space constraints, we need to refer the reader to the technical report [9] for an introduction to DLs and for the proofs of the claims presented here.

2 Typicality Models for Rational Entailment

In order to achieve rational entailment for quantified concepts, defeasible concept inclusion statements (DCIs) need to hold for concepts in (nested) existential restrictions. We want to decide subsumption between the $\mathcal{EL}_\bot$-concepts C and D w.r.t. the $\mathcal{EL}_\bot$-DKB $\mathcal{K} = (\mathcal{T}, \mathcal{D})$. In order to deal with possible inconsistencies between DBox $\mathcal{D}$ and TBox $\mathcal{T}$, the reasoning procedure needs to consider subsets of the DBox. We use the same sequence of subsets of the DBox as in the materialisation-based approach [3]. Let $\overline{\mathcal{D}} = \bigsqcap_{E \mathrel{\underset{\sim}{\sqsubset}} F \in \mathcal{D}} (\neg E \sqcup F)$ be the materialisation of $\mathcal{D}$. For a DKB $\mathcal{K}$, the set of exceptional DCIs from $\mathcal{D}$ w.r.t. $\mathcal{T}$ is defined as $\mathcal{E}(\mathcal{D}) = \{C \mathrel{\underset{\sim}{\sqsubset}} D \in \mathcal{D} \mid \mathcal{T} \models \overline{\mathcal{D}} \sqcap C \sqsubseteq \bot\}$. The *sequence of DBox subsets* is defined inductively as $\mathcal{D}_0 = \mathcal{D}$ and $\mathcal{D}_i = \mathcal{E}(\mathcal{D}_{i-1})$ for $i \geq 1$. If $\mathcal{D}$ is finite, there exists a least n with $\mathcal{D}_n = \mathcal{D}_{n+1}$, which we call the *rank of* $\mathcal{D}$ ($rk(\mathcal{D}) = n$). If $\mathcal{D}_n = \emptyset$, then the DKB $\mathcal{K}$ is called *well-separated*, since the set of DCIs can turn consistent by iteratively removing consistent DCIs. A given DKB $\mathcal{K} = (\mathcal{T}, \mathcal{D})$ can be transformed into a well-separated DKB $\mathcal{K}' = (\mathcal{T} \cup \{C \sqsubseteq \bot \mid C \mathrel{\sqsubset\!\!\!\sim} D \in \mathcal{D}_n\}, \mathcal{D} \setminus \mathcal{D}_n)$ by deciding a quadratic number of subsumption tests in the size of $\mathcal{D}$. We assume that a given DKB $\mathcal{K} = (\mathcal{T}, \mathcal{D})$ is well-separated and thus the sequence $\mathcal{D}_0, \dots, \mathcal{D}_n$ for $n = rk(\mathcal{D})$ ends with $\mathcal{D}_n = \emptyset$. The index of a DBox $\mathcal{D}_i$ in this sequence is referred to as its (level of) *typicality*: a lower i indicates higher typicality of $\mathcal{D}_i$.

Canonical models for classical $\mathcal{EL}$ [1], represent every concept F that occurs in an existential restriction in $\mathcal{T}$, i.e., $F \in Qc(\mathcal{K})$ by a single domain element $d_F \in \Delta$. One concept F can be used in several existential restrictions inducing different role-successors with different typicality. There are up to $rk(\mathcal{D}) = n$ different levels of typicality to be reflected in the model.

Definition 1. *Let $\mathcal{K} = (\mathcal{T}, \mathcal{D})$ be a DKB with $rk(\mathcal{D}) = n$. A* complete typicality domain *is defined as $\Delta^{\mathcal{K}} = \bigcup_{i=0}^{n} \{d_F^i \mid F \in Qc(\mathcal{K})\}$. A domain Δ with $\{d_F^n \mid F \in Qc(\mathcal{K})\} \subseteq \Delta \subseteq \Delta^{\mathcal{K}}$ is a* typicality domain*. An interpretation over a typicality domain is a* typicality interpretation*. A typicality interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ is a* model of $\mathcal{K}$ *(written $\mathcal{I} \models \mathcal{K}$) iff $\mathcal{I} \models \mathcal{T}$ and $d_F^i \in G^{\mathcal{I}} \implies d_F^i \in H^{\mathcal{I}}$ for all $G \mathrel{\sqsubset\!\!\!\sim} H \in \mathcal{D}_i$, $0 \leq i \leq rk(\mathcal{D})$, $F \in Qc(\mathcal{K})$.*

We specify when defeasible subsumption relationships hold in typicality interpretations.

Definition 2. *Let $\mathcal{I}$ be a typicality interpretation. Then $\mathcal{I}$* satisfies a defeasible subsumption *$C \mathrel{\sqsubset\!\!\!\sim} D$ (written $\mathcal{I} \models C \mathrel{\sqsubset\!\!\!\sim} D$) iff $d_C^i \in D^{\mathcal{I}}$ for $0 \leq i \leq n$ s.t. $d_C^i \in \Delta^{\mathcal{I}}$ and $d_C^{i-1} \notin \Delta^{\mathcal{I}}$.*

To construct a model for $\mathcal{K}$ by means of a TBox, the auxiliary names from $N_C^{aux} \subseteq N_C \setminus sig(\mathcal{K})$ introduce representatives for $F \in Qc(\mathcal{K})$ on each level of typicality. We use $F_{\mathcal{D}} \in N_C^{aux}$ to define the *extended TBox* of concept F w.r.t. $\mathcal{D}$: $\mathcal{T}_{\mathcal{D}}(F) = \mathcal{T} \cup \{F_{\mathcal{D}} \sqsubseteq F\} \cup \{F_{\mathcal{D}} \sqcap G \sqsubseteq H \mid G \mathrel{\sqsubset\!\!\!\sim} H \in \mathcal{D}\}$. Here $\{F_{\mathcal{D}} \sqsubseteq F\}$ propagates all constraints on F to $F_{\mathcal{D}}$. The last set of GCIs is an equivalent $\mathcal{EL}_{\bot}$-rewriting of $\{F_{\mathcal{D}} \sqsubseteq \neg G \sqcup H \mid G \mathrel{\sqsubset\!\!\!\sim} H \in \mathcal{D}\}$. We have shown in [9] that subsumptions w.r.t. classical TBoxes and subsumptions for auxiliary names w.r.t. TBoxes extended by $D_n = \emptyset$ coincide.

Proposition 3. *Let $sig(G) \cap N_C^{aux} = \emptyset$. Then $F \sqsubseteq_{\mathcal{T}} G$ iff $F_{\emptyset} \sqsubseteq_{\mathcal{T}_{\emptyset}(F)} G$.*

To use typicality interpretations for reasoning under materialisation-based rational entailment, $\mathcal{D}_i$ needs to be satisfied at the elements on typicality level i, but not (necessarily) for their role successors. It is indeed sufficient to construct a typicality interpretation with minimal typical role successors induced by existential restrictions that satisfy only $\mathcal{T}$ and $\mathcal{D}_n = \emptyset$.

Definition 4. *The* minimal typicality model *of $\mathcal{K}$ is $\mathfrak{L}_{\mathcal{K}} = (\Delta^{\mathfrak{L}_{\mathcal{K}}}, \cdot^{\mathfrak{L}_{\mathcal{K}}})$, where the domain is $\Delta^{\mathfrak{L}_{\mathcal{K}}} = \{d_F^i \in \Delta^{\mathcal{K}} \mid F_{\mathcal{D}_i} \not\sqsubseteq_{\mathcal{T}_{\mathcal{D}_i}(F)} \bot\}$ and $\cdot^{\mathfrak{L}_{\mathcal{K}}}$ satisfies for all elements $d_F^i \in \Delta^{\mathfrak{L}_{\mathcal{K}}}$ with $0 \leq i \leq n = rk(\mathcal{D})$ both of the following conditions: (i) $d_F^i \in A^{\mathfrak{L}_{\mathcal{K}}}$ iff $F_{\mathcal{D}_i} \sqsubseteq_{\mathcal{T}_{\mathcal{D}_i}(F)} A$, for $A \in sig_{N_C}(\mathcal{K})$ and (ii) $(d_F^i, d_G^n) \in r^{\mathfrak{L}_{\mathcal{K}}}$ iff $F_{\mathcal{D}_i} \sqsubseteq_{\mathcal{T}_{\mathcal{D}_i}(F)} \exists r.G$, for $r \in sig_{N_R}(\mathcal{K})$.*

Typicality models need not use the complete typicality domain due to inconsistencies. They are models of DKBs (cf. Def. 1) as we have shown in [9]. Proposition 3 implies that $\mathfrak{L}_{\mathcal{K}}$, restricted to elements of typicality n, is the canonical model for an $\mathcal{EL}_{\bot}$-TBox $\mathcal{T}$. Next, we characterise different entailment relations based on different kinds of typicality models which vary in the typicality admitted for required role successors. First, we use minimal typicality models to characterize entailment of propositional nature $\models_p$, where all role successors are uniformly non-typical.

Definition 5. *Let $\mathcal{K}$ be a DKB. $\mathcal{K}$* propositionally entails *a defeasible subsumption relationship $C \sqsubsim D$ (written $\mathcal{K} \models_p C \sqsubsim D$) iff $\mathfrak{L}_{\mathcal{K}} \models C \sqsubsim D$.*

Deciding entailments of propositional nature by computing the extended TBox for a concept F and deriving its minimal typicality model, coincides with enriching F with the materialisation of $\mathcal{D}$.

Lemma 6. *Let $sig(X) \cap N_C^{aux} = \emptyset$ for $X \in \{\mathcal{T}, \mathcal{D}, C, D\}$. Then $\overline{\mathcal{D}} \sqcap C \sqsubseteq_{\mathcal{T}} D$ iff $C_{\mathcal{D}} \sqsubseteq_{\mathcal{T}_{\mathcal{D}}(C)} D$.*

Proof (sketch). The proof is by induction on $|\mathcal{D}|$. In the base case $\mathcal{D} = \emptyset$ and Prop. 3 holds. For the induction step, let $\mathcal{D}' = \mathcal{D} \cup \{G \sqsubsim H\}$ and distinguish: (i) $\overline{\mathcal{D}} \sqcap C \sqsubseteq G$ and (ii) $\overline{\mathcal{D}} \sqcap C \not\sqsubseteq G$. For case (i), we show: reasoning with $C \sqcap H$, yields the same consequences as with C. All elements in $C \sqcap H$ satisfying G, already satisfy H. Hence, $G \sqsubsim H$ can be removed from $\mathcal{D}'$ and the induction hypothesis holds. In case (ii) $G \sqsubsim H$ has no effect on reasoning: since $\overline{\mathcal{D}} \sqcap C \not\sqsubseteq G$, some element in C, satisfying all DCIs in D, does not satisfy G. Thus, $G \sqsubsim H$ does not affect reasoning and allows application of the induction hypothesis directly.

We denote the result of the reasoning algorithm from [3] as *materialisation-based rational entailment* (written $\mathcal{K} \models_m C \sqsubsim D$). This algorithm and typicality interpretations use the same DBox sequence. Although $\models_p$ and $\models_m$ are defined on different semantics, they derive the same subsumption relationships.

Theorem 7. *$\mathcal{K} \models_p C \sqsubsim D$ iff $\mathcal{K} \models_m C \sqsubsim D$.*

The crucial point in the proof of Theorem 7 is the use of Lemma 6. We have established an alternative characterisation for materialisation-based rational entailment by minimal typicality models. It ignores defeasible knowledge relevant to existential restrictions which motivates the extension of typicality models with role successors of the same concept, but of varying typicality.

3 Maximal Typicality Models for Rational Entailment

In materialisation-based rational entailment concepts implying $\exists r.C$ are *all* represented by a single element d_C^i allowing for one degree of (non-)typicality. Now, to obtain models where each role successor is of the highest (consistent) level of typicality, we transform minimal typical models s.t. each role edge is copied, but its endpoint, say d_H^i, is exchanged for a more typical representative d_H^{i-1}.

Definition 8. *Let $\mathcal{I}$ be a typicality interpretation over a DKB $\mathcal{K}$. The* set of more typical role edges *for a given role r is defined as* $TR_{\mathcal{I}}(r) = \{(d^i_G, d^j_H) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \setminus r^{\mathcal{I}} \mid \exists k > 0.\ (d^i_G, d^{j+k}_H) \in r^{\mathcal{I}}\}$. *Let $\mathcal{I}$ and $\mathcal{J}$ be typicality interpretations. $\mathcal{J}$ is a* typicality extension *of $\mathcal{I}$ iff (i) $\Delta^{\mathcal{J}} = \Delta^{\mathcal{I}}$, (ii) $A^{\mathcal{J}} = A^{\mathcal{I}}$ (for $A \in N_C$), (iii) $r^{\mathcal{J}} = r^{\mathcal{I}} \cup R$ (for $r \in sig_{N_R}(\mathcal{K})$ and $R \subseteq TR_{\mathcal{I}}(r)$), and (vi) $\exists r \in sig_{N_R}(\mathcal{K}).\ r^{\mathcal{I}} \subset r^{\mathcal{J}}$. The* set of all typicality extensions *of $\mathcal{I}$ is $typ(\mathcal{I})$.*

Unfortunately, typicality extensions do not preserve the property of being a typicality model, since the increased typicality of the successor can invoke new concept memberships of the role *pre*decessor which in turn can necessitate new additions in order to be compliant with all GCIs from $\mathcal{T}$. We formalize the required additions to obtain a model again by *model completions*.

Definition 9. *Let $\mathcal{K}$ be a DKB with $n = rk(\mathcal{D})$ and Δ a typicality domain. An interpretation $\mathcal{I} = (\Delta, \cdot^{\mathcal{I}})$ is a* model completion *of an interpretation $\mathcal{J} = (\Delta, \cdot^{\mathcal{J}})$ iff (i) $\mathcal{J} \subseteq \mathcal{I}$, (ii) $\mathcal{I} \models \mathcal{K}$, and (iii) $\forall E \in Qc(\mathcal{K}).d^i_F \in (\exists r.E)^{\mathcal{I}} \implies (d^i_F, d^n_E) \in r^{\mathcal{I}}$ (for any $F \in Qc(\mathcal{K})$ and $0 \leq i \leq n$). The* set of all model completions *of $\mathcal{J}$ is denoted as $mc(\mathcal{J})$.*

Note, that model completions introduce role successors only on typicality level n which can necessitate typicality extensions again. Thus typicality extensions and model completions are applied alternately until a fixpoint is reached. Clearly neither Definition 8 nor 9 yield a unique extension or completion. Thus, one (full) upgrade step is executed by an operator T working on sets of typicality interpretations. E.g. applying T to the singleton set $\{\mathcal{I}\}$ results in a set of all possible model completions of all possible typicality extensions of $\mathcal{I}$ (one upgrade step). The fixpoint of T is denoted as $typ^{max}()$, it collects the maximal typicality interpretations for a given input (c.f. [9]). There are several ways to use these sets of obtained maximal typicality models for our new entailment. Since in classical DL reasoning entailment considers *all* models, we employ cautious reasoning. We use a single model that is canonical in the sense that it is contained in all maximal typicality models obtained from $\mathfrak{L}_{\mathcal{K}}$. The *rational canonical model* $\mathfrak{R}_{\mathcal{K}}$ is defined as $\mathfrak{R}_{\mathcal{K}} = \bigcap_{\mathcal{I} \in typ^{max}(\{\mathfrak{L}_{\mathcal{K}}\})} \mathcal{I}$. This intersection is well-defined as $typ^{max}(\{\mathfrak{L}_{\mathcal{K}}\})$ is finite and $\mathfrak{L}_{\mathcal{K}} \in mc(\mathfrak{L}_{\mathcal{K}})$ is not empty, see [9].

Lemma 10. *The rational canonical model $\mathfrak{R}_{\mathcal{K}}$ is a model of the DKB $\mathcal{K}$.*

Basis of the proof of Lemma 10 is that the intersection of models satisfying Conditions (i)–(iii) in Def. 9 will satisfy the same conditions. The rational canonical model is used to decide nested rational entailments of the form $\exists r.C \sqsubsetapprox_{\mathcal{K}} \exists r.D$, which requires to propagate DCIs to the instances of C and D. For a DKB $\mathcal{K}$, we capture (quantifier aware) *nested rational entailment* as $\mathcal{K} \models_q C \sqsubsetapprox D$ *iff* $\mathfrak{R}_{\mathcal{K}} \models C \sqsubsetapprox D$.

Theorem 11. *Let $\mathcal{K}$ be a DKB and C,D concepts. Then (i) $\mathcal{K} \models_m C \sqsubsetapprox D \implies \mathcal{K} \models_q C \sqsubsetapprox D$, and (ii) $\mathcal{K} \models_m C \sqsubsetapprox D \not\Longleftarrow \mathcal{K} \models_q C \sqsubsetapprox D$.*

To show Claim (i) of Theorem 11 observe that $\mathcal{J} \in typ^{max}(\{\mathfrak{L}_{\mathcal{K}}\})$ implies that $\mathfrak{L}_{\mathcal{K}} \subseteq \mathcal{J}$ and thus $\mathfrak{L}_{\mathcal{K}} \subseteq \mathfrak{R}_{\mathcal{K}}$. Hence by Definition 2, $\mathfrak{L}_{\mathcal{K}} \models C \sqsubsetapprox D$ implies $\mathfrak{R}_{\mathcal{K}} \models C \sqsubsetapprox D$. Claim (ii) is shown by an example in [9].

Theorem 11 shows that our approach produces results that satisfy the rational reasoning postulates as introduced by KLM [7] and lifted to DLs as presented in [3], as it gives strictly more entailments than the materialisation approach. The additional entailments are compliant with the argument of Lehmann and Magidor [8] that implications inferred from conditional knowledge bases should *at least* satisfy the postulates for rational reasoning.

4 Concluding Remarks

We have proposed a new approach to characterize entailment under rational closure (for deciding subsumption) in the DDL $\mathcal{EL}_\bot$ motivated by the fact that earlier reasoning procedures do not treat existential restrictions adequately. The key idea is to extend canonical models such that for each concept from the DKB, several copies representing different typicality levels of the respective concept are introduced. In minimal typicality models the role successors are "non-typical" in the sense that they satisfy only the GCIs from the TBox. Such models can be computed by a reduction to classical TBox reasoning by extended TBoxes. We showed that the entailments obtained from minimal typical models coincide with those obtained by materialisation. For maximal typicality models, where role successors are of "maximal typicality", DCIs are propagated to each role successor individually, thus allowing for more entailments. Existential restrictions are disregarded in several materialisation-based algorithms for defeasible reasoning in DLs, such as lexicographic [5] and relevant [3] closure. It is future work to extend our approach to these more sophisticated semantics.

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