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Bird - Meertens - Formalismus

MINIMAX - PROBLEM

Minimax - Problem

Berechnung des Minimum der Maxima von Listen

Beispiel

$$minimax\ [[1,2,3],[4,5,6],[7,8],[9]] \ \rightarrow\ 3$$

Definition

$$minimax \ = \ (min \circ map\ max) \tag{1}$$

Fold-Map-Fusion

$$\begin{aligned}
minimax &= (min \circ map\ max) && (2)\\
&= ((foldl\ (\downarrow)\ \infty) \circ map\ max) && (3)\\
((foldl\ (\downarrow)\ \infty) \circ map\ max) &= (foldl\ (\hat{f})\ \infty),\ \text{mit} && (4)\\
\hat{f}\ x\ xs &= x \downarrow (max\ xs)\ \text{und} && (5)\\
\downarrow, \uparrow &\quad \text{Min und Max binär} && (6)
\end{aligned}$$

Für $\downarrow$ und $\uparrow$ gilt das Distributivgesetz:

$$x \downarrow (y \uparrow z) \ = \ (x \downarrow z) \uparrow (x \downarrow y). \tag{7}$$

Damit gilt ebenfalls

$$\begin{aligned}
x \downarrow (max\ xs) &= max\ (map\ (x\downarrow)\ xs) && (8)\\
&= (max \circ (map\ (x\downarrow))\ xs && (9)\\
&= ((foldl\ (\uparrow)\ (-\infty)) \circ map\ (x\downarrow))\ xs && (10)
\end{aligned}$$

Fold-Map-Fusion

$$\begin{aligned}
((foldl\ (\uparrow)\ (-\infty)) \circ map\ (x\downarrow))\ xs &= foldl\ (\hat{g})\ (-\infty)\ xs && (11)\\
\hat{g}\ y\ z &= y \uparrow (x \downarrow)\ z && (12)\\
&= y \uparrow (x \downarrow z) && (13)
\end{aligned}$$

Damit ist

$$\begin{aligned}
minimax &= foldl\ (\hat{f})\ \infty,\ \text{mit} && (14)\\
\hat{f}\ x\ xs &= foldl\ (\hat{g})\ (-\infty)\ xs,\ \text{wobei} && (15)\\
\hat{g}\ y\ z &= y \uparrow (x \downarrow z)\ . && (16)
\end{aligned}$$