Description Logics That Count, and What They Can and Cannot Count (Extended Abstract)*

Franz Baader and Filippo De Bortoli

Institute of Theoretical Computer Science, TU Dresden, Germany
franz.baader@tu-dresden.de, filippo.de_bortoli@tu-dresden.de

Simple counting quantifiers that can be used to compare the number of role successors of an individual or the cardinality of a concept with a fixed natural number have been employed in Description Logics (DLs) for more than two decades, under the respective names of *number restrictions* [8,11,10] and *cardinality restrictions on concepts* (CRs) [5,17]. The exact complexity of concept satisfiability in $\mathcal{ALCQ}$ [10] has been shown to be PSpace-complete without *concept inclusions* (CIs) and ExpTime-complete w.r.t. CIs, independently from the encoding (unary or binary) of the numbers occurring in the restriction [16,18]. For the DL $\mathcal{ALCQ}$, checking consistency w.r.t. CIs is ExpTime-complete [18], whereas consistency w.r.t. CRs is NExpTime-complete if the numbers occurring in the CRs are assumed to be encoded in binary [17]. With unary coding of numbers, consistency stays ExpTime-complete even w.r.t. CRs [17]. It should be noted that both qualified number restrictions and CRs (which generalize CIs) can be expressed in $\mathcal{C}^2$, the two-variable fragment of first-order logic with counting quantifiers [9,14], whose satisfiability problem is known to be NExpTime-complete [15].

In recent work [1], we have extended $\mathcal{ALCQ}$ by allowing the statement of restrictions on role successors using the quantifier-free fragment of Boolean Algebra with Presburger Arithmetic (QFBAPA) [12], in which one can express Boolean combinations of set and numerical constraints comparing the cardinalities of finite sets. The resulting logic, called $\mathcal{ALCSCC}$, strictly extends the expressive power of $\mathcal{ALCQ}$. In [1] it is shown that the constraint $succ(|r| = |s|)$, which describes individuals having the same number of r-successors as s-successors, cannot be expressed in $\mathcal{ALCQ}$. In [4], the constraint $succ(|r \cap A| = |r \cap \neg A|)$, describing individuals whose number of r-successors belonging to A is the same as the number of r-successors not belonging to A, is shown to be not even expressible in first-order logic. In spite of this considerable increase in expressive power, we were able to show in [1] that there is no increase in complexity: like for $\mathcal{ALCQ}$, the complexity of the satisfiability problem in $\mathcal{ALCSCC}$ is PSpace-complete without CIs and ExpTime-complete w.r.t. CIs.

Just like classical number restrictions, CRs can only relate the cardinality of a concept to a *fixed* number. In [7] we have introduced and investigated more a generalization of CRs, which we called *extended cardinality constraints*. The main idea was again to use QFBAPA to formulate and combine these constraints. In

* Supported by DFG in TRR 248 (CPEC, grant 389792660) and RTG 1763 (QuantLA).

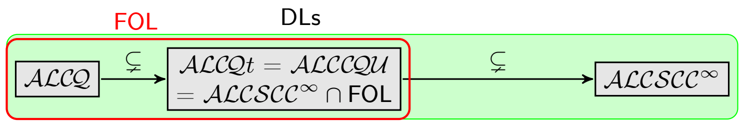

Fig. 1. The relative expressivity of the DLs $\mathcal{ALCQ}$, $\mathcal{ALCQ}t$, $\mathcal{ALCCQU}$, and $\mathcal{ALCSCC}^\infty$.

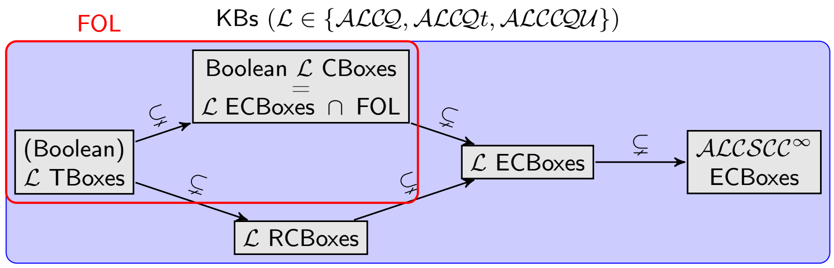

Fig. 2. The relative expressivity of boxes.

[7] it is shown that, in the DL $\mathcal{ALC}$, the complexity of reasoning w.r.t. extended cardinality constraints (NExpTime for binary coding of numbers), is the same as for reasoning w.r.t. CRs. In addition, the paper introduces a restricted version of this formalism, which can express CIs, but not CRs, and shows that this way the complexity can be lowered to ExpTime.

In [2,3], we combined the work in [1] and [7] by considering extended cardinality constraints in $\mathcal{ALCSCC}$. This turned out to be non-trivial since the local cardinality constraints of $\mathcal{ALCSCC}$ may interact with the global ones in the extended cardinality constraints. Nevertheless, we were able to show that the complexity results (NExpTime-complete in general, and ExpTime-complete in the restricted case) hold not only for $\mathcal{ALC}$, but also for $\mathcal{ALCSCC}$.

The purpose of the present paper is twofold. On the one hand, we give a compact representation of the known complexity results for the DLs with extended counting facilities mentioned above, and transfer them to a setting where arbitrary rather than just finite models are considered. On the other hand, we investigate the expressive power of these DLs in detail. A first step in this direction was already made in [4], where the expressive power of concept descriptions was examined using appropriate bisimulation relations. Here, we recall these results, and then extend them to TBoxes, CRs, and extended cardinality constraints, by adapting methods and ideas from [13]. As in [4], we consider variants of QF-BAPA and $\mathcal{ALCSCC}$ that allow for possibly infinite sets and interpretations, respectively. This change has no influence on the complexity of reasoning, but it eases the comparison with classical DLs, for which one usually employs arbitrary models rather than finite ones when defining the semantics. The diagrams in Figure 1 and Figure 2 summarize our results.

The results herein illustrated were published in [6].

References

1. Franz Baader. A new description logic with set constraints and cardinality constraints on role successors. In Clare Dixon and Marcelo Finger, editors, *Proc. of the 11th Int. Symposium on Frontiers of Combining Systems (FroCoS'17)*, volume 10483 of *Lecture Notes in Computer Science*, pages 43–59, Brasília, Brazil, 2017. Springer-Verlag.
2. Franz Baader. Expressive cardinality constraints on $\mathcal{ALCSCC}$ concepts. In *Proc. of the 34th ACM/SIGAPP Symposium On Applied Computing (SAC'19)*. ACM, 2019.
3. Franz Baader. Expressive cardinality restrictions on concepts in a description logic with expressive number restrictions. *ACM SIGAPP Applied Computing Review*, 19:5–17, 2019.
4. Franz Baader and Filippo De Bortoli. On the expressive power of description logics with cardinality constraints on finite and infinite sets. In Andreas Herzig and Andrei Popescu, editors, *Proc. of the 12th Int. Symposium on Frontiers of Combining Systems (FroCoS'19)*, volume 11715 of *Lecture Notes in Computer Science*. Springer-Verlag, 2019.
5. Franz Baader, Martin Buchheit, and Bernhard Hollunder. Cardinality restrictions on concepts. *Artificial Intelligence*, 88(1–2):195–213, 1996.
6. Franz Baader and Filippo De Bortoli. Description logics that count, and what they can and cannot count. In Laura Kovacs, Konstantin Korovin, and Giles Reger, editors, *ANDREI-60. Automated New-era Deductive Reasoning Event in Iberia*, volume 68 of *EPiC Series in Computing*, pages 1–25. EasyChair, 2020.
7. Franz Baader and Andreas Ecke. Extending the description logic $\mathcal{ALC}$ with more expressive cardinality constraints on concepts. In *Proc. of the 3rd Global Conf. on Artificial Intelligence (GCAI'17)*, volume 50 of *EPiC Series in Computing*, pages 6–19. EasyChair, 2017.
8. Alexander Borgida, Ronald J. Brachman, Deborah L. McGuinness, and Lori Alperin Resnick. CLASSIC: A structural data model for objects. In *Proc. of the ACM SIGMOD Int. Conf. on Management of Data*, pages 59–67, 1989.
9. Erich Grädel, Martin Otto, and Eric Rosen. Two-variable logic with counting is decidable. In *Proc. of the 12th IEEE Symp. on Logic in Computer Science (LICS'97)*, pages 306–317. IEEE Computer Society Press, 1997.
10. Bernhard Hollunder and Franz Baader. Qualifying number restrictions in concept languages. In *Proc. of the 2nd Int. Conf. on the Principles of Knowledge Representation and Reasoning (KR'91)*, pages 335–346, 1991.
11. Bernhard Hollunder, Werner Nutt, and Manfred Schmidt-Schauß. Subsumption algorithms for concept description languages. In *Proc. of the 9th Eur. Conf. on Artificial Intelligence (ECAI'90)*, pages 348–353, London (United Kingdom), 1990. Pitman.
12. Viktor Kuncak and Martin Rinard. Towards efficient satisfiability checking for boolean algebra with presburger arithmetic. In Frank Pfenning, editor, *Automated Deduction – CADE-21*, Lecture Notes in Computer Science, pages 215–230. Springer Berlin Heidelberg, 2007.
13. Carsten Lutz, Robert Piro, and Frank Wolter. Description Logic TBoxes: Model-Theoretic Characterizations and Rewritability. In *IJCAI 2011, Proceedings of the 22nd International Joint Conference on Artificial Intelligence, Barcelona, Catalonia, Spain, July 16-22, 2011*, pages 983–988, 2011. Long version available at `https://arxiv.org/pdf/1104.2844.pdf`.

14. Leszek Pacholski, Wieslaw Szwast, and Lidia Tendera. Complexity of two-variable logic with counting. In *Proc. of the 12th IEEE Symp. on Logic in Computer Science (LICS'97)*, pages 318–327. IEEE Computer Society Press, 1997.
15. Ian Pratt-Hartmann. Complexity of the two-variable fragment with counting quantifiers. *J. of Logic, Language and Information*, 14(3):369–395, 2005.
16. Stephan Tobies. A PSPACE algorithm for graded modal logic. In Harald Ganzinger, editor, *Proc. of the 16th Int. Conf. on Automated Deduction (CADE'99)*, volume 1632 of *Lecture Notes in Artificial Intelligence*, pages 52–66. Springer-Verlag, 1999.
17. Stephan Tobies. The complexity of reasoning with cardinality restrictions and nominals in expressive description logics. *J. of Artificial Intelligence Research*, 12:199–217, 2000.
18. Stephan Tobies. *Complexity Results and Practical Algorithms for Logics in Knowledge Representation*. PhD thesis, LuFG Theoretical Computer Science, RWTH-Aachen, Germany, 2001.