TECHNISCHE UNIVERSITÄT DRESDEN

CPEC
CENTER FOR PERSPICUOUS COMPUTING

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Finding Proofs for Description Logic Entailments in Practice

Based on "Finding Small Proofs for Description Logic Entailments—Theory and Practice" (LPAR'20) // Explainable Logic-Based Knowledge Representation (XLoKR 2020), September 14, 2020

Description Logics and Ontologies

Syntax of DL $\mathcal{ALC}$

Concepts: $C \ ::= \ A \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \exists r.C \mid \forall r.C$

Axioms: $\alpha \ ::= \ C \sqsubseteq C \mid C \equiv C$

Description Logics

- Well-established formalism for specifying terminological knowledge in Ontologies
- Used for many large-scale ontologies
  - SNOMED CT: over 300,000 concepts
  - BioPortal: repository of bio-medical ontologies, currently hosting 889 ontologies defining 12,084,317 terms
  - MOWLCorp: ontologies obtained by web-crawling, containing 21,000 ontologies

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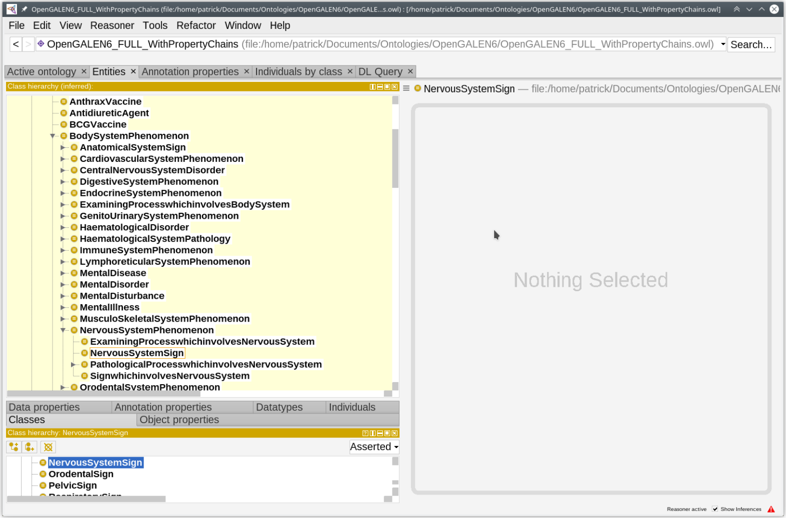
Description Logics

- Well-established formalism for specifying terminological knowledge in Ontologies
- Used for many large-scale ontologies
- With increasing complexity of the ontology, understanding entailments becomes both crucial and difficult
- One typical reasoning task is classification
  - compute all entailed axioms of form $A \sqsubseteq B$
  - obtain concept hierarchy

Current Tool of Choice: Justifications



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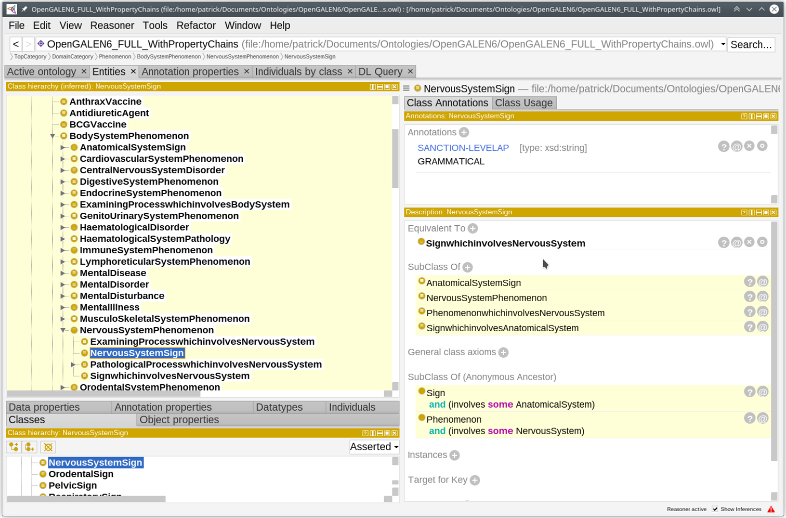

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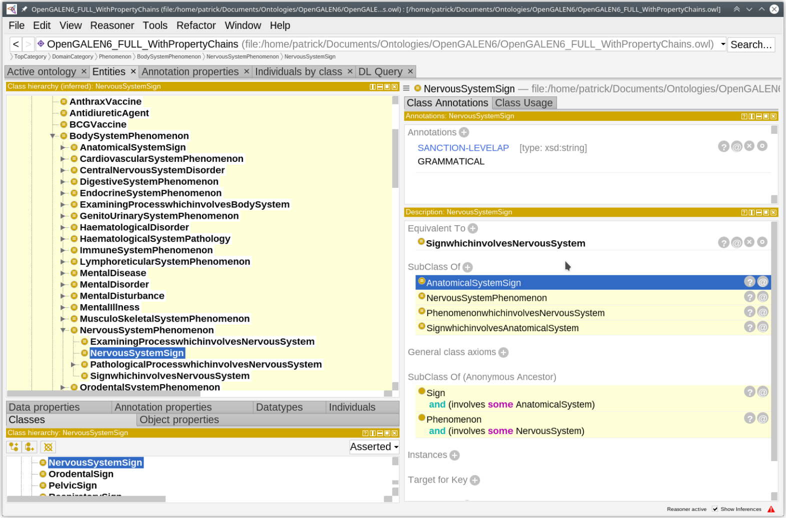

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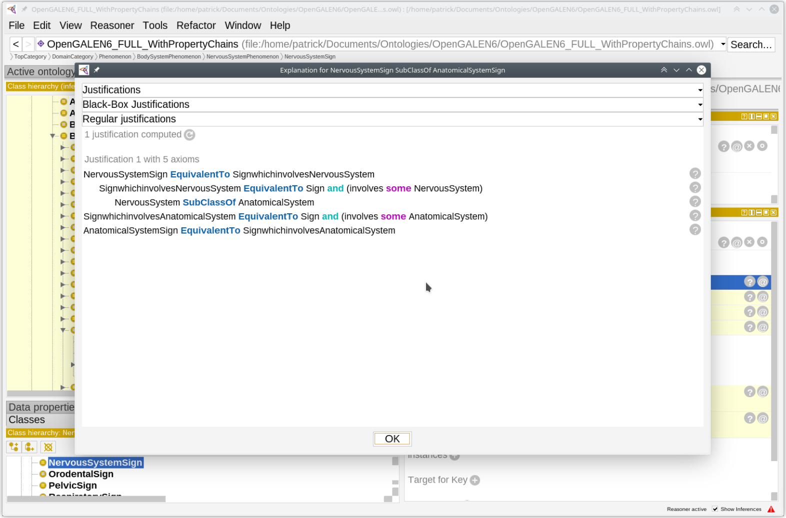


Justifications

Justifications: Minimal subsets entailing given subsumption

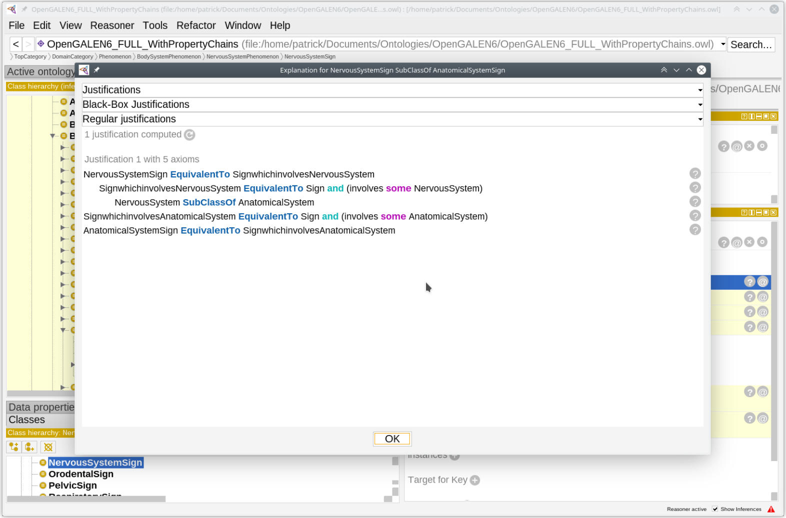
In practice often insufficient :

- can be large
- inferences often not obvious

Showing *how* to obtain the inference would be better

- simple reasoning steps leading to conclusion
- generally known as proof

Proofs for ELK in Protege



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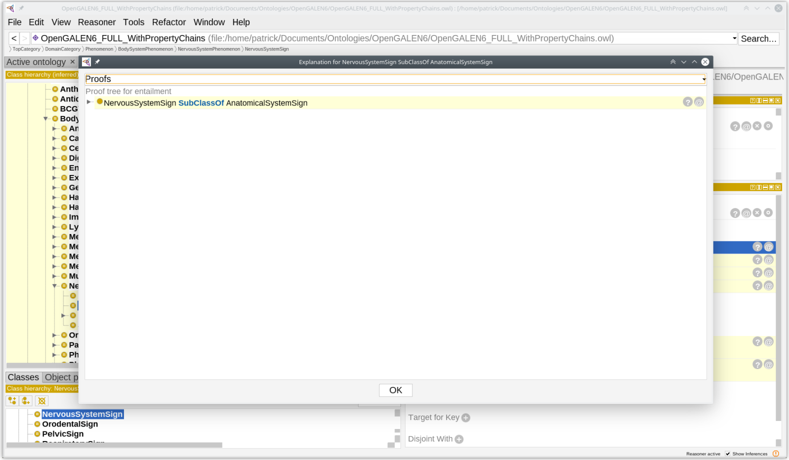

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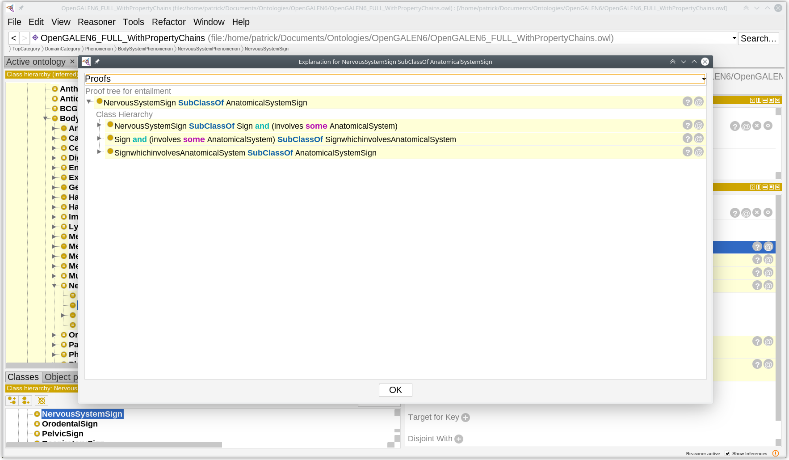

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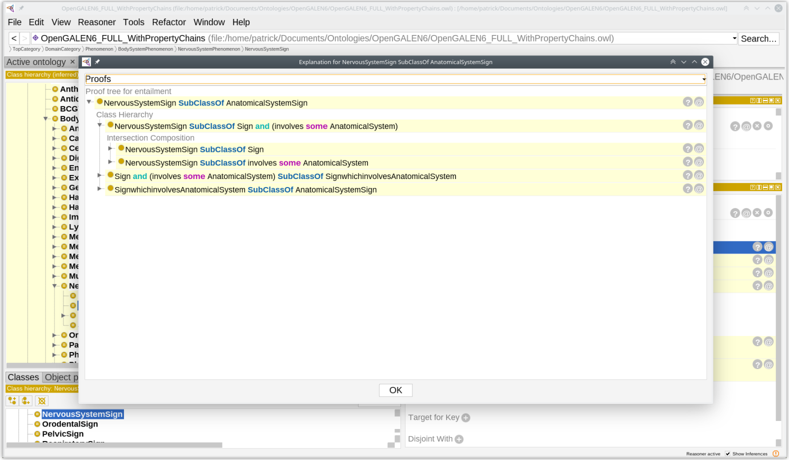

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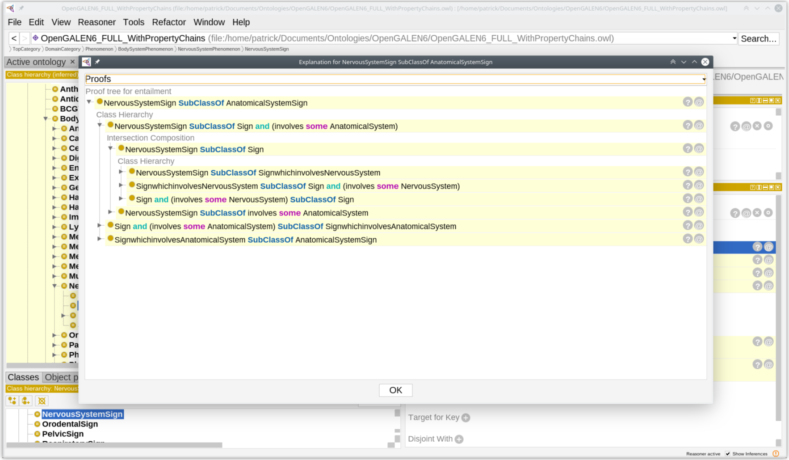
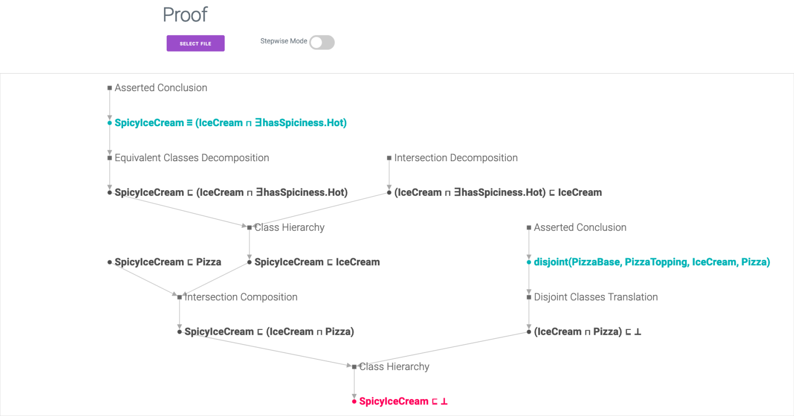


Proofs using Evonne (work in progress)



C. Alrabbaa, F. Baader, R. Dachselt, T. Flemisch, P. Koopmann: *Visualizing Proofs and the Modular Structure for Ontology Repair*, DL 2020.

Proofs with ELK

- ELK using a *consequence-based* reasoning procedure

⇒ inferences performed using a calculus

$$\mathbf{R}_0 \frac{}{C \sqsubseteq C} \qquad \mathbf{R}_\top \frac{}{C \sqsubseteq \top} \qquad \mathbf{R}_\sqcap^- \frac{C \sqsubseteq D \sqcap E}{C \sqsubseteq D \quad C \sqsubseteq E}$$

$$\mathbf{R}_\sqcap^+ \frac{C \sqsubseteq D \quad C \sqsubseteq E}{C \sqsubseteq D \sqcap E} \qquad \mathbf{R}_\exists \frac{C \sqsubseteq \exists r.D \quad D \sqsubseteq E}{C \sqsubseteq \exists r.E}$$

$$\mathbf{R}_\bot \frac{C \sqsubseteq \exists r.D \quad D \sqsubseteq \bot}{C \sqsubseteq \bot} \qquad \mathbf{R}_\sqsubseteq \frac{C \sqsubseteq D}{C \sqsubseteq E} : D \sqsubseteq E \in \mathcal{O}$$

$$\mathbf{R}_\circ \frac{C_0 \sqsubseteq \exists r_1.C_1, C_1 \sqsubseteq \exists r_2.C_2 \ \ldots \ C_{n-1} \sqsubseteq \exists r_n.C_n}{C_0 \sqsubseteq \exists r.C_n} : r_1 \circ \ldots \circ r_n \sqsubseteq r \in \mathcal{O}$$

⇒ proofs generated as part of reasoning process

Proofs For More Expressive DLs

- Currently, ELK is the only DL reasoner supporting proof generation
- ELK supports only a limited fragment of OWL, OWL EL
- More expressive reasoner often use other reasoning procedures
  - for a long time prominent: tableau reasoning
  - less convenient for understanding entailments
- Existing consequence-based reasoning for expressive DLs
  - often involved in complex systems
  - often combined with other reasoning paradigms
  - may use normal forms requiring different syntax

  $\Rightarrow$ generation of proofs not obvious
- Can we generate proofs without a calculus?

Justification Based Proofs

Justification-Based Proofs (Matthew Horridge 2011)

- derive intermediate steps between conclusion and justification
- consider all axioms of some predefined shapes
- justification-relation between allows to construct a proof
- involved ranking function allows to select axioms to be used in proof

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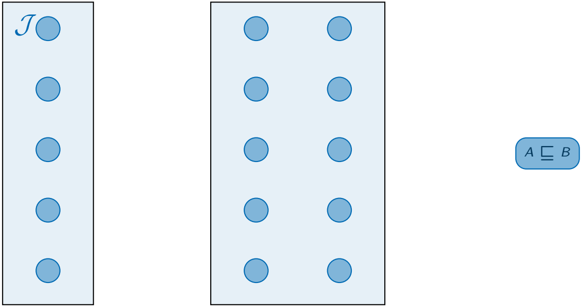

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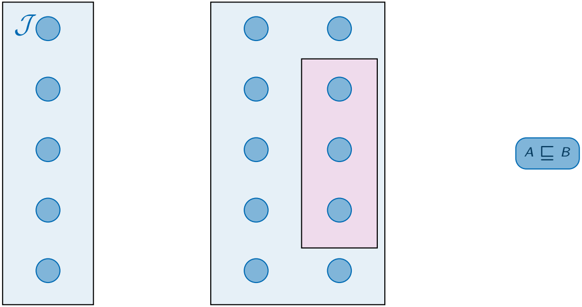

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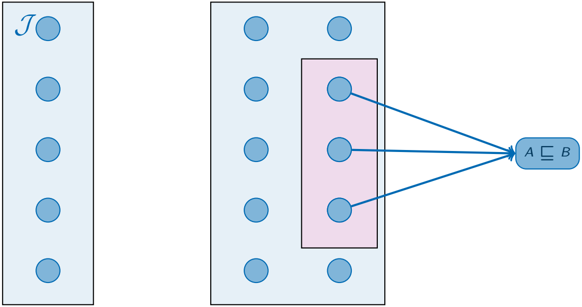

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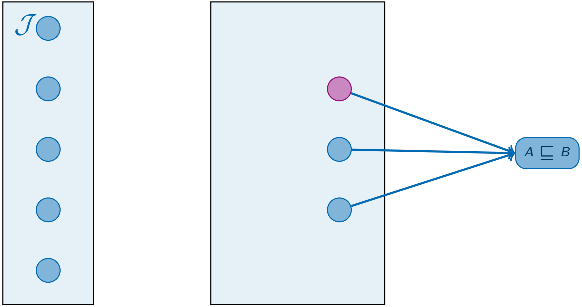

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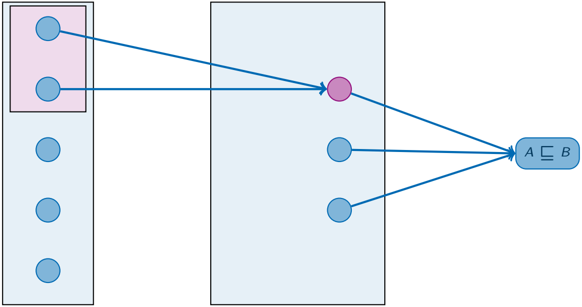

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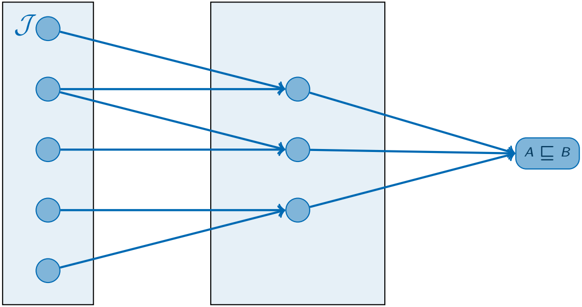

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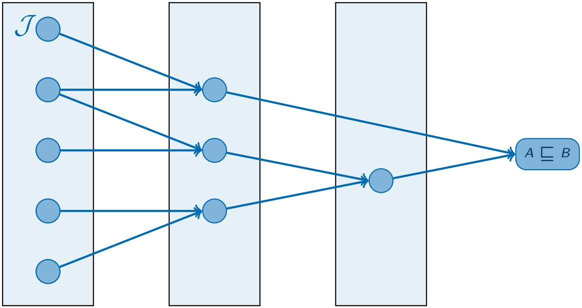

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Advantage of approach:

- generates best proof according to ranking

Disadvantage of approach:

- no clear inference principle
- hard to implement
- strongly depends on ranking function

Forgetting-Based Proofs

Forgetting Based Proofs

- Idea from propositional resolution:

$$\frac{q_1 \vee p \qquad q_2 \vee \neg p}{p_1 \vee p_2}$$

$\Rightarrow$ inference through elimination of p

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- Idea from propositional resolution:

$$\frac{q_1 \vee p \qquad q_2 \vee \neg p}{p_1 \vee p_2}$$

$\Rightarrow$ inference through elimination of p

- Decide satisfiability by eliminating names one after the other:

$$b \qquad a \vee b \qquad \neg b \vee c \qquad \neg b \vee \neg c \qquad \neg a \vee c$$

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$$A \sqsubseteq B$$

Forgetting

Definition

Let $\mathcal{O}$ be an ontology and X a predicate name. Then, $\mathcal{O}^{-X}$ is a *result of forgetting* X iff

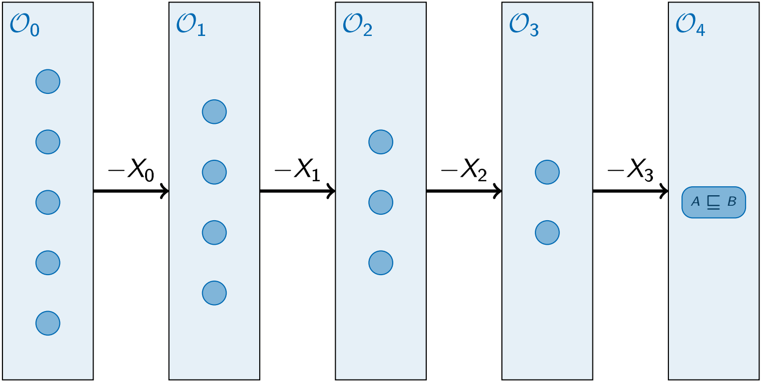
- X does not occur in $\mathcal{O}^{-X}$
- for every axiom α in which X does not occur, $\mathcal{O} \models \alpha$ iff $\mathcal{O}^{-X} \models \alpha$

$\Rightarrow$ strongest ontology without X entailed by $\mathcal{O}$

- which α to preserve also depends on underlying DL

Forgetting based proofs

- Use forgetting to produce sequence of ontologies



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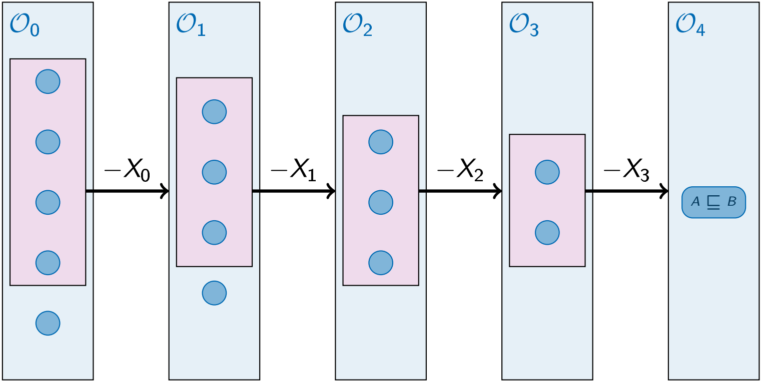

- In each step, recompute justification for $A \sqsubseteq B$

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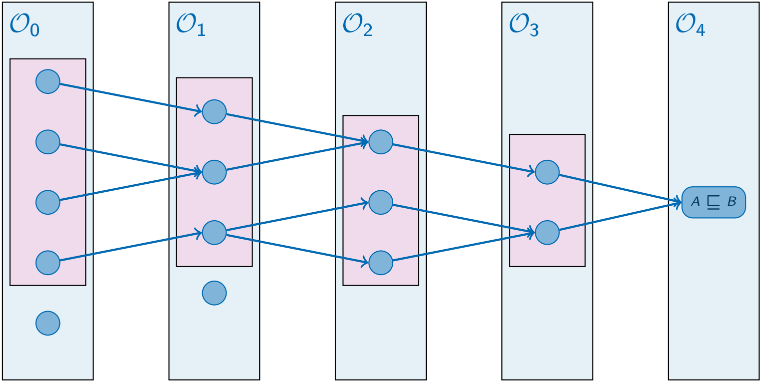

- In each step, recompute justification for $A \sqsubseteq B$
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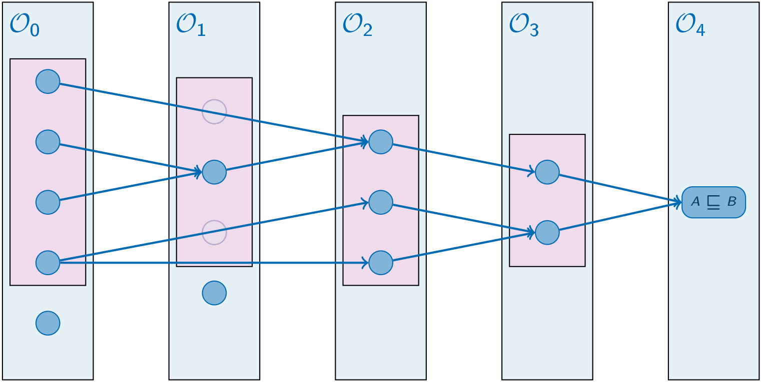


- In each step, recompute justification for $A \sqsubseteq B$
- Finally, reconstruct proof using justifications, skipping steps if it makes sense

Forgetting Based Proof

$$
(C)\,\dfrac{A \sqsubseteq C \qquad (D)\,\dfrac{C \sqsubseteq \exists r.D}{C \sqsubseteq \exists r.\top}}{(r)\,\dfrac{A \sqsubseteq \exists r.\top \qquad\qquad \exists r.\top \sqsubseteq B}{A \sqsubseteq B}}
$$

Forgetting Order

Forgetting order, as well as selection of justification, affects proof

- Forgetting D first:

$$(r)\ \dfrac{(C)\ \dfrac{A \sqsubseteq C \qquad (D)\ \dfrac{C \sqsubseteq \exists r.D}{C \sqsubseteq \exists r.\top}}{A \sqsubseteq \exists r.\top} \qquad \exists r.\top \sqsubseteq B}{A \sqsubseteq B}$$

- Forgetting r first:

$$(C)\ \dfrac{A \sqsubseteq C \qquad (r)\ \dfrac{C \sqsubseteq \exists r.D \qquad \exists r.\top \sqsubseteq B}{C \sqsubseteq B}}{A \sqsubseteq B}$$

Forgetting Order

To obtain nicer proofs *practically*, we process names using the following heuristics:

- role with non-trivial fillers last:
  - otherwise may hide most of inference:

$$(r)\ \frac{A \sqsubseteq \exists r.B \qquad A \sqsubseteq \forall r.C \sqcup D \qquad B \sqsubseteq \exists s.D \qquad C \sqsubseteq \forall s.\neg D}{A \sqsubseteq D}$$

- unnested names first
  - delay complex inferences
- less frequent names first
  - delay expensive forgetting operations
  - (also used by existing forgetting procedures)

Evaluation

- Implemented approach in modular fashion
  - easy exchange of different forgetting procedures, provided they produce OWL ontologies
  - easy comparison with proofs generated by ELK
    - Dijkstra-based search to extract shortest proof
  - use 2 forgetting tools in the evaluation
    - $\mathcal{ALCH}$ variant of LETHE 0.6
    - $\mathcal{ALCOI}$ variant of FAME 1.0[1]

[1] there is a much improved version FAME 2.0, but it often creates ontologies outside of the OWL-standard

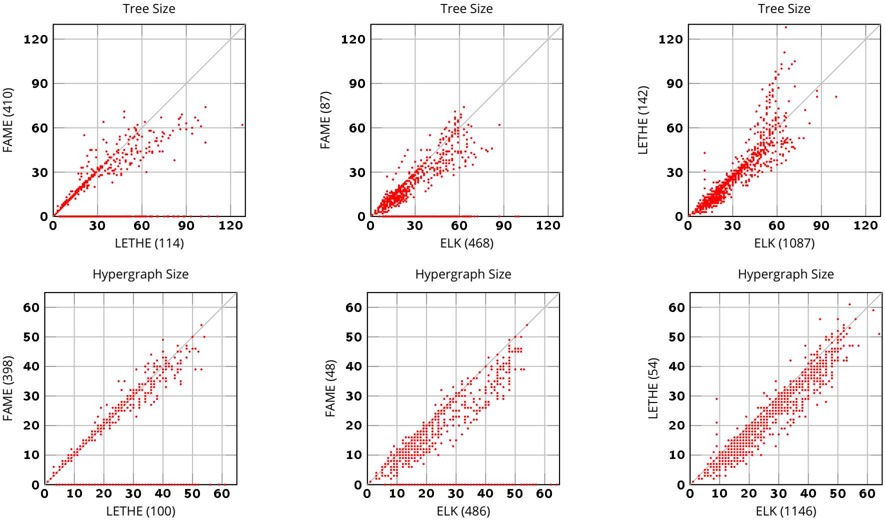
Evaluation: Corpus

- Focus on proofs in $\mathcal{ELH}$
  - to be able to compare with ELK
  - easier extraction of *justification patterns* (see below)
- Use ontologies from the OWL Reasoner Evaluation 2015, OWL EL Classification Track
  - well-balanced mix of ontologies from different repositories
- Extracted 1,573 *justification patterns*
  - all entailments of form $A \sqsubseteq B$ or $A \equiv B$
  - all justifications for these entailments
  - abstract away concept and role names
  - remove resulting duplicates

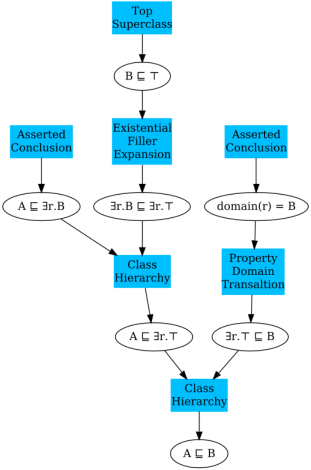
Evaluation: Metrics

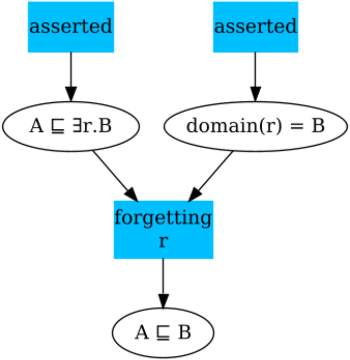
- Hypergraph-size
  - number of distinct axioms used in the proof
- Tree-size
  - sub-proofs count as often as they are used
- Justification Complexity
  - Matthew et al. 2013: "Toward cognitive support for OWL justifications"
  - attempt to measure cognitive complexity of justification
  - provides value for each proof step
  - we measured maximum and sum for each proof

Evaluation: Proof size

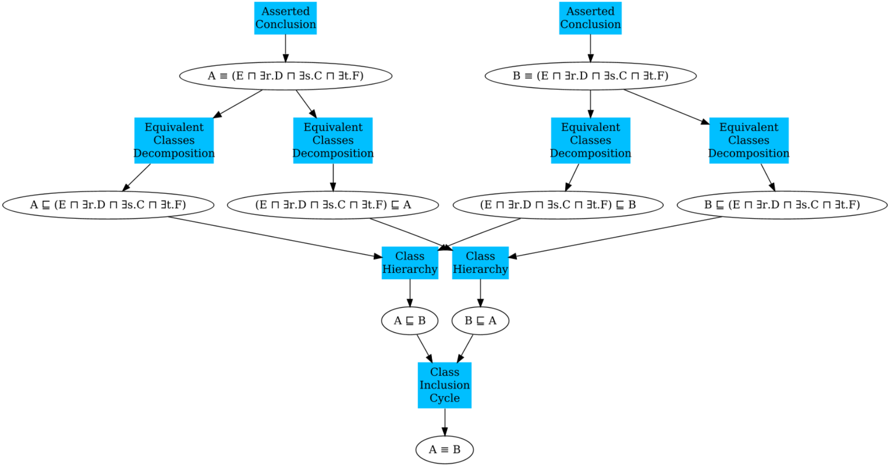


Evaluation: ELK vs. Forgetting-Based Proofs

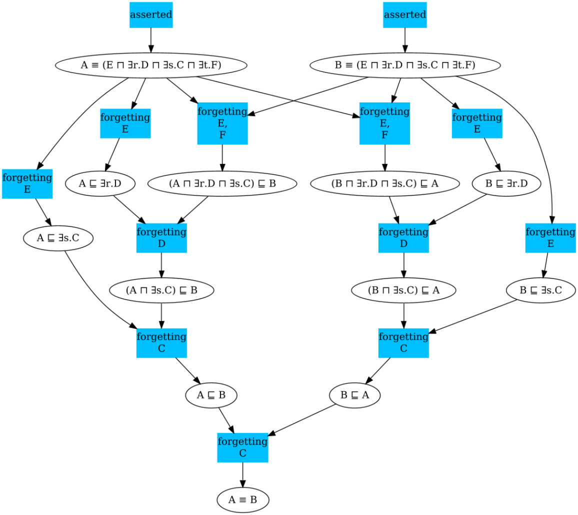




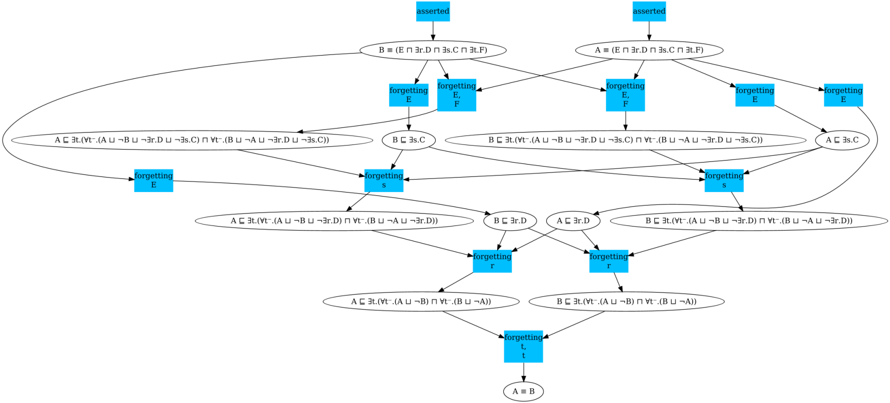
Evaluation: ELK Proof



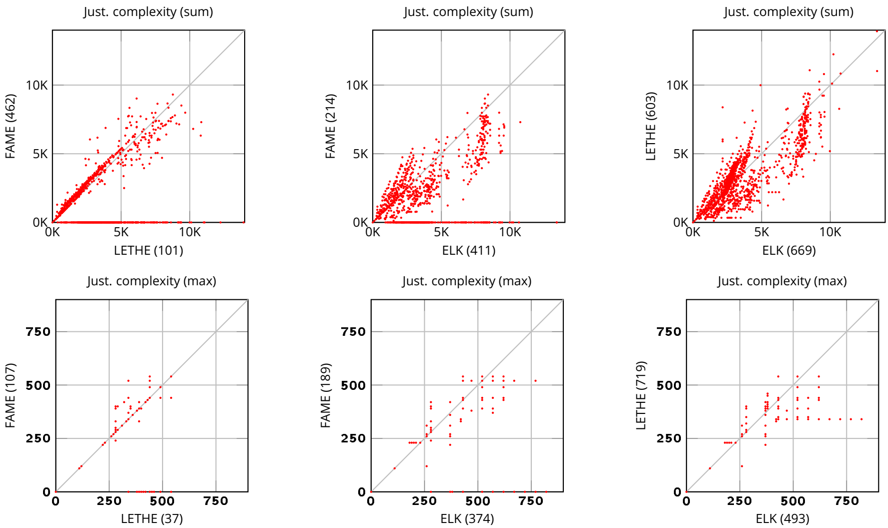
Evaluation: LETHE Proof



Evaluation: FAME Proof



Evaluation: Justification Complexity



Evaluation: Complexity of Inferences

- Both LETHE and FAME may use logical operators outside of $\mathcal{EL}$
- Large number of distinct "inference rules" was used:
  - LETHE: 362 different rules
  - FAME: 281 different rules

Conclusion

- New proof generation procedure based on forgetting
- Generate proof by repeated use of forgetting and justification
- Proofs for expressive DLs without calculus
- Can sometimes even compete with ELK
- Several possibilities to improve:
  - better heuristics on forgetting order or when to skip steps
  - dynamic selection of forgetting order
  - use learned “rules” to shorten proof computation times
  - integrate newer version of FAME