Description Logics for the Representation of Aggregated Objects

Ulrike Sattler[1]

Abstract. Aggregated objects play an important role in many knowledge representation applications. For the adequate representation of aggregated objects, it is crucial to represent part-whole relations. We discuss properties of part-whole relations and extend the description logic $\mathcal{ALC}$ with means for the adequate representation of part-whole relations and thus of aggregated objects.

1 Motivation

Description logics are a family of knowledge representation formalisms well-suited for the representation of and reasoning about configurations [27, 21], ontologies [19], and database schemata, where they can support schema design, evolution, and query optimisation [4, 7], source integration in heterogeneous databases/data warehouses [5, 6], and conceptual modeling of multidimensional aggregation [11].

In all these applications, *aggregated objects* play a central role, that is, objects that are composed of various parts, which again can be composite, etc. It is natural to describe an aggregated object by means of its parts and vice versa, to describe parts by means of the aggregate they belong to. For example, the following statements describe a control rod and a reactor core by means of their parts and wholes, where $\sqsubseteq$ is a subsumption (implication) relationship:

$$\begin{aligned}\texttt{Control-rod} &\sqsubseteq \texttt{Device} \sqcap \\ &\quad \exists\,\texttt{part-of}\,.\texttt{Reactor-core} \\ \texttt{Reactor-core} &\sqsubseteq \texttt{Device} \sqcap \exists\,\texttt{has-part}\,.\texttt{Control-rod} \sqcap \\ &\quad \exists\,\texttt{part-of}\,.\texttt{Nuclear-reactor}\end{aligned}$$

Referring to wholes a part belongs to, we use the *part-whole relation* (written `part-of` and abbreviated pw-relation). Vice versa, to refer to the parts of an object, we use the has-part relation, which is the inverse of the pw-relation, is written `has-part`, and abbreviated hp-relation. It is commonly believed [1] that only a formalism with very high expressive power can represent pw-relations and aggregated objects adequately. In this paper, we argue in how far the high expressive power of the description logic $\mathcal{SHIQ}$ is crucial for the adequate representation of aggregated objects. Despite $\mathcal{SHIQ}$'s high expressiveness, there is a practicable reasoning algorithm which decides inference problems such as satisfiability and subsumption of $\mathcal{SHIQ}$ concepts w.r.t. to (possibly cyclic) terminological knowledge bases.

2 Some properties of Part-Whole Relations

In contrast to, for example, the relation `likes`, the pw-relation has a variety of properties. For a complete collection of these properties, we refer to [25]. Most importantly, the general pw-relation is a strict partial order, i.e., it is *transitive* and *asymmetric* (and hence irreflexive). That is, if x `part-of` y and y `part-of` z, then x `part-of` z, and if x `part-of` y, then not y `part-of` x. Moreover, an aggregated object has at least two parts where none is a part of the other. Next, we might consider to assume that two objects consisting of the same parts are identical. As a last example, we might assume the existence of atoms, i.e., indivisible objects of which all other objects are composed. This is equivalent to assume that `has-part` is well-founded and thus to exclude infinite chains x_0 `has-part` x_1 `has-part` $x_2 \ldots$

Sub-Relations of the General Part-Whole Relation Besides the properties mentioned above, the pw-relation is assumed to have various sub-relations, like, for example, the relation between a *component* and its *composite* (e.g. between a motor and the car the motor is in), the relation between *stuff* and an *object* containing this stuff (e.g. between metal and a car), or the relation between a *member* and a *collection* it belongs to (e.g. between a tree and the forest this tree belongs to).

These pw-relations are subject of several investigations and discussions; see, for example, [28, 12, 18]. However, various questions concerning pw-relations are still open. Let $\texttt{part-of}_i$ denote sub-relations of the general pw-relation:

- How are pw-relations defined, i.e., what is a necessary and sufficient condition $\Phi_i(x, y)$ for a part x of y to be in the relation $\texttt{part-of}_i$ with y, i.e., which Φ_i satisfies

$$x\ \texttt{part-of}_i\ y \Leftrightarrow (\Phi_i(x, y) \land x\ \texttt{part-of}\ y)?$$

- What is the interrelationship between pw-relations? Is one a specialisation of the other, or are they all independent?
- How do they interact with each other and with the general pw-relation? I.e., for which i, j, k does the following implication hold:

$$(x\ \texttt{part-of}_i\ y \land y\ \texttt{part-of}_j\ z) \Rightarrow x\ \texttt{part-of}_k\ z?$$

- What is a complete collection of pw-relations? And which of these relations are of importance in a specific application?

To help answering some of these questions, we present a scheme to structure pw-relations in Figure 2. At the top, you find the general pw-relation `part-of`. The only property we impose on `part-of` is that it is a strict partial ordering. Then `part-of` is specialised along three dimensions[2].

[1] LuFG Theoretical Computer Science, RWTH Aachen, Germany, email: sattler@cs.rwth-aachen.de

[2] The specialisations are not assumed to be disjoint.

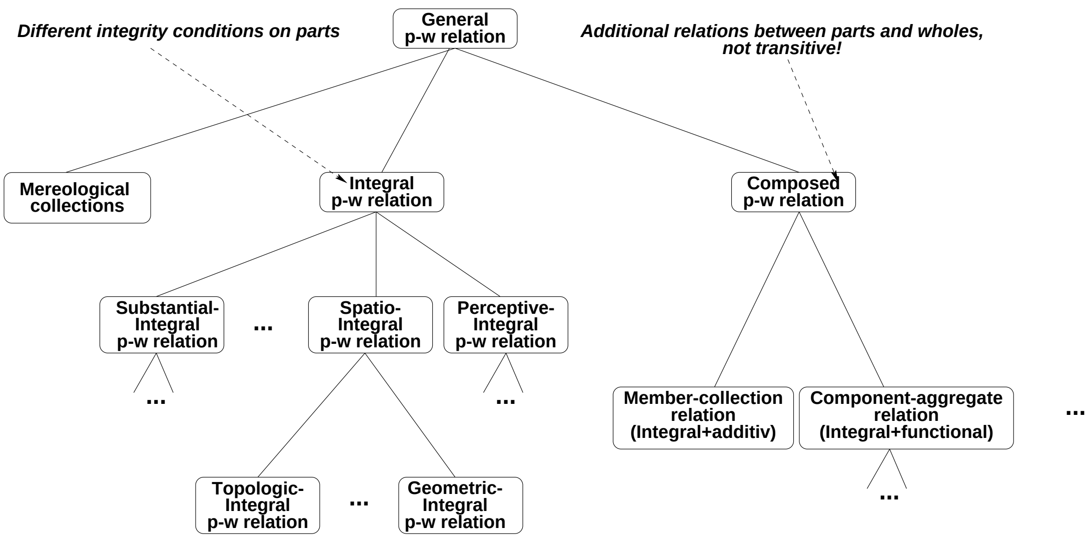

Figure 1. A Taxonomy of Part-Whole Relations.

Mereological collections refers to the pw-relation in the rather strict sense of classical mereology [25]. Besides being a strict partial order, mereological collections must satisfy a variety of properties. These properties determine the structures one accepts as models and those one wants to exclude from being models. For example, the *supplementation principle* says that if a whole has a strict part, then it also has another part that is independent from the first one. Another condition, *atomicity*, excludes structures with infinite ascending chains from being models.

Integral pw-relations are sub-relations of the general part-whole relation which involve certain *integrity conditions* on the parts. For x to be a part of y with respect to an integral pw-relation, x has to be a part of y, and x has to satisfy the integrity condition associated with this relation. Integral pw-relations are specialised in a natural way by specialising the associated integrity conditions.

For example, for the *substantial-integral pw-relation* to hold between x and y, x must be a part of y and x must be integral with respect to the substantial aspect, i.e., x must consist completely of some substance, which is, since x is also a part of y, also present in y. Another example to mention is the *geometric-integral pw-relation*, which holds between x and y if x is a part of y and x is geometrically integral, for example, a convex body. It is important to note that all these integrity conditions are imposed on the parts only, independent of the whole.

Composed pw-relations are characterised by an additional relation which has to hold between a part and its whole. For example, the *component-aggregate relation* holds between x and y if x is a part of y, x is an integral object (with respect to a certain integrity condition) and x is *functional* for y, i.e., the functioning of y depends on x. They are specialised by specialising the associated additional relations.

This additional relation (such as "being functional for" in the previous example) is the reason why composed pw-relations are, in general, not transitive: For a composed pw-relation to be transitive, the additional relation must be transitive.

This way of structuring pw-relations has two advantages: First of all, the interaction between pw-relations is given through their definition and the definition of the integrity conditions and additional relations. For example, the following implications are immediate consequences of the above definitions, where $\mathtt{int}_i$ are integrity conditions associated with integral pw-relations $\mathtt{int}{-}i{-}\mathtt{part}{-}\mathtt{of}$, and $\mathtt{comp}_i$ are additional relations for composed pw-relations $\mathtt{comp}{-}i{-}\mathtt{part}{-}\mathtt{of}$.

$$x \ \mathtt{integral{-}part{-}of} \ y \wedge y \ \mathtt{part{-}of} \ z \\ \Rightarrow x \ \mathtt{integral{-}part{-}of} \ z$$

$$x \ \mathtt{int{-}1{-}part{-}of} \ y \wedge (\forall z.\mathtt{int}_1(z) \Rightarrow \mathtt{int}_2(z)) \\ \Rightarrow x \ \mathtt{int{-}2{-}part{-}of} \ y$$

$$x \ \mathtt{comp{-}1{-}part{-}of} \ y \wedge \\ (\forall z_1, z_2.\mathtt{comp}_1(z_1, z_2) \Rightarrow \mathtt{comp}_2(z_1, z_2)) \\ \Rightarrow x \ \mathtt{comp{-}2{-}part{-}of} \ y$$

Secondly, the pw-relations identified in the literature can be placed in our taxonomy. For example, the classification of pw-relations from [28] into our taxonomy is given in Table 1.

3 Introduction to Description Logics

We briefly introduce syntax and semantics of the well-known, basic description logic $\mathcal{ALC}$ [24].

Definition 1 *Let* $\mathbf{C}$ *be a set of* concept names *and let* $\mathbf{R}$ *be a set of* role names. *The set of* $\mathcal{ALC}$-concepts *is the smallest set such that*

1. *every concept name is a concept and*
2. *if C and D are concepts and R is a role name, then $(C \sqcap D)$, $(C \sqcup D)$, $(\neg C)$, $(\forall R.C)$, $(\exists R.C)$ are concepts.*

The semantics is given by an interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, *which consists of a set* $\Delta^{\mathcal{I}}$, *called the* domain *of* $\mathcal{I}$, *and a function* $\cdot^{\mathcal{I}}$ *which maps every concept to a subset of* $\Delta^{\mathcal{I}}$ *and every role to a subset of*

part-whole relations in [28]	Example	Classification w.r.t. our taxonomy
Component → Composite	`motor → car`	Component-Aggregate relation (integral and functional)
Stuff → Object	`metal → car`	Substantial-Integral PW-relation
Member → Collection	`tree → forest`	Member-Collection PW-relation (integral and additive)
Portion → Mass	`slice → pie`	Spatio-Integral PW-relation
Feature → Activity	`paying→shopping`	Temporal Component-Aggregate Relation (integral and functional)
Place→ Area	`oasis → desert`	Geograph.-Integral PW-rel.

Table 1. Classification of the Part-Whole Relations from [28] w.r.t. our Taxonomy.

$\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ *such that*

$$\begin{aligned}
(C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}},\\
(C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}},\\
\neg C^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}},\\
(\exists R.C)^{\mathcal{I}} &= \{d \in \Delta^{\mathcal{I}} \mid \textit{There exists an } e \in \Delta^{\mathcal{I}} \textit{ with } (d,e) \in R^{\mathcal{I}}\\
&\qquad \textit{and } e \in C^{\mathcal{I}}\},\\
(\forall R.C)^{\mathcal{I}} &= \{d \in \Delta^{\mathcal{I}} \mid \textit{For all } e \in \Delta^{\mathcal{I}}, \textit{ if } (d,e) \in R^{\mathcal{I}},\\
&\qquad \textit{then } e \in C^{\mathcal{I}}\}.
\end{aligned}$$

A concept C is called satisfiable *iff there is some interpretation $\mathcal{I}$ such that $C^{\mathcal{I}} \neq \emptyset$. Such an interpretation is called a* model *of C. A concept D* subsumes *a concept C (written $C \sqsubseteq D$) iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ holds for each interpretation $\mathcal{I}$.*

So far, $\mathcal{ALC}$ allows us to describe concepts relevant in our application domain. For example, the following concept describes a cooled mixer-reactor:

$$\begin{aligned}
\texttt{Mat-Object} \sqcap (&\exists \texttt{has-part.Cooler}) \sqcap\\
(&\exists \texttt{has-part.Mixer}) \sqcap\\
(&\forall \texttt{contains.}(\texttt{Hom-phase} \sqcup \texttt{Inhom-phase}))
\end{aligned}$$

In [24], it was shown that reasoning in $\mathcal{ALC}$ (i.e., subsumption and satisfiability of $\mathcal{ALC}$-concepts) is decidable, more precisely, it is PSPACE-complete. Although this is far more complex than what is commonly assumed to be tractable, it turned out that the corresponding algorithms are amenable to optimisation and behave quite well in practice [3, 14, 13].

The terminological knowledge of an application domain can be fixed in a so-called *terminology*.

Definition 2 *A* terminological axiom *is an expression of the form $C \sqsubseteq D$, where C and D are concepts. A* terminology *is a finite set of terminological axioms. We use $C \doteq D$ as abbreviation for $C \sqsubseteq D$ and $D \sqsubseteq C$.*

An interpretation $\mathcal{I}$ satisfies *a terminological axiom $C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$, and it* satisfies *a terminology $\mathcal{T}$ iff it satisfies each axiom in it. Such an interpretation is called a* model *of $\mathcal{T}$. A concept C is* subsumed by *a concept D* with respect to a terminology *$\mathcal{T}$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ holds for each model $\mathcal{I}$ of $\mathcal{T}$. A concept C is* satisfiable with respect to a terminology *$\mathcal{T}$ iff $C^{\mathcal{I}} \neq \emptyset$ holds for some model $\mathcal{I}$ of $\mathcal{T}$.*

For example, in Section 1, you find two terminological axioms, one describing a control rod, the other describing a reactor core. Please note that this terminology contains a *cycle*: the description of control rod refers to reactor core, whose description refers to control rod. Besides the underlying description logic, knowledge representation systems based on description logics also differ in whether they allow for those *cyclic* terminologies or whether they restrict the left hand side of terminologies to concept names and disallow cycles.

4 Description Logics for Part-Whole Relations

In this section, we gradually extend $\mathcal{ALC}$ to give it more of the expressive power required for the representation of aggregated objects.

Transitivity of `part-of` One shortcoming of $\mathcal{ALC}$, when used for the representation of aggregated objects, is that it does not provide any means for the representation of *transitive* relations. For example, in $\mathcal{ALC}$, the concept

$$\begin{aligned}
\texttt{Device} \sqcap \exists\, \texttt{has-part}\,.(\,&\texttt{Reactor-core} \sqcap\\
&\exists\, \texttt{has-part}\,.\texttt{Control-rod})
\end{aligned}$$

is *not* subsumed by

$$\texttt{Device} \sqcap \exists\, \texttt{has-part}\,.\texttt{Control-rod},$$

although the first concept is a specialisation of the second one under the assumption that `has-part` is interpreted as a transitive relation.

There are basically three possibilities to overcome this shortcoming: We extend $\mathcal{ALC}$ with

1. the *transitive closure* of roles [2]. That is, for a role name R, we allow the use of its transitive closure R^+ in concepts of the form $\exists R^+.C$ and $\forall R^+.C$, and define interpretations to interpret R^+ as the transitive closure of $R^{\mathcal{I}}$.
   Unfortunately, this extension leads to EXPTIME-completeness of reasoning [22]—even with respect to empty terminologies.
2. the *transitive orbit* of roles [20], whose syntax is defined analogously to the one of the transitive closure. An interpretation is then defined to interpret the transitive orbit $R^{\oplus}$ of a role name R as *some* transitive relation containing $R^{\mathcal{I}}$. Although this seems to be much weaker an extension than the one by the transitive closure, it has the same consequences for the computational complexity, i.e., reasoning in $\mathcal{ALC}$ with transitive orbits is EXPTIME-complete [20].
3. *transitive roles*, i.e., we allow the user to specify a subset $\mathbf{R}_+ \subseteq \mathbf{R}$ of transitive role names, and define interpretations to interpret transitive role names $S \in \mathbf{R}_+$ as transitive relations. Reasoning in $\mathcal{ALC}$ with transitive roles could be shown to be in the same complexity class as pure $\mathcal{ALC}$, namely PSPACE [20].

The way of decomposing aggregated objects strongly depends on the individual taste, aims, and circumstances. Hence modeling aggregated objects using a *direct* hp-relation `has-d-part` is problematic. For example, defining $\texttt{Human} \sqsubseteq \exists\texttt{has-d-part.Abdomen}$ is as sensible as defining $\texttt{Human} \sqsubseteq \exists\texttt{has-d-part.Stomach}$ and $\texttt{Stomach} \sqsubseteq \exists\texttt{has-d-part.Abdomen}$. However, this yields models where a human has an abdomen as a direct part, and where the abdomen is also a part of the human's stomach—which clearly clashes with our intuition of *direct* pw-relations. Hence we believe that the third and "cheapest" extension is sufficient for most applications. By $\mathcal{S}$, we refer to the description logic $\mathcal{ALC}$ extended with transitive roles.[3]

Obviously, $\mathcal{S}$ provides the means to represent the general pw-relation as a transitive relation by asserting $\texttt{part-of} \in \mathbf{R}_+$. Additionally, since $\mathcal{S}$ has the tree-model property, for each model $\mathcal{I}$, we can construct a model $\mathcal{I}'$ in which $\texttt{part-of}^{\mathcal{I}'}$ is a strict partial ordering. Hence in $\mathcal{S}$ and all its extensions, we can model the general pw-relation. Moreover, we can also represent integral pw-relations:

Let $C_{\texttt{int}}$ be a concept describing a certain integrity condition,
replace $\exists\texttt{int-part-of}.C$ by $C_{\texttt{int}} \sqcap \exists\,\texttt{part-of}\,.C$ and
replace $\forall\texttt{int-part-of}.C$ by $\neg C_{\texttt{int}} \sqcup \forall\,\texttt{part-of}\,.C$.

Obviously, this substitution yields a concept in which `int-part-of` is interpreted according to the intended meaning, i.e., each instance of $\exists\texttt{int-part-of}.C$ is integral w.r.t. to the condition $C_{\texttt{int}}$. Please note that different kinds integrity conditions require different expressive power—possibly more than $\mathcal{S}$ provides.

Either `part-of` or `has-part`? When describing concepts of an application domain using $\mathcal{S}$ and using the pw- as well as the hp-relation, we risk that the description is not adequate: `part-of` is the *inverse* of `has-part` (and vice versa), a fact that cannot be expressed in $\mathcal{S}$. For example, extending the terminology in Section 1 with

$$\texttt{Nuclear\_reactor} \sqcap \exists\texttt{has\_part.Faulty} \sqsubseteq \texttt{Dangerous},$$

we would assume that $\texttt{Control\_rod} \sqcap \texttt{Faulty}$ is subsumed by $\exists\texttt{part-of.Dangerous}$ w.r.t. to this terminology—which is only the case if `part-of` were the inverse of `has-part`. Hence in $\mathcal{S}$, we must decide whether (1) we use either `part-of` or `has-part`, (2) we use `part-of` and `has-part` and live with the fact that our model is inadequate in the sense of the previous example, or (3) extend $\mathcal{S}$ with inverse roles. We have decided to choose option 3:

Definition 3 *The description logic $\mathcal{SI}$ is obtained from $\mathcal{S}$ by allowing, additionally, for* inverse role names R^- *with $R \in \mathbf{R}$ to occur in the place of role names. An interpretation must satisfy, additionally,*

$$(R^-)^{\mathcal{I}} := \{(e,d) \mid (d,e) \in R^{\mathcal{I}}\}.$$

Hence in $\mathcal{SI}$, we can describe both objects by means of the wholes they belong to and by means of the parts they have. Substituting `has-part` by $\texttt{part-of}^-$ in the last example yields that $\texttt{Control\_rod} \sqcap \texttt{Faulty}$ is indeed subsumed by $\exists\texttt{part-of.Dangerous}$.

Fortunately, it could be shown that reasoning in $\mathcal{SI}$ is still PSPACE-complete [16].

Composed Sub-Part-Whole Relations To additionally represent composed pw-relations, we extend $\mathcal{SI}$ with *role-hierarchies*, which allow the user to represent composed part-whole relations as *sub-roles* of the general pw-relation.

Definition 4 *A* role inclusion axiom *is an expression of the form $R \sqsubseteq S$, where R and S are (possibly inverse) roles. A* role hierarchy *is a finite set of role inclusion axioms. An interpretation $\mathcal{I}$* satisfies *a role hierarchy $\mathcal{R}$ iff $R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$ for each $R \sqsubseteq S$ in $\mathcal{R}$. Such an interpretation is called a* model *of $\mathcal{R}$.*

Satisfiability and subsumption w.r.t. role hierarchies are defined in the obvious way. $\mathcal{SHI}$ is the extension of $\mathcal{SI}$ with role hierarchies.

For a role hierarchy $\mathcal{R}$, the sub-role relation *is the transitive closure of $\sqsubseteq$ on $\mathcal{R} \cup \{R^- \sqsubseteq S^- \mid R \sqsubseteq S \in \mathcal{R}\}$.*[4]

Adding role hierarchies to $\mathcal{SI}$ has mainly two consequences: First, we can specify *composed pw-relations*, i.e., we can introduce (possibly transitive—depending on the additional relation) role names $\texttt{comp-}i\texttt{-part-of}$ and add role inclusion axioms

$$\begin{array}{lcl}\texttt{comp-}i\texttt{-part-of} & \sqsubseteq & \texttt{part-of} \text{ or} \\ \texttt{comp-}i\texttt{-part-of} & \sqsubseteq & \texttt{comp-}j\texttt{-part-of}.\end{array}$$

However, we cannot define (in the sense of axiomatise) these additional relations. For example, we cannot specify what conditions a part and a whole must satisfy for the component-aggregate relation to hold between them.

Second, $\mathcal{SHI}$ (as well as $\mathcal{SH}$ and $\mathcal{SHIQ}$) has the expressive power for the *internalisation* of terminologies [2, 15]. This technique polynomially reduces reasoning w.r.t. a (possibly cyclic) terminology to pure concept reasoning. First, we introduce a new transitive role name $U \in \mathbf{R}_+$ and specify that U is a super-role of all roles and their respective inverses. Then, a concept C is satisfiable w.r.t. $\{C_i \sqsubseteq D_i \mid 1 \leq i \leq n\}$ iff $C \sqcap \bigsqcap_{1 \leq i \leq n} \neg C_i \sqcup D_i$ is satisfiable.

Number Restrictions In general, when modeling an application domain, it seems to be natural to describe an object by restricting the number of objects it is related to via a certain relation. For example, the first of the following concepts describes pipes as those connections having exactly 1 input and 1 output, whereas the second concept describes forks as those connections having 1 input and at least 2 outputs:[5]

$$\begin{array}{l}\texttt{Connection} \sqcap (=1\ \texttt{c-part-of}^-\ \texttt{In}) \sqcap (=1\ \texttt{c-part-of}^-\ \texttt{Out}) \\ \texttt{Connection} \sqcap (=1\ \texttt{c-part-of}^-\ \texttt{In}) \sqcap (\geq 2\ \texttt{c-part-of}^-\ \texttt{Out})\end{array}$$

Since number restrictions are mostly "harmless" from an algorithmic point of view [26], we have added them to $\mathcal{SHI}$.

Definition 5 *A (possibly inverse) role is called* simple *if it is neither transitive nor has a transitive sub-role.*

$\mathcal{SHIQ}$ is obtained from $\mathcal{SHI}$ by allowing, additionally, for concepts of the form $(\geqslant nR.C)$ and $(\leqslant nR.C)$ for n a non-negative integer, R a simple role, and C a $\mathcal{SHIQ}$-concept.

Let $\sharp M$ denote the cardinality of a set M. An interpretation must satisfy, additionally,

$$\begin{array}{lcl}(\geqslant nR.C)^{\mathcal{I}} & = & \{x \mid \sharp\{y.\langle x,y\rangle \in R^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\} \geqslant n\} \text{ and} \\ (\leqslant nR.C)^{\mathcal{I}} & = & \{x \mid \sharp\{y.\langle x,y\rangle \in R^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\} \leqslant n\}.\end{array}$$

5 Discussion

Computational Properties of $\mathcal{SHIQ}$: It is known that reasoning for the highly expressive description logic $\mathcal{CIQ}$[6] is EXPTIME-complete [9]. Now $\mathcal{SHIQ}$ is less expressive than $\mathcal{CIQ}$ and in the same complexity class, namely EXPTIME-complete (this is a consequence of

[3] The logic $\mathcal{S}$ has previously been called $\mathcal{ALC}_{R^+}$, but this becomes too cumbersome when adding letters to represent additional features.

[4] We assume that $R^{--} = R$.

[5] We use $(=1\ R)$ as a short hand for $(\leq 1\ R) \sqcap (\geq 1\ R)$

[6] Basically, $\mathcal{CIQ}$ is obtained from $\mathcal{SHIQ}$ by allowing for regular expressions of roles in concepts of the form $\exists R.C$ or $\forall R.C$. The main difference between $\mathcal{CIQ}$ and $\mathcal{SHIQ}$ is that the latter has the transitive closure operator on roles, whereas $\mathcal{SHIQ}$ only has transitive roles and role hierarchies.

the fact that $\mathcal{SHIQ}$ is a fragment of $\mathcal{CIQ}$, which is in EXPTIME, and the fact that $\mathcal{SHIQ}$ is an extension of $\mathcal{ALC}$ with transitive orbits, which is EXPTIME-hard [20]—which is basically due to the fact that we can introduce a transitive super-role of all other roles). Hence a question naturally arising here is why we should be interested in $\mathcal{SHIQ}$. The answer is that there is a direct tableau-based decision procedure for the satisfiability and subsumption of $\mathcal{SHIQ}$-concepts with respect to role hierarchies and possibly cyclic terminologies [16]. After first experiments with the extension IFaCT of the description logic system FaCT [14], we believe that this algorithm is as practicable and well-behaved in practice as the one implemented in FaCT. One reason for this good behaviour could be that a large fragment of $\mathcal{SHIQ}$, namely the one obtained by omitting role hierarchies (and thus the ability to internalise terminologies) is PSPACE-complete [16]. Another nice property of the tableau-based algorithm for $\mathcal{SHIQ}$ is that it does not have an equivalent to the *analytic cut rule* [10], a rule that introduces a large amount of non-determinism and that is used in satisfiability algorithms in the presence of the transitive closure operator and inverse roles.

Expressive power of $\mathcal{SHIQ}$: In $\mathcal{SHIQ}$ and $\mathcal{CIQ}$, we can internalise terminologies, hence polynomially reduce reasoning w.r.t. terminologies to pure concept reasoning.

Like $\mathcal{CIQ}$, $\mathcal{SHIQ}$ lacks the *finite model property* (i.e., some concepts only have infinite models), and has the *tree model property* (i.e., each satisfiable concept has a tree model).

If we assume the existence of atoms, the $\mathcal{SHIQ}$ reasoning algorithm in [16] is not yet satisfactory since it admits models where—even though `part-of` is interpreted as a strict partial ordering—`part-of` is not well-founded. Since `part-of` should be interpreted as an asymmetric relation, having a reasoning algorithm for *finite* models would neither be satisfactory. Instead, atomicity requires a reasoning algorithm that decides the existence of a model where `part-of` is a *well-founded*, strict partial ordering [8].

Finally, since $\mathcal{SHIQ}$ has the tree model property, we cannot model what is called "inheritance along pw-relations". For example, we cannot model that the owner of the reactor is also the owner of the mixer and the cooler, i.e., that the property `owner` is "inherited" along the hp-relation (in [1], some approaches are discussed that have this kind of expressive power). Using *role value maps* (i.e., concepts like `has-component` $\circ$ `owner` $\sqsubseteq$ `owner`) to model this "inheritance" is not a good idea: even for very weak description logics, the extension with role-value maps leads to the undecidability of subsumption and satisfiability [23]. However, there are some restrictions on or variants of role value maps that should be investigated since they might yield decidability of these inference problems.

Summing up from what we have said above, $\mathcal{SHIQ}$ is able to model the general pw-relation as well as various integral pw-relations (depending on the integrity condition), and composed pw-relations. All these relations can be modeled in both directions, and their interaction—as implied by the integrity conditions and role hierarchy—are taken into account. Unfortunately, some properties of pw-relations cannot be modeled, e.g., we can neither impose atomicity nor express inheritance along pw-relations. However, we believe that $\mathcal{SHIQ}$ is a highly expressive logic with good computational properties and well-suited for the representation of aggregated objects. To overcome the shortcomings mentioned above will be part of future work.

Acknowledgements I would like to thank Alessandro Artale, Franz Baader, Enrico Franconi, Nicola Guarino, Carsten Lutz, and Stephan Tobies for inspiring and valuable discussions.

REFERENCES

[1] A. Artale, E. Franconi, N. Guarino, and L. Pazzi, 'Part-whole relations in object-centered systems: An overview', *Data and Knowledge Engineering*, **20**(3), (1996).

[2] F. Baader, 'Augmenting concept languages by transitive closure of roles: An alternative to terminological cycles', in *Proc. of IJCAI-91*, (1991).

[3] F. Baader, E. Franconi, B. Hollunder, B. Nebel, and H.J. Profitlich, 'An empirical analysis of optimization techniques for terminological representation systems, or: Making KRIS get a move on', *Applied Artificial Intelligence*, **4**, (1994).

[4] D. Calvanese, G. De Giacomo, and M. Lenzerini, 'On the decidability of query containment under constraints', in *Proc. of PODS'98*, (1998).

[5] D. Calvanese, G. De Giacomo, M. Lenzerini, D. Nardi, and R. Rosati, 'Description logic framework for information integration', in *Proc. of KR-98*, (1998).

[6] D. Calvanese, G. De Giacomo, and R. Rosati, 'Data integration and reconciliation in data warehousing: Conceptual modeling and reasoning support', *Network and Information Systems*, **4**(2), (1999).

[7] D. Calvanese, M. Lenzerini, and D. Nardi, 'Description logics for conceptual data modeling', in *Logics for Databases and Information Systems*, eds., Jan Chomicki and Günter Saake, 229–263, Kluwer Academic Publisher, (1998).

[8] D. Calvanese, 'Finite model reasoning in description logics', in *Proc. of KR-96*. M. Kaufmann, Los Altos, (1996).

[9] G. De Giacomo and M. Lenzerini, 'Tbox and Abox reasoning in expressive description logics', in *Proc. of KR-96*, (1996).

[10] G. De Giacomo and F. Massacci, 'Combining deduction and model checking into tableaux and algorithms for converse-pdl', *Information and Computation*, (1998).

[11] E. Franconi and U. Sattler, 'A data warehouse conceptual data model for multidimensional aggregation: a preliminary report', *Italian Association for Artificial Intelligence AI*IA Notizie*, **1**, (1999).

[12] Peter Gerstl and Simone Pribbenow, 'Midwinters, end games and body-parts', in *Intern. Workshop on Formal Ontology-93*, eds., N. Guarino and R. Poli, (1993).

[13] V. Haarslev and R. Möller, 'RACE system description', in *Proc. of DL'99*, (1999). CEUR.

[14] I. Horrocks, 'Using an Expressive Description Logic: FaCT or Fiction?', in *Proc. of KR-98*, (1998).

[15] I. Horrocks and U. Sattler, 'A description logic with transitive and inverse roles and role hierarchies', *J. of Logic and Computation*, (1999).

[16] I. Horrocks, U. Sattler, and S. Tobies, 'Practical reasoning for expressive description logics', in *Proc. of LPAR'99*, nr. 1705 of *LNAI*, Springer-Verlag, (1999).

[17] L. Padgham and P. Lambrix, 'A Framework for Part-of Hierarchies in Terminological Logics', in *Proc. of KR-94*, (1994).

[18] S. Pribbenow, 'Modeling physical objects: Reasoning about (different kinds of) parts', in *Time, Space, and Movement Workshop 95*, Bonas, France, (1995).

[19] A. Rector and I. Horrocks, 'Experience building a large, re-usable medical ontology using a description logic with transitivity and concept inclusions', in *Proc. of AAAI'97*. AAAI Press, Menlo Park, California, (1997).

[20] U. Sattler, 'A concept language extended with different kinds of transitive roles', in *20. Deutsche Jahrestagung für KI*, volume 1137 of *LNAI*. Springer-Verlag, (1996).

[21] U. Sattler, 'Knowledge representation in process engineering', in *Proc. of WRKP'96*. DFKI GmbH, (1996).

[22] K. Schild, 'A correspondence theory for terminological logics: Preliminary report', in *Proc. of IJCAI-91*, Sydney, (1991).

[23] M. Schmidt-Schauss, 'Subsumption in KL-ONE is undecidable', in *Proc. of KR-89*, Boston (USA), (1989).

[24] M. Schmidt-Schauß and G. Smolka, 'Attributive concept descriptions with complements', *Artificial Intelligence*, **48**(1), (1991).

[25] P. M. Simons, *Parts. A study in Ontology*, Oxford: Clarendon, 1987.

[26] S. Tobies, 'PSPACE reasoning for graded modal logics', *J. of Logic and Computation*, (2000). To appear.

[27] H. Wache and G. Kamp, 'Using description logic for configuration problems', in *Proc. of WRKP'96*. DFKI GmbH, (1996).

[28] M.E. Winston, R. Chaffin, and D. Herrmann, 'A taxonomy of part whole relations', *Cognitive Science*, **11**, (1987).