

# Query-Based Access Control for Ontologies

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**Abstract.** Role-based access control is a standard mechanism in information systems. Based on the role a user has, certain information is kept from the user even if requested. For ontologies representing knowledge, deciding what can be told to a user without revealing secrets is more difficult as the user might be able to infer secret knowledge using logical reasoning. In this paper, we present two approaches to solving this problem: query rewriting vs. axiom filtering, and show that while both approaches prevent the unveiling of secret knowledge, axiom filtering is more complete in the sense that it does not suppress knowledge the user is allowed to see while this happens frequently in query rewriting. Axiom filtering requires that each axiom carries a label representing its access level. We present methods to find an optimal axiom labeling to enforce query-based access restrictions and report experiments on real world data showing that a significant number of results are retained using the axiom filtering method.

## 1 Motivation

Access control is an essential operation in standard information systems to prevent unauthorized access and use of information from the system. In a traditional information system, where all the available information is stored explicitly, it is possible to simply label information items with the roles, a user must have, to be allowed to receive this particular information. With knowledge represented in ontologies, this approach does not work anymore, because new knowledge can be derived, leading to the '*inference problem*' [5]: avoiding a situation where a user can infer knowledge he should not have access to using knowledge he is allowed to access. To make the problem well defined, we assume that the user has the same ability to derive knowledge as the system.

In this paper, we compare two existing proposals for solving the inference problem: query rewriting vs. axiom filtering. For both, we start from an access restriction given in the form of a query, whose result is a set of axioms that shall be protected. Such a query could, for example, address knowledge about a concept and all subconcepts in order to restrict knowledge along the subsumption hierarchy comparable to information systems restricting access to files in a directory and all subdirectories. Conflict resolution mechanism might be necessary then since a concept might have multiple superconcepts. The query rewriting approach proposed in [4] is based on the idea of rewriting user queries

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based on the role a user has in such a way that the result to the rewritten query only returns knowledge the user is allowed to see. The axiom filtering approach proposed in [1] assumes an a priori labeling of axioms in the ontology to consistently derive labels for implicit consequences. Axioms and consequences are delivered based on a comparison of user label and axiom label. Our assessment of the two approaches concludes that axiom filtering is independent of the ontology language and more complete in the sense that it does not suppress knowledge the user is allowed to see.

However axiom filtering requires an a priori labeling of axioms and it is not clear from previous work how to create an access labeling from query-based access restrictions. Our main contributions are (1) algorithms to repair a given axiom labeling in an optimal way so that a query-based access restriction is enforced to explicit and implicit knowledge, (2) conflict resolution strategies for cases where query-based access restrictions contain conflicts, (3) empirical results for our algorithms with practical ontologies. Our main result is that axiom filtering provides higher availability of knowledge compared to query rewriting.

## 2 Preliminaries

### 2.1 Ontologies

Ontologies are formal descriptions of the terminology used in an application domain. A number of logical languages have been proposed for representing ontologies. In this paper, we only consider sublanguages of the Web Ontology Language (OWL) that can be translated to Description Logics (DL).

Formally, an *ontology*  $O$  is a finite set, whose elements are called *axioms*, such that every subset of an ontology is itself an ontology. If  $O' \subseteq O$  and  $O$  is an ontology, then  $O'$  is called a *sub-ontology* of  $O$ . One can distinguish ABox axioms  $A$  and TBox axioms  $T$  and let  $O = T \cup A$ . An ontology language specifies which sets of axioms are admitted as ontologies. For instance, given a Description Logic  $\mathcal{L}$  (e.g., the DL  $\mathcal{SHOIN}(\mathbf{D})$  underlying OWL DL), an ontology is a finite set of general concept inclusion axioms (GCIs) of the form  $C \sqsubseteq D$ , concept assertion axioms of the form  $C(a)$  and role assertion axioms of the form  $R(a, b)$  for  $\mathcal{L}$ -concept descriptions  $C, D$ , role  $R$  and individuals  $a, b$ . In order not to mix user roles and DL roles, we stick to the OWL lingo and call DL roles from now on *properties*. The signature  $\text{sig}(O)$  of an ontology is the set of all concept and role names occurring in its axioms. Given an ontology language, a *monotone consequence relation*  $\models$  is a binary relation between ontologies  $O$  and *consequences*  $c$  such that if  $O \models c$ , then for every ontology  $O' \supseteq O$  it holds that  $O' \models c$ . If  $O \models c$ , we say that  $c$  *follows from*  $O$  or that  $O$  *entails*  $c$ . Often, a consequence  $c$  already follows from a subset  $S \subseteq O$  of the axioms in the ontology. We call such a subset an *explanation* for  $O \models c$  if there is no subset  $S' \subset S$  such that  $S' \models c$ . Note that for one consequence there might be multiple explanations.

A query to an ontology is a conjunction  $Q = A_1, \dots, A_n$  of OWL axioms over  $\text{sig}(O)$ , but not necessarily from  $O$ , containing variables. For a concrete definition of the form of axioms see [12]. The set of variables occurring in  $Q$  is denoted as  $\text{var}(Q)$ . Let  $\text{ind}(O)$  be the set of individuals in  $O$ , then the result of a query is the set of all

mappings  $\mu : \text{var}(Q) \rightarrow \text{ind}(O)$  assigning individuals from  $O$  to variables in  $Q$ . An answer  $\mu(Q)$  to a query  $Q$  is an instantiation of all variables in the query, so that  $O \models \mu(Q)$  [12]. Note that there might be several possible  $\mu$  for one query.

## 2.2 Access Control

Access control systems enable the regulation of access to protected resources (i.e. objects) in distributed systems by subjects such as users or system processes. They can be categorized in discretionary access control (DAC), mandatory access control (MAC), and role-based access control (RBAC) models. In DAC-based systems, the permissions to access an object are defined by its owner. In MAC models, the system determines the access to objects either by utilizing access rules or lattices for assigning permissions to subjects. It thus removes the ability of the users to control access to their resources. RBAC systems finally remove the explicit use of subjects within access rules or lattices and replace them with roles, which form a logical group of a number of subjects. In fact, permissions are assigned to roles and the subjects are assigned members of a number of roles. Thus changes of single subjects do not necessarily have consequences in the actual access control policies. On the most fine-grained level, permissions can be defined on the level of axioms, or on the level of query responses.

## 2.3 Access Restrictions as Queries

Assume we want customers and employees to query knowledge from a product ontology. From Example 1, employees have full access and we do not want customers to see if any product gets an increased price soon. This restriction could be defined by enumerating all query responses except the price increase as permissions and assigning them to the respective user role. There are two problems with this approach. First of all, the price increase can still be inferred if the axioms of  $O$  can be queried. Further, enumerating all query responses, however, is not feasible in practice and asks for more efficient ways of specifying these restrictions, e.g. by means of a query.

*Example 1.* Let  $O$  be an ontology from a marketplace in the Semantic Web with the following axioms

- $a_1 : EUecoService \sqcap HighperformanceService(ecoCalculatorV1)$
- $a_2 : HighperformanceService$   
 $\sqsubseteq ServiceWithLowCustomerNr \sqcap LowProfitService$
- $a_3 : EUecoService \sqsubseteq ServiceWithLowCustomerNr \sqcap LowProfitService$
- $a_4 : ServiceWithLowCustomerNr \sqsubseteq ServiceWithComingPriceIncrease$
- $a_5 : LowProfitService \sqsubseteq ServiceWithComingPriceIncrease$

The consequence  $c_1 : ServiceWithComingPriceIncrease(ecoCalculatorV1)$  follows from each of the explanations  $\{a_1, a_2, a_4\}, \{a_1, a_2, a_5\}, \{a_1, a_3, a_4\}, \{a_1, a_3, a_5\}$ . The consequence  $c_2 : LowProfitService(ecoCalculatorV1)$  follows from each of the explanations  $\{a_1, a_2\}, \{a_1, a_3\}$ . Three more instance assertions of individual  $ecoCalculatorV1$  to the concept names  $EUecoService$ ,  $HighperformanceService$ ,  $ServiceWithLowCustomerNr$  are consequences of  $O$ .

A way is to define permissions intentionally in terms of queries over the signature of the ontology. More specifically, we can describe facts that should not be accessible by a certain role in terms of a set of axioms - the same kinds of axioms used in queries - whose instantiations should not be derivable from query results. In the case of the example above, we could formulate the following access restriction for customers:

$$ServiceWithComingPriceIncrease(x)$$

stating that for no instantiation of the variable  $x$  it should be possible to infer that it is an instance of *ServiceWithComingPriceIncrease*.

### 3 Enforcing Access Restrictions

There are different ways for implementing access control for ontological knowledge. While query rewriting extends a user's query to include all access restrictions, axiom filtering only allows a subset of the ontology to be used to answer the unchanged query.

#### 3.1 Access Control by Query Rewriting

One option for enforcing access restrictions is by means of query rewriting. This approach has been proposed in [4] as a suitable way for enforcing access restrictions in the context of SPARQL queries, while the TBox is assumed to be completely public. Similar approaches are also allowing to hide TBox parts [7], or to define not the restrictions but the permissions by a query [3]. The idea in [4] is to automatically add filter conditions to the query that suppress such answers the user is not supposed to see. Given a Query  $Q$  and a set of access restrictions  $\{AR_1, \dots, AR_n\}$  that apply to the current user, the query can be rewritten to a new query that is defined as:

$$Q \wedge \neg AR_1 \wedge \dots \wedge \neg AR_n$$

Where the junction of two queries  $Q_1 \wedge Q_2$  is the junction of all contained query axioms  $\bigwedge_{q \in Q_1} q \wedge \bigwedge_{q \in Q_2} q$  [12]. This way of rewriting the query based on the access restrictions of the individual users effectively prevents the system from giving away restricted knowledge. In particular, using query rewriting, the effective answer to a query is

$$\{\mu(Q)|O \models \mu(Q \wedge \neg AR_1 \wedge \dots \wedge \neg AR_n)\}$$

It however comes with a problem: it hides more knowledge than necessary. In particular, in the example above where we want to hide from customers that some product is increased in price, the query rewriting approach hides too much knowledge. If a customer for instance asks the system for all high performance services, thus  $Q = HighperformanceService(x)$ , this query will be rewritten to  $HighperformanceService(x) \wedge \neg ServiceWithComingPriceIncrease(x)$ . This query will only return high performance services which will not be increased in price. This is unfortunate, because the knowledge that *ecoCalculatorV1* is a high performance service was not supposed to be hidden. Similarly querying for instances of the remaining four concept names in *sig(O)* are filtered, resulting in five queries without an answer.

### 3.2 Access Control by Axiom Filtering

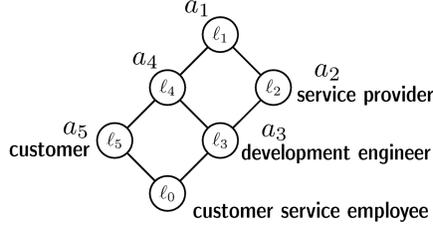
A framework to control access to an ontology's axioms is introduced in [1]. In contrast to the query rewriting approach above, the TBox is not assumed to be completely public. The idea is to label each axiom with a certain access restriction. Users are labeled with the restrictions they are allowed to see. The approach is to use a labeling lattice  $(L, \leq)$ ; i. e. a set  $L$  of labels together with a partial order  $\leq$  such that every finite set of labels has a join ( $\oplus$ , supremum, least upper bound) and a meet ( $\otimes$ , infimum, greatest lower bound) w.r.t.  $\leq$ . Every axiom  $a$  in the ontology  $O$  is assumed to have a label  $\text{lab}(a) \in L$ , and each user receives also a label  $\ell \in L$ . The sub-ontology to which a user with label  $\ell$  has access is defined as

$$O_{\geq \ell} := \{a \in O \mid \text{lab}(a) \geq \ell\}.$$

The sub-ontologies  $O_{\geq \ell}$ ,  $O_{\leq \ell}$  etc. can be defined analogously. Applied to our scenario with the user roles customer ( $\ell_C$ ) and employee ( $\ell_E$ ), let the labeling lattice be  $(L, \leq)$  with  $L = \{\ell_C, \ell_E\}$  and  $\leq = \{(\ell_E, \ell_C)\}$ . Let the labeling function  $\text{lab}$  assign label  $\ell_C$  to axioms  $a_1, a_2, a_3$  and  $\ell_E$  to axioms  $a_4, a_5$ . Employees can see  $O_{\geq \ell_E} = \{a_1, a_2, a_3, a_4, a_5\}$ , i.e. the complete ontology. Customers can see  $O_{\geq \ell_C} = \{a_1, a_2, a_3\}$ . Intuitively, the access restriction to a consequence, called boundary, should be based on the access restriction of its implying axioms. The access restriction for a consequence with multiple explanations should be the least restrictive of all explanations and within one explanation the most restrictive of all axioms. Formally, a consequence  $c$  with  $n$  explanations  $S_1, \dots, S_n$  has boundary  $\bigoplus_{i=1}^n \bigotimes_{a \in S_i} \text{lab}(a)$ . In our example, each of the four explanations for  $c_1$  has label  $(\ell_C \otimes \ell_C \otimes \ell_E) = \ell_E$ , thus the boundary is  $\ell_E$ , i.e. employees can see it but customers not. Consequence  $c_2$  has boundary  $\ell_C$ , i.e. employees and customers can see it. Apart from  $c_1, c_2$ , instance relationships to the three remaining concepts in  $\text{sig}(O)$  have boundary  $\ell_C$  as can be verified easily. A customer querying for instances of the five concept names in the ontology will get no answer for  $Q = \text{ServiceWithComingPriceIncrease}(x)$  but will get an answer for the four remaining queries. So axiom filtering provides 4/5 answers, while query rewriting provides 0/5 answers.

### 3.3 Discussion

As we have seen, query rewriting and axiom filtering are approaches of ensuring that no classified knowledge is given to users that do not have the permission to see it. Both approaches do neither require to track the history of queries nor disallow query askers of the same user role to share any knowledge. We have seen that query rewriting is suboptimal with respect to availability in the sense of preserving maximal access to non-restricted knowledge. Axiom filtering provides a higher availability and is more general since it is independent of the concrete ontology language which makes the approach preferable in many situations. However it requires an a priori axiom labeling, and it is not clear how to enforce query-based access restrictions. Previous work on labeled ontologies focused on computing a consequence's label based on axiom labels [1] and on repairing the axiom labeling in order to determine one consequence's label [9, 10]. However, access restrictions in the form of queries might require changing labels



**Fig. 1.** Lattice  $(L, \leq)$  with 4 user labels and an assignment of 5 axioms to labels

of multiple consequences simultaneously. Such a mechanism will be presented in the next section. Our main quality criterion for the algorithms is availability. In the empirical evaluation we measure how many knowledge is additionally accessible with axiom filtering compared to query rewriting.

#### 4 Optimal Axiom Labeling for implementing Access Control

In the last section we have only shown that there is an axiom labeling to enforce access restrictions for a selected example. Now we will elaborate how to compute it in general. We are starting from an arbitrary label assignment, and change it in a minimal way so that a given access restriction is enforced.

*Example 2.* We continue Example 1. Let  $(L, \leq)$  be the lattice shown in Figure 1, where valid user labels are  $l_0, l_2, l_3, l_5$  which represent user roles as illustrated. The condition for a valid user label is the join prime property discussed in [1]. Let  $O$  of Example 1 be a labeled ontology where the function  $\text{lab}$  assigns to each axiom  $a_i$  the label  $l_i$  as shown in Figure 1. The computed boundary is  $l_3$  for  $c_1$ , since  $= (l_1 \otimes l_2 \otimes l_4) \oplus (l_1 \otimes l_2 \otimes l_5) \oplus (l_1 \otimes l_3 \otimes l_4) \oplus (l_1 \otimes l_3 \otimes l_5)$ . It is  $l_2$  for  $c_2$ , since  $= (l_1 \otimes l_2) \oplus (l_1 \oplus l_3)$ . For users  $l_0$  and  $l_3$ , consequences  $c_1$  and  $c_2$  are visible. For user  $l_2$ , only  $c_2$  is visible.

We now define a notion for changing an axiom label assignment. Beforehand, we define the function  $\text{lbl}$  in order to address computed boundaries of consequences in a convenient way.

**Definition 1 (Consequence Labeling Function).** Let  $O$  be a labeled ontology,  $(L, \leq)$  a labeling lattice,  $\text{lab} : O \rightarrow L$  a labeling function. The consequence labeling function  $\text{lbl} : \{c \mid O \models c\} \rightarrow L$  assigns labels to consequences and is defined as  $\text{lbl}(c) =$  computed boundary of  $c$ .

**Definition 2 (MCS).** Let  $O$  be an ontology,  $c$  any consequence of  $O$ ,  $(L, \leq)$  a lattice,  $\text{lab}$  a labeling function,  $G$  a set of goals of the form  $(c, \ell_g)$  with goal label  $\ell_g$  for consequence  $c$ ,  $M$  a set of assignments  $(a, \ell)$  of label  $\ell$  to axiom  $a$ . The modified assignment  $\text{lab}_M$  is defined to be

$$\text{lab}_M(a) = \begin{cases} \ell, & \text{if } (a, \ell) \in M, \\ \text{lab}(a), & \text{otherwise.} \end{cases}$$

The respective consequence labeling function  $\text{lbl}_M$  is given by Definition 1. The set  $M$  is called multiple change set (MCS) iff for any  $c, (c, \ell_g) \in G : \text{lbl}_M(c) = \ell_g$  and there is no  $M' \subset M$  with  $\text{lbl}_{M'}(c) = \ell_g$ .

Whether we can find a  $\text{lab}_M$  fulfilling a given goal set is independent of the label assignment  $\text{lab}$  we start from. For default deny-all behavior, we start with all axioms assigned to the bottom lattice element. For default allow-all behavior, we start with all axioms assigned to the top lattice element. We will now introduce the computation of a change set for one goal and building on that introduce the computation of a MCS.

#### 4.1 Computing a Change Set for one Goal Label

If  $G$  is the singleton set of only one tuple  $(c, \ell)$ , computing a multiple change set boils down to computing a change set (CS) which has been introduced in our prior work in [10, 9]. For every CS  $S \subseteq O$  there is a MCS  $M := \{(a, \ell_g) \mid a \in S\}$  and  $\text{lbl}_M(c) = \ell_g$  holds. The computation of a CS exploited main ideas from axiom-pinpointing [8, 2] and we presented a black-box approach that yields the desired set. Intuitively, a consequence  $c$  needs to be made more public if  $\ell_g > \text{lbl}(c)$  or less public if  $\ell_g < \text{lbl}(c)$ . From the perspective of the target users who see  $O_{\geq \ell_g}$ , the former is achieved by including an axiom set IAS to their ontology and the latter by removing an axiom set RAS from other user's ontologies. The definition of an IAS (RAS) is a generalization of the definition of a MinA (diagnosis) [10].

**Definition 3 (IAS,RAS).** A minimal inserted axiom set (IAS) for  $\ell_g$  is a subset  $I \subseteq O_{\geq \ell_g}$  such that  $O_{\geq \ell_g} \cup I \models c$  and for every  $I' \subset I : O_{\geq \ell_g} \cup I' \not\models c$ . A minimal removed axiom set (RAS) for  $\ell_g$  is a subset  $R \subseteq O_{\leq \ell_g}$  such that  $O_{\leq \ell_g} \setminus R \not\models c$  and for every  $R' \subset R : O_{\leq \ell_g} \setminus R' \models c$ .

A CS is either an IAS, a RAS, or union of both. As elaborated in [10], computing IAS and RAS is tightly related to computing explanations (also called MinA) and diagnoses. The computation by a Hitting Set Tree (HST) algorithm [11] is repeated here only briefly. The HST algorithm makes repeated calls to an auxiliary procedure that computes one CS. A tree is built, where each node is labeled with a CS and each edge with an axiom. If the CS labeling a node has  $n$  axioms ( $S := \{a_1, \dots, a_n\}$ ), then this node is expanded with  $n$  children: the edge to the  $i$ -th child labeled with  $a_i$ , the child labeled with a CS that is not allowed to contain neither  $a_i$  nor any ancestor's edge label. This ensures that each node is labeled with a CS distinct from those of its predecessors.

HST optimizations such as *early termination* and *node reuse* avoid redundant computations and are included in current implementations. Another optimization is putting a *cardinality limit*, applicable when not all, but only the CS of minimal cardinality  $|S|$  is of interest. Then nodes might contain partial solutions, called *partial CS*, in the sense that some axioms are missing, but still the smallest CS is proven to be found [10, 9].

*Example 3.* We continue Example 2. Assume we want to make  $c_1$  as private as possible, i.e.  $G = \{(c_1, \ell_0)\}$ . All RAS are  $\{a_1\}, \{a_2, a_3\}, \{a_4, a_5\}$ , so the smallest MCS is  $M_1 = \{(a_1, \ell_0)\}$  and we get  $\text{lbl}_{M_1}(c_1) = \ell_0$ . As second example assume we want to make  $c_2$  as public as possible, i.e.  $G = \{(c_2, \ell_1)\}$ . All IAS are  $\{a_2\}, \{a_3\}$ , so one of the smallest MCS is  $M_2 = \{(a_3, \ell_1)\}$  and we get  $\text{lbl}_{M_2}(c_2) = \ell_1$ .

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**Algorithm 1** Extract cMCS with optimizations CS reuse (switch off: remove Line 11) and cardinality limit (switch off: in Line 7 replace “ $n - |M|$ ” by “ $\infty$ ”)

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**Procedure** `init-cMCS-extraction`( $O, \text{lab}, (L, \leq), G$ )

**Input:**  $O, \text{lab}$ : labeled ontology;  $(L, \leq)$ : lattice;  $G$ : goal set

- 1: **Global:**  $O, \text{lab}, G' := \{(c, \ell_g, \text{is}_I, \text{is}_R, C_S) \mid (c, \ell_g) \in G,$   
 $\text{is}_I := \ell_g \not\prec \text{lbl}(c) \wedge O_{\geq \ell_g} \not\models c,$  (decision to compute IAS)  
 $\text{is}_R := \ell_g \not\prec \text{lbl}(c) \wedge O_{\not\geq \ell_g} \models c,$  (decision to compute RAS)  
 $C_S := \emptyset\}$  (reuse set for CS)

**Procedure** `extract-partial-cMCS`( $K, n$ )

**Input:**  $K$ : prohibited label changes;  $n$ : cardinality limit

**Output:** first  $n$  elements of a cMCS

- 1:  $M := \emptyset$
  - 2: **for** each goal  $(c, \ell_g, \text{is}_I, \text{is}_R, C_S) \in G'$  **do**
  - 3:    $H := \{a \mid (a, \ell_g) \in K\}$  (set of axioms not allowed to be labelled with  $\ell_g$ )
  - 4:   **if**  $\exists S' \in C_S : \emptyset = S' \cap H$  **then**
  - 5:      $S := S'$  (CS reuse)
  - 6:   **else**
  - 7:      $S := \text{extract-partial-CS}(O, \text{lab}, c, \ell_g, \text{is}_I, \text{is}_R, H, n - |M|)$  (defined by [9])
  - 8:     **if**  $\emptyset = S$  **then**
  - 9:       **return**  $\emptyset$  (HST normal termination for one goal fires for complete goal set)
  - 10:    **if**  $|S| \neq n - |M|$  **then**
  - 11:      $C_S := C_S \cup \{S\}$  (remember only non-partial CS)
  - 12:     $M := M \cup \{(a, \ell_g) \mid a \in S\}$
  - 13: **return**  $M$
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## 4.2 Computing a Multiple Change Set for Multiple Goal Labels

An MCS for several goals consists of CS for each of the individual goals. However, it is no solution to compute single CS and combine them since this might not yield the smallest MCS or they might even conflict.

*Example 4.* We combine both goals of Example 3 simultaneously, i.e. we want to make  $c_1$  as private as possible and  $c_2$  as public as possible,  $G = \{(c_1, \ell_0), (c_2, \ell_1)\}$ . Just concatenating the above mentioned MCS to  $M = M_1 \cup M_2 = \{(a_1, \ell_0), (a_3, \ell_1)\}$  is no MCS since  $\text{lbl}_M(c_2) = \ell_0 \neq \ell_1$ . However,  $M = \{(a_4, \ell_0), (a_5, \ell_0), (a_2, \ell_1)\}$  is an MCS.

For this reason we call any combination of CS a *candidate MCS* (cMCS). To compute the shortest MCS, we introduce Algorithm 2 which is similar to the HST algorithm for computing the shortest CS in [9]. The only difference is that each call to the auxiliary procedure computes a (partial) cMCS instead of a (partial) CS which is assigned to a node in the search tree, and edges are not labeled with an axiom but with a tuple  $(a, \ell)$  which is not allowed in the child node’s (partial) cMCS.

A (partial) cMCS is computed by a call `extract-partial-cMCS`( $K, n$ ) to the auxiliary procedure in Algorithm 1, where  $K$  is the set of prohibited label changes, i.e. all tuples at edges to ancestors in the HST, and  $n$  is the size of the currently known shortest MCS. The procedure comes with 2 optimizations: *CS reuse* and *cardinality limit*.

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**Algorithm 2** HST algorithm to find smallest MCS for  $G$ 

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**Procedure** hst-extract-smallest-MCS( $O, \text{lab}, (L, \leq), G, K$ )**Input:**  $O, \text{lab}$ : labeled ontology;  $(L, \leq)$ : lattice;  $G$ : goal set;  $K$ : prohibited label changes**Output:** MCS of minimum cardinality

- 1: **Global**  $M_{\min} := \emptyset, n := \infty, G$
- 2: **init-cMCS-extraction**( $O, \text{lab}, (L, \leq), G$ )
- 3: **expand-hst-MCS**( $K$ )
- 4: **return**  $M_{\min}$

**Procedure** expand-hst-MCS( $K$ )

**Input:**  $K$ : prohibited label changes

**Side effects:** modifications to  $M_{\min}$  and  $n$

- 1:  $M := \text{extract-partial-cMCS}(K, n)$
- 2: **if**  $M = \emptyset$  **then**
- 3:     **return** (HST normal termination)
- 4: **if**  $|M| < n$  **then**
- 5:     **if**  $(a, \ell_1), (a, \ell_2) \in M \implies \ell_1 = \ell_2$  **then**
- 6:         **if**  $\forall (c, \ell_g) \in G : \text{lbl}_M(c) = \ell_g$  **then**
- 7:              $M_{\min} := M$
- 8:              $n := |M_{\min}|$
- 9:         **else**
- 10:             ... (semantic conflict resolution)
- 11:         **else**
- 12:             ... (syntactic conflict resolution)
- 13: **for** the first  $(n - 1)$  label changes  $(a, \ell) \in M$  **do**
- 14:     **expand-hst-MCS**( $K \cup \{(a, \ell)\}$ )

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As any cMCS is a combination of CS, one CS might be contained in several cMCS. Instead of computing it anew for every cMCS, the first optimization reuses it. Putting a cardinality limit is a second optimization which computes a cMCS or stops once this has reached a size  $n$  and returns a potentially partial cMCS. Computing partial CS for one goal turned out to reduce execution time [9]. In a partial cMCS, the last contained CS is partial. Partial CS are not reused.

Turning to Algorithm 2, whenever a cMCS  $M$  is found with  $|M| < n$ , it is shorter than our currently known shortest MCS and we can be sure that it is not partial. The question remains if it is a MCS or only a cMCS, which is checked in Line 6: neither is an axiom allowed to have multiple labels assigned (*syntactic conflict*) nor might a change set for one goal influence any other goal which is the case if any computed boundary does not equal the goal label (*semantic conflict*). Only after passing both checks, we update our globally known shortest known MCS  $M_{\min}$  in Line 7. Loosening the constraints of a goal set, the semantic conflicts can be resolved in Line 10 or syntactic conflicts can be resolved in Line 12 which is explained in the next section.

We now show correctness of both optimizations, CS reuse and cardinality limit. Reuse of CS is correct, since the only non-constant parameter to extract a CS in Line 7 is the set of prohibited axioms  $H$  and Line 4 ensures  $H$  and the reused CS are disjoint.

**Theorem 1 (Cardinality Limit Optimization).** *Let  $O, \text{lab}$  be a labeled ontology and  $G$  a goal set. If  $m$  is the minimum cardinality of all MCS for  $G$ , the HST Algorithm 2 outputs a MCS  $M$  such that  $|M| = m$ .*

*Proof.* The described algorithm outputs a MCS since the globally stored and finally returned  $M_{\min}$  is only modified when the output of `extract-partial-cMCS` has size strictly smaller than the limit  $n$ , has neither any syntactic nor any semantic conflict and hence only when this is indeed a MCS itself. Suppose now that the output MCS  $M_{\min}$  is such that  $m < |M_{\min}|$ , and let  $M_0$  be a MCS such that  $|M_0| = m$ , which exists by assumption. Then, every MCS, i.e. every cMCS free of syntactic and semantic conflicts, obtained by calls to `extract-partial-cMCS` has size strictly greater than  $m$ , since otherwise,  $M_{\min}$  and  $n$  would be updated. Consider now an arbitrary MCS  $M'$  found during the execution through a call to `extract-partial-cMCS`, and let  $M'_n := \{(a_1, \ell_1), \dots, (a_n, \ell_n)\}$  be the first  $n$  assignments of  $M'$ . Since  $M'$  is a (partial) MCS, it must be the case that  $M_0 \not\subseteq M'_n$  since every returned MCS is minimal in the sense that no label change might be removed to obtain another MCS. Then, there must be an  $i, 1 \leq i \leq n$  such that  $(a_i, \ell_i) \notin M_0$ . But then,  $M_0$  will still be a MCS (and a cMCS anyway) after label change  $\{(a_i, \ell_i)\}$  has been removed. Since this argument is true for all nodes, it is in particular true for all leaf nodes, but then they should not be leaf nodes, since a new cMCS, namely  $M_0$  can still be found by expanding the HST, which contradicts the fact that  $M_{\min}$  is the output of the algorithm.  $\square$

### 4.3 Conflict resolution

We already elaborated on syntactic and semantic conflicts which might prevent a cMCS from being a MCS. It might be the case that for a goal set, no MCS can be found.

*Example 5.* We continue Example 2. Assume  $G = \{(c_1, \ell_4), (c_2, \ell_3)\}$ . For the goal  $(c_1, \ell_4)$  all IAS are  $\{a_2\}, \{a_3\}$ . For the goal  $(c_2, \ell_3)$  all RAS are  $\{a_1\}, \{a_2\}$ . The cMCS  $M_1 = \{(a_2, \ell_4), (a_2, \ell_3)\}$  is obviously no MCS due to a syntactic conflict. But also the remaining cMCS  $M_2 = \{(a_2, \ell_4), (a_1, \ell_3)\}, M_3 = \{(a_3, \ell_4), (a_1, \ell_3)\}, M_4 = \{(a_3, \ell_4), (a_2, \ell_3)\}$  are no MCS due to semantic conflicts, since  $\text{lbl}_{M_2}(c_1) = \text{lbl}_{M_3}(c_1) = \ell_3 \neq \ell_4$  and  $\text{lbl}_{M_4}(c_2) = \ell_4 \neq \ell_3$ .

For these cases we introduce a generalization of an MCS called *Relaxed MCS* (RMCS) where the goal set is only partially satisfied according to a defined strategy. For the special case of no conflict, the RMCS equals the MCS. We identified 4 strategies to resolve conflicts, where we focus on syntactic conflict resolution only:

1. **Overrestrictive:** accept lower labels for a minimal number of consequences than specified by the goal label. Formally,  $\forall (c, \ell_g) \in G : \text{lbl}_M(c) \neq \ell_g \implies |\text{lbl}_M(c) < \ell_g \text{ and cardinality } |\{(c, \ell_g) \in G \mid \text{lbl}_M(c) \neq \ell_g\}| \text{ is minimal. Applied to the above example, } \{(a_2, \ell_3)\}$  is a RMCS.
2. **Overpermissive:** accept higher labels for a minimal number of consequences than specified by the goal label. Formally,  $\forall (c, \ell_g) \in G : \text{lbl}_M(c) \neq \ell_g \implies |\text{lbl}_M(c) > \ell_g \text{ and cardinality } |\{(c, \ell_g) \in G \mid \text{lbl}_M(c) \neq \ell_g\}| \text{ is minimal. Applied to the above example, } \{(a_2, \ell_4)\}$  is a RMCS.

---

**Algorithm 3** Computing a RMCS, overpermissive strategy (for overrestrictive strategy: replace “ $\oplus$ ” with “ $\otimes$ ” in Line 3, “ $\geq$ ” with “ $\leq$ ” in Line 4, “ $>$ ” with “ $<$ ” in Line 5)

---

Basis is the Algorithm 2. In Procedure `hst-extract-smallest-MCS`, add global variables  $N := \emptyset$ ,  $r := \infty$ , and add before Line 4:

```
1: if  $\emptyset = M_{\min}$  then
2:   return  $N$ 
```

In Procedure `expand-hst-MCS`, replace Line 12 for syntactic conflict resolution with:

```
1:  $N' := M$ 
2: for each  $a : (a, \ell_1), (a, \ell_2) \in N' \wedge \ell_1 \neq \ell_2$  do
3:    $N' := N' \setminus \{(a, \ell_1), (a, \ell_2)\} \cup \{(a, \ell_1 \oplus \ell_2)\}$ 
4: if  $\forall (c, \ell_g) \in G : \text{lbl}_{N'}(c) \geq \ell_g$  then (fulfills overpermissive strategy)
5:    $r' := |\{(c, \ell_g) \in G \mid \text{lbl}_{N'}(c) > \ell_g\}|$ 
6:   if  $r' < r$  then
7:      $N := N'$ 
8:      $r := r'$ 
```

---

3. Override strategy: The goal  $G$  set is split up into fragments  $G_i$  so that  $G = G_1 \cup \dots \cup G_n$  for which individual MCS  $M_i$  can be computed. The changed label assignment  $((\text{lab}_{M_1}) \dots)_{M_n}$  is obtained by sequentially applying each MCS  $M_i$ , where the order can be chosen based on some prioritization. This implies that labels changed by one MCS might be changed again by any subsequent MCS. Applied to the above example, splitting up  $G$  into  $G_1$  and  $G_2$ ,  $G_1 = \{(c_1, \ell_4)\}$  yields MCS  $M_5 = \{(a_2, \ell_4)\}$ , subsequently  $G_2 = \{(c_2, \ell_3)\}$  yields MCS  $M_6 = \{(a_2, \ell_3)\}$ .

Strategy 3 although easy to implement has an unacceptable drawback, conflicting our RMCS definition: even if there is a MCS for the union of all goal subsets, a sequentially applied MCS for one goal subset might override a previous for another goal subset since they are computed independently of each other. For this reason we focus on strategies 1 and 2 for resolution of syntactic conflicts.

Algorithm 3 describes the resolution of syntactic conflicts. It is an adapted version of Algorithm 2, where additionally the global variable  $r$  stores the minimal number of overpermissive (overrestrictive) consequence labels and  $N$  stores the RMCS with minimal  $r$ . Again this Algorithm relies on the cMCS extraction Algorithm 1 and the optimization of reusing CS can be applied. The cardinality limit optimization is of no use here since if no MCS is found, then no cardinality limit is set and the HST is fully expanded.

There are goal sets yielding semantic conflicts but no syntactic conflicts in cMCS. These are not solved by syntactic conflict resolution. For these cases not only IAS and RAS, but complete explanations and diagnoses need to be taken into account, as the following example shows.

*Example 6.* We continue Example 2. Assume  $G = \{(c_1, \ell_2), (c_2, \ell_5)\}$ . For the goal  $(c_1, \ell_2)$  all IAS are  $\{a_4\}, \{a_5\}$ . For the goal  $(c_2, \ell_5)$  all IAS are  $\{a_2\}, \{a_3\}$ , all RAS are  $\{a_1\}, \{a_2, a_3\}$ . Obviously no combination of CS for both goals yields a syntactic conflict. Nevertheless there is no MCS since every combination of CS has a semantic conflict. After conflict resolution, an overpermissive RMCS is  $N_{OP} =$

$\{(a_4, \ell_2), (a_2, \ell_2 \oplus \ell_5 = \ell_1), (a_3, \ell_5)\}$ , yielding  $\text{lbl}_{N_{OP}}(c_1) = \ell_1, \text{lbl}_{N_{OP}}(c_2) = \ell_1$ . An overrestrictive RMCS is  $N_{OR} = \{(a_4, \ell_2), (a_2, \ell_2 \otimes \ell_5 = \ell_0), (a_3, \ell_5)\}$ , yielding  $\text{lbl}_{N_{OR}}(c_1) = \ell_5, \text{lbl}_{N_{OR}}(c_2) = \ell_5$ .

## 5 Experiments

We implemented and evaluated our algorithms empirically with large practical ontologies. The following sections describe our test setting and the results.

### 5.1 Test Procedure and Test Data

We test on a PC with 2GB RAM and Intel Core Duo CPU 3.16GHz. We implemented all approaches with Java 1.6, Pellet 2.0 and OWL API trunk revision 1150. As labeling lattice  $(L, \leq)$  we use the one introduced in Figure 1. We use the top lattice element  $\ell_1$  for public knowledge,  $\ell_2$  for intermediate knowledge and  $\ell_3$  for top secret knowledge.

Our test ontologies  $O^{\text{GEO}1}$ ,  $O^{\text{MGED}2}$ ,  $O^{\text{PROCESS}3}$  are selected ontologies from the TONES Ontology Repository<sup>4</sup> with a high number of individuals. At time of their download on March 25th 2010, they had the characteristics given in Table 1. The test ontology  $O^{\text{FUNCT}}$  is an OWL ontology for functional description of mechanical engineering solutions presented in [6].

In a first experimental setting we tested the availability of access control by query rewriting vs. access control by axiom filtering. Initially each ontology axiom is labeled  $\ell_1$  so that the complete ontology is public. This reflects default allow-all behavior of a security policy. Then for each concept  $C$  in the ontology, we apply access restriction  $AR = C(x)$  by including each query result  $c = \mu(AR)$  with goal label  $\ell_3$  in the goal set. The computed MCS is used to create a newly labeled ontology, on which we perform the following queries. We count for every  $C$ -instance the instance relationships to concepts other than  $C$  which are available for public users ( $\ell_1$ ). With query rewriting their count is 0. With axiom filtering their count is the availability gain of axiom filtering vs. query rewriting. For cMCS extraction defined by Algorithm 1, we tested both optimizations CS reuse and cardinality limit separately and their combination. In this setting every cMCS is automatically an MCS since there are no conflicting goals. Although not included in Algorithm 2 for transparency reasons, the mentioned usual HST optimizations *early termination* and *node reuse* are included in our implementation.

In a second experimental setting we tested conflict resolution strategies in cases where multiple goals conflict each other so that no MCS can be computed without relaxing one of the goals. We test the overrestrictive conflict resolution approach vs. the overpermissive conflict resolution approach of Algorithm 3 with the same ontologies. Only the CS reuse optimization of the auxiliary procedure in Algorithm 1 to extract cMCS is used, cardinality limit is not used for reasons explained in Section 4.3. First all axioms are labeled with intermediate security level, i.e.  $\ell_2$ . A goal set is created

<sup>1</sup> <http://i2geo.net/ontologies/dev/ontology.owl>

<sup>2</sup> <http://mged.sourceforge.net/ontologies/MGEDOntology.owl>

<sup>3</sup> <http://sweet.jpl.nasa.gov/ontology/process.owl>

<sup>4</sup> <http://owl.cs.manchester.ac.uk/repository/>

| Ontology DL expressivity |                                 | #logical axioms | #concepts | #individuals | #goal sets | #goals per goal set |
|--------------------------|---------------------------------|-----------------|-----------|--------------|------------|---------------------|
| $O^{\text{FUNCT}}$       | $\mathcal{ALCOIN}(\mathbf{D})$  | 3189            | 115       | 545          | 102        | 12.2                |
| $O^{\text{GEOM}}$        | $\mathcal{ALCHOIN}(\mathbf{D})$ | 8803            | 589       | 2010         | 571        | 14.1                |
| $O^{\text{PROCESS}}$     | $\mathcal{ALCHOFF}(\mathbf{D})$ | 2578            | 1537      | 150          | 40         | 20.9                |
| $O^{\text{MGED}}$        | $\mathcal{ALEOF}(\mathbf{D})$   | 1387            | 234       | 681          | 125        | 28.8                |

**Table 1.** Test sets consisting of ontologies and goal sets

for each concept  $C$  containing the same consequences described above, but now one half of this set has goal label  $\ell_1$  and the other half  $\ell_3$ . Some of the resulting goal sets are contradictory. We test Algorithm 3 to compute a RMCS with overpermissive vs. overrestrictive conflict resolution strategy for the same goal set and we count the number of overpermissive/overrestrictive consequence labels.

For both experiments the test data characteristics are given in Table 1. The number of goal sets and of goals per goal set are the same for both experiments since they contain the assertions to each of the ontology’s concepts, only with different goal labels. In order to limit runtime we compute in maximum 10 cMCS before the HST Algorithms 2 and 3 return, so there might be MCS or RMCS of lower cardinality.

## 5.2 Empirical Results

The experimental results for the first experiment are given in Table 2. It compares availability of access control by query rewriting vs. access control by axiom filtering and it compares performance of both optimizations cardinality limit vs. CS reuse. The given total number of CS includes reused CS. The number of cMCS is equal to the number of MCS since the goals contain no conflicts with the first experiment. The number of gained assertions confirms that our ideas improve availability of knowledge when using axiom filtering instead of query rewriting. While the number of gained assertions is comparable between the optimizations applied, their runtime differs significantly. CS reuse alone, and also in combination with cardinality limit runs significantly faster compared to using cardinality limit optimization only. Testing  $O^{\text{MGED}}$  with cardinality limit optimization did not terminate after 4 days, so no results are provided.

The experimental results for the second experiment comparing conflict resolution with overrestrictive strategy vs. overpermissive strategy are given in Table 3. Only some of the goal sets constructed as described above are conflicting, and results are only given for those. Only the given percentage of the goals in one goal set are enforced, the remaining consequences have overpermissive/overrestrictive labels making them more public/private than intended by the goal set. The runtime limit of 10 cMCS was hit in every case, making the HST algorithm stop so there might be RMCS with less overpermissive/overrestrictive consequence labels when relaxing this runtime limit.

| Test set             | optimization | Results (averages per goal set) |            |                 |      |                      |                       |
|----------------------|--------------|---------------------------------|------------|-----------------|------|----------------------|-----------------------|
|                      |              | #CS                             | #reused CS | #cMCS<br>= #MCS | MCS  | runtime<br>(minutes) | #gained<br>assertions |
| $O^{\text{FUNCT}}$   | card. limit  | 131.8                           | 0.0        | 3.9             | 23.9 | 3.6                  | 28.5                  |
|                      | CS reuse     | 135.2                           | 118.4      | 3.9             | 24.0 | 0.7                  | 28.6                  |
|                      | both         | 132.6                           | 115.7      | 3.9             | 24.1 | 0.6                  | 28.4                  |
| $O^{\text{GEOM}}$    | card. limit  | 146.9                           | 0.0        | 2.6             | 9.2  | 24.0                 | 43.4                  |
|                      | CS reuse     | 148.9                           | 132.9      | 2.5             | 9.3  | 4.2                  | 43.3                  |
|                      | both         | 147.3                           | 131.1      | 2.6             | 9.3  | 4.2                  | 43.3                  |
| $O^{\text{PROCESS}}$ | card. limit  | 199.3                           | 0.0        | 6.9             | 12.0 | 2.3                  | 92.6                  |
|                      | CS reuse     | 250.9                           | 217.8      | 6.7             | 12.2 | 0.6                  | 91.8                  |
|                      | both         | 197.9                           | 165.0      | 6.8             | 12.2 | 0.6                  | 91.9                  |
| $O^{\text{MGED}}$    | card. limit  | n/a                             | n/a        | n/a             | n/a  | n/a                  | n/a                   |
|                      | CS reuse     | 286.4                           | 253.4      | 2.9             | 15.1 | 115.9                | 53.9                  |
|                      | both         | 265.1                           | 232.4      | 3.0             | 15.1 | 114.3                | 54.1                  |

**Table 2.** Gained assertions compared to query rewriting, performance of optimizations

## 6 Conclusions

We considered scenarios where different parts of a given ontology should be visible for different users. We introduced access restrictions intentionally defined by means of a query. The answer to that query is the set of those axioms and consequences of the ontology, which have to be access restricted. We compared two basic approaches to enforce those access restrictions: query rewriting vs. axiom filtering. Compared to query rewriting, axiom filtering allows higher availability in the sense of more answers delivered to a user without unveiling any secret and is independent of any ontology language.

Axiom filtering relies on an axiom labeling. The problem solved by this paper is to find an optimal axiom labeling to enforce given access restrictions. Given a query-generated goal set containing consequences and intended labels, our algorithms compute a minimal change set defining a new axiom labeling. We show that a change set does not always exist since a goal set might contain conflicts, and we provide 2 conflict resolution strategies to relax the goal set so that a change set can be computed. Our experimental results show that our algorithms behave well in practical scenarios.

As future work we will look at other criteria for the minimality of change sets for example not counting the amount of changed axiom labels but the distance of the new from the old label in the lattice, the amount of other consequence’s labels changed, or the amount of affected users. We will also look at resolution of semantic conflicts and study a more expressive goal language to define for each single goal of a goal set whether it may be lowered or lifted in case of conflicts.

| Test set             | #goal sets confl. | #goals strat- per egy confl. goal set | Results (averages per conflicting goal set) |       |       |                    |                   |                     |
|----------------------|-------------------|---------------------------------------|---|-------|-------|--------------------|-------------------|---------------------|
|                      |                   |                                       | #cMCS                                       | #RMCS | RMCS  | runtime (min-utes) | #OR/OP cons. labs | % of enforced goals |
| $O^{\text{FUNCT}}$   | 19                | 50.3 OR                               | 10.0  | 10.0  | 101.4 | 2.2                | 19.5              | 61%                 |
|                      |                   | OP                                    | 10.0  | 10.0  | 110.0 | 2.0                | 20.3              | 60%                 |
| $O^{\text{GEOM}}$    | 39                | 150.7 OR                              | 10.0  | 10.0  | 139.4 | 45.4               | 63.3              | 58%                 |
|                      |                   | OP                                    | 10.0  | 10.0  | 140.4 | 37.0               | 52.1              | 65%                 |
| $O^{\text{PROCESS}}$ | 23                | 31.0 OR                               | 10.0  | 10.0  | 32.3  | 0.9                | 12.7              | 59%                 |
|                      |                   | OP                                    | 10.0  | 10.0  | 32.6  | 0.8                | 11.0              | 64%                 |
| $O^{\text{MGED}}$    | 16                | 165.8 OR                              | 10.0  | 10.0  | 140.4 | 814.6              | 75.6              | 54%                 |
|                      |                   | OP                                    | 10.0  | 10.0  | 141.6 | 780.8              | 51.9              | 69%                 |

**Table 3.** Conflict resolution with overrestrictive (OR) strategy vs. overpermissive (OP) strategy

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