Towards Reverse Engineering Temporal Queries: Generalizing $\mathcal{EL}$ Concepts with Next and Global

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Abstract. Stream reasoning systems often rely on complex temporal queries that are answered over enriched data sources. While there are systems for answering such queries, automated support for building temporal queries is rare. We present in this paper initial results on how to derive a temporalized concept from a set of examples. The resulting concept can then be used to retrieve objects that change over time. We consider the temporalized description logic that extends the description Logic $\mathcal{EL}$ with the LTL operators next ($\mathbf{X}$) and global ($\mathbf{G}$) and we present an approach that extends generalization inferences for classical $\mathcal{EL}$.

1 Introduction

Ontology-based data access is an established approach to perform complex event recognition in the context of stream reasoning [6,5]. The description of the complex situation to be recognized is often given by a temporalized conjunctive query (TCQ). While there are many results on answering such TCQs [5,1], there is hardly any support for generating such TCQs and this task is left to the knowledge engineer to be done manually. In this paper we provide first steps towards addressing this issue. We consider the problem of learning temporalized query concepts from a set of examples. The examples are individuals from the data stream, i.e. in our case from a sequence of ABoxes. The kind of query concept to be leaned here is a tree-shaped TCQ with one answer variable.

The *bottom-up* approach [3] provides a method for such learning problems by reverse engineering the query. It uses two generalization inferences: the *most specific concept* (msc) [3,7], which derives a concept for an ABox individual, and the *least common subsumer* (lcs) [3,9] which generalizes a set of concepts into one by computing their commonalities. If each individual from the set of examples is generalized into a concept and then all of these concepts are generalized into a single concept, then a query concept for the set of examples is obtained.

We investigate the temporalized description logic $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$ whose concepts constructors are a combination of standard $\mathcal{EL}$-concept constructors [4,2] and temporal operators that are used in propositional linear temporal logic (LTL), $\mathbf{X}$ (next) and $\mathbf{G}$ (global/always) [8]. Intuitively, the semantics of a concept is interpreted in two dimensions. The $\mathcal{EL}$-constructors express relations to other elements in object domain, while the LTL-operators express the evolution of objects in the temporal dimension.

Now, to extend the bottom-up approach to the 2-dimensional DL $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$, requires to address two challenges in comparison to $\mathcal{EL}$. The first is how to extend the msc and the lcs to the temporal setting, such that the interaction of the temporal operators are treated correctly in the generalizations. The second is to adapt to the structure of the examples, as we are learning from a sequence of ABoxes that captures observations made over time and that refers to different time points. In this paper we provide a characterization of subsumption, of instance checking, and of the lcs in $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$. As for $\mathcal{EL}$, the msc need not exist, if the ABox is cyclic. It is common to use an approximation of the msc by limiting its role depth. We provide such approximations for the msc in $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$.

2 Preliminaries

We briefly recall the DL $\mathcal{EL}$ and propositional LTL and define the temporal DL $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$ that extends $\mathcal{EL}$-concepts with $\mathbf{X}$ (next) and $\mathbf{G}$ (global) from LTL.

Description Logic $\mathcal{EL}$. Let $\mathsf{N_C}$, $\mathsf{N_R}$, $\mathsf{N_I}$ be sets of *concept names*, *role names* and *individual names*, respectively. $\mathcal{EL}$-*concepts* are defined by the following grammar:

$$C, D ::= A \mid C \sqcap D \mid \exists r.C \mid \top$$

An *interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of a nonempty domain $\Delta^{\mathcal{I}}$ and a function $\cdot^{\mathcal{I}}$ that maps every concept names $A \in \mathsf{N_C}$ to a subset $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, every role name $r \in \mathsf{N_R}$ to a relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, and every individual name $a \in \mathsf{N_R}$ to an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. The interpretation $\cdot^{\mathcal{I}}$ is lifted to complex $\mathcal{EL}$-concepts as follows: $(\top)^{\mathcal{I}} = \Delta$, $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$, and $(\exists r.C)^{\mathcal{I}} = \{x \in \Delta \mid \exists y. y \in \Delta^{\mathcal{I}}$ s.t. $(x, y) \in r^{\mathcal{I}}$ and $y \in C^{\mathcal{I}}\}$. Let C and D be $\mathcal{EL}$-concepts, $r \in \mathsf{N_R}$, and $a, b \in \mathsf{N_I}$. An $\mathcal{EL}$-*concept inclusion* (CI) is of the form $C \sqsubseteq D$. An $\mathcal{EL}$-*concept assertion* is of the forms $C(a)$ and a *role assertion* of the form $r(a, b)$. An interpretation $\mathcal{I}$ satisfies: a concept assertion $C(a)$ if $a^{\mathcal{I}} \in C^{\mathcal{I}}$; a role assertion $r(a, b)$ if $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$; and a CI $C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$.

A Fragment of Linear Temporal Logic: $\text{LTL}^{\mathbf{X},\mathbf{G}}$. Let P be a set of propositional variables. LTL-formulae are defined inductively as follows: every propositional variable is a LTL-formula; and if ϕ and ψ are LTL-formulae, then $\phi \wedge \psi$ (conjunction), $\mathbf{X}\,\phi$ (next), and $\mathbf{G}\,\phi$ (global) are LTL-formulae. The semantics of LTL formulae is based on the notion of a LTL-structure. A LTL-*structure* is a sequence $\mathfrak{I} = (w_i)_{i \geq 0}$ of worlds $w_i \subseteq P$. Intuitively, w_i is a set of propositional variables that are true at time point i. The validity of a LTL-formula ϕ in LTL-structure $\mathfrak{I}$ at time point $i \geq 0$ (denoted $\mathfrak{I}, i \models \phi$) is defined inductively:

- $\mathfrak{I}, i \models p$ for $p \in P$ iff $p \in w_i$;

- $\mathfrak{I}, i \models \phi \wedge \psi$ iff $\mathfrak{I}, i \models \phi$ and $\mathfrak{I}, i \models \psi$;
- $\mathfrak{I}, i \models \mathbf{X}\,\phi$ iff $\mathfrak{I}, i+1 \models \phi$;
- $\mathfrak{I}, i \models \mathbf{G}\,\phi$ iff $\mathfrak{I}, j \models \phi$ for all $j \geq i$.

A LTL-formula ϕ is satisfiable if there exists a LTL-structure $\mathfrak{I}$ s.t. $\mathfrak{I}, 0 \models \phi$. Deciding satisfiability for LTL is PSPACE-complete, but it is trivial for $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}$.

The Temporal Description Logic $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$. Let $A \in \mathsf{N_C}$ and $r \in \mathsf{N_R}$. $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-*concepts* are defined by the following grammar:

$$C, D ::= A \mid C \sqcap D \mid \exists r.C \mid \mathbf{X}\,C \mid \mathbf{G}\,C \mid \top.$$

A *TBox* a finite set of *concept inclusions* (CIs) $C \sqsubseteq D$. An *ABox* is a finite set of *concept assertions* $C(a)$ and *role assertions* $r(a,b)$ where $a, b \in \mathsf{N_I}$. An *axiom* is either a CI or an assertion.

The semantics of $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$ is based on the notion of temporal interpretation, which extends LTL structures. A *temporal interpretation* is a sequence $\mathfrak{I} = (\mathcal{I}_i)_{0 \leq i}$ of interpretations $\mathcal{I}_i = (\Delta, \cdot^{\mathcal{I}_i})$ over a common domain Δ and that respects rigid individual names, i.e., $a^{\mathcal{I}_i} = a^{\mathcal{I}_j}$ for all $a \in \mathsf{N_I}$ and $i, j \geq 0$. The interpretation $\cdot^{\mathcal{I}_i}$ is lifted to complex $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$ concepts as follows:

- $(\top)^{\mathcal{I}_i} = \Delta$
- $(C \sqcap D)^{\mathcal{I}_i} = C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i}$
- $(\exists r.C)^{\mathcal{I}_i} = \{x \in \Delta \mid \exists y, y \in \Delta^{\mathcal{I}_i} \text{ such that } (x,y) \in r^{\mathcal{I}_i} \text{ and } y \in C^{\mathcal{I}_i}\}$
- $(\mathbf{X}\,C)^{\mathcal{I}_i} = \{x \in \Delta \mid x \in C^{\mathcal{I}_{i+1}}\}$
- $(\mathbf{G}\,C)^{\mathcal{I}_i} = \{x \in \Delta \mid x \in C^{\mathcal{I}_j} \text{ for all } j \geq i\}$

A temporal interpretation $\mathfrak{I}$ at time point i *satisfies* an axiom α (denoted $\mathfrak{I}, i \models \alpha$) of the form: GCI $C \sqsubseteq D$ iff $C^{\mathcal{I}_i} \subseteq D^{\mathcal{I}_i}$; concept assertion $C(a)$ iff $a^{\mathcal{I}_i} \in C^{\mathcal{I}_i}$; and role assertion $r(a,b)$ iff $(a^{\mathcal{I}_i}, b^{\mathcal{I}_i}) \in r^{\mathcal{I}_i}$.

We say that $\mathfrak{I} = (\mathcal{I}_i)_{0 \leq i}$ is a *model of a concept* C if C is satisfied at time point 0, i.e., $C^{\mathcal{I}_0} \neq \emptyset$. $\mathfrak{I}$ is a *model of* $C \sqsubseteq D$ iff $\mathfrak{I}, i \models C \sqsubseteq D$ for all $i \geq 0$. $\mathfrak{I}$ is a *model of an ABox* $\mathcal{A}_i$ at time point i iff it is a model of all assertions in $\mathcal{A}_i$ at time point i, and is a *model of a sequence of ABoxes* $(\mathcal{A}_i)_{0 \leq i \leq n}$ iff it is a model of all ABox $\mathcal{A}_i$, $0 \leq i \leq n$.

Every $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-concept is satisfiable, as in $\mathcal{EL}$ and $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}$. We use two reasoning services to build the generalization inferences on. C *subsumes* D ($C \sqsubseteq D$) iff for all interpretations $\mathfrak{I}$, $\mathfrak{I}$ is a model of $C \sqsubseteq D$. Given $\mathcal{A}$, C and a, *instance checking* tests whether $a \in C^{\mathcal{I}}$ holds for all models of $\mathcal{A}$. We denote a sequence of ABoxes $(\mathcal{A}_i)_{0 \leq i \leq n}$ with $\vec{\mathcal{A}_n}$. We say $\vec{\mathcal{A}_n}, i \models C(a)$ iff for all models $\mathfrak{I} \models \vec{\mathcal{A}_n}$, we have that $\mathfrak{I}, i \models C(a)$.

Definition 1 (LCS & MSC). *A* $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$ *concept D is the* least common subsumer *(lcs) of the concepts $C_1, \ldots, C_n$ ($\mathbf{lcs}(C_1, \ldots, C_n)$ for short) iff it satisfies*

- *$C_i \sqsubseteq D$ for all $i = 1, \ldots, n$, and*
- *D is the least* $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$ *concept with this property, i.e., if* $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$ *concept E satisfies $C_i \sqsubseteq E$ for all $i = 1, \ldots, n$, then $D \sqsubseteq E$.*

A $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$ *concept* D *is the* most specific concept *(msc) of the individual* a *w.r.t. the sequence of ABoxes* $\vec{\mathcal{A}}_n$ *at time point* i *(*$msc_i(a)$ *for short) iff*

- $\vec{\mathcal{A}}_n, i \models D(a)$*, and*
- D *is the least* $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$ *concept satisfying this property, i.e., if* E *is a* $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$ *concept satisfying* $\vec{\mathcal{A}}_n, i \models E(a)$*, then* $D \sqsubseteq E$*.*

In combination, the msc and the lcs facilitate the learning of a $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$ query concept from a set of positive instances of individuals from a sequence of ABoxes.

3 Representing $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-concepts

In preparation for characterizing the lcs and the msc in $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$, we need a characterization of subsumption (and of instance) in this DL. We extend the approach for $\mathcal{EL}$ from [3], where concepts are represented by $\mathcal{EL}$-description trees. The subsumption test is then simply deciding the existence of a homomorphism between such trees. Our goal is use this test to decide subsumption in $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$.

3.1 $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-description Trees

We extend $\mathcal{EL}$-description trees to $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-description trees by accommodating temporal operators $\mathbf{X}$ and $\mathbf{G}$. Each element in the domain of a temporal interpretation is connected to exactly one element "in the next time point"—namely itself. This justifies to combine the concepts using the $\mathbf{X}$ and $\mathbf{G}$ operators each, if they refer to the same element. Furthermore, if we consider $\mathbf{X}$ as a special role that generates an infinite chain, then $\mathbf{G}$ is the transitive closure of $\mathbf{X}$. Another concern is the non-local behavior of $\mathbf{G}$. To handle this, we introduce a role that represents the information that holds globally at every time point of an element. This $\mathbf{G}$ node represents the concept that needs to be satisfied from that point onward. We implement these ideas in the following normal form for $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$ concepts. Note that any domain element is an instance of $\mathbf{G}\top$.

Definition 2 (Normal form). *An* $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$*-concept* C *is in* normal form *if it is of the form*

$$C = A_1 \sqcap \ldots \sqcap A_n \sqcap \exists r_1.D_1 \sqcap \ldots \sqcap \exists r_m.D_m \sqcap \mathbf{X}\,E \sqcap \mathbf{G}\,F \ ,$$

where $A_1, \ldots, A_n$ *are concept names;* $D_1, \ldots, D_m, E, F$ *are* $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$ *concepts in normal form;* F *does have neither* $\mathbf{X}$ *nor* $\mathbf{G}$ *on the top-level conjunction.*

Intuitively, this normal form captures four aspects that need to be satisfied by an instance a of concept C. First, $A_1, \ldots, A_n$ is the set of concept names that a needs to be an instance of. Second, $\exists r_1.D_1 \sqcap \ldots \sqcap \exists r_m.D_m$ is the set of r_i successors that a requires. Third, a needs to be an instance of E at the next time point. Fourth, a must be an instance of F from the current time point on and onward. We define a set of transformation rules to convert a $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$ concept C into normal form:

$$
\begin{array}{rcll}
\mathbf{X}\,C_1 \sqcap \mathbf{X}\,C_2 & \rightsquigarrow & \mathbf{X}(C_1 \sqcap C_2) & \text{[MergeX]} \\
\mathbf{G}\,C_1 \sqcap \mathbf{G}\,C_2 & \rightsquigarrow & \mathbf{G}(C_1 \sqcap C_2) & \text{[MergeG]} \\
\mathbf{G}(\mathbf{G}\,C_1) & \rightsquigarrow & \mathbf{G}\,C_1 & \text{[FlattenG]} \\
\mathbf{G}(\mathbf{X}\,C_1) & \rightsquigarrow & \mathbf{X}(\mathbf{G}\,C_1) & \text{[MoveG]} \\
\mathbf{G}\,C_1 & \rightsquigarrow & C_1 \sqcap \mathbf{G}\,C_1 & \text{[DistributeG]} \\
\mathbf{X}\,C_1 \sqcap \mathbf{G}\,C_2 & \rightsquigarrow & \mathbf{X}(C_1 \sqcap \mathbf{G}\,C_2) \sqcap \mathbf{G}\,C_2 & \text{[PropagateG]}
\end{array}
$$

The rules are applied exhaustively to a concept C, but with some priorization of the rules. First, the rules MergeX, MergeG, FlattenG and MoveG have to be applied. Then, we need to apply the rules to the subconcepts before the root concept for DistributeG and PropagateG. The normalization might cause an exponential blow-up due to PropagateG. Propagating information from concept $\mathbf{G}\,C$ in $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$ copies C along the chain of $\mathbf{X}$-successors. We denote with C^* the $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-concept resulting from applying those rules to an $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-concept C.

Proposition 3. *Let C be a $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-concept. Then, it holds that $C \equiv C^*$. The size of C^* can be exponential in the size of C.*

Proof Sketch. It is not hard to show that the rules are equivalence preserving. Thus C^* is equivalent to C. The exponential blow-up comes from the interaction between rules DistributeG and PropagateG. However, since there is no rule that extends a $\mathbf{X}$ chain, PropagateG is only applicable as many times as the length of the longest $\mathbf{X}$-chain in C. This ensures the termination of the procedure.

From now on, we assume that all $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-concepts are in normal form unless otherwise stated. W.l.o.g. we assume $\mathbf{X}, \mathbf{G} \notin \mathsf{N_R}$.

Definition 4 ($\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-description tree). *An $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-description tree is of the form $\mathcal{G} = (V, \mathcal{E}, v_0, \ell)$, where $\mathcal{G}$ is a tree with root v_0 where*

- *the edges $vrw \in \mathcal{E}$ are labeled with a role name $r \in \mathsf{N_R} \cup \{\mathbf{X}, \mathbf{G}\}$; and*
- *the nodes $v \in V$ are labeled with sets $\ell(v)$ of concept names from $\mathsf{N_C}$. The empty label corresponds to $\top$.*

Any $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$ concept C can be translated into a $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-description tree $\mathcal{G}_C = (V, E, v_0, \ell)$. Intuitively, concepts of the form $\exists r.C$, $\mathbf{X}\,C$, and $\mathbf{G}\,C$ give rise to successor nodes, while (conjunctions of) concept names induce complex labels. The reverse construction of a $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-concept $C_{\mathcal{G}}$ from a $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-description tree $\mathcal{G}$ is done in the obvious manner. For a description tree $\mathcal{G}_C$ and a node w, we denote the subtree of $\mathcal{G}_C$ with root w by $\mathcal{G}_C(w)$.

Example 5 ($\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-description tree). The $\text{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-concept description

$$C := P \sqcap \exists r.(P \sqcap \mathbf{X}\,Q) \sqcap \mathbf{X}(P \sqcap Q \sqcap \mathbf{G}(P \sqcap Q)) \sqcap \mathbf{G}\,P$$

corresponds to the description tree $\mathcal{G}_C$ depicted in Figure 1.

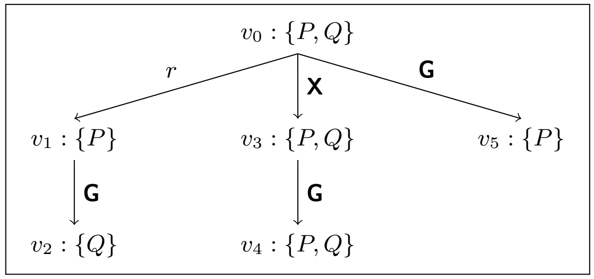

Fig. 1. $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-description tree.

Note, that equivalent concepts can result in different normalized concepts. An $\mathbf{X}$-path in a description tree is a path, where each edge is labelled with $\mathbf{X}$.

Lemma 6. *Let C be a $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-concept in normal form and $\mathcal{G}_C = (V, \mathcal{E}, v_0, \ell)$ be the $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-description tree of C.*

1. *$C = C_{\mathcal{G}_C}$ up to commutativity and associativity of conjunction, and $\mathcal{G}_C = \mathcal{G}_{C_{\mathcal{G}_C}}$ up to renaming nodes.*
2. *For each node $v \in V$, v has at most one outgoing edge labeled $\mathbf{X}$ and at most one outgoing edge labeled $\mathbf{G}$.*
3. *Let $v \,\mathbf{G}\, w \in \mathcal{E}$. Then, there does not exists $x \in V$ such that either $w \,\mathbf{X}\, x \in \mathcal{E}$ or $w \,\mathbf{G}\, x \in \mathcal{E}$,i.e., $C_{\mathcal{G}}$ does not have a subconcept of the form $\mathbf{G}\,\mathbf{X}\, D$ or $\mathbf{G}\,\mathbf{G}\, D$.*
4. *Let $v \,\mathbf{G}\, w \in \mathcal{E}$ and D denote the $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-concept corresponding to the subtree of $\mathcal{G}_C$ with root w, then for any w' where there is a $\mathbf{X}$-path from v to w'. Let D' be $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-concept corresponding to the subtree of $\mathcal{G}_C$ with root w', we have that $D' \sqsubseteq D$.*

In order to characterize subsumption, we need to establish a connection between description trees and the semantics of $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-concepts given by temporal interpretations. More precisely, we describe in the next subsection how to obtain a temporal interpretation from a description tree.

3.2 Canonical Interpretation of $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-concepts

Observe, that there are different kinds of nodes in description trees, depending on whether they are a $\mathbf{X}$-successor or the successor for a role from $\mathsf{N_C}$. In Figure 2 the nodes are classified according to their distance from the root v_0 in terms of time steps. We call the *temporal depth* of v ($\mathrm{td}(v)$) the number of $\mathbf{X}$-edges that occur in the path from v_0 to v. Furthermore note, that if a node is connected by $\mathbf{X}$-edges then they represent the same element in the domain but at different points in time. We define some notions to distinguish nodes in V. In a $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-description tree $\mathcal{G}_C = (V, \mathcal{E}, v_0, \ell)$ a node $v \in V$ is called a

- *next copy* if there exists $w \,\mathbf{X}\, v \in \mathcal{E}$,
- *global copy*, if $w \,\mathbf{G}\, v \in \mathcal{E}$,

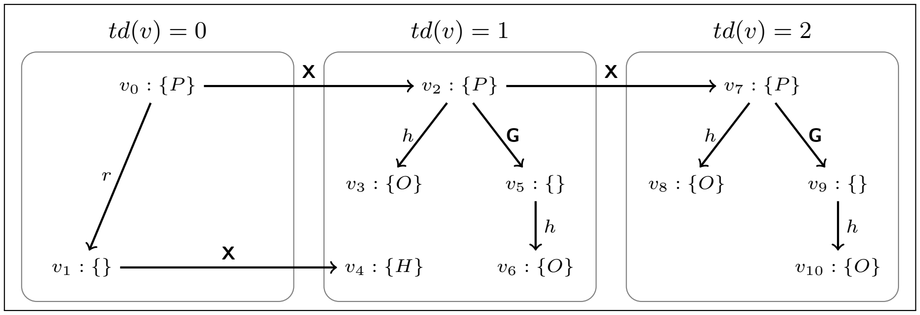

Fig. 2. $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-description tree for $P \sqcap \exists r.(\mathbf{X}\,Q) \sqcap (\mathbf{X}(P \sqcap \mathbf{X}\,P) \sqcap \mathbf{G}(\exists s.O))$.

- *temporal copy* iff v is either a next copy or a global copy, and
- *temporal root* iff v is not a temporal copy.

We use the following sets: V_X for next copies, V_G for global copies, and V_R for temporal roots. Given a temporal copy $v \in V_G$, $w \in V_R$ is the *temporal root of* v ($w = \mathrm{tr}(v)$) if $w\,\mathbf{G}\,v \in \mathcal{E}$ or there is a $\mathbf{X}$-path from w to v in $\mathcal{G}_C$. A temporal root has itself as a temporal root. Recall, that C is normalized.

Definition 7 (Canonical Interpretation). *Let C be a $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-concept and $\mathcal{G}_C = (V, E, v_0, \ell)$ the $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-description tree of C. The* canonical interpretation *of C is the temporal interpretation $\mathfrak{I}_C = ((\mathcal{I}_C)_i)_{0\leq i}$, where*

- $(\mathcal{I}_C)_i = (\Delta_C, \cdot^{(\mathcal{I}_C)_i})$ *for each* $i \geq 0$,
- $\Delta_C = V_R$,
- *for each* $A \in \mathsf{N_C}$ *and* $i \geq 0$
$$\begin{aligned}A^{(\mathcal{I}_C)_i} := {} & \{v \in V_R \mid A \in \ell(v) \text{ and } \mathrm{td}(v) = i\} \cup \\ & \{v \in V_R \mid \exists w \in V_X \text{ s.t. } v = \mathrm{tr}(w),\ A \in \ell(w) \text{ and } \mathrm{td}(w) = i\} \cup \\ & \{v \in V_R \mid \exists w \in V_G \text{ s.t. } v = \mathrm{tr}(w),\ A \in \ell(w) \text{ and } \mathrm{td}(w) \leq i\}, \text{ and}\end{aligned}$$
- *for* $r \in \mathsf{N_R}$ *and* $i \geq 0$
$$\begin{aligned}r^{(\mathcal{I}_C)_i} := {} & \{(v, w) \mid v \in V_R, vrw \in \mathcal{E} \text{ and } \mathrm{td}(v) = i\} \cup \\ & \{(v, w) \mid v = \mathrm{tr}(x) \text{ where } w \in V_X,\ xrw \in \mathcal{E} \text{ and } \mathrm{td}(w) = i+1\} \cup \\ & \{(v, w) \mid v = \mathrm{tr}(x) \text{ where } x \in V_G,\ xrw \in \mathcal{E} \text{ and } \mathrm{td}(x) \leq i\}.\end{aligned}$$

Lemma 8. *Let C be a $\mathrm{LTL}_{\mathcal{EL}}^{\mathbf{X},\mathbf{G}}$-concept, v_0 the root of $\mathcal{G}_C$, and $\mathfrak{I}_C = ((\mathcal{I}_C)_i)_{0\leq i}$ its canonical interpretation. Then $v_0 \in C^{\mathfrak{I}_C}$ holds.*

Proof Sketch. We show by induction on the depth of $\mathcal{G}_C$ that $v_0 \in (C_{\mathcal{G}_C})^{\mathfrak{I}_C}$. The proof for the base case and case of r-successors (with $r \in \mathsf{N_R}$) are rather similar. For the case of $\mathbf{X}\,E$, we show there exists $w \in \Delta_C$ such that $\mathrm{tr}(w) = v_0$ and $\mathrm{td}(w) = \mathrm{td}(v_0) + 1$. Furthermore, since $w \in (E_{\mathcal{G}_C(w)})^{\mathfrak{I}_C}$, we have that $v_0 \in (E_{\mathcal{G}_C})^{\mathfrak{I}_C}$. For the case of $\mathbf{G}\,F$, we show there exists an element w such that $\mathrm{tr}(w) = v_0$ and $\mathrm{td}(w) = \mathrm{td}(v_0)$. Then, we use the fact that the existence of $w \in (F_{\mathcal{G}_C(w)})^{\mathfrak{I}_j}$ propagates that $v_0 \in (F_{\mathcal{G}_C})^{\mathcal{I}_j}$ for all time points $j \geq 0$ due to the construction of $\mathfrak{I}_C$.

4 Characterization of Subsumption and LCS

In order to decide subsumption, it needs to be fixed to which depth to consider the concept in the X-chain. Obviously, such chains can get arbitrarily long when repeatedly propagating a G part onto a X. For a subsumption $C_1 \sqsubseteq C_2$ to hold, $\mathcal{G}_{C_1}$ needs to have a temporal height greater or equal to the one of $\mathcal{G}_{C_2}$, to employ homomorphisms for the comparison. If concept C_1 describes "less time points" than C_2, then padding is needed.

Definition 9 ($\mathrm{LTL}_{\mathcal{EL}}^{\mathsf{X},\mathsf{G}}$-concept padding). *Let C_1 and C_2 be $\mathrm{LTL}_{\mathcal{EL}}^{\mathsf{X},\mathsf{G}}$-concepts in normal form. A function to pad C_1 w.r.t. C_2 (denoted by $pad_{C_2}(C_1)$) proceeds as follows:*

- *for each $\exists r.D_1$ in the top-level of C_1, replace $\exists r.D_1$ with $\exists r.pad_{D_j}(D_1)$ recursively for all $\exists r.D_j$ in the top-level of C_2;*
- *if there exists $\mathsf{X}\, E_2$ in the top-level conjunction of C_2, then*
  - *if there exists $\mathsf{X}\, E_1$ in the top-level of C_1, then replace it with $\mathsf{X}\, pad_{E_2}(E_1)$*
  - *otherwise:*
    - *if there exists $\mathsf{G}\, F$ in the top-level of C_1, replace C_1 with $C_1 \sqcap \mathsf{X}(pad_{E_2}(F \sqcap \mathsf{G}\, F))$*
    - *otherwise, replace C_1 with $C_1 \sqcap \mathsf{X}(pad_{E_2}(\top))$*

Furthermore, we say that C_1 is aligned *w.r.t. C_2 if $pad_{C_2}(C_1) = C_1$.*

The padding function preserves equivalence of $\mathrm{LTL}_{\mathcal{EL}}^{\mathsf{X},\mathsf{G}}$-concepts. Now, using the notion of aligned concepts, we can ensure that the description tree for C_1 is deep enough in the temporal dimension to be compared with the one for C_2. We can use homomorphisms between two description trees to characterize subsumption.

Definition 10 (Homomorphism between $\mathrm{LTL}_{\mathcal{EL}}^{\mathsf{X},\mathsf{G}}$-description trees). *Let $\mathcal{H} = (V_H, \mathcal{E}_H, w_0, \ell_H)$ and $\mathcal{G} = (V_G, \mathcal{E}_G, v_0, \ell_G)$ be $\mathrm{LTL}_{\mathcal{EL}}^{\mathsf{X},\mathsf{G}}$-description trees. A homomorphism from $\mathcal{H}$ into $\mathcal{G}$ is a mapping $\varphi : V_H \mapsto V_G$ where*

1. *$\varphi(w_0) = v_0$;*
2. *$\ell_H(v) \subseteq \ell_G(\varphi(v))$ for all $v \in V_H$; and*
3. *$\varphi(v) r \varphi(w) \in \mathcal{E}_G$ for all $vrw \in \mathcal{E}_H$.*

Theorem 11. *Let C, D be $\mathrm{LTL}_{\mathcal{EL}}^{\mathsf{X},\mathsf{G}}$ concepts and C is aligned w.r.t. D. Then, we have that $C \sqsubseteq D$ iff there exists homomorphism from $\mathcal{G}_D$ to $\mathcal{G}_C$.*

Proof Sketch. For the if-direction, we prove $x_0 \in D^{\mathfrak{I}}$ by induction on the number of $|V_D|$ of nodes in $\mathcal{G}_D$. In the induction step, we prove that if $a \in C^{\mathfrak{I}}$, i.e., a is a model of each top-level conjunct in C has an appropriate successor for each top-level conjunct in D. We separate the case depending on the type of the conjunct and utilize the existence of the homomorphism to show each successor exists.

For the only-if-direction, we prove inductively on depth(D) by constructing an appropriate homomorphism on-the-fly. For the base case this is straighforward, since $\ell_D \subseteq \ell_C$. In the induction step, we show for each $w_0 r w \in \mathcal{E}_D$ there exists an appropriate $v_0 r v$ for each $r \in \mathsf{N_R} \cup \{\mathsf{X}, \mathsf{G}\}$ and $C_{\mathcal{G}_{C(v)}} \sqsubseteq C_{\mathcal{G}_{D(v)}}$. Then, we map w_0 to v_0 in the construction of the homomorphism inductively.

This characterization of subsumption is used to show correctness of our lcs construction. The latter is given by the product of description trees.

Definition 12 (Product of description trees). *Let* $\mathcal{G} = (V_G, \mathcal{E}_G, v_0, \ell_G)$ *and* $\mathcal{H} = (V_H, \mathcal{E}_H, w_0, \ell_H)$ *be* $\text{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$*-description trees. The* product of $\mathcal{G}$ and $\mathcal{H}$ *is* $\mathcal{G} \times \mathcal{H} := (V, \mathcal{E}, (v_0, w_0), \ell)$ *with the following components. Node* (v_0, w_0) *is the root of* $\mathcal{G} \times \mathcal{H}$*, labeled with* $\ell_G(v_0) \cap \ell_H(v_0)$*. For each* r*-successor* ($r \in \mathsf{N_R} \cup \{\mathbf{X}, \mathbf{G}\}$) v *of* v_0 *in* $\mathcal{G}$ *and* w *of* w_0 *in* $\mathcal{H}$*, there is an* r*-successor* (v, w) *of* (v_0, w_0) *in* $\mathcal{G} \times \mathcal{H}$ *that is the root of* $\mathcal{G}(v) \times \mathcal{H}(w)$*.*

Theorem 13. *Let* C_1, C_2 *be* $\text{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$ *concepts in normal form and* C_1 *is aligned w.r.t.* C_2 *and* C_2 *is aligned w.r.t.* C_1*. Then,* $C_{\mathcal{G}_{C_1} \times \mathcal{G}_{C_2}}$ *is the lcs of* C_1 *and* C_2*.*

Proof Sketch. We need to show these statements hold: (1) $C_1 \sqsubseteq C_{\mathcal{G}_{C_1} \times \mathcal{G}_{C_2}}$, (2) $C_2 \sqsubseteq C_{\mathcal{G}_{C_1} \times \mathcal{G}_{C_2}}$, and (3) for each C' with $C_1 \sqsubseteq C'$ and $C_2 \sqsubseteq C'$, we have that $C_{\mathcal{G}_{C_1} \times \mathcal{G}_{C_2}} \sqsubseteq C'$. The statements (1) and (2) can be proven by showing that the product of description trees captures, by construction, properties of both C_1 and C_2. Due to this, there exists the required homomorphisms which in turn shows that both subsumption relationships hold.

Since C' is a common subsumer of C_1 and C_2, there exists homomorphisms φ_1 from $\mathcal{G}_{C'}$ to $\mathcal{G}_{C_1}$ and φ_2 from $\mathcal{G}_{C'}$ to $\mathcal{G}_{C_2}$. Then, (3) can be shown by defining a mapping $\varphi := < \varphi_1, \varphi_2 >$ from $\mathcal{G}_{C'}$ to $\mathcal{G}_{C_1} \times \mathcal{G}_{C_2}$ as the product of φ_1 and φ_2, i.e., $\varphi(v') := (\varphi_1(v'), \varphi_2(v'))$ for all $v' \in V'$. Then, φ is well-defined, i.e., $\varphi(v') \in V$ for all $v' \in V'$ by induction. Finally, we show φ is a homomorphism from $\mathcal{G}_{C'}$ to $\mathcal{G}_{C_1} \times \mathcal{G}_{C_2}$ due to the construction of the product of $\text{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$ description trees.

5 Characterization of the Instance Relationship and MSC

In this section, we develop a method for computing a $\text{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-concept that describes an individual from a sequence of ABoxes best, i.e. a computation method for the msc in $\text{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$. To show the correctness of the method, we need a characterization of the instance relationship in $\text{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$. The approach used in this section follows [7] closely, but extends it by the temporal operators and adopts a sequence of ABoxes as part of the input.

5.1 Characterization of the Instance Relationship

Since there are n ABoxes, we have to consider information from each time point on the input individual a and combine it. For $\text{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$, we can use the $\mathbf{X}$ operator to describe the temporal information in one concept. Intuitively, we construct a concept C that represents a from the start of the observations on. Let $Ind(\vec{\mathcal{A}}_n)$ denote the set of all individuals occurring in $\vec{\mathcal{A}}_n$. For each $a \in Ind(\vec{\mathcal{A}}_n)$, we define $C_a := \bigsqcap_{0 \leq i \leq n} \big(\bigsqcap_{D(a) \in \mathcal{A}_i} \mathbf{X}^i D\big)$, if there exists an assertion $D(a) \in \mathcal{A}_i$ for any i; and $\top$ otherwise. We assume that each C_a for all a is normalized.

As the relational structure in the ABoxes can be arbitrary, we need to represent the information on a by graphs instead of trees. An $\text{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$*-description*

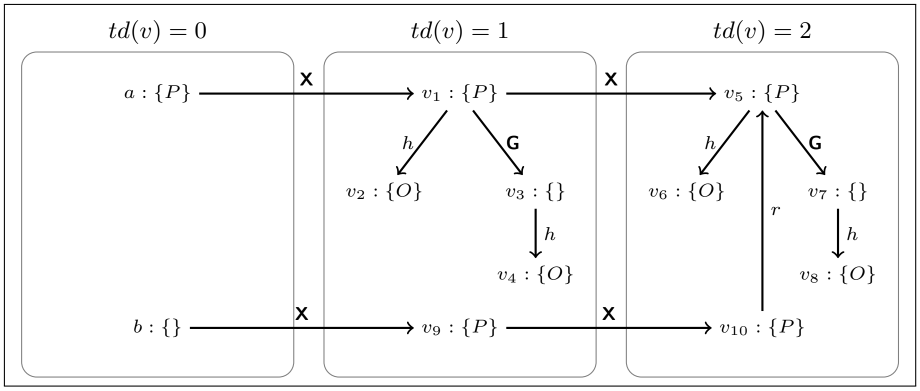

Fig. 3. $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-description graph for $\vec{\mathcal{A}}_3$.

graph is a labeled graph of the form $\mathcal{G} = (V, \mathcal{E}, \ell)$ whose edges $vrw \in \mathcal{E}$ are labeled with role names $r \in \mathsf{N_R} \cup \{\mathbf{X}, \mathbf{G}\}$ and whose nodes $v \in V$ are labeled with sets $\ell(v) \subseteq \mathsf{N_C}$. Let $\mathcal{G}_{C_a} = (V_a, \mathcal{E}_a, a, \ell_a)$ denote the $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-description tree for C_a. Assume that the sets V_a for all $a \in Ind(\vec{\mathcal{A}}_n)$ are pairwise disjoint. Given an individual $a \in Ind(\vec{\mathcal{A}}_n)$, we define the *temporal copy of a at time point i* as $\mathrm{tc}_i(a) = v$, where $v \in V_a$ such that there is a $\mathbf{X}$-path of length i from a to v in $\mathcal{G}_{C_a}$. The description graph of a sequence of ABoxes is $\mathcal{G}(\vec{\mathcal{A}}_n) = (V, \mathcal{E}, \ell)$, with:

- $V = \bigcup_{a \in (\vec{\mathcal{A}}_n)} V_a$;
- $\mathcal{E} = \{xry \mid r(a,b) \in \mathcal{A}_i, x = \mathrm{tc}_i(a) \text{ and } y = \mathrm{tc}_i(b)\} \cup \bigcup_{a \in Ind(\vec{\mathcal{A}}_n)} \mathcal{E}_a$; and
- $\ell(v) = \ell_a(v)$ for all $v \in V_a$.

Example 14 (Description graph of a sequence of ABoxes). Let $\vec{\mathcal{A}}_3 = (\mathcal{A}_i)_{0 \le i \le 3}$ be a sequence of the ABoxes: $\mathcal{A}_0 = \{P(a)\}$, $\mathcal{A}_1 = \{(P \sqcap \mathbf{G}(\exists h.O))(a),\ P(b)\}$, and $\mathcal{A}_2 = \{P(a),\ P(b),\ r(a,b)\}$. Then $\mathcal{G}(\vec{\mathcal{A}}_3)$ is as depicted in Figure 3. Observe, although b does not occur in $\mathcal{A}_0$, $\mathcal{G}(\vec{\mathcal{A}}_3)$ contains b at time point 0.

For a sequence of interpretations, one might be interested in concept membership at certain time point instead of only the beginning. We use this idea and characterize the instance relationship using $\mathcal{G}(\vec{\mathcal{A}}_n)$ and $\mathcal{G}_C$. To put i into consideration, we place the temporal copy $\mathrm{tc}_i(a)$ as the root instead of a.

Lemma 15. *Let $\vec{\mathcal{A}}_n$ be a sequence of ABoxes, $a \in Ind(\vec{\mathcal{A}}_n)$, C be a $\mathrm{LTL}^{\mathbf{X},\mathbf{G}}_{\mathcal{EL}}$-concept, and i a time point where $i \ge 0$. Then, $\vec{\mathcal{A}}_n, i \models C(a)$ if there exists a homomorphism φ from $\mathcal{G}_C$ into $\mathcal{G}(\vec{\mathcal{A}}_n)$ such that $\varphi(v_0) = \mathrm{tc}_i(a)$, where v_0 is the root of $\mathcal{G}_C$.*

Proof Sketch. We show by induction that the corresponding subtree with root $\mathrm{tc}_i(a)$ together with existence of homomorphism yield $\mathrm{tc}_i(a) \in C^{\mathcal{I}_i}$ of the canonical interpretation. Let $C_{\mathrm{tc}_i(a)} := \bigsqcap_{0 \le i \le n} \big(\bigsqcap_{D(\mathrm{tc}_i(a) \in \mathcal{A}_i} \mathbf{X}^i\, D \big)$, then $\mathfrak{I} \models \vec{\mathcal{A}}_n$ implies $a \in C^{\mathfrak{I}}_{\mathrm{tc}_i(a)}$ due to the construction of $\mathcal{G}(\vec{\mathcal{A}}_n)$. Then, we show $a^{\mathfrak{I}} \in C^{\mathfrak{I}}$ by induction on $\mathrm{depth}(C)$ by utilizing the existence of homomorphism.

5.2 k-MSC of $\text{LTL}_{\mathcal{EL}}^{\mathsf{X},\mathsf{G}}$ Concepts

The *role depth* of C (rdepth(C)) is the maximum number of nested quantifiers in C. The msc in $\text{LTL}_{\mathcal{EL}}^{\mathsf{X},\mathsf{G}}$ suffers the same problem as the one in $\mathcal{EL}$: cycles in the description graph cause infinite role depth in the msc [7] and thus the msc need not exist as concepts are finite. A common approach to remedy this, is to use k-approximation of the msc, i.e. to limit the role depth of it to $k \in \mathbb{N}$.

Definition 16. *Let $\vec{\mathcal{A}_n}$ be a sequence of ABoxes, $a \in Ind(\vec{\mathcal{A}_n})$, C a $\text{LTL}_{\mathcal{EL}}^{\mathsf{X},\mathsf{G}}$-concept, and $i, k \geq 0$. Then, C is the k-msc of a w.r.t. $\vec{\mathcal{A}_n}$ at time point i (k-$\mathbf{msc}_i(a)$) iff*

- $\vec{\mathcal{A}_n}, i \models C(a)$;
- $\text{rdepth}(C) \leq k$; *and*
- $C \sqsubseteq C'$ *for all* $\text{LTL}_{\mathcal{EL}}^{\mathsf{X},\mathsf{G}}$*-concepts* C' *with* $\vec{\mathcal{A}_n}, i \models C'(a)$ *and* $\text{rdepth}(C') \leq k$.

The computation of the k-msc performs the following steps. First, employ a tree unraveling of $\mathcal{G}(\vec{\mathcal{A}_n})$ with root a to obtain a $\text{LTL}_{\mathcal{EL}}^{\mathsf{X},\mathsf{G}}$-description tree $\mathcal{T}(a, \mathcal{G}(\vec{\mathcal{A}_n}))$. The starting point is $\text{tc}_i(a)$ as the root instead of a. Second, prune all paths to (non-temporal) length k to obtain $\mathcal{T}_k(a, \mathcal{G}(\vec{\mathcal{A}_n}))$. We assume w.l.o.g. that all C_a are pairwise aligned which can easily be achieved by the padding function. Then, we summarize this fact and characterization of the existence of the msc in following theorem. Notice that the characterization is sound, but remains open in the completeness as in the characterization of msc in $\mathcal{ALE}$.

Theorem 17. *Let $\vec{\mathcal{A}_n}$ be a sequence of ABoxes, $a \in Ind(\vec{\mathcal{A}_n})$, let $i, k \geq 0$, and $T = \mathcal{T}_k(\text{tc}_i(a), \mathcal{G}(\vec{\mathcal{A}_n}))$. Then*

1. *C_T is the k-msc of a w.r.t. $\vec{\mathcal{A}_n}$ at time point i.*
2. *If no cyclic path is reachable from $\text{tc}_i(a)$ in $\mathcal{G}(\vec{\mathcal{A}_n})$, then C_T is the msc of individual a w.r.t. $\vec{\mathcal{A}_n}$ at time i; otherwise no such msc exists.*

Proof Sketch. Proving 1. can be seen as an extension of Lemma 15. We can map $\text{tc}_i(a)$ of $\mathcal{G}_{C_T}$ to $\text{tc}_i(a)$ of $\mathcal{G}_{\vec{\mathcal{A}_n}}$ to obtain a homomorphism, since it $\mathcal{G}_{\vec{\mathcal{A}_n}}$ contains $\mathcal{G}_{C_T}$ with root $\text{tc}_i(a)$. This yields that a is an instance of C_T. Then, if C is a $\text{LTL}_{\mathcal{EL}}^{\mathsf{X},\mathsf{G}}$-concept such that a is an instance of C with $\text{depth}(C) \leq k$, we can construct a homomorphism from $\mathcal{G}_C$ to $\mathcal{G}_{C_T}$. Then, we have that $C_T \sqsubseteq C$ and finally C_T is the k-msc.

To prove 2., consider the case where the depth of unraveled tree is finite, e.g., k'. Then, the length of all k-mscs are bounded by $k \geq k'$, i.e., all concept with a larger role depth (k'-msc) are equivalent to the k-msc. Now consider the case where the unraveled tree has infinite depth. Assume that there exists k-msc that also serves as msc of a and call it C_k. Then, it is easy to see that there always exists $k+1$-msc such that $C_{k+1} \sqsubseteq C_k$ due to the cycle. Then, we have to construct an infinitely large k-msc of C for a. Since a concept description only has a fixed and finite depth, a cannot have an msc.

6 Conclusion

In this work, we devised a method how to derive a temporalized query concept from examples occurring in a data stream. We investigated the case of $\mathrm{LTL}^{\mathsf{X},\mathsf{G}}_{\mathcal{EL}}$-concepts, which extend $\mathcal{EL}$ with temporal operators next (X) and global (G). From a given stream of data in the form of a sequence of $\mathrm{LTL}^{\mathsf{X},\mathsf{G}}_{\mathcal{EL}}$-ABoxes, our methods can generalize the given set of example individuals into a $\mathrm{LTL}^{\mathsf{X},\mathsf{G}}_{\mathcal{EL}}$-concept by applying the (k-)msc and then the lcs. The result is a $\mathrm{LTL}^{\mathsf{X},\mathsf{G}}_{\mathcal{EL}}$-concept that captures the shared properties of all individuals while keeping it as least general as possible. While extending DLs with temporal operators is often problematic, we show this fragment is rather well-behaved.

This study is a rather preliminary investigation of reverse engineering of temporal queries. Obvious extensions are to learn w.r.t. a general TBox or to use rigid concepts or even rigid roles. In the longer run, we would like to use a bigger fragment of LTL and to investigate the case of reverse engineering temporalized conjunctive queries.

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