Optimal Repairs in Ontology Engineering as Pseudo-Contractions in Belief Change

Franz Baader
TU Dresden, Institute of Theoretical Computer Science
Dresden, Germany
Center for Scalable Data Analytics and Artificial Intelligence (ScaDS.AI)
Dresden/Leipzig, Germany
franz.baader@tu-dresden.de

ABSTRACT

The question of how a given knowledge base can be modified such that certain unwanted consequences are removed has been investigated in the area of knowledge engineering under the name of repair and in the area of belief change under the name of contraction. Whereas in the former area the emphasis was more on designing and implementing concrete repair algorithms, the latter area concentrated on characterizing classes of contraction operations by certain postulates they satisfy. In the classical setting, repairs and contractions are subsets of the knowledge base that no longer have the unwanted consequence. This makes these approaches syntax-dependent and may result in removal of more consequences than necessary. To alleviate this problem, gentle repairs and pseudo-constractions have been introduced in the respective research areas, and their connections have been investigated in recent work. Optimal repairs preserve a maximal amount of consequences, but they may not always exist. We show that, if they exist, then they can be obtained by certain pseudo-contraction operations, and thus they comply with the postulates that these operations satisfy. Conversely, under certain conditions, pseudo-contractions are guaranteed to produce optimal repairs.

CCS CONCEPTS

• **Theory of computation** → **Description logics**; • **Computing methodologies** → **Ontology engineering**; **Nonmonotonic, default reasoning and belief revision**;

KEYWORDS

belief change, ontology repair, description logic

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1 INTRODUCTION

Representing knowledge in a logic-based knowledge representation language allows one to draw implicit consequences from the explicitly represented knowledge. If such a consequence is deemed to be incorrect or no longer wanted for some reason, then it is often not obvious how to modify the knowledge base to get rid of this consequence. In ontology engineering, the knowledge base usually defines the important notions of the application domain as background knowledge in the terminology, and then uses these notions to represent a specific application situation. Modelling errors are detected when the reasoner generates a consequence that formally follows from the knowledge base, but is incorrect in the sense that it does not hold in the application domain that is supposed to be modelled. The question is then how to repair the knowledge base such that no new consequences are added, the unwanted consequence no longer follows, and other consequences are not lost unnecessarily. The classical approaches for ontology repair consider as repairs maximal subsets of the ontology (viewed as a set of logical sentences) that do not have the unwanted consequence, and employ methods inspired by model-based diagnosis [22] to compute these sets [6, 20, 24], which are called optimal classical repairs in [5]. While these approaches preserve as many of the sentences in the ontology as possible, they need not preserve a maximal amount of consequences (see [5] as well as the examples at the end of Section 2 and the beginning of Section 4 of the present paper). To overcome this problem, more gentle repair approaches have been introduced, e.g., in [5, 17, 25], but these methods still need not produce optimal repairs, i.e., ones that preserve a maximal set of consequences. In general, such optimal repairs need not exist [5]. In the setting of repairing ABoxes of the description logic $\mathcal{EL}$ w.r.t. static $\mathcal{EL}$ TBoxes, methods for computing optimal repairs (if they exist) are available [4].

In belief change [10], one usually assumes that the knowledge base represents the beliefs of a rational agent. These beliefs may change if the agent receives new information, and the question is how this can be reflected by a change of the knowledge base. Removing (implied) information is called contraction in this setting. Instead of directly constructing contraction operations, the belief change community has formulated properties (called postulates) that should be satisfied by reasonable contraction operations, and then developed approaches for constructing contraction operations that capture exactly those contraction operations that satisfy a certain combination of postulates. This approach, which was pioneered in [1], is called the AGM approach. The original AGM approach works with *belief sets*, which are assumed to be closed under consequences. From a practical point of view, it makes more sense to work with non-deductively closed (and ideally finite) representations of belief sets, called belief bases [9, 12, 14]. Similar to classical repairs, the original approaches for belief base contraction consider subsets of the knowledge base as possible contractions. For the same reasons as for repairs, operations that preserve more consequences, called pseudo-contractions, have been introduced in the belief change literature [11, 13, 18, 19, 23].

Although contractions and classical repairs as well as pseudo-contractions and repairs tackle basically the same problems, there has until recently been little interaction between the two communities, and thus the connections between the developed approaches remained unclear. The papers [18, 19] address this problem, with an emphasis on showing connections between gentle repairs and certain pseudo-contraction approaches called partial meet and kernel pseudo-contractions. In the present paper, we concentrate on optimal repairs, both in the classical and the general sense. We show that, under certain conditions, operations that compute optimal (classical) repairs can be obtained as partial meet and kernel pseudo-contractions (contractions), and vice versa. This shows, on the one hand, that the approaches developed in ontology engineering satisfy the postulates required in belief change. On the other hand, under certain conditions the approaches developed in belief change yield optimal (classical) repairs. We instantiate our results using the setting of repairing ABoxes of the description logic $\mathcal{EL}$ w.r.t. static $\mathcal{EL}$ TBoxes.

The main novelty of this work is that we consider the relationship of contraction operations from belief change with *optimal* repairs (both in the classical and the general sense), i.e., repairs that are maximal subsets of the knowledge base to be repaired (classical case) or repairs that are entailed by the knowledge base to be repaired and preserve a maximal amount of consequences (general case). This notion of optimality usually does not play an important rôle in belief change (there is no optimality postulate), but under the assumption that the repair process should not lose consequences unnecessarily, it is important for ontology engineering. In [18, 19], classical repairs and gentle repairs are respectively set in relationship with contraction and pseudo-contraction operations, but optimal repairs are not considered. Work on revision and contraction for DLs [21] usually adapts the approaches from the belief change community to DLs as underlying logical formalism, but does not compare them with other ontology repair approaches, and in particular not with optimal repairs.

The next section introduces the general notion of a logical consequence operator, and then instantiates it with entailment from $\mathcal{EL}$ ABoxes w.r.t. an $\mathcal{EL}$ TBox. The definitions of contractions and repairs in the subsequent sections will be formulated in the general setting, with the concrete instance providing us with (counter)-examples. In Section 3, we first review relevant notions from belief change. In particular, we introduce partial meet and kernel contractions, and recall the postulates they satisfy. We then show that certain partial meet and kernel contractions always yield optimal classical repairs. Conversely, we note that a contraction operation that always returns an optimal classical repair (in case there is any repair) satisfies three of the four postulates characterizing partial meet contractions, but not the fourth (called *uniformity*). In Section 4, we introduce pseudo-contractions and in particular the "pseudo-versions" of partial meet and kernel contraction [19, 23]. Roughly speaking, we show that there always exists a partial meet pseudo-contraction that produces optimal repairs whenever such repairs exist, and optimal classical repairs otherwise. In general, however, partial meet pseudo-contractions need not yield optimal repairs (even if they exist) unless an additional property is satisfied.

2 PRELIMINARIES

Following [18], we assume that a logic is given by its language $\mathfrak{L}$, i.e., the set of sentences one can build in it, and its consequence operator $\mathsf{Cn} : 2^{\mathfrak{L}} \to 2^{\mathfrak{L}}$, which maps each set of sentences $\mathcal{X}$ to the set of its consequences $\mathsf{Cn}(\mathcal{X})$. Usually, $\mathfrak{L}$ will consist of certain first-order sentences, such as sentences expressed in some description logic, and Cn is first-order consequence restricted to $\mathfrak{L}$. Given sets of sentences $\mathcal{X}, \mathcal{Y} \subseteq \mathfrak{L}$ (a sentence $\alpha \in \mathfrak{L}$), we write $\mathcal{X} \models \mathcal{Y}$ ($\mathcal{X} \models \alpha$) to indicate that $\mathcal{Y} \subseteq \mathsf{Cn}(\mathcal{X})$ ($\alpha \in \mathsf{Cn}(\mathcal{X})$). In general, we only assume that Cn satisfies the following properties:

- $\mathcal{X} \subseteq \mathsf{Cn}(\mathcal{X})$ (inclusion),
- $\mathcal{X} \subseteq \mathcal{Y}$ implies $\mathsf{Cn}(\mathcal{X}) \subseteq \mathsf{Cn}(\mathcal{Y})$ (monotonicity),
- $\mathsf{Cn}(\mathsf{Cn}(\mathcal{X})) = \mathsf{Cn}(\mathcal{X})$ (idempotency),
- $\alpha \in \mathsf{Cn}(\mathcal{X})$ implies that there is a *finite* set $\mathcal{X}' \subseteq \mathcal{X}$ such that $\alpha \in \mathsf{Cn}(\mathcal{X}')$ (compactness).

These four properties are satisfied by first-order consequence, and thus also for most description logics.

As a concrete example, we consider ABoxes of the description logic $\mathcal{EL}$ as (finite) sets of sentences and consequence w.r.t. an $\mathcal{EL}$ TBox as the consequence operator. Our introduction of $\mathcal{EL}$ concepts, TBoxes, and ABoxes follows the presentation in [4].

The name space available for defining $\mathcal{EL}$ concepts and ABox assertions is given by a *signature* Σ, which is the disjoint union of sets Σ_I, Σ_C, and Σ_R of *individual names*, *concept names*, and *role names*. Starting with concept names and the top concept $\top$, $\mathcal{EL}$ *concepts* are defined inductively: if C, D are $\mathcal{EL}$ concepts and r is a role name, then $C \sqcap D$ (conjunction) and $\exists r.C$ (existential restriction) are also $\mathcal{EL}$ concepts. An $\mathcal{EL}$ *general concept inclusion (GCI)* is of the form $C \sqsubseteq D$, an $\mathcal{EL}$ *concept assertion* is of the form $C(a)$, and a *role assertion* is of the form $r(a, b)$, where C, D are $\mathcal{EL}$ concepts, $r \in \Sigma_\mathsf{R}$, and $a, b \in \Sigma_\mathsf{I}$. An $\mathcal{EL}$ *assertion* is a concept or a role assertion. An $\mathcal{EL}$ *TBox* is a finite set of $\mathcal{EL}$ GCIs and an $\mathcal{EL}$ *ABox* is a finite set of $\mathcal{EL}$ concept assertions and role assertions. Since, in this paper, we consider only one description logic, we sometimes omit the prefix $\mathcal{EL}$, and write assertion, ABox, etc. in place of $\mathcal{EL}$ assertion, $\mathcal{EL}$ ABox, etc.

The semantics of the syntactic entities introduced above can either be defined directly using interpretations or by a translation into first-order logic (FO) [2]. To make the connection to FO clearer, we choose here the latter approach. In the translation, the elements of Σ_I, Σ_C, and Σ_R are respectively viewed as constant symbols, unary predicate symbols, and binary predicate symbols. $\mathcal{EL}$ concepts C are inductively translated into FO formulas $\phi_C(x)$ with one free variable x:

- concept A for $A \in \Sigma_\mathsf{C}$ is translated into $A(x)$ and $\top$ into $A(x) \vee \neg A(x)$ for an arbitrary $A \in \Sigma_\mathsf{C}$;
- if C, D are translated into $\phi_C(x), \phi_D(x)$, then $C \sqcap D$ is translated into $\phi_C(x) \wedge \phi_D(x)$ and $\exists r.C$ into $\exists y.\,(r(x, y) \wedge \phi_C(y))$,

where $\phi_C(y)$ is obtained from $\phi_C(x)$ by replacing the free variable x by a variable y not occurring in $\phi_C(x)$.

GCIs $C \sqsubseteq D$ yield sentences $\phi_{C\sqsubseteq D} := \forall x.\,(\phi_C(x) \to \phi_D(x))$ and TBoxes $\mathcal{T}$ sets of sentences $\Phi_{\mathcal{T}} := \{\phi_{C\sqsubseteq D} \mid C \sqsubseteq D \in \mathcal{T}\}$. Concept assertions $C(a)$ are translated into $\phi_{C(a)} := \phi_C(a)$, role assertions $r(a,b)$ stay the same, i.e., $\phi_{r(a,b)} := r(a,b)$, and ABoxes $\mathcal{A}$ are translated into sets of sentences $\Phi_{\mathcal{A}} := \{\phi_\alpha \mid \alpha \in \mathcal{A}\}$.

The assertion α is a *consequence* of the set of assertions $\mathcal{A}$ w.r.t. the TBox $\mathcal{T}$ (written $\mathcal{A} \models^{\mathcal{T}} \alpha$) if ϕ_α is a consequence of the set of sentences $\Phi_{\mathcal{A}} \cup \Phi_{\mathcal{T}}$ according to the semantics of FO. This yields the consequence operator $\mathsf{Cn}_{\mathcal{T}}$, which takes as input a set of assertions $\mathcal{A}$, is parameterized with an $\mathcal{EL}$ TBox $\mathcal{T}$, and yields the following set of assertions as consequences:

$$\mathsf{Cn}_{\mathcal{T}}(\mathcal{A}) = \{\alpha \mid \mathcal{A} \models^{\mathcal{T}} \alpha \text{ where } \alpha \text{ is an } \mathcal{EL} \text{ assertion}\}.$$

Since its semantics is based on first-order consequence, $\mathsf{Cn}_{\mathcal{T}}$ clearly satisfies inclusion, monotonicity, idempotency, and compactness.

As an example, consider a situation where our rational agent believes that Ben has a parent called Jerry, who is both rich and famous. The agent also believes that people that have a rich and famous parent are arrogant. The former belief is represented in the ABox

$$\mathcal{A} := \{\, \mathit{has\_parent}(\mathit{BEN}, \mathit{JERRY}),\ \mathit{Famous}(\mathit{JERRY}),\ \mathit{Rich}(\mathit{JERRY}) \,\}$$

whereas the latter is expressed in the TBox

$$\mathcal{T} := \{\, \exists\, \mathit{has\_parent}.\,(\mathit{Famous} \sqcap \mathit{Rich}) \sqsubseteq \mathit{Arrogant} \,\}.$$

Clearly, we have $\mathit{Arrogant}(\mathit{BEN}) \in \mathsf{Cn}_{\mathcal{T}}(\mathcal{A})$. Now assume that the agent actually meets Ben and notices that he is not arrogant. Since the agent insists on sticking with the prejudice that children of rich and famous people are arrogant, the unwanted consequence $\mathit{Arrogant}(\mathit{BEN})$ can only be removed by modifying the ABox. In the classical repair approach, this can be achieved by removing one of its three assertions from $\mathcal{A}$. Let us assume that the agent decides to remove $\mathit{Famous}(\mathit{JERRY})$. This removes the unwanted consequence $\mathit{Arrogant}(\mathit{BEN})$, but also the consequence $\exists\, \mathit{has\_parent}.\,\mathit{Famous}(\mathit{BEN})$. Removing $\mathit{Famous}(\mathit{JERRY})$ from $\mathcal{A}$, but adding the assertion $\exists\, \mathit{has\_parent}.\,\mathit{Famous}(\mathit{BEN})$ to the ABox yields a repair that retains more consequences than the classical repair. This improved repair corresponds to the agent's new belief that Jerry is only rich, and that Ben has another famous parent, whose name is not known to the agent.

3 CLASSICAL REPAIRS AND CONTRACTIONS

The classical notions of contraction and repair resort to subsets of the given knowledge base to remove an unwanted consequence. Following [18, 19, 23], we first define contractions and recall two approaches for constructing them. Then, we describe their connection to classical repairs.

3.1 Contractions in Belief Change

Let $\mathfrak{L}$ be a logical language and Cn a monotone, idempotent, and compact consequence operator satisfying inclusion. A *belief base* is an arbitrary subset of $\mathfrak{L}$. Contractions get rid of unwanted consequences of a belief base by removing some of its sentences. More formally, a *contraction operation* ctr accepts a belief base $\mathcal{B} \subseteq \mathfrak{L}$ and a sentence $\alpha \in \mathfrak{L}$ as input, and produces as output a belief base $\mathrm{ctr}(\mathcal{B}, \alpha)$ that satisfies the following two postulates:

- $\mathrm{ctr}(\mathcal{B}, \alpha) \subseteq \mathcal{B}$ (inclusion),
- if $\alpha \notin \mathsf{Cn}(\emptyset)$, then $\alpha \notin \mathsf{Cn}(\mathrm{ctr}(\mathcal{B}, \alpha))$ (success).

In the belief change literature, reasonable contraction operations are usually assumed to satisfy additional postulates. This is the case for contractions obtained by applying one of the following two prominent approaches for constructing contraction operations: partial meet contraction [1, 14] and kernel contraction [15]. To define the former, we must introduce remainders, remainder sets, and selection functions. Let $\mathcal{B}$ be a belief base and α a sentence.

- A *remainder* of $\mathcal{B}$ with respect to α is a maximal subset $\mathcal{X}$ of $\mathcal{B}$ such that $\alpha \notin \mathsf{Cn}(\mathcal{X})$. We denote the set of all remainders of $\mathcal{B}$ with respect to α as $\mathsf{rem}(\mathcal{B}, \alpha)$.
- A *selection function* γ for $\mathcal{B}$ takes such sets of remainders as input and satisfies the following properties for each $\alpha \in \mathcal{B}$:
  - If $\mathsf{rem}(\mathcal{B}, \alpha) \neq \emptyset$, then $\emptyset \neq \gamma(\mathsf{rem}(\mathcal{B}, \alpha)) \subseteq \mathsf{rem}(\mathcal{B}, \alpha)$.
  - If $\mathsf{rem}(\mathcal{B}, \alpha) = \emptyset$, then $\gamma(\mathsf{rem}(\mathcal{B}, \alpha)) = \{\mathcal{B}\}$.

Note that the value returned by the selection function does not depend of α itself, but on the set $\mathsf{rem}(\mathcal{B}, \alpha)$. In case this set is non-empty, this value is a non-empty subset of $\mathsf{rem}(\mathcal{B}, \alpha)$. Otherwise, the set consisting of $\mathcal{B}$ is returned. This second case occurs iff $\alpha \in \mathsf{Cn}(\emptyset)$. Each selection function γ induces a *partial meet contraction* operation ctr_γ as follows:

$$\mathrm{ctr}_\gamma(\mathcal{B}, \alpha) := \bigcap \gamma(\mathsf{rem}(\mathcal{B}, \alpha)).$$

As shown by Hansson in [14], the operation ctr_γ satisfies *inclusion* and *success*, and thus is a contraction operation, and additionally the following postulates:

- if $\beta \in \mathcal{B} \setminus \mathrm{ctr}(\mathcal{B}, \alpha)$, then there is $\mathcal{B}'$ such that $\mathrm{ctr}(\mathcal{B}, \alpha) \subseteq \mathcal{B}' \subseteq \mathcal{B}$, $\alpha \notin \mathsf{Cn}(\mathcal{B}')$, and $\alpha \in \mathsf{Cn}(\mathcal{B}' \cup \{\beta\})$ (relevance),
- if $\alpha \in \mathsf{Cn}(\mathcal{B}')$ iff $\beta \in \mathsf{Cn}(\mathcal{B}')$ holds for all $\mathcal{B}' \subseteq \mathcal{B}$, then $\mathrm{ctr}(\mathcal{B}, \alpha) = \mathrm{ctr}(\mathcal{B}, \beta)$ (uniformity).

Hansson [14] also shows that any contraction operation that satisfies the postulates *inclusion*, *success*, *relevance*, and *uniformity* can be obtained as a partial meet contraction. In [15] he introduces another construction for obtaining contraction operations, which is based on the notions of kernels and incision functions.

- The *kernel* $\mathsf{ker}(\mathcal{B}, \alpha)$ of $\mathcal{B}$ with respect to α consists of the minimal subsets $\mathcal{X}$ of $\mathcal{B}$ satisfying $\alpha \in \mathsf{Cn}(\mathcal{X})$.
- An *incision function* σ for $\mathcal{B}$ takes such kernel sets as input and satisfies the following properties for each $\alpha \in \mathcal{B}$:
  - $\sigma(\mathsf{ker}(\mathcal{B}, \alpha)) \subseteq \bigcup \mathsf{ker}(\mathcal{B}, \alpha)$,
  - If $\mathcal{X}$ is a non-empty element of $\mathsf{ker}(\mathcal{B}, \alpha)$, then $\mathcal{X} \cap \sigma(\mathsf{ker}(\mathcal{B}, \alpha)) \neq \emptyset$.

Like selection functions, incision functions depend only on the kernel set $\mathsf{ker}(\mathcal{B}, \alpha)$, and not on the sentence α itself. It is easy to see that $\emptyset \in \mathsf{ker}(\mathcal{B}, \alpha)$ iff $\mathsf{ker}(\mathcal{B}, \alpha) = \{\emptyset\}$ iff $\alpha \in \mathsf{Cn}(\emptyset)$. Each incision function σ induces a *kernel contraction* operation ctr_σ as follows:

$$\mathrm{ctr}_\sigma(\mathcal{B}, \alpha) := \mathcal{B} \setminus \sigma(\mathsf{ker}(\mathcal{B}, \alpha)).$$

As shown by Hansson in [15], the operation ctr_σ satisfies *inclusion*, *success*, and *uniformity*, but *relevance* needs to be replaced by the following weaker postulate:

- if $\beta \in \mathcal{B} \setminus \mathsf{ctr}(\mathcal{B},\alpha)$, then there is $\mathcal{B}' \subseteq \mathcal{B}$ such that $\alpha \notin \mathsf{Cn}(\mathcal{B}')$ and $\alpha \in \mathsf{Cn}(\mathcal{B}' \cup \{\beta\})$ (core-retainment).

Any contraction operation that satisfies the postulates *inclusion*, *success*, *core-retainment*, and *uniformity* can be obtained as a kernel contraction [15].

3.2 Classical Repairs in Ontology Engineering

Knowledge bases in ontology engineering are usually assumed to be finite. Thus, given a logical language $\mathfrak{L}$ and a monotone, idempotent, and compact consequence operator Cn satisfying inclusion, a *knowledge base* is a *finite* subset of $\mathfrak{L}$.

A classical repair is then just a contraction, i.e., given a knowledge base $\mathcal{B} \subseteq \mathfrak{L}$ and a sentence $\alpha \in \mathfrak{L}$, a *classical repair* of $\mathcal{B}$ with respect to α is (by definition) a subset $\mathcal{X}$ of $\mathcal{B}$ that satisfies $\alpha \notin \mathsf{Cn}(\mathcal{X})$ [5]. Thus, if we consider an operation $\mathsf{ctr}_{\mathsf{rep}}$ that, on input $\mathcal{B}$ and α, returns a classical repair of $\mathcal{B}$ with respect to α if $\alpha \notin \mathsf{Cn}(\emptyset)$, and $\mathcal{B}$ otherwise, then $\mathsf{ctr}_{\mathsf{rep}}$ satisfies *inclusion* and *success*, and thus is a contraction operation (see Proposition 3 in [18]).

In ontology engineering, one usually wants to remove a minimal amount of information to eliminate an unwanted consequence. Thus, one is interested in computing optimal classical repairs. Given a knowledge base $\mathcal{B} \subseteq \mathfrak{L}$ and a sentence $\alpha \in \mathfrak{L}$, an *optimal classical repair* of $\mathcal{B}$ with respect to α is a *maximal* subset $\mathcal{X}$ of $\mathcal{B}$ satisfying $\alpha \notin \mathsf{Cn}(\mathcal{X})$. Obviously, the notions *optimal classical repair* and *remainder* coincide, which yields the following proposition.

Proposition 3.1. *Let $\mathcal{B}$ be a knowledge base, α a sentence, and γ a selection function for $\mathcal{B}$ such that $|\gamma(\mathsf{rem}(\mathcal{B},\alpha))| = 1$ for all $\alpha \in \mathfrak{L}$. Then $\mathsf{ctr}_\gamma(\mathcal{B},\alpha)$ is an optimal classical repair of $\mathcal{B}$ with respect to α for all sentences α satisfying $\alpha \notin \mathsf{Cn}(\emptyset)$, and $\mathsf{ctr}_\gamma(\mathcal{B},\alpha) = \mathcal{B}$ if $\alpha \in \mathsf{Cn}(\emptyset)$.*

In [1, 18], a partial meet contraction operation defined using a selection function γ satisfying $|\gamma(\mathsf{rem}(\mathcal{B},\alpha))| = 1$ for all sentences α is called a *maxichoice* contraction operation. Thus, one can rephrase the statement of Proposition 3.1 as follows.

Corollary 3.2. *If ctr is a maxichoice contraction operation and $\mathcal{B}$ has a classical repair with respect to α, then $\mathsf{ctr}(\mathcal{B},\alpha)$ is an optimal classical repair of $\mathcal{B}$ with respect to α.*

Optimal classical repairs can also be obtained as kernel contractions. In fact, in ontology engineering, optimal classical repairs are often constructed using justifications and Reiter's hitting set duality [22]. Before we can describe this approach, we must introduce the relevant notions. Let $\mathcal{B}$ be a knowledge base and α a sentence.

- A *justification* of α in $\mathcal{B}$ is a minimal subset $\mathcal{X}$ of $\mathcal{B}$ such that $\alpha \in \mathsf{Cn}(\mathcal{X})$. We denote the set of all justifications of α in $\mathcal{B}$ as $\mathsf{jus}(\mathcal{B},\alpha)$. Note that $\mathsf{jus}(\mathcal{B},\alpha) = \emptyset$ if $\alpha \notin \mathsf{Cn}(\mathcal{B})$, and $\mathsf{jus}(\mathcal{B},\alpha) = \{\emptyset\}$ if $\alpha \in \mathsf{Cn}(\emptyset)$.
- Given a collection $\{\mathcal{X}_1,\ldots,\mathcal{X}_k\}$ of subsets $\mathcal{X}_i$ of $\mathcal{B}$, a *hitting set* $\mathcal{H}$ of this collection is a subset of $\mathcal{X}_1 \cup \ldots \cup \mathcal{X}_k$ such that $\mathcal{H} \cap \mathcal{X}_i \neq \emptyset$ for all $i = 1,\ldots,k$. This hitting set is *minimal* if no other hitting set is strictly contained in it. Note: if the collection is empty (i.e., if $k = 0$), then $\emptyset$ is a minimal hitting set; if it contains the empty set (i.e., if $\mathcal{X}_i = \emptyset$ for some $i, 1 \leq i \leq k$), then it has no hitting set.

It is well-known [5, 22] that the optimal classical repairs of $\mathcal{B}$ with respect to α are exactly the sets $\mathcal{B} \setminus \mathcal{H}$ where $\mathcal{H}$ ranges over the minimal hitting sets of $\mathsf{jus}(\mathcal{B},\alpha)$. Note that this characterization also works in the following borderline cases. If $\alpha \in \mathsf{Cn}(\emptyset)$, then there is no optimal classical repair, and neither is there a hitting set of $\mathsf{jus}(\mathcal{B},\alpha) = \{\emptyset\}$. If $\alpha \notin \mathsf{Cn}(\mathcal{B})$, then $\mathcal{B}$ is the only optimal classical repair, and $\mathsf{jus}(\mathcal{B},\alpha) = \emptyset$ has $\emptyset$ as its only minimal hitting set.

Obviously, the set of all justifications of α in $\mathcal{B}$ coincides with $\mathsf{ker}(\mathcal{B},\alpha)$, i.e., $\mathsf{jus}(\mathcal{B},\alpha) = \mathsf{ker}(\mathcal{B},\alpha)$. In addition, if $\alpha \notin \mathsf{Cn}(\emptyset)$, then $\sigma(\mathsf{ker}(\mathcal{B},\alpha))$ is a hitting set of $\mathsf{jus}(\mathcal{B},\alpha) = \mathsf{ker}(\mathcal{B},\alpha)$ for every incision function σ. We call an incision function *minimal* if $\sigma(\mathsf{ker}(\mathcal{B},\alpha))$ is a minimal hitting set of $\mathsf{ker}(\mathcal{B},\alpha)$ for all α with $\alpha \notin \mathsf{Cn}(\emptyset)$, and $\sigma(\mathsf{ker}(\mathcal{B},\alpha)) = \emptyset$ if $\alpha \in \mathsf{Cn}(\emptyset)$.

Proposition 3.3. *Let $\mathcal{B}$ be a knowledge base, α a sentence, and σ a minimal incision function for $\mathcal{B}$. Then $\mathsf{ctr}_\sigma(\mathcal{B},\alpha)$ is an optimal classical repair of $\mathcal{B}$ with respect to α for all sentences α satisfying $\alpha \notin \mathsf{Cn}(\emptyset)$, and $\mathsf{ctr}_\sigma(\mathcal{B},\alpha) = \mathcal{B}$ if $\alpha \in \mathsf{Cn}(\emptyset)$.*

Using Reiter's hitting set duality [22], it is easy to see that every maxichoice partial meet contraction can be obtained as a kernel contraction induced by a minimal incision function, and vice versa (see [8] for details).

Now, consider a special case $\mathsf{ctr}_{\mathsf{orep}}$ of the contraction operation $\mathsf{ctr}_{\mathsf{rep}}$ introduced above, where we require that $\mathsf{ctr}_{\mathsf{orep}}(\mathcal{B},\alpha)$ is an *optimal* classical repair of $\mathcal{B}$ with respect to α if $\alpha \notin \mathsf{Cn}(\emptyset)$.

Proposition 3.4. *The operation* $\mathsf{ctr}_{\mathsf{orep}}$ *satisfies* inclusion*,* success*, and* relevance*, but it need not satisfy* uniformity.

Proof. We already know that *inclusion* and *success* are satisfied even in the more general setting where an arbitrary repair, rather than an optimal one, is chosen. To show *relevance*, assume that $\beta \in \mathcal{B} \setminus \mathsf{ctr}_{\mathsf{orep}}(\mathcal{B},\alpha)$. If we take $\mathcal{B}' := \mathsf{ctr}_{\mathsf{orep}}(\mathcal{B},\alpha)$, then $\mathsf{ctr}_{\mathsf{orep}}(\mathcal{B},\alpha) \subseteq \mathcal{B}' \subseteq \mathcal{B}$ is satisfied. Maximality of $\mathsf{ctr}_{\mathsf{orep}}(\mathcal{B},\alpha)$ yields $\alpha \in \mathsf{Cn}(\mathcal{B}' \cup \{\beta\})$.

Without additional assumptions on how the optimal repairs are chosen, *uniformity* need not be satisfied. This is demonstrated by the example presented below. □

Example 3.5. Consider the logical language that consists of $\mathcal{EL}$ assertions and the consequence operator $\mathsf{Cn}_\mathcal{T}$ for the $\mathcal{EL}$ TBox $\mathcal{T} := \{A \sqcap B \sqsubseteq C, A \sqcap B \sqsubseteq D\}$, and set $\mathcal{B} := \{A(a), B(a)\}$, $\alpha := C(a)$, and $\beta := D(a)$. Then $\alpha \in \mathsf{Cn}_\mathcal{T}(\mathcal{B}')$ iff $\beta \in \mathsf{Cn}_\mathcal{T}(\mathcal{B}')$ holds for all $\mathcal{B}' \subseteq \mathcal{B}$. In fact, for $\mathcal{B}' = \mathcal{B}$, both α and β belong to $\mathsf{Cn}_\mathcal{T}(\mathcal{B}')$, whereas for $\mathcal{B}' \subset \mathcal{B}$ neither α nor β belongs to $\mathsf{Cn}_\mathcal{T}(\mathcal{B}')$. However, our contraction operation $\mathsf{ctr}_{\mathsf{orep}}$ could choose the optimal classical repair $\{A(a)\}$ for α and $\{B(a)\}$ for β, thus violating *uniformity*.

The problem in this example is caused by the fact that α and β produce the same sets of optimal classical repairs. If in such a situation we insist that $\mathsf{ctr}_{\mathsf{orep}}$ chooses the same element of this set for both α and β, then $\mathsf{ctr}_{\mathsf{orep}}$ is actually a maxichoice partial meet contraction, and thus also satisfies *uniformity*.

4 OPTIMAL REPAIRS AND PSEUDO-CONTRACTIONS

The classical notions of contraction and repair have the disadvantage that they are syntax-dependent in the sense that a contraction (repair) can only use the sentences that are explicitly present in the belief (knowledge) base. This may lead to removal of more consequences than is necessary to get rid of the unwanted one. For example, consider the ABoxes $\mathcal{A} := \{(A \sqcap B)(a)\}$ and $\mathcal{B} := \{(A(a), B(a)\}$, and let the unwanted consequence be $\alpha := A(a)$. The two ABoxes $\mathcal{A}$, $\mathcal{B}$ are equivalent (i.e., $\mathsf{Cn}_\emptyset(\mathcal{A}) = \mathsf{Cn}_\emptyset(\mathcal{B})$). However, with respect to α, the ABox $\mathcal{A}$ has the empty ABox as only optimal classical repair for the consequence operator $\mathsf{Cn}_\emptyset$, whereas $\mathcal{B}$ has the optimal classical repair $\{B(a)\}$. Thus, the latter repair retains the consequence $B(a)$, whereas the former does not. Pseudo-contractions and optimal repairs try to overcome this problem.

4.1 Pseudo-Contractions in Belief Change

The problem of syntax-dependency is caused by the *inclusion* postulate. In the definition of pseudo-contractions, this postulate is replaced by *logical inclusion* [11, 13]:

- $\mathsf{Cn}(\mathsf{ctr}(\mathcal{B}, \alpha)) \subseteq \mathsf{Cn}(\mathcal{B})$ (logical inclusion).

The operation $\mathsf{ctr} : 2^{\mathfrak{L}} \times \mathfrak{L} \to 2^{\mathfrak{L}}$ is a *pseudo-contraction* operation if it satisfies *success* and *logical inclusion.*

To construct pseudo-contractions that retain more consequences than contractions, one can first add some of the logical consequences of $\mathcal{B}$ to the given belief base $\mathcal{B}$, and then apply the partial meet or the kernel contraction approach to the resulting extended belief base [18, 19, 23]. In the cited literature, both one-place and two-place extension functions Cn^* are considered, where the former add consequences independently of the unwanted sentence α, whereas the latter also take α into account. Here, we consider the two-place setting since it makes it easier to obtain a connection with optimal repairs. A *two-place consequence operator* is a function $\mathsf{Cn}^* : 2^{\mathcal{L}} \times \mathcal{L} \to 2^{\mathcal{L}}$. We call such an operator a *two-place extension function* with respect to Cn if it satisfies $\mathcal{B} \subseteq \mathsf{Cn}^*(\mathcal{B}, \alpha) \subseteq \mathsf{Cn}(\mathcal{B})$ for all belief bases $\mathcal{B}$ and sentences α. This operator is further called *finite* if $\mathsf{Cn}^*(\mathcal{B}, \alpha)$ is finite whenever its first argument $\mathcal{B}$ is finite. In case the value returned by Cn^* does not depend on the second argument, we write $\mathsf{Cn}^*(\mathcal{B})$ in place of $\mathsf{Cn}^*(\mathcal{B}, \alpha)$ and call Cn^* a *one-place extension function* with respect to Cn.

Example 4.1. In the $\mathcal{EL}$ ABox setting, one can define a one-place extension function with respect to $\mathsf{Cn}_\mathcal{T}$ by breaking conjunctions in concept assertions into their conjuncts, i.e., if $C(a) \in \mathcal{B}$ and $C = C_1 \sqcap \ldots \sqcap C_n$ where the C_i are existential restrictions or concept names, then $C_1(a), \ldots, C_n(a)$ are added to $\mathsf{Cn}^*(\mathcal{B})$ (see Example 6 in [19]). Clearly, this yields a finite extension function with respect to $\mathsf{Cn}_\mathcal{T}$. In our introductory example, for $\mathcal{A} = \{(A \sqcap B)(a)\}$, we obtain $\mathsf{Cn}^*_\emptyset(\mathcal{A}) = \mathcal{A} \cup \{(A(a), B(a)\}$.

Another possibility is to add assertions entailed by the TBox. To keep the extension function finite, we can, e.g., restrict this to concept assertions for concept names: for every concept name $A \in \Sigma_\mathsf{C}$ and every individual name a occurring in $\mathcal{B}$, add $A(a)$ to $\mathsf{Cn}^*(\mathcal{B})$ if $\mathcal{B} \models^\mathcal{T} A(a)$. This yields a finite extension function since it is easy to see that $A(a)$ can only be entailed if the concept name A occurs in $\mathcal{B}$ or $\mathcal{T}$.

The idea is now to apply the partial meet or the kernel contraction approach to $\mathsf{Cn}^*(\mathcal{B}, \alpha)$ rather than to $\mathcal{B}$. A Cn^* *partial meet pseudo-contraction* is thus obtained by considering remainders and selection functions of $\mathsf{Cn}^*(\mathcal{B}, \alpha)$. Given the set of remainders $\mathsf{rem}(\mathsf{Cn}^*(\mathcal{B}, \alpha), \alpha)$ and a selection function γ^* of $\mathsf{Cn}^*(\mathcal{B}, \alpha)$, the Cn^* partial meet pseudo-contraction induced by γ^* is then defined as

$$ctr^*_{\gamma^*}(\mathcal{B}, \alpha) := \bigcap \gamma^*(\mathsf{rem}(\mathsf{Cn}^*(\mathcal{B}, \alpha), \alpha)).$$

For the ABox $\mathcal{A} = \{(A \sqcap B)(a)\}$ of Example 4.1, the only remainder of $\mathsf{Cn}^*_\emptyset(\mathcal{A}) = \mathcal{A} \cup \{(A(a), B(a)\}$ with respect to $\alpha = A(a)$ is $\{B(a)\}$, and thus the selection function γ^* must choose this remainder. This shows that $\mathsf{ctr}^*_{\gamma^*}(\mathcal{A}, \alpha) = \{B(a)\}$.

Cn^* *kernel pseudo-contractions* are defined analogously, by using kernels and incision functions for $\mathsf{Cn}^*(\mathcal{B}, \alpha)$ rather than for $\mathcal{B}$. In the example, the kernel set of $\mathsf{Cn}^*_\emptyset(\mathcal{B}, \alpha)$ consists of the sets $\{(A \sqcap B)(a)\}$ and $\{A(a)\}$, and thus the only hitting set is $\{(A \sqcap B)(a), (A(a)\}$, which thus must be chosen by the incision function δ^*. This shows that the $\mathsf{Cn}^*_\emptyset$ kernel pseudo-contractions $\mathsf{ctr}^*_{\delta^*}(\mathcal{A}, \alpha)$ is in this case also equal to $\{B(a)\}$.

Basically, these pseudo-contractions inherit the postulates satisfied by the underlying contraction operations, but they need to be formulated in a "starred" variant that takes the application of Cn^* into account, and they may depend also on properties of Cn^* (like monotonicity). More details regarding postulates can be found in [18, 19, 23]. Here, we only point out that, as an obvious consequence of the definition of *extension function* and the fact that kernel and partial meet contractions satisfy *inclusion* and *success*, the Cn^* kernel and partial meet pseudo-contractions introduced above satisfy *logical inclusion* and *success*, and thus are indeed pseudo-contractions.

4.2 Optimal Repairs in Knowledge Engineering

Given a knowledge base $\mathcal{A}$ and a sentence α, a *repair* of $\mathcal{A}$ with respect to α is a knowledge base $\mathcal{B}$ that satisfies $\mathcal{B} \subseteq \mathsf{Cn}(\mathcal{A})$ and $\alpha \notin \mathsf{Cn}(\mathcal{B})$ [5]. Thus, like pseudo-contractions, repairs need to satisfy *logical inclusion* and *success.* Since the repair must again be a knowledge base, a pseudo-contraction ctr only yields repairs if it additionally satisfies the following postulate:

- if $\mathcal{B}$ is finite, then $\mathsf{ctr}(\mathcal{B}, \alpha)$ is also finite (finiteness).

Contractions satisfy finiteness since they yield a subset of the input set $\mathcal{B}$. Since Cn^* partial meet or kernel pseudo-contractions yield contractions of $\mathsf{Cn}^*(\mathcal{B}, \alpha)$, their output is finite if $\mathsf{Cn}^*(\mathcal{B}, \alpha)$ is finite.

Proposition 4.2. *If* Cn^* *is finite,* $\mathcal{B}$ *is a knowledge base, and* α *is a sentence, then* $\mathsf{ctr}(\mathcal{B}, \alpha)$ *is a repair whenever* ctr *is a* Cn^* *partial meet or kernel pseudo-contraction.*

To obtain repairs that preserve more consequences than classical repairs, an approach similar to the one described in the previous subsection is used, e.g., in [7, 16]. In these papers, a specific syntactic structural transformation is applied to the axioms in an ontology, which replaces them by sets of logically weaker axioms. The knowledge bases obtained by this approach are then repaired using the classical approach. There are also repair methods that directly apply weakening operations to axioms to construct a repair, such as the ones described in [5, 17, 25]. The connection between such "gentle repairs" and pseudo-contractions has been investigated in [18, 19].

Here, we concentrate on optimal repairs instead. Given a knowledge base $\mathcal{A}$ and a sentence α, the repair $\mathcal{B}$ of $\mathcal{A}$ with respect to α is *optimal* if there is no repair C of $\mathcal{A}$ with respect to α such that

$\mathcal{C} \models \mathcal{B}$ and $\mathcal{B} \not\models \mathcal{C}$ [5]. As shown in [5], optimal repairs need not exists even if there are repairs.

Example 4.3 ([5]). Consider the logical language that consists of $\mathcal{EL}$ assertions and the consequence operator $\mathsf{Cn}_\mathcal{T}$ for the $\mathcal{EL}$ TBox $\mathcal{T} := \{A \sqsubseteq \exists r.A, \exists r.A \sqsubseteq A\}$, and set $\mathcal{A} := \{A(a)\}$ and $\alpha := A(a)$. The empty ABox is clearly a repair in this case. However, as shown in the proof of Proposition 2 in [5], $\mathcal{A}$ does not have an optimal repair. Intuitively, the reason for this is that any ABox of the form $\mathcal{A}_n := \{(\exists r.)^n\top(a)\}$ for $n \geq 1$ is a repair, but any fixed repair can entail only finitely many of them. Thus, if $\mathcal{B}$ is a repair, then there is an n such that $\mathcal{B} \not\models^\mathcal{T} \mathcal{A}_n$. But then $\mathcal{B} \cup \mathcal{A}_n$ is a repair that entails $\mathcal{B}$, but is not entailed by $\mathcal{B}$, which shows that $\mathcal{B}$ cannot be optimal.

Moreover, even if optimal repairs exist, they need not cover all repairs in a sense to be made more precise below. First, note that optimal classical repairs cover all classical repairs in the sense that every classical repair is contained in an optimal classical repair. For general repairs, the notion of containment needs to be replaced by entailment, i.e., containment of the consequence sets. We say that the set of all optimal repairs of $\mathcal{A}$ with respect to α *covers* all repairs of $\mathcal{A}$ with respect to α if, for every repair $\mathcal{B}$ of $\mathcal{A}$ with respect to α, there is an optimal repair $\mathcal{C}$ of $\mathcal{A}$ with respect to α such that $\mathcal{C} \models \mathcal{B}$.

Example 4.4 ([4]). Consider the ABox $\mathcal{A} := \{A(a), r(a,b), B(b)\}$, the TBox $\mathcal{T} := \{B \sqsubseteq \exists r.B, \exists r.B \sqsubseteq B\}$, and the sentence $\alpha := (A \sqcap \exists r.B)(a)$. As shown in [4] (Example 12), for the consequence operator $\mathsf{Cn}_\mathcal{T}$, the ABox $\mathcal{C} := \{r(a,b), B(b)\}$ is the only optimal repair of $\mathcal{A}$ with respect to α. However, the ABox $\mathcal{B} := \{A(a), r(a,b), (\exists r.\exists r.\top)(b)\}$ is also a repair of $\mathcal{A}$ with respect to α, but it is not entailed by $\mathcal{C}$. Thus, in this example, the set of optimal repairs does not cover all repairs.

In the remainder of this section, we investigate the connection between optimal repairs and partial meet and kernel pseudo-contractions. For this, we first need to define an appropriate consequence operator Cn^*. Let $\mathcal{A}$ be a knowledge base and α a sentence. We define $\mathsf{Orep}(\mathcal{A}, \alpha)$ to consists of the optimal repairs of $\mathcal{A}$ with respect to α,[1] and set

$$\mathsf{Cn}^*(\mathcal{A}, \alpha) := \mathcal{A} \cup \bigcup \mathsf{Orep}(\mathcal{A}, \alpha).$$

This operator is a two-place extension function w.r.t. Cn since $\mathcal{A} \subseteq \mathsf{Cn}^*(\mathcal{A}, \alpha)$ by definition and $\mathsf{Cn}^*(\mathcal{A}, \alpha) \subseteq \mathsf{Cn}(\mathcal{A})$ holds because every repair of $\mathcal{A}$ is entailed by $\mathcal{A}$. This extension function is finite iff $\mathsf{Orep}(\mathcal{A}, \alpha)$ is finite for all knowledge bases $\mathcal{A}$ and sentences α. This condition is satisfied in our ABox setting.

Proposition 4.5. *Let $\mathcal{T}$ be an $\mathcal{EL}$ TBox and $\mathsf{Cn}_\mathcal{T}$ the induced consequence operator on $\mathcal{EL}$ ABoxes. Then $\mathsf{Cn}^*_\mathcal{T}$ is a finite extension function that can effectively be computed.*

Proof. It remains to show that $\mathsf{Cn}^*_\mathcal{T}$ is finite and computable. This is an easy consequence of the results proved in [4]. In fact, it is shown there that the optimal ABox repairs of $\mathcal{A}$ with respect to α can be computed by first computing the optimal quantified ABox (qABox) repairs of $\mathcal{A}$ with respect to α for IRQ-entailment. This set is finite and can effectively be computed. The optimal ABox repairs are obtained from this set by computing, for each qABox in this set, its optimal ABox approximation, if it exists. Existence of this approximation is decidable, and if it exists, then the approximation can be computed. □

The next lemma yields a connection between optimal repairs and the notion of a remainder.

Lemma 4.6. *Let $\mathcal{A}$ be a knowledge base and α a sentence. If $\mathcal{B} \in \mathsf{Orep}(\mathcal{A}, \alpha)$, then $\mathcal{B}$ is equivalent to a remainder of $\mathsf{Cn}^*(\mathcal{A}, \alpha)$ with respect to α.*

Proof. We must show that $\mathcal{B} \in \mathsf{Orep}(\mathcal{A}, \alpha)$ is equivalent to a maximal subset of $\mathsf{Cn}^*(\mathcal{A}, \alpha)$ that does not have the consequence α. Since it is a repair with respect to α, it does not have the consequence α. Assume that $\mathcal{B}$ is not maximal, i.e., there is $\mathcal{B} \subset \mathcal{B}' \subseteq \mathsf{Cn}^*(\mathcal{A}, \alpha)$ such that $\alpha \notin \mathsf{Cn}(\mathcal{B}')$. We can assume without loss of generality that $\mathcal{B}'$ is a remainder.[2] If $\mathcal{B}'$ is equivalent to $\mathcal{B}$, then we are done. Otherwise, we obtain a contradiction to our assumption that $\mathcal{B}$ is an optimal repair. □

The following result is an easy consequence of this lemma.

Theorem 4.7. *Let $\mathcal{A}$ be a knowledge base and α a sentence. Then there exists a Cn^* partial meet pseudo-contraction $\mathsf{ctr}^*_{\gamma^*}$ such that $\mathsf{ctr}^*_{\gamma^*}(\mathcal{A}, \alpha)$ is an optimal repair of $\mathcal{A}$ w.r.t. α if $\mathsf{Orep}(\mathcal{A}, \alpha) \neq \emptyset$, and an optimal classical repair of $\mathcal{A}$ w.r.t. α if $\mathsf{Orep}(\mathcal{A}, \alpha) = \emptyset$ and $\alpha \notin \mathsf{Cn}(\emptyset)$.*

Proof. Define γ^* such that it chooses an element of $\mathsf{Orep}(\mathcal{A}, \alpha)$ if this set is non-empty, and an arbitrary remainder of $\mathsf{Cn}^*(\mathcal{A}, \alpha)$ otherwise. By Lemma 4.6, this indeed yields a selection function for $\mathsf{Cn}^*(\mathcal{A}, \alpha)$. In case $\mathsf{Orep}(\mathcal{A}, \alpha) = \emptyset$, we know that $\mathsf{Cn}^*(\mathcal{A}, \alpha) = \mathcal{A}$, and thus a remainder is an optimal classical repair in this case, unless there is no repair. □

In general, remainders of $\mathsf{Cn}^*(\mathcal{A}, \alpha)$ need not be optimal repairs even if $\mathsf{Orep}(\mathcal{A}, \alpha) \neq \emptyset$.

Example 4.8. Consider the ABox $\mathcal{A}$, the TBox $\mathcal{T}$, and the sentence α of Example 4.4. Since in this case the only optimal repair is a subset of $\mathcal{A}$, we have $\mathsf{Cn}^*_\mathcal{T}(\mathcal{A}, \alpha) = \mathcal{A}$. The ABox $\mathcal{B}' := \{A(a), r(a,b)\}$ is a remainder of $\mathsf{Cn}^*_\mathcal{T}(\mathcal{A}, \alpha)$, but it is not optimal since $\mathcal{B} = \{A(a), r(a,b), (\exists r.\exists r.\top)(b)\}$ is a repair that strictly entails $\mathcal{B}'$.

This problem cannot occur if $\mathsf{Orep}(\mathcal{A}, \alpha)$ covers all repairs.

Lemma 4.9. *Let $\mathcal{A}$ be a knowledge base and α a sentence such that $\mathsf{Orep}(\mathcal{A}, \alpha)$ covers all repairs of $\mathcal{A}$ w.r.t. α. If $\mathcal{B}$ is a remainder of $\mathsf{Cn}^*(\mathcal{A}, \alpha)$ w.r.t. α, then $\mathcal{B}$ is an optimal repair of $\mathcal{A}$ w.r.t. α.*

Proof. First, note that $\mathcal{B}$ is entailed by $\mathcal{A}$ since $\mathcal{B} \subseteq \mathsf{Cn}^*(\mathcal{A}, \alpha) \subseteq \mathsf{Cn}(\mathcal{A})$. In addition, $\alpha \notin \mathsf{Cn}(\mathcal{B})$ holds by the definition of a remainder. Thus, $\mathcal{B}$ is a repair of $\mathcal{A}$ with respect to α.

Assume that $\mathcal{B}$ is not optimal. Then there is a repair $\mathcal{B}'$ of $\mathcal{A}$ with respect to α that strictly entails $\mathcal{B}$. Since $\mathsf{Orep}(\mathcal{A}, \alpha)$ covers all repairs, there is an element $\mathcal{C}$ of $\mathsf{Orep}(\mathcal{A}, \alpha)$ that entails $\mathcal{B}'$,

[1] More precisely, we assume that $\mathsf{Orep}(\mathcal{A}, \alpha)$ contains one representative of every equivalence class of optimal repairs, where two knowledge bases are equivalent if they entail each other.

[2] If $\mathsf{Cn}^*(\mathcal{A}, \alpha)$ is finite, then this is trivial. Otherwise, one needs to use transfinite induction and the fact that Cn is compact.

and thus strictly entails $\mathcal{B}$. Consequently, there is $\beta \in C$ that is not entailed by $\mathcal{B}$. Thus, $\beta \in \mathsf{Cn}^*(\mathcal{A}, \alpha)$, but $\beta \notin \mathcal{B}$, which shows that $\mathcal{B} \subset \mathcal{B} \cup \{\beta\} \subseteq \mathsf{Cn}^*(\mathcal{A}, \alpha)$. This yields a contradiction to our assumption that $\mathcal{B}$ is a remainder of $\mathsf{Cn}^*(\mathcal{A}, \alpha)$ with respect to α if we can show that $\alpha \notin \mathsf{Cn}(\mathcal{B} \cup \{\beta\})$. This finishes the proof since $\alpha \notin \mathsf{Cn}(\mathcal{B} \cup \{\beta\})$ is an easy consequence of the facts that $\mathcal{B} \subseteq \mathsf{Cn}(C)$, $\beta \in C$, and $\alpha \notin \mathsf{Cn}(C)$. □

As a consequence of this lemma, we can show that maxichoice Cn^* partial meet pseudo-contractions (i.e., ones where the selection function returns a singleton set) always produce optimal repairs in case $\mathsf{Orep}(\mathcal{A}, \alpha)$ covers all repairs.

Theorem 4.10. *Let $\mathcal{A}$ be a knowledge base and α a sentence such that* $\mathsf{Orep}(\mathcal{A}, \alpha)$ *covers all repairs of $\mathcal{A}$ with respect to α. If* $\mathsf{ctr}^*_{\gamma^*}$ *is a maxichoice* Cn^* *partial meet pseudo-contraction, then* $\mathsf{ctr}^*_{\gamma^*}(\mathcal{A}, \alpha)$ *is an optimal repair of $\mathcal{A}$ with respect to α.*

Proof. In the maxichoice case, the selection function returns a remainder of $\mathsf{Cn}^*(\mathcal{A}, \alpha)$ with respect to α. By Lemma 4.9, this remainder is an optimal repair of $\mathcal{A}$ with respect to α. □

Since every kernel contraction induced by a minimal incision function can be obtained as a maxichoice partial meet contraction, the theorem also holds if we replace "maxichoice Cn^* partial meet pseudo-contraction" with "Cn^* kernel pseudo-contraction induced by a minimal incision function."

Corollary 4.11. *Let $\mathcal{A}$ be a knowledge base and α a sentence such that* $\mathsf{Orep}(\mathcal{A}, \alpha)$ *covers all repairs of $\mathcal{A}$ with respect to α. If* $\mathsf{ctr}^*_{\delta^*}$ *is a* Cn^* *kernel pseudo-contraction induced by a minimal incision function δ^*, then* $\mathsf{ctr}^*_{\delta^*}(\mathcal{A}, \alpha)$ *is an optimal repair of $\mathcal{A}$ with respect to α.*

In the ABox repair setting, the condition that $\mathsf{Orep}(\mathcal{A}, \alpha)$ covers all repairs of $\mathcal{A}$ with respect to α is satisfied if we restrict the ABox to being acyclic and the TBox to being cycle-restricted. The ABox $\mathcal{A}$ is called *cyclic* if, for some $n \geq 1$, there are role names $r_1, \ldots, r_n$ and individual names $a_0, a_1, \ldots, a_n$ such that the role assertions $r_1(a_0, a_1), \ldots, r_n(a_{n-1}, a_n)$ belong to $\mathcal{A}$ and $a_0 = a_n$. Otherwise, $\mathcal{A}$ is called *acyclic*. The $\mathcal{EL}$ TBox $\mathcal{T}$ is called *cycle-restricted* if there is no $\mathcal{EL}$ concept C such that $C \sqsubseteq^{\mathcal{T}} \exists r_1. \cdots \exists r_k.C$ for $k \geq 1$ and role names $r_1, \ldots, r_k$.[3]

Proposition 4.12 ([4], Corollary 20). *If $\mathcal{A}$ is acyclic and $\mathcal{T}$ is cycle-restricted, then* $\mathsf{Orep}(\mathcal{A}, \alpha)$ *covers all repairs of $\mathcal{A}$ w.r.t. α.*

We have already seen in Example 4.4 that the proposition need not hold if the TBox is not cycle-restricted. The following example demonstrates why acyclicity of $\mathcal{A}$ is needed.

Example 4.13. Assume that $\mathcal{T} = \emptyset$ and consider the cyclic ABox $\mathcal{A} := \{A(a), r(a, a)\}$. If we set $\alpha := \exists r.A(a)$, then $\mathcal{B} := \{A(a)\}$ is a repair of $\mathcal{A}$ with respect to α. Assume that C is an optimal repair of $\mathcal{A}$ with respect to α that entails $\mathcal{B}$. Then C cannot contain the role assertion $r(a, a)$, and thus it can entail $(\exists r.)^n \top(a)$ only for finitely many n. Hence, there is an n such that C does not entail $(\exists r.)^n \top(a)$, which implies that $C \cup \{(\exists r.)^n \top(a)\}$ is a repair that strictly entails C. This contradicts our assumption that C is optimal.

[3] $C \sqsubseteq^{\mathcal{T}} D$ holds if $\phi_{C \sqsubseteq D}$ is a consequence of $\Phi_{\mathcal{T}}$ according to the semantics of FO.

5 CONCLUSION

The results shown is this paper complement recent results [18, 19] on the relationship between gentle repairs and pseudo-contractions by demonstrating that there are close connections between optimal repairs and certain pseudo-contraction operations. We have illustrated these results on the use case of repairing $\mathcal{EL}$ ABoxes with respect to static $\mathcal{EL}$ TBoxes, where optimal repairs can effectively be computed (if they exists) [4].

In [3], it was shown that optimal repairs always exist and cover all repairs if one uses quantified ABoxes (where some of the individuals can be anonymized by representing them as existentially quantified variables) in place of ABoxes. Extending the result of the present paper to this setting poses new challenges since the first-order translation of a quantified ABox is not a set of sentences, but a single one, which starts with an existential quantifier prefix. Thus, considering subsets when constructing contractions does not make sense. We conjecture that this problem can be overcome by introducing an "inclusion" relation on quantified ABoxes that shares enough properties with set inclusion for the constructions and proofs regarding (pseudo-)contractions to continue working.

On a more conceptual level, there are certain differences between repair approaches in ontology engineering and contraction approaches in belief change that are worth investigating. On the one hand, the work on optimal repairs [3, 4] usually considers a single repair problem and does not investigate the relationship between repairs for different unwanted consequences, whereas postulates like *uniformity* in belief change make statements on how results for different unwanted consequences should be connected under certain conditions on these consequences. It would be interesting to see whether and how postulates like uniformity and their variants in the context of pseudo-contractions [19, 23] can be satisfied by methods that compute optimal repairs. On the other hand, contraction and pseudo-contraction operators produces a single belief base as output, whereas work on optimal repairs is also concerned with how to compute the set of all such repairs and investigates properties of this set (like whether it covers all repairs or not). In contrast, on the belief change side, there are no postulates about the sets of all pseudo-contractions that can be obtained be applying a certain approach (e.g., in the partial meet case, if one looks at all possible selection functions). It would be interesting to see whether taking this "set view" can lead to interesting kinds of new postulates.

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