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Contractions Based on Optimal Repairs

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Abstract

Removing unwanted consequences from a knowledge base has been investigated in belief change under the name contraction and is called repair in ontology engineering. Simple repair and contraction approaches based on removing statements from the knowledge base (respectively called belief base contractions and classical repairs) have the disadvantage that they are syntax-dependent and may remove more consequences than necessary. Belief set contractions do not have these problems, but may result in belief sets that have no finite representation if one works with logics that are not fragments of propositional logic. Similarly, optimal repairs, which are syntax-independent and maximize the retained consequences, may not exist. In this paper, we want to leverage advances in characterizing and computing optimal repairs of ontologies based on the description logics $\mathcal{EL}$ to obtain contraction operations that combine the advantages of belief set and belief base contractions. The basic idea is to employ, in the partial meet contraction approach, optimal repairs instead of optimal classical repairs as remainders. We introduce this new approach in a very general setting, and prove a characterization theorem that relates the obtained contractions with well-known postulates. Then, we consider several interesting instances, not only in the standard repair/contraction setting where one wants to get rid of a consequence, but also in other settings such as variants of forgetting in propositional and description logic. We also show that classical belief set contraction is an instance of our approach.

1 Introduction

Representing knowledge in a logic-based knowledge representation language allows one to derive implicit consequences from a given knowledge base (KB). Modifying a given KB such that a certain unwanted consequence no longer follows is a nontrivial task, which has been investigated in the area of belief change under the name of contraction (Alchourrón, Gärdenfors, and Makinson 1985) and in ontology engineering under the name of repair (Kalyanpur et al. 2006; Schlobach et al. 2007; Baader et al. 2018; Troquard et al. 2018). Whereas research in ontology engineering was mainly concerned with designing, implementing, and testing concrete repair algorithms, research in belief change concentrated on characterizing reasonable classes of contraction operations by formulating certain properties, called postulates, they are supposed to satisfy. Connections between these two areas have, e.g., been investigated in (Flouris, Plexousakis, and Antoniou 2005; Qi and Yang 2008; Ribeiro and Wassermann 2009; Nikitina, Rudolph, and Glimm 2012; Euzenat 2015; Matos et al. 2019; Baader 2023).

The purpose of the present paper is to leverage recent advances in characterizing and computing optimal repairs (Baader, Koopmann, and Kriegel 2023) of ontologies based on Description Logics (DLs) (Baader et al. 2017) to obtain contraction operations that combine the advantages of belief set (Alchourrón, Gärdenfors, and Makinson 1985) and belief base (Hansson 1992) contractions. To be more precise, we will introduce a general framework for constructing contraction operations satisfying certain well-known postulates, which generalizes the partial meet contraction approach. Like base contraction approaches, it has the advantage that (under certain conditions) it can work with finite KBs. However, unlike base contraction, it is syntax independent and loses less consequences.

Partial meet contraction is a well-know approach for constructing contraction operations that satisfy a collection of reasonable postulates. For belief sets, i.e., KBs that are closed under logical consequence, this approach was investigated in the seminal AGM paper (Alchourrón, Gärdenfors, and Makinson 1985). Basically, it considers all maximal subsets of the given belief set that do not *contain* a certain undesired consequence, selects a non-empty collection of these maximal subsets, and then builds their intersection (i.e., the "meet"). This results in a very elegant theory with intuitive postulates, but has the disadvantage that the belief sets obtained by applying this operation may not be representable as the logical closure of a finite KB, even if one starts with belief sets that are finitely representable. To overcome this problem, Nebel (1989) and Hansson (1992) use finite KBs (called belief bases), take their maximal subsets that do not *entail* the undesired consequence, and again builds the intersection of a non-empty collection of these maximal subsets. In the belief change literature, these maximal subsets are called remainders, whereas they are called optimal classical repairs in the DL community (Baader et al. 2018). Both partial meet contractions in the belief base setting and optimal classical repairs in ontology engineering have the disadvantage that these operations are syntax-dependent and may remove too many consequences (Hansson 1993; Baader et al. 2018;

Santos et al. 2018; Matos et al. 2019; Baader 2023).

On the DL side, optimal repairs have been introduced, which maximize the set of consequences of the knowledge base rather than the set of its explicit statements, while still being representable by a finite KB (Baader et al. 2018). In general, such optimal repairs need not exists even in cases where there is a repair (see Proposition 2 in (Baader et al. 2018)). In cases where they exist (Baader et al. 2021a; Baader et al. 2022; Baader and Kriegel 2022; Baader, Koopmann, and Kriegel 2023), optimal repairs yield a syntax-independent repair approach that does not lose consequences unnecessarily. The main idea underlying the approach proposed in this paper is to replace, in the partial meet contraction approach, remainders (i.e., optimal classical repairs) with optimal repairs. This approach has been used in (Rienstra, Schon, and Staab 2020; Baader 2023) in the context of designing contraction operations for concepts of the DL $\mathcal{EL}$, though there it was not phrased in this way.

Instead of introducing and applying this new approach in a specific instance, we consider here a very general setup, which clarifies the basic properties needed to apply it. Basically, we consider an entailment relation between KBs, without making explicit assumption on the structure of the KBs and their semantics. For a start, we only require that entailment is reflexive and transitive. In addition, we abstract from non-entailment of a certain consequence as repair goal and only require that the set of repairs is closed under entailment. To apply a variant of the partial meet contraction approach in this setting, we need to make some additional assumptions. First, we assume that operations akin to (but not necessarily equal to) conjunction and disjunction are available, which we will respectively call sum and product. These operations correspond to union and intersection of belief sets, but are performed on (possibly finite) KBs representing them. From a technical point of view, sum is needed to formulate some of the relevant postulates whereas product plays the role of meet in the construction of the contraction operation. In addition, we require the existence of remainders, which are optimal repairs in our setting. An important property needed in the proof of the characterization theorem (i.e., the theorem that states the connections between the constructed contraction operations and the postulates) is that finitely many of these optimal repairs cover all repairs in the sense that every repair is entailed by an optimal one.

In the next section, we describe the general setup and illustrate it with two simple examples, one describing a standard repair/contraction setting, where the repair goal is non-entailment of a certain consequence, and the other one inspired by variable forgetting in propositional logic (Lin and Reiter 1994; Lang, Liberatore, and Marquis 2003; Sauerwald, Beierle, and Kern-Isberner 2024). Then, we introduce our new contraction approach (called partial product contractions since the product is used as the meet operation), and state the characterization theorem. Finally, we introduce several concrete instances of the general approach where KBs are formulated using the DL $\mathcal{EL}$. Due to space constraints, we cannot give detailed proofs of our results and detailed descriptions of all instances here. They can be found in (Baader and Wassermann 2024).

2 The General Setup

We assume that we are given a set of *knowledge bases* (KBs) and an *entailment relation* between knowledge bases. We usually write KBs as $\mathcal{K}$, possibly primed ($\mathcal{K}'$) or with an index ($\mathcal{K}_i$), and entailment as $\models$, i.e., $\mathcal{K} \models \mathcal{K}'$ means that $\mathcal{K}$ *entails* $\mathcal{K}'$, or equivalently that $\mathcal{K}'$ is *entailed by* $\mathcal{K}$. We assume that entailment satisfies the following properties:

- $\mathcal{K} \models \mathcal{K}$ (reflexivity),
- $\mathcal{K} \models \mathcal{K}'$ and $\mathcal{K}' \models \mathcal{K}''$ implies $\mathcal{K} \models \mathcal{K}''$ (transitivity).

We define $\mathrm{Con}(\mathcal{K}) := \{\mathcal{K}' \mid \mathcal{K} \models \mathcal{K}'\}$, and also call an element of $\mathrm{Con}(\mathcal{K})$ a *consequence* of $\mathcal{K}$. Clearly, reflexivity and transitivity of $\models$ yield the following properties of the Con operator:

- $\mathcal{K} \in \mathrm{Con}(\mathcal{K})$ (inclusion),
- $\mathcal{K} \models \mathcal{K}'$ iff $\mathrm{Con}(\mathcal{K}') \subseteq \mathrm{Con}(\mathcal{K})$ (correspondence).

We call two knowledge bases $\mathcal{K}$ and $\mathcal{K}'$ *equivalent* (and write $\mathcal{K} \equiv \mathcal{K}'$) if $\mathrm{Con}(\mathcal{K}) = \mathrm{Con}(\mathcal{K}')$. Obviously, this is the case iff $\mathcal{K} \models \mathcal{K}'$ and $\mathcal{K}' \models \mathcal{K}$. We say that $\mathcal{K}$ *strictly entails* $\mathcal{K}'$ if $\mathcal{K} \models \mathcal{K}'$, but $\mathcal{K}' \not\models \mathcal{K}$. In this case we write $\mathcal{K} \models_s \mathcal{K}'$. The relation $\equiv$ on KBs is indeed an equivalence relation, and we write the equivalence class of a KB $\mathcal{K}$ as $[\mathcal{K}]$, i.e., $[\mathcal{K}] := \{\mathcal{K}' \mid \mathcal{K} \equiv \mathcal{K}'\}$. Note that $\mathrm{Con}(\mathcal{K})$ uniquely determines the equivalence class of $\mathcal{K}$.

To illustrate the notions introduced in this section, we use a very simple example. More practically relevant examples dealing with KBs for the Description Logic $\mathcal{EL}$ are presented in Section 4.2.

Example 1. *Given a countably infinite set of propositional variables V, a knowledge base is a finite conjunction of such variables, where the empty conjunction is the always true constant $\top$. Entailment $\models$ between KBs is then classical entailment in propositional logic, which obviously satisfies reflexivity and transitivity. For such a KB $\mathcal{K}$, we denote the set of variables occurring in it with* $\mathrm{Var}(\mathcal{K})$. *It is easy to see that $\mathcal{K} \models \mathcal{K}'$ iff* $\mathrm{Var}(\mathcal{K}') \subseteq \mathrm{Var}(\mathcal{K})$. *Consequently, $\mathcal{K} \equiv \mathcal{K}'$ iff* $\mathrm{Var}(\mathcal{K}') = \mathrm{Var}(\mathcal{K})$.

In the general case, we make no assumptions on the inner structure of knowledge bases, but we assume that we have operations sum and product available that are akin to conjunction and disjunction.

Definition 2. *We call the operations $\oplus$ and $\otimes$ on finite, non-empty sets of KBs* sum *and* product *operations, respectively, if they satisfy the following properties for each finite, non-empty set of KBs $\mathfrak{K}$:*

- $\mathrm{Con}(\oplus\mathfrak{K}) \supseteq \mathrm{Con}(\mathcal{K})$ *for all $\mathcal{K} \in \mathfrak{K}$ and $\oplus\mathfrak{K}$ is the least KB satisfying this property, i.e., if $\mathcal{K}'$ is a KB satisfying* $\mathrm{Con}(\mathcal{K}') \supseteq \mathrm{Con}(\mathcal{K})$ *for all $\mathcal{K} \in \mathfrak{K}$, then* $\mathrm{Con}(\oplus\mathfrak{K}) \subseteq \mathrm{Con}(\mathcal{K}')$.
- $\mathrm{Con}(\otimes\mathfrak{K}) \subseteq \mathrm{Con}(\mathcal{K})$ *for all $\mathcal{K} \in \mathfrak{K}$ and $\otimes\mathfrak{K}$ is the greatest KB satisfying this property, i.e., if $\mathcal{K}'$ is a KB satisfying* $\mathrm{Con}(\mathcal{K}') \subseteq \mathrm{Con}(\mathcal{K})$ *for all $\mathcal{K} \in \mathfrak{K}$, then* $\mathrm{Con}(\otimes\mathfrak{K}) \supseteq \mathrm{Con}(\mathcal{K}')$.

Readers familiar with the definition of product and coproduct (sum) in category theory (Barr and Wells 1990) should be aware of the fact that, viewed from the categorical point of view, we assume that the entailment $\mathcal{K} \models \mathcal{K}'$ yields a morphism $\mathcal{K}' \to \mathcal{K}$ in the other directions. With this translation, our product and sum coincide with the corresponding notions in category theory. The reason for turning the arrow around is motivated by the fact that this is what happens in several instances of our framework (see Section 4.2). For example, subsumption $C \sqsubseteq^{\emptyset} D$ between concepts of the DL $\mathcal{EL}$ can be characterized by the existence of a homomorphism from the description tree representation of D to the description tree representation of C. The least common subsumer operation, generalizing disjunction in a logic that does not have disjunction as a constructor, is then obtained by building the direct product of the description trees (Baader, Küsters, and Molitor 1999).

Note that "least" and "greatest" in the above definition must be read modulo equivalence of KBs. In fact, it is easy to see that the above conditions imply that sum and product of a finite set of KBs are unique up to equivalence. If $\mathfrak{K} = \{\mathcal{K}\}$ is a singleton set, then $\oplus\mathfrak{K} \equiv \mathcal{K} \equiv \otimes\mathfrak{K}$. If $\mathfrak{K} = \{\mathcal{K}_1, \ldots, \mathcal{K}_n\}$ for $n \geq 2$, then we will sometimes write its sum as $\mathcal{K}_1 \oplus \ldots \oplus \mathcal{K}_n$ and its product as $\mathcal{K}_1 \otimes \ldots \otimes \mathcal{K}_n$. The following are easy consequences of Definition 2.

Lemma 3. *Let $\mathcal{K}$ be a KB and $\mathfrak{K}$ a finite, non-empty set of KBs. Then the following holds:*

1. $\oplus\mathfrak{K} \models \mathcal{K}'$ *and* $\mathcal{K}' \models \otimes\mathfrak{K}$ *for all* $\mathcal{K}' \in \mathfrak{K}$.
2. $\mathcal{K} \models \oplus\mathfrak{K}$ *iff* $\mathcal{K} \models \mathcal{K}'$ *for all* $\mathcal{K}' \in \mathfrak{K}$.
3. $\otimes\mathfrak{K} \models \mathcal{K}$ *iff* $\mathcal{K}' \models \mathcal{K}$ *for all* $\mathcal{K}' \in \mathfrak{K}$.

Example 1 (continued). *It is easy to see that sum corresponds to conjunction of KBs, and thus to the union of the corresponding variable sets. Dually, product corresponds to the intersection of the variable sets. Thus, we define*

$$\oplus\mathfrak{K} := \mathrm{KB}\left(\bigcup_{\mathcal{K}\in\mathfrak{K}} \mathrm{Var}(\mathcal{K})\right), \ \otimes\mathfrak{K} := \mathrm{KB}\left(\bigcap_{\mathcal{K}\in\mathfrak{K}} \mathrm{Var}(\mathcal{K})\right),$$

where, for a finite set $P \subseteq V$, we denote the conjunction of its elements as $\mathrm{KB}(P)$. E.g.: $p \wedge q \wedge r \oplus q \wedge s = p \wedge q \wedge r \wedge s$ and $p \wedge q \wedge r \otimes q \wedge s = q$. It is easy to see that the product and sum operations defined this way satisfy the properties required by Definition 2 (see (Baader and Wassermann 2024) for details).

When defining repairs, we assume that we have additional syntactic entities called repair requests.

Definition 4. *Given a KB $\mathcal{K}$, a* repair request *α determines a set of KBs $\mathrm{Rep}(\mathcal{K}, \alpha)$ such that*

- *$\mathcal{K} \models \mathcal{K}'$ holds for every element $\mathcal{K}' \in \mathrm{Rep}(\mathcal{K}, \alpha)$, and*
- *$\mathcal{K}' \in \mathrm{Rep}(\mathcal{K}, \alpha)$ and $\mathcal{K}' \models \mathcal{K}''$ imply $\mathcal{K}'' \in \mathrm{Rep}(\mathcal{K}, \alpha)$.*

We call the elements of $\mathrm{Rep}(\mathcal{K}, \alpha)$ repairs *of $\mathcal{K}$ for α. Two repair requests α and α' are* equivalent w.r.t. $\mathcal{K}$ *($\alpha \equiv_{\mathcal{K}} \alpha'$) if they induce the same repairs of $\mathcal{K}$, i.e., $\mathrm{Rep}(\mathcal{K}, \alpha) = \mathrm{Rep}(\mathcal{K}, \alpha')$.*

Example 1 (continued). *In this example, we consider a standard repair setting, where each KB can also be used as a repair request. Given a KB $\mathcal{K}$ and a repair request α, the goal then is to find a KB entailed by $\mathcal{K}$ that does not entail α, i.e., the induced set of repairs is defined as $\mathrm{Rep}(\mathcal{K}, \alpha) := \{\mathcal{K}' \mid \mathcal{K} \models \mathcal{K}', \mathcal{K}' \not\models \alpha\}$, where $\mathcal{K}'$ range over KBs. The first condition on repair sets of Definition 4 is satisfied by definition and the second by transitivity of $\models$.*

Continuing with presenting our general setup, we additionally assume the *optimal repair property*, which says that, for every pair $\mathcal{K}, \alpha$ consisting of a KB and a repair request (called a *repair problem*), there exists a finite set of KBs $\mathrm{Orep}(\mathcal{K}, \alpha)$ satisfying

- $\mathrm{Orep}(\mathcal{K}, \alpha) \subseteq \mathrm{Rep}(\mathcal{K}, \alpha)$ (repair property),
- every element $\mathcal{K}'$ of $\mathrm{Orep}(\mathcal{K}, \alpha)$ is *optimal*, i.e., there is no $\mathcal{K}'' \in \mathrm{Rep}(\mathcal{K}, \alpha)$ such that $\mathcal{K}'' \models_s \mathcal{K}'$ (optimality),
- $\mathrm{Orep}(\mathcal{K}, \alpha)$ *covers* all repairs, i.e., for every $\mathcal{K}'' \in \mathrm{Rep}(\mathcal{K}, \alpha)$ there is $\mathcal{K}' \in \mathrm{Orep}(\mathcal{K}, \alpha)$ such that $\mathcal{K}' \models \mathcal{K}''$ (coverage).

Example 1 (continued). *In this example, the optimal repair property is satisfied. Let $\mathcal{K}$ and α be KBs. If $\mathcal{K} \not\models \alpha$, then we set $\mathrm{Orep}(\mathcal{K}, \alpha) := \{\mathcal{K}\}$, which in this case clearly is a set of optimal repairs that covers all repairs. If $\alpha = \top$, then there is no repair, and we can set $\mathrm{Orep}(\mathcal{K}, \alpha) := \emptyset$. Finally, assume that $\mathcal{K} \models \alpha$ and $\alpha \neq \top$, which means that $\emptyset \neq \mathrm{Var}(\alpha) \subseteq \mathrm{Var}(\mathcal{K})$. For every $p \in \mathrm{Var}(\alpha)$ we define $\mathcal{K}^{-p} := \mathrm{KB}(\mathrm{Var}(\mathcal{K}) \setminus \{p\})$. It is easy to see that each such KB $\mathcal{K}^{-p}$ is a repair of $\mathcal{K}$ for α, i.e., is entailed by $\mathcal{K}$ and does not entail α. It is not hard to show that $\mathrm{Orep}(\mathcal{K}, \alpha) := \{\mathcal{K}^{-p} \mid p \in \mathrm{Var}(\alpha)\}$ is a set of optimal repairs of $\mathcal{K}$ for α that covers all repairs (see (Baader and Wassermann 2024) for details).*

We conclude this section with a simple example that considers repair requests that do not require non-entailment. It is inspired by variable forgetting in propositional logic (Lang, Liberatore, and Marquis 2003).

Example 5. *Given a countably infinite set of propositional variables V, a knowledge base is a formula of propositional logic (built using the connectives $\wedge$, $\vee$, $\neg$, and the truth constants $\top$ and $\bot$). Entailment $\models$ between KBs is the following restriction of classical entailment $\models_{\mathsf{PL}}$ in propositional logic: $\mathcal{K} \models \mathcal{K}'$ if $\mathcal{K} \models_{\mathsf{PL}} \mathcal{K}'$ and additionally $\mathrm{Var}(\mathcal{K}) \supseteq \mathrm{Var}(\mathcal{K}')$ is satisfied. This entailment relation is clearly reflexive and transitive. As* repair requests*, we consider finite subsets of the set of propositional variables V. Given a KB $\mathcal{K}$ and a repair request α, the induced set of repairs is defined as $\mathrm{Rep}(\mathcal{K}, \alpha) := \{\mathcal{K}' \mid \mathcal{K} \models \mathcal{K}', \mathrm{Var}(\mathcal{K}') \cap \alpha = \emptyset\}$. Due to the additional requirement on entailment, the second condition of Definition 4 is satisfied.*

Given a repair problem $\mathcal{K}, \alpha$, we construct the associated set of optimal repairs *as follows. For every mapping $\tau : \alpha \to \{\top, \bot\}$, let $\mathcal{K}^{\tau}$ be the propositional formula obtained from $\mathcal{K}$ by replacing every variable $p \in \alpha$ with $\tau(p)$. We set $\mathrm{Orep}(\mathcal{K}, \alpha) := \{\mathcal{K}^{-\alpha}\}$, where $\mathcal{K}^{-\alpha}$ is the disjunction of the formulas $\mathcal{K}^{\tau}$ with τ ranging over all mappings from α to $\{\top, \bot\}$. Clearly, the formulas $\mathcal{K}^{\tau}$ do not contain any of the variables of α, and thus the same is true for $\mathcal{K}^{-\alpha}$. To show optimality and coverage, it is proved in (Baader* *and Wassermann 2024) that every repair $\mathcal{K}'$ of $\mathcal{K}$ for α is entailed by $\mathcal{K}^{-\alpha}$.*

It is easy to see that the sum *operation again corresponds to conjunction, i.e., $\mathcal{K}_1 \oplus \ldots \oplus \mathcal{K}_n := \mathcal{K}_1 \wedge \ldots \wedge \mathcal{K}_n$. For the* product*, one could be tempted to use the disjunction operation of propositional logic. While disjunction behaves correctly w.r.t. $\models_{\mathsf{PL}}$, there is a problem with the containment condition for the variables. The set of variables occurring in a disjunction is again the union of the set of variables occurring in its disjuncts, but we would need it to be the intersection. We overcome this problem by repairing the disjunction. To be more precise, consider KBs $\mathcal{K}_1, \ldots, \mathcal{K}_n$, and set $\beta := \bigcup_{1\leq i\leq n} \mathrm{Var}(\mathcal{K}_i) \setminus \bigcap_{1\leq i\leq n} \mathrm{Var}(\mathcal{K}_i)$. We define $\mathcal{K}_1 \otimes \ldots \otimes \mathcal{K}_n := (\mathcal{K}_1 \vee \ldots \vee \mathcal{K}_n)^{-\beta}$. It is easy to see that, with this definition, the properties required for the product are satisfied (see (Baader and Wassermann 2024)).*

3 Partial Product Contractions

In this section, we assume that we are given a set of KBs, a set of repair requests inducing repair sets that satisfy the conditions in Definition 4, and an entailment relation $\models$ with the associated consequence operator Con such that all the properties introduced in the previous section are satisfied. In the following, we adapt the partial meet contraction approach to this setting, but call the resulting approach the partial product contraction (PPC) approach since intersection (meet) is replaced with the product. Since the properties of entailment relations introduced in the previous section are needed for this contraction approach to work, we call such entailment relations PPC enabling.

Definition 6. *Given a set of knowledge bases (KBs), a set of repair requests inducing repair sets, and a binary relation $\models$ between KBs, we call $\models$* PPC enabling *if it is reflexive and transitive, has sum and product operations $\oplus$ and $\otimes$ satisfying the properties stated in Definition 2, and for every repair problem $\mathcal{K}, \alpha$ the induced set of repairs $\mathrm{Rep}(\mathcal{K}, \alpha)$ satisfies the conditions in Definition 4 and has a finite subset $\mathrm{Orep}(\mathcal{K}, \alpha)$ that consists of optimal repairs and covers all repairs.*

Let $\mathcal{K}$ be a KB and $\mathrm{Orep}(\mathcal{K}, \alpha)$ for each repair request α the corresponding set of optimal repairs, which covers all repairs of $\mathcal{K}$ for α. A *selection function* γ for $\mathcal{K}$ takes such sets of optimal repairs as input and satisfies the following properties, for each repair request α:

- If $\mathrm{Orep}(\mathcal{K}, \alpha) \neq \emptyset$, then the selected set $\gamma(\mathrm{Orep}(\mathcal{K}, \alpha))$ satisfies $\emptyset \neq \gamma(\mathrm{Orep}(\mathcal{K}, \alpha)) \subseteq \mathrm{Orep}(\mathcal{K}, \alpha)$.
- If $\mathrm{Orep}(\mathcal{K}, \alpha) = \emptyset$, then $\gamma(\mathrm{Orep}(\mathcal{K}, \alpha)) = \{\mathcal{K}\}$.

Note that coverage of $\mathrm{Orep}(\mathcal{K}, \alpha)$ implies that this set is empty iff $\mathrm{Rep}(\mathcal{K}, \alpha) = \emptyset$.

In addition, we require that selection functions are *invariant under equivalence* of their input sets, where we say that two sets $\mathfrak{K}$ and $\mathfrak{K}'$ of knowledge bases are *equivalent* (written $\mathfrak{K} \equiv \mathfrak{K}'$) if they induce the same sets of equivalence classes, i.e., $\{[\mathcal{K}] \mid \mathcal{K} \in \mathfrak{K}\} = \{[\mathcal{K}'] \mid \mathcal{K}' \in \mathfrak{K}'\}$. More formally, the third condition on selection functions requires that, for all repair requests α and α', the following property is satisfied:

- If $\mathrm{Orep}(\mathcal{K}, \alpha) \equiv \mathrm{Orep}(\mathcal{K}, \alpha')$, then $\gamma(\mathrm{Orep}(\mathcal{K}, \alpha)) \equiv \gamma(\mathrm{Orep}(\mathcal{K}, \alpha'))$.

Each selection function γ induces a *PPC operation* ctr_γ: $\mathrm{ctr}_\gamma(\mathcal{K}, \alpha) := \otimes\gamma(\mathrm{Orep}(\mathcal{K}, \alpha))$.

A PPC operation defined using a selection function γ satisfying $|\gamma(\mathrm{Orep}(\mathcal{K}, \alpha))| = 1$ for all repair requests α is called a *MaxiChoice* PPC operation. In this setting, the selection function returns a singleton set consisting of $\mathcal{K}$ (if there is no repair) or an optimal repair (otherwise). In the latter case, $\mathrm{ctr}_\gamma(\mathcal{K}, \alpha)$ is then this optimal repair.

In the AGM setting, MaxiChoice operations have been criticized for producing belief sets that are too large (Alchourrón, Gärdenfors, and Makinson 1985). However, this only happens when dealing with logics that contain full propositional logic. In some cases, it is the most appropriate way to define contractions (Makinson 1987; Wassermann 2000). Another criticism of the *MaxiChoice* approach is that, from a purely logical point of view, the choice of a single optimal repair may seem *arbitrary* (Fermé and Hansson 2018). In the context of using optimal repairs in ontology engineering, however, non-arbitrariness is achieved by how the selection function is obtained. Basically, the ontology engineer (which is assumed to be a domain expert) chooses a single optimal repair by answering a polynomial number of questions regarding whether certain statements hold in the application domain (this interactive approach for choosing an optimal repair is briefly sketched in (Baader and Kriegel 2022), and in more detail in the accompanying technical report).

Postulates

We show that each PPC operation ctr satisfies the following postulates:

- $\mathrm{ctr}(\mathcal{K}, \alpha) \in \mathrm{Con}(\mathcal{K})$ (logical inclusion),
- $\mathrm{ctr}(\mathcal{K}, \alpha) \in \mathrm{Rep}(\mathcal{K}, \alpha)$ if $\mathrm{Rep}(\mathcal{K}, \alpha) \neq \emptyset$ (success),
- $\mathrm{ctr}(\mathcal{K}, \alpha) \equiv \mathcal{K}$ if $\mathrm{Rep}(\mathcal{K}, \alpha) = \emptyset$ (failure),
- if $\mathcal{K} \in \mathrm{Rep}(\mathcal{K}, \alpha)$, then $\mathrm{ctr}(\mathcal{K}, \alpha) \equiv \mathcal{K}$ (vacuity),
- if $\alpha \equiv_{\mathcal{K}} \alpha'$, then $\mathrm{ctr}(\mathcal{K}, \alpha) \equiv \mathrm{ctr}(\mathcal{K}, \alpha')$ (preservation),
- if $\mathcal{K}' \in \mathrm{Con}(\mathcal{K})$ and $\mathcal{K}' \notin \mathrm{Con}(\mathrm{ctr}(\mathcal{K}, \alpha))$, then there is $\mathcal{K}''$ such that $\mathcal{K} \models_s \mathcal{K}'' \models \mathrm{ctr}(\mathcal{K}, \alpha)$, $\mathcal{K}'' \in \mathrm{Rep}(\mathcal{K}, \alpha)$, and $\mathcal{K}'' \oplus \mathcal{K}' \notin \mathrm{Rep}(\mathcal{K}, \alpha)$ (relevance).

MaxiChoice PPC operations also satisfy the postulate *fullness*, which is stronger than *relevance*:

- if $\mathcal{K}' \in \mathrm{Con}(\mathcal{K})$ and $\mathcal{K}' \notin \mathrm{Con}(\mathrm{ctr}(\mathcal{K}, \alpha))$, then $\mathrm{ctr}(\mathcal{K}, \alpha) \oplus \mathcal{K}' \notin \mathrm{Rep}(\mathcal{K}, \alpha)$ (fullness).

It is easy to see that, in the presence of *logical inclusion*, *success*, and *failure*, the postulate *fullness* implies *relevance*.

Proposition 7. *Let γ be a selection function. Then the PPC operation ctr_γ induced by γ satisfies the postulates* logical inclusion*,* success*,* failure*,* vacuity*,* preservation*, and* relevance*. If γ is such that $|\gamma(\mathrm{Orep}(\mathcal{K}, \alpha))| = 1$ for all repair requests α, then ctr_γ additionally satisfies* fullness*.*

The proof of this proposition is similar to standard proofs of such results from the belief change community, and in particular to the proof of the corresponding result in (Rienstra, Schon, and Staab 2020) for the special case of concept contraction in the DL $\mathcal{EL}$. It is nevertheless important to have a detailed proof of this proposition since one needs to check that such a proof also goes through under the sparse assumptions made by our framework. Such a detailed proof can be found in (Baader and Wassermann 2024).

The postulates *logical inclusion*, *success*, *vacuity*, and *preservation* are variants of the original AGM postulates for belief set contraction (Alchourrón, Gärdenfors, and Makinson 1985), but adapted to a setting where the belief set is represented by a KB $\mathcal{K}$ and the goal of the contraction may be different from getting rid of an unwanted consequence (see Example 5). In case the repair request α is itself a knowledge base, and $\mathrm{Rep}(\mathcal{K}, \alpha)$ consists of the KBs entailed by $\mathcal{K}$, but not entailing α, the AGM *recovery* postulate can be formulated in our setting as

- $\mathrm{Con}(\mathcal{K}) \subseteq \mathrm{Con}(ctr(\mathcal{K}, \alpha) \oplus \alpha)$ (recovery).

However, even in this restricted setting, it need not hold. It is replaced by *failure* and *relevance* (or *fullness* for the MaxiChoice case), which are adaptations of postulates employed in the belief base setting (Hansson 1992). For the simple instance of our setup introduced in Example 1, *recovery* does actually hold. In the setting of Example 5, writing $\mathrm{ctr}_\gamma(\mathcal{K}, \alpha) \oplus \alpha$ does not even make sense since α is not a KB. An instance where formulating *recovery* make sense, but nevertheless *recovery* fails, is concept contraction in the DL $\mathcal{EL}$ (Rienstra, Schon, and Staab 2020).

Characterization theorem

We now show that, modulo equivalence, the converse of Proposition 7 holds as well. We say that two contraction operations ctr and ctr′ are equivalent if $\mathrm{ctr}(\mathcal{K}, \alpha) \equiv \mathrm{ctr}'(\mathcal{K}, \alpha)$ holds for all KBs $\mathcal{K}$ and repair requests α. The following theorem states this result simultaneously for the general and the MaxiChoice setting.

Theorem 8. *Assume that $\models$ is PPC enabling, and let* ctr *be an operation that receives as input a KB and a repair request, and returns as output a KB. Then the following are equivalent:*

1. *The operation* ctr *satisfies* logical inclusion*,* success*,* failure*,* vacuity*,* preservation*, and* relevance *(*fullness*).*
2. *The operation* ctr *is equivalent to a (MaxiChoice) PPC operation.*

Proof. (sketch) The implication "2 ⇒ 1" is an immediate consequence of Proposition 7.

To prove "1 ⇒ 2," we first consider the MaxiChoice case. Thus, assume that ctr satisfies the postulates *logical inclusion*, *success*, *failure*, *vacuity*, *preservation*, and *fullness*. To show that ctr is a MaxiChoice PPC operation, we define an appropriate selection function. For a KB $\mathcal{K}$ and repair request α, we set

$$\gamma(\mathrm{Orep}(\mathcal{K}, \alpha)) := \begin{cases} \{\mathcal{K}'\} & \text{if there is } \mathcal{K}' \in \mathrm{Orep}(\mathcal{K}, \alpha) \\ & \text{such that } \mathcal{K}' \equiv \mathrm{ctr}(\mathcal{K}, \alpha), \\ \{\mathcal{K}\} & \text{otherwise.} \end{cases}$$

It is shown in (Baader and Wassermann 2024) that this definition yields a well-defined selection function γ satisfying $|\gamma(\mathrm{Orep}(\mathcal{K}, \alpha))| = 1$ and $\mathrm{ctr} \equiv \mathrm{ctr}_\gamma$.

For the general case, we assume that ctr satisfies the postulates *logical inclusion*, *success*, *failure*, *vacuity*, *preservation*, and *relevance*. To show that ctr is a PPC operation, we again define an appropriate selection function. For a KB $\mathcal{K}$ and repair request α, we set

$$\gamma(\mathrm{Orep}(\mathcal{K}, \alpha)) := \begin{cases} \{\mathcal{K}' \in \mathrm{Orep}(\mathcal{K}, \alpha) \mid \mathcal{K}' \models \mathrm{ctr}(\mathcal{K}, \alpha)\} \\ \qquad \text{if } \mathrm{Orep}(\mathcal{K}, \alpha) \neq \emptyset, \\ \{\mathcal{K}\} \quad \text{otherwise.} \end{cases}$$

The proof that this definition yields a well-defined selection function γ satisfying $\mathrm{ctr} \equiv \mathrm{ctr}_\gamma$ can again be found in (Baader and Wassermann 2024). □

4 Instances of the General Setup

First, we show that under weak assumptions on the underlying logic, partial meet contractions for belief sets are an instance of our framework. Then, we consider instances with finite KBs defined using the DL $\mathcal{EL}$. The main result in each of the following subsections is that the entailment relation under consideration is PPC enabling, and thus Theorem 8 applies.

4.1 Belief Set Contraction as Instance

Contraction operations and in particular partial meet contractions were introduced in the seminal AGM paper (Alchourrón, Gärdenfors, and Makinson 1985) for belief sets, i.e., sets of formulas that are closed under the inference relation of an underlying logic. We show that this can be seen as an instance of the approach introduced in this paper. However, the instance we investigate here is more general than the original AGM setting (Alchourrón, Gärdenfors, and Makinson 1985) since we make less assumptions on the underlying logic.

We assume that we are given a set of formulas $\mathcal{F}$ (without any assumptions on their syntactic form) and a *closure operator* Cl mapping sets of formulas to sets of formulas (which generalizes inference closure w.r.t. some logic). A *belief set* $\mathcal{B}$ is a *closed* subset of $\mathcal{F}$, i.e. $\mathrm{Cl}(\mathcal{B}) = \mathcal{B} \subseteq \mathcal{F}$. The closure operator is assumed to satisfy the following properties (for all $\mathcal{A}, \mathcal{A}' \subseteq \mathcal{F}$):

- $\mathcal{A} \subseteq \mathrm{Cl}(\mathcal{A})$ (inclusion),
- $\mathcal{A} \subseteq \mathcal{A}'$ implies $\mathrm{Cl}(\mathcal{A}) \subseteq \mathrm{Cl}(\mathcal{A}')$ (monotonicity),
- $\mathrm{Cl}(\mathrm{Cl}(\mathcal{A})) = \mathrm{Cl}(\mathcal{A})$ (idempotency),
- $\varphi \in \mathrm{Cl}(\mathcal{A})$ implies that there is a *finite* set $\mathcal{E} \subseteq \mathcal{A}$ such that $\varphi \in \mathrm{Cl}(\mathcal{E})$ (compactness).

The first three properties imply that, for every set of formulas $\mathcal{A}$, its closure $\mathrm{Cl}(\mathcal{A})$ is the least belief set containing $\mathcal{A}$. These are exactly the conditions needed for a closure operator to be compliant with the relevance postulate (Ribeiro et al. 2013), and they are satisfied by Tarskian logics (Flouris 2006; Falakh, Rudolph, and Sauerwald 2022). We use $\mathcal{F}$ and a closure operator Cl satisfying inclusion, monotonicity, idempotency, and compactness to define the following instance of our general framework:

- *Knowledge bases* are belief sets, i.e., subsets of $\mathcal{F}$ that are closed under Cl.
- *Entailment* is the superset relation between belief sets, i.e., $\mathcal{B}_1$ entails $\mathcal{B}_2$ (written $\mathcal{B}_1 \models_\supseteq \mathcal{B}_2$) if $\mathcal{B}_1 \supseteq \mathcal{B}_2$.
- *Repair requests* are formulas $\varphi \in \mathcal{F}$, and they induce the repair sets $\mathrm{Rep}(\mathcal{B}, \varphi) := \{\mathcal{B}' \mid \mathcal{B} \supseteq \mathcal{B}' \text{ and } \varphi \notin \mathcal{B}'\}$.

Note that the consequence operator $\mathrm{Con}^{\supseteq}$ induced by $\models_\supseteq$ does not coincide with Cl. The operator Cl applies to arbitrary sets of formulas and defines what we consider to be KBs (i.e., sets that are closed under Cl). The operator $\mathrm{Con}^{\supseteq}$ applies to KBs and yields all KBs that are subsets of its input KB. Since the superset relation is *reflexive* and *transitive*, the entailment relation $\models_\supseteq$ satisfies these two properties required by our framework. The repair operator Rep satisfies the first condition of Definition 4 by definition and the second one since $\varphi \notin \mathcal{B}' \supseteq \mathcal{B}''$ clearly implies $\varphi \notin \mathcal{B}''$.

As sum operation on belief sets we define $\mathcal{B}_1 \oplus \ldots \oplus \mathcal{B}_n := \mathrm{Cl}(\mathcal{B}_1 \cup \ldots \cup \mathcal{B}_n)$ and as product operation on belief sets we take intersection, i.e., $\mathcal{B}_1 \otimes \ldots \otimes \mathcal{B}_n := \mathcal{B}_1 \cap \ldots \cap \mathcal{B}_n$.

Lemma 9. *The operation $\oplus$ ($\otimes$) on belief sets satisfies the properties of sum (product) for the entailment relation $\models_\supseteq$.*

Regarding repairs, given a belief set $\mathcal{B}$ and a repair request φ, we define $\mathrm{Orep}(\mathcal{B}, \varphi)$ to consist of the maximal subsets of $\mathcal{B}$ whose closure does not contain φ.

Lemma 10. *The set $\mathrm{Orep}(\mathcal{B}, \varphi)$ consists of belief sets that are optimal repairs of $\mathcal{B}$ for φ, and it covers all repairs of $\mathcal{B}$ for φ.*

Proofs of these two lemmas can be found in (Baader and Wassermann 2024). For Lemma 10, this proof is similar to standard proofs in the belief change literature, but again one must check in detail whether it works under the assumptions made here.

Summing up, we have thus shown that the entailment relation $\models_\supseteq$ on belief sets induced by a closure operator Cl that satisfies the conditions introduced above fulfills all the properties introduced in Section 2.

Theorem 11. *Consider as knowledge bases belief sets that are closed w.r.t. a closure operator* Cl *that satisfies inclusion, monotonicity, idempotency, and compactness, and as repair requests single formulas with associated repair sets of the form* $\mathrm{Rep}(\mathcal{B}, \varphi) := \{\mathcal{B}' \mid \mathcal{B} \supseteq \mathcal{B}' \text{ and } \varphi \notin \mathcal{B}'\}$. *Then the entailment relation $\models_\supseteq$ corresponding to the superset relation between belief sets is PPC enabling.*

As a consequence, we can use the PPC approach introduced in Section 3 to obtain contraction operations for belief sets that satisfy the postulates *logical inclusion*, *success*, *failure*, *vacuity*, *preservation*, and *relevance* (and additionally *fullness* in the MaxiChoice case). Since in this case product is intersection and optimal repairs are obtained as maximal sets that do not have the consequence, the construction of partial product contractions as described in Section 3 coincides with the construction of the partial meet contractions for belief sets introduced in the seminal AGM paper. Nevertheless, our postulates do not coincide with the ones given there. In particular, instead of *recovery* we have *relevance* or *fullness*. The reason is that Alchourrón, Gärdenfors, and Makinson make additional assumptions on the formulas and the closure operator. Their proof of *recovery* actually employs the fact that their closure operator corresponds to logical consequence for a logic that has negation and disjunction. The setting introduced in this subsection does not make any assumptions on the formulas, and only requires the closure operator to satisfy inclusion, monotonicity, idempotency, and compactness. For example, we could use as formulas Horn implications or more generally concepts of the Description Logic $\mathcal{EL}$, and as closure operator logical consequence for Horn formulas or subsumption between $\mathcal{EL}$ concepts. In these setting, recovery does not hold (Delgrande and Wassermann 2013; Zhuang and Pagnucco 2009). Intuitionistic Logic (Heyting 1956) is another example where recovery does not hold (Ribeiro et al. 2013). A detailed study of the postulates recovery and relevance for logics that do not satisfy all the assumptions of the original AGM paper can be found in (Ribeiro et al. 2013)

4.2 Instances with Finite KBs

Considering belief sets as knowledge bases has the disadvantage that, for logics that are not fragments of propositional logic, belief sets may be infinite. This is unproblematic as long as such belief sets can be represented by a finite KB (i.e., are the closure of a finite set of formulas). However, there are cases where the optimal repairs, and thus also the belief sets produced by applying the contraction operator, may become infinite without appropriate finite representation, even if one starts with finitely generated belief sets (see, e.g., Proposition 2 in (Baader et al. 2018)).

As practically relevant instances of the general setup for which KBs are finite, we consider KBs and entailment relations connected with the DL $\mathcal{EL}$ (Baader et al. 2017). In this setting, when showing that a set of KBs, repair requests, and an entailment relation satisfy the properties required for the entailment relation to be PPC enabling, the most challenging task is to prove that the properties related to optimal repairs are satisfied. Fortunately, in most of the cases considered below, this task has already been solved by recent work on optimal repairs in $\mathcal{EL}$. Nevertheless, the overall task of showing that the considered entailment relations are PPC enabling remains non-trivial since we must prove the existence of appropriate product and sum operations.

$\mathcal{EL}$ Concept Contraction and Forgetting In this setting, knowledge bases are $\mathcal{EL}$ concepts and entailment is subsumption w.r.t. an $\mathcal{EL}$ TBox (Baader et al. 2017).

$\mathcal{EL}$ concepts are built inductively, starting with concept names A from a set N_C of such names, and using the concept constructors $\top$ (top concept), $C \sqcap D$ (conjunction), and $\exists r.C$ (existential restriction), where C, D are $\mathcal{EL}$ concepts and r belongs to a set N_R of role names. A general concept inclusion (GCI) of $\mathcal{EL}$ is of the form $C \sqsubseteq D$ for $\mathcal{EL}$ concepts C, D, and an $\mathcal{EL}$ TBox is a finite set of such GCIs. Given an $\mathcal{EL}$ concept C, its signature $\mathrm{Sig}(C)$ consists of the concept and role names occurring in C.

The semantics of $\mathcal{EL}$ is defined in a model-theoretic way, using the notion of an *interpretation* $\mathcal{I}$, which is a pair

$\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where the domain $\Delta^{\mathcal{I}}$ is a non-empty set and the interpretation function $\cdot^{\mathcal{I}}$ maps each concept name $A \in N_C$ to $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ and each role name $r \in N_R$ to a binary relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. The interpretation of an $\mathcal{EL}$ concept is defined inductively as follows: $\top^{\mathcal{I}} := \Delta^{\mathcal{I}}$, $(C \sqcap D)^{\mathcal{I}} := C^{\mathcal{I}} \cap D^{\mathcal{I}}$, and $(\exists r.C)^{\mathcal{I}} := \{d \in \Delta^{\mathcal{I}} \mid \exists e \in \Delta^{\mathcal{I}}$ such that $(d,e) \in r^{\mathcal{I}}$ and $e \in C^{\mathcal{I}}\}$. A model $\mathcal{I}$ of the $\mathcal{EL}$ TBox $\mathcal{T}$ is an interpretation that satisfies all its GCIs, i.e., $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ holds for all $C \sqsubseteq D \in \mathcal{T}$. Given $\mathcal{EL}$ concepts C, D and an $\mathcal{EL}$ TBox $\mathcal{T}$, we say that C is *subsumed by* D *w.r.t.* $\mathcal{T}$ (and write $C \sqsubseteq^{\mathcal{T}} D$) if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ holds in all models $\mathcal{I}$ of $\mathcal{T}$. The $\mathcal{EL}$ concepts C, D are equivalent (written $C \equiv^{\mathcal{T}} D$) if $C \sqsubseteq^{\mathcal{T}} D$ and $D \sqsubseteq^{\mathcal{T}} C$.

For a given $\mathcal{EL}$ TBox $\mathcal{T}$, we obtain the following *two instances* of our general framework:

- *Knowledge bases* are $\mathcal{EL}$ concepts.
- *Entailment* is given by the subsumption relation w.r.t. $\mathcal{T}$, i.e., C entails D (written $C \models_{\sqsubseteq^{\mathcal{T}}} D$) iff $C \sqsubseteq^{\mathcal{T}} D$.
- *Repair requests* are $\mathcal{EL}$ concepts, and repairs are defined as $\mathrm{Rep}^{\mathcal{T}}_{\text{ent}}(C, D) := \{C' \mid C \sqsubseteq^{\mathcal{T}} C', C' \not\sqsubseteq^{\mathcal{T}} D\}$, or
- *Repair requests* are finite sets of concept and role names, and repairs are defined as
$\mathrm{Rep}^{\mathcal{T}}_{\text{for}}(C, \alpha) := \{C' \mid C \sqsubseteq^{\mathcal{T}} C', \mathrm{Sig}(C') \cap \alpha = \emptyset\}$.

The *first instance* has first been considered in (Rienstra, Schon, and Staab 2020) for subsumption w.r.t. the empty TBox ($\sqsubseteq^{\emptyset}$) and was then extended to subsumption w.r.t. a cycle-restricted $\mathcal{EL}$ TBox $\mathcal{T}$ ($\sqsubseteq^{\mathcal{T}}$) in (Baader 2023).

It is easy to see that the sum operation for the entailment relation $\models_{\sqsubseteq^{\mathcal{T}}}$ is conjunction of concepts, and the product is the least common subsumer (lcs) w.r.t. the TBox $\mathcal{T}$:

- the $\mathcal{EL}$ concept C is a *least common subsumer* of the $\mathcal{EL}$ concepts $C_1, \dots, C_n$ w.r.t. $\mathcal{T}$ if $C_i \sqsubseteq^{\mathcal{T}} C$ for all $i = 1, \dots, n$, and C is the least $\mathcal{EL}$ concept (for $\sqsubseteq^{\mathcal{T}}$) with this property, i.e., if D is an $\mathcal{EL}$ concept satisfying $C_i \sqsubseteq^{\mathcal{T}} D$ for all $i = 1, \dots, n$, then $C \sqsubseteq^{\mathcal{T}} D$.

Obviously, if it exists, then such an lcs is unique up to equivalence $\equiv^{\mathcal{T}}$. For the case of the empty TBox, the lcs in $\mathcal{EL}$ always exists (Baader, Küsters, and Molitor 1999), but this is not the case w.r.t. an arbitrary $\mathcal{EL}$ TBox. The characterization of the existence of the lcs w.r.t. an $\mathcal{EL}$ TBox given in (Zarrieß and Turhan 2013) implies that the lcs always exists for cycle-restricted TBoxes:

- The $\mathcal{EL}$ TBox $\mathcal{T}$ is *cycle-restricted* if there is no $\mathcal{EL}$ concept C and $m \geq 1$ (not necessarily distinct) role names $r_1, \dots, r_m$ such that $C \sqsubseteq^{\mathcal{T}} \exists r_1. \cdots \exists r_m.C$.

As stated in (Baader, Borgwardt, and Morawska 2012), it can be decided in polynomial time whether a given $\mathcal{EL}$ TBox is cycle-restricted or not. Cycle-restrictedness is also required to obtain the necessary repair properties. As explained in more detail in (Baader 2023), satisfaction of these properties is an easy consequence of the results on optimal ABox repairs shown in (Baader et al. 2022).

Theorem 12. *Let $\mathcal{T}$ be a cycle-restricted $\mathcal{EL}$ TBox and $\models_{\sqsubseteq^{\mathcal{T}}}$ subsumption w.r.t. $\mathcal{T}$ between $\mathcal{EL}$ concepts, and consider $\mathcal{EL}$ concepts as repair requests inducing repair sets defined as $\mathrm{Rep}^{\mathcal{T}}_{ent}(C, D) := \{C' \mid C \sqsubseteq^{\mathcal{T}} C', C' \not\sqsubseteq^{\mathcal{T}} D\}$. Then $\models_{\sqsubseteq^{\mathcal{T}}}$ is PPC enabling.*

The *second instance* has as repair goal the removal of concepts and role names, which is usually called *forgetting*. In the DL literature, different versions of forgetting have been investigated (see, e.g., (Konev, Walther, and Wolter 2009; Lutz and Wolter 2011; Ludwig and Konev 2014; Koopmann and Schmidt 2015; Sakr and Schmidt 2021), but the variant considered here seems to be new.

To ensure that the second condition of Definition 4 is satisfied, we must impose an additional restriction on repair requests: α must be compatible with $\mathcal{T}$. A finite set α of concept and role names is *compatible with* $\mathcal{T}$ if $\mathrm{Sig}(E) \cap \alpha = \emptyset$ implies $\mathrm{Sig}(F) \cap \alpha = \emptyset$ for all GCIs $E \sqsubseteq F$ in $\mathcal{T}$. The following lemma shows that, with this additional restriction, the second condition of Definition 4 is indeed satisfied.

Lemma 13. *Let α be a repair request and D an $\mathcal{EL}$ concept with $\mathrm{Sig}(D) \cap \alpha = \emptyset$. If $D \sqsubseteq^{\mathcal{T}} D'$, then $\mathrm{Sig}(D') \cap \alpha = \emptyset$.*

By adapting results for entailment between qABoxes from (Baader et al. 2021a) (see (Baader and Wassermann 2024) and the next subsection), we obtain the following characterization of subsumption w.r.t. a cycle-restricted $\mathcal{EL}$ TBox.

Lemma 14. *Let $\mathcal{T}$ be a cycle-restricted $\mathcal{EL}$ TBox and C an $\mathcal{EL}$ concept. Then one can compute in at most exponential time an $\mathcal{EL}$ concept $\mathrm{sat}^{\mathcal{T}}(C)$ such that $C \sqsubseteq^{\mathcal{T}} D$ iff $\mathrm{sat}^{\mathcal{T}}(C) \sqsubseteq^{\emptyset} D$ holds for all $\mathcal{EL}$ concepts D.*

We have already seen above that $\models_{\sqsubseteq^{\mathcal{T}}}$ has products and sums. Thus, it remains to prove that the optimal repair property is satisfied as well. Given a cycle-restricted $\mathcal{EL}$ TBox $\mathcal{T}$, an $\mathcal{EL}$ concept C, and a finite set α of concept and role names, we first saturate C w.r.t. $\mathcal{T}$, i.e., compute the concept $\mathrm{sat}^{\mathcal{T}}(C)$. Then we remove from $\mathrm{sat}^{\mathcal{T}}(C)$ all concept names occurring in α and all existential restrictions of the form $\exists r.E$ for $r \in \alpha$. We denote the resulting concept as $\mathrm{sat}^{\mathcal{T}}(C)^{-\alpha}$ and set $\mathrm{Orep}(C, \alpha) := \{\mathrm{sat}^{\mathcal{T}}(C)^{-\alpha}\}$.

Example 15. *Let $\mathcal{T} := \{A \sqsubseteq B \sqcap \exists r.B\}$, $C := A$, and $\alpha := \{A, r\}$. Then α is compatible with $\mathcal{T}$, and $\mathrm{sat}^{\mathcal{T}}(C) = A \sqcap B \sqcap \exists r.B$. Removing A and $\exists r.B$ from this concept yields $\mathrm{sat}^{\mathcal{T}}(C)^{-\alpha} = B$, and thus $\mathrm{Orep}(C, \alpha) = \{B\}$.*

That fact that $\mathrm{Orep}(C, \alpha)$ consists of optimal repairs and covers all repairs is an immediate consequence of the following lemma, whose proof can be found in (Baader and Wassermann 2024).

Lemma 16. *The concept $\mathrm{sat}^{\mathcal{T}}(C)^{-\alpha}$ is a repair of C for α that entails every repair of C for α.*

Summing up, we have thus shown that the entailment relation $\models_{\sqsubseteq^{\mathcal{T}}}$ satisfies also all the properties introduced in Section 2 if we use $\mathrm{Rep}_{\text{for}}$ to construct repair sets.

Theorem 17. *Let $\models_{\sqsubseteq^{\mathcal{T}}}$ be subsumption w.r.t. a cycle-restricted $\mathcal{EL}$ TBox $\mathcal{T}$, and consider as repair requests finite sets of concept and role names that are compatible with $\mathcal{T}$ and induce repair sets defined as $\mathrm{Rep}_{for}(C, \alpha) := \{D \mid C \sqsubseteq^{\mathcal{T}} D$ and $\mathrm{Sig}(D) \cap \alpha = \emptyset\}$. Then $\models_{\sqsubseteq^{\mathcal{T}}}$ is PPC enabling.*

Contractions for Quantified ABoxes ABoxes of $\mathcal{EL}$ are finite sets of concept assertions $C(a)$ and role assertions $r(a,b)$, where C is an $\mathcal{EL}$ concept, r a role name, and a, b are individuals from a set N_I. In the presence of an ABox, an interpretation $\mathcal{I}$ additionally interprets individuals a as elements $a^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$. The interpretation $\mathcal{I}$ is a model of the $\mathcal{EL}$ ABox $\mathcal{A}$ if $a^{\mathcal{I}} \in C^{\mathcal{I}}$ and $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$ respectively holds for all concept and role assertions $C(a)$ and $r(a,b)$ in $\mathcal{A}$.

Quantified ABoxes were first introduced in (Baader et al. 2020) since they allow for the existence of optimal repairs in situations where this would not be the case if only ABoxes were used. Basically, they are variants of ABoxes where some of the individual names are assumed to be anonymous, which we express by writing them as existentially quantified variables. More formally, a *quantified ABox (qABox)* $\exists X.\mathcal{A}$ consists of a finite set X of variables, which is disjoint with N_I, and a matrix $\mathcal{A}$, which is a finite set of concept assertions $A(u)$ and role assertions $r(u,v)$, where $A \in N_C$, $r \in N_R$ and $u, v \in N_I \cup X$. Thus, the matrix is an ABox built using the extended set of individuals $N_I \cup X$, but cannot contain complex concept descriptions. Semantically, the latter is not a restriction since it is easy to see that a concept assertions $C(a)$ for a complex $\mathcal{EL}$ concept C can be expressed by a qABox.

The interpretation $\mathcal{I}$ is a model of the qABox $\exists X.\mathcal{A}$ if there is a variable assignment $\mathcal{Z} : X \to \Delta^{\mathcal{I}}$ such that the augmented interpretation $\mathcal{I}[\mathcal{Z}]$ that additionally maps each variable x to $\mathcal{Z}(x)$ is a model of the matrix $\mathcal{A}$, i.e, $u^{\mathcal{I}[\mathcal{Z}]} \in A^{\mathcal{I}}$ for each $A(u) \in \mathcal{A}$ and $(u^{\mathcal{I}[\mathcal{Z}]}, v^{\mathcal{I},\mathcal{Z}}) \in r^{\mathcal{I}}$ for each $r(u,v) \in \mathcal{A}$. The qABox $\exists X.\mathcal{A}$ entails the qABox $\exists Y.\mathcal{B}$ w.r.t. the $\mathcal{EL}$ TBox $\mathcal{T}$ (written $\exists X.\mathcal{A} \models^{\mathcal{T}} \exists Y.\mathcal{B}$) if every model of $\exists X.\mathcal{A}$ and $\mathcal{T}$ is also a model of $\exists Y.\mathcal{B}$. Note that this also defines entailment of a concept assertion $C(a)$ by a qABox w.r.t. an $\mathcal{EL}$ TBox since $C(a)$ can be expressed by a qABox. For the empty TBox, we write the entailment relation as $\models$ rather than $\models^{\emptyset}$.

The entailment relation $\models$ between qABoxes can be characterized using the notion of a homomorphism. Given qABoxes $\exists X.\mathcal{A}$ and $\exists Y.\mathcal{B}$, a *homomorphism* from $\exists X.\mathcal{A}$ to $\exists Y.\mathcal{B}$ is a mapping h from the objects (i.e., variables or individuals) of $\mathcal{A}$ to the objects of $\mathcal{B}$ such that

- $h(a) = a$ for all individuals a,
- $A(u) \in \mathcal{A}$ implies $A(h(u)) \in \mathcal{B}$,
- $r(u,v) \in \mathcal{A}$ implies $r(h(u), h(v)) \in \mathcal{B}$.

The following characterization of entailment was shown in (Baader et al. 2020): $\exists Y.\mathcal{B} \models \exists X.\mathcal{A}$ iff there is a homomorphism from $\exists X.\mathcal{A}$ to $\exists Y.\mathcal{B}$.[1] This characterization also works in the setting with a background TBox $\mathcal{T}$ if one first saturates the qABox $\exists Y.\mathcal{B}$ w.r.t. $\mathcal{T}$. However, a finite saturation only exists if the TBox is *cycle-restricted*. Given a qABox $\exists Y.\mathcal{B}$ and a cycle-restricted TBox $\mathcal{T}$, one can compute the saturation $\mathrm{sat}^{\mathcal{T}}(\exists Y.\mathcal{B})$ of $\exists Y.\mathcal{B}$ w.r.t. $\mathcal{T}$ in exponential time, and this saturation satisfies $\exists Y.\mathcal{B} \models^{\mathcal{T}} \exists X.\mathcal{A}$ iff $\mathrm{sat}^{\mathcal{T}}(\exists Y.\mathcal{B}) \models \exists X.\mathcal{A}$ for each qABox $\exists X.\mathcal{A}$ (Baader et al. 2021a). Thus, we have the following characterization of entailment w.r.t. a cycle-restricted TBox.

Lemma 18. *Let $\exists X.\mathcal{A}, \exists Y.\mathcal{B}$ be qABoxes, and $\mathcal{T}$ a cycle-restricted $\mathcal{EL}$ TBox. Then the following are equivalent:*

- $\exists Y.\mathcal{B} \models^{\mathcal{T}} \exists X.\mathcal{A}$,
- $\mathrm{sat}^{\mathcal{T}}(\exists Y.\mathcal{B}) \models \exists X.\mathcal{A}$,
- *there is a homomorphism from $\exists X.\mathcal{A}$ to $\mathrm{sat}^{\mathcal{T}}(\exists Y.\mathcal{B})$.*

The saturation of a qABox is of at most exponential size, and there are examples showing that this size-bound is tight (see Example III in (Baader et al. 2021b)). Nevertheless, as pointed out in (Baader et al. 2021a), deciding the entailment relation $\models^{\mathcal{T}}$ is an NP-complete problem (where hardness already holds without TBox).

In the following, we use qABoxes as KBs, $\models^{\mathcal{T}}$ for a cycle-restricted TBox $\mathcal{T}$ as entailment, and finite sets of $\mathcal{EL}$ concept assertions as repair requests. Since repair requests are not single concept assertions, but sets of them, there are (at least) two options for how to define repairs, corresponding to choice and package contraction in the belief change literature (Fuhrmann and Hansson 1994; Fermé, Saez, and Sanz 2003; Resina, Ribeiro, and Wassermann 2014). *Package repairs* are defined as $\mathrm{Rep}^{\mathcal{T}}_p(\exists X.\mathcal{A}, \alpha) := \{\exists Y.\mathcal{B} \mid \exists X.\mathcal{A} \models^{\mathcal{T}} \exists Y.\mathcal{B},\ \exists Y.\mathcal{B} \not\models^{\mathcal{T}} C(a) \text{ for all } C(a) \in \alpha\}$, i.e., all the assertions of α must be removed from the consequence set. For a *choice repair*, it is sufficient to remove at least one element of α, i.e., $\mathrm{Rep}^{\mathcal{T}}_c(\exists X.\mathcal{A}, \alpha) := \{\exists Y.\mathcal{B} \mid \exists X.\mathcal{A} \models^{\mathcal{T}} \exists Y.\mathcal{B},\ \exists Y.\mathcal{B} \not\models^{\mathcal{T}} C(a) \text{ for some } C(a) \in \alpha\}$. We show that in both cases we obtain an entailment relation such that all the properties introduced in Section 2 are satisfied, i.e., we show that $\models^{\mathcal{T}}$ is PPC enabling.

Reflexivity and transitivity of $\models^{\mathcal{T}}$ are obvious. Next, we introduce an appropriate sum operation. For a singleton set $\mathfrak{K} = \{\exists X.\mathcal{A}\}$, its sum is simply $\exists X.\mathcal{A}$ itself. Given a set of $n \geq 2$ qABoxes $\mathfrak{K} = \{\exists X_1.\mathcal{A}_1, \ldots, \exists X_n.\mathcal{A}_n\}$, we construct its disjoint union as follows: we first rename the qABoxes in $\mathfrak{K}$ into equivalent ones $\exists X'_1.\mathcal{A}'_1, \ldots, \exists X'_n.\mathcal{A}'_n$ with pairwise disjoint sets of variables $X'_1, \ldots, X'_n$, and then set $\uplus\mathfrak{K} := \exists(X'_1 \cup \ldots \cup X'_n).(\mathcal{A}'_1 \cup \ldots \cup \mathcal{A}'_n)$.

Lemma 19. *Disjoint union $\uplus$ of qABoxes satisfies the properties of sum for $\models^{\mathcal{T}}$.*

The product of a set of qABoxes $\mathfrak{K} = \{\exists X_1.\mathcal{A}_1, \ldots, \exists X_n.\mathcal{A}_n\}$ is $\exists X_1.\mathcal{A}_1$ if $n = 1$. For $n \geq 2$, we consider the saturations $\exists Y_1.\mathcal{B}_1 := \mathrm{sat}^{\mathcal{T}}(\exists X_1.\mathcal{A}_1), \ldots, \exists Y_n.\mathcal{B}_n := \mathrm{sat}^{\mathcal{T}}(\exists X_n.\mathcal{A}_n)$ of $\exists X_1.\mathcal{A}_1, \ldots, \exists X_n.\mathcal{A}_n$. Let Ind be the set of individuals occurring in at least one of the ABoxes $\mathcal{B}_1, \ldots, \mathcal{B}_n$ and $\mathrm{Obj}_i := Y_i \cup \mathrm{Ind}$ for $i = 1, \ldots, n$. We set $\mathrm{Ind}^{\times} := \{(a, \ldots, a) \mid a \in \mathrm{Ind}\}$ and $Y := \mathrm{Obj}_1 \times \ldots \times \mathrm{Obj}_n \setminus \mathrm{Ind}^{\times}$, and define $\otimes\mathfrak{K} := \exists Y.\mathcal{B}$ where

$$\begin{aligned}\mathcal{B} := \{A(u_1, \ldots, u_n) \mid A(u_i) \in \mathcal{B}_i \quad & \text{for } i = 1, \ldots, n\} \cup \\ \{r((u_1, \ldots, u_n), (v_1, \ldots, v_n)) \mid\ & r(u_i, v_i) \in \mathcal{B}_i \\ & \text{for } i = 1, \ldots, n\}.\end{aligned}$$

In this qABox, each tuple $(a, \ldots, a) \in \mathrm{Ind}^{\times}$ is viewed as representing the individual $a \in \mathrm{Ind}$.

Lemma 20. *The product $\otimes$ of qABoxes satisfies the properties of product for $\models^{\mathcal{T}}$.*

[1]Note that checking for the existence of homomorphisms between qABoxes is an NP-complete problem (Baader et al. 2020).

Proofs of these two lemmas can be found in (Baader and Wassermann 2024). They make heavy use of the homomorphism characterization of entailment.

Package repairs of qABoxes w.r.t. cycle-restricted TBoxes for repair requests given as finite sets of $\mathcal{EL}$ concept assertions have been investigated in (Baader et al. 2021a). It is shown there that, up to equivalence, the set of all optimal repairs of a qABox for a repair request w.r.t. a cycle-restricted TBox can be computed in exponential time using an NP oracle (Theorem 9 in (Baader et al. 2021a)). To be more precise, the paper introduces the notion of canonical repairs induced by repair seed functions. There are at most exponentially many such canonical repairs, each of which is of at most exponential size. These canonical repairs are indeed repairs, and the set of canonical repairs covers all repairs (see Proposition 8 in (Baader et al. 2021a)). As a consequence, up to equivalence, this set contains all optimal repairs, which can be obtained by removing elements that are strictly entailed by another element.[2] The coverage property for the obtained set of optimal repairs $\mathrm{Orep}_p^{\mathcal{T}}(\exists X.\mathcal{A}, \alpha)$ is then an easy consequence of the coverage property for the set of canonical repairs. Summing up, we have thus shown for the package repair case that $\models^{\mathcal{T}}$ for a cycle-restricted TBox $\mathcal{T}$ as entailment satisfies all the properties introduced in Section 2.

Theorem 21. *Let $\mathcal{T}$ be a cycle-restricted TBox and $\models^{\mathcal{T}}$ entailment w.r.t. $\mathcal{T}$ between qABoxes, and consider as repair requests finite sets of $\mathcal{EL}$ concept assertions inducing repair sets according to the package approach. Then $\models^{\mathcal{T}}$ is PPC enabling.*

The same result holds if we use the choice approach for defining repairs. To show this we must demonstrate that the optimal repair property is satisfied in this setting. Given a qABox $\exists X.\mathcal{A}$ and a repair request $\alpha = \{C_1(a_1), \ldots, C_n(a_n)\}$, we consider the union of the sets $\mathrm{Orep}_p^{\mathcal{T}}(\exists X.\mathcal{A}, \{C_i(a_i)\})$. It is easy to see that this set covers $\mathrm{Rep}_c^{\mathcal{T}}(\exists X.\mathcal{A}, \alpha)$. Thus, the set of all optimal repairs in the choice setting is obtained by removing elements that are strictly entailed by another elements.

Corollary 22. *Let $\mathcal{T}$ be a cycle-restricted TBox and $\models^{\mathcal{T}}$ entailment w.r.t. $\mathcal{T}$ between qABoxes, and consider as repair requests finite sets of $\mathcal{EL}$ concept assertions inducing repair sets according to the choice approach. Then $\models^{\mathcal{T}}$ is PPC enabling.*

As a consequence, in both the package and the choice setting, we can use the PPC approach to obtain contraction operations for qABoxes w.r.t. cycle-restricted TBoxes that satisfy the postulates *logical inclusion*, *success*, *failure*, *vacuity*, *preservation*, and *relevance* (and additionally *fullness* in the MaxiChoice case).

Additional Instances In (Baader and Wassermann 2024), we describe three additional instances of our framework.

First, we consider a variant of contraction for qABoxes where classical entailment is replaced with IQ-entailment. In fact, if one is only interested in answering instance queries (i.e., checking which concept assertions a qABox entails), then it makes sense to compare qABoxes w.r.t. the instance relationships they entail rather than w.r.t. the models they have or (equivalently) w.r.t. the conjunctive queries they entail (as classical entailment does) (Baader, Koopmann, and Kriegel 2023). We say that $\exists X.\mathcal{A}$ IQ-*entails* $\exists Y.\mathcal{B}$ w.r.t. the TBox $\mathcal{T}$ (written $\exists X.\mathcal{A} \models^{\mathcal{T}}_{\mathsf{IQ}} \exists Y.\mathcal{B}$) if every concept assertion entailed by $\exists Y.\mathcal{B}$ w.r.t. $\mathcal{T}$ is also entailed by $\exists X.\mathcal{A}$ w.r.t. $\mathcal{T}$. IQ-entailment has the advantage that it can be decided in polynomial time and that the restriction to cycle-restricted TBoxes can be dispensed with since finite IQ-saturations always exist. In the IQ-setting, the product can be defined as in the case of classical entailment. However, the sum is not simply disjoint union, it requires a more sophisticated construction. Using results on how to compute optimal IQ-repairs (Baader et al. 2021a), we can show that IQ-entailment is PPC enabling. This holds both for the package and the choice approach for defining repairs and does not require any restriction on the $\mathcal{EL}$ TBox.

Second, following (Kriegel 2022), we consider contraction and repair for $\mathcal{EL}$ TBoxes, where the goal is to get rid of an entailed GCI, but keep the left-hand sides of GCIs in the TBox intact in the repair process. Interestingly, this requires the use of two different entailment relations, one for comparing TBoxes and one for defining the repair goal. This situation can nicely by covered by our general notion of repair requests and the repairs induced by them.

Third, we illustrate the generality of our approach by considering a setting where KBs define formal languages and entailment corresponds to the superset relation between languages. The repair request is then a finite language α, and the repair goal is to remove at least one element (choice approach) or all elements (package approach) of α. We show that the superset relation is PPC enabling if KBs are finite automata, linear bounded automata, or Turing machines. However, if we use context-free grammars instead, then this entailment relation is not PPC enabling since the product need not exist.

5 Conclusion

We have shown that the partial meet contraction approach can be generalized to the setting of a reflexive and transitive entailment relation between KBs with associated sum and product operations generalizing conjunction and disjunction. The main novelty of the approach is that we employ optimal repairs in place of remainders. Under the additional assumption that the optimal repairs cover all repairs, we were able to prove a characterization theorem linking the obtained contraction operations, called partial product contraction (PPC) operations, with reasonable postulates, both for the MaxiChoice and the general case. In contrast to belief base contractions, our PPC operations are syntax-independent and usually preserve more consequences. Though PPC operations can express belief set contractions, they also work in settings where finite KBs generating the belief sets are required. In these settings, the main challenge is usually to show that the required repair properties are satisfied. In Section 4.2 we were able to use recent results on optimal repairs for the DL $\mathcal{EL}$ to obtain instances of our approach that are relevant for ontology engineering.

[2]The NP oracle is used to realize these entailment tests.

A second important novelty of our approach is that it generalizes the notion of contraction and repair towards repair goals different from non-entailment of a certain formula or knowledge base. This allows us, for instance, to treat different approaches to multiple contraction, such a choice and package contraction, in a uniform way. Additionally, we have shown that certain notions of variable forgetting in propositional logic (see Example 5) and concept and role forgetting in DLs (see the corresponding subsection in Section 4.2) can be seen as instances of our approach, and thus satisfy the same postulates as the more standard contraction approaches that have non-entailment as a goal.

One interesting direction for future research is to identify instances of our approach also for other logics, or for repair goals other than non-entailment or signature forgetting. Another is to determine whether other contraction approaches, such as kernel contractions (Hansson 1994), can be generalized in a similar way. Finally, the relationship to previous work on forgetting, both in the DL community (Konev, Walther, and Wolter 2009; Lutz and Wolter 2011; Ludwig and Konev 2014; Koopmann and Schmidt 2015; Sakr and Schmidt 2021) and in the belief change community (Lang and Marquis 2010; Delgrande 2017; Kern-Isberner et al. 2019a; Kern-Isberner et al. 2019b) needs to be investigated in more detail.

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