The Unification Type of an Equational Theory May Depend on the Instantiation Preorder (Extended Abstract)

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Abstract

The unification type of an equational theory is defined using a preorder on substitutions, called the instantiation preorder, whose scope is either restricted to the variables occurring in the unification problem, or unrestricted such that all variables are considered. Most of the results on the unification type of equational theories were shown for the restricted setting. In this extended abstract, we recall our recent results on how the unification type may change when going from the restricted to the unrestricted setting.

1 Introduction

Syntactic unification of terms seems to have appeared first in the work of Herbrand [17]. It was later independently rediscovered by Robinson [33] and Knuth and Bendix [25] as a tool for computing resolvents in resolution-based theorem proving and critical pairs in the completion of term rewriting systems. Both showed the important result that any solvable unification problem has a most general unifier (mgu). In these papers, a substitution θ is defined to be an instance of a substitution σ if there is a substitution λ such that $\lambda(\sigma(x)) = \theta(x)$ holds for all variables x in the countably infinite set of variables V available for building terms. In [5, 6] and in this extended abstract, we call the preorder on substitutions obtained this way the *unrestricted instantiation preorder* and write it as $\sigma \leq_{\emptyset}^{V} \theta$, where the index $\emptyset$ indicates that terms are to be made syntactically equal.

In his seminal paper [31], Plotkin proposed to build certain equational theories (such as associativity or commutativity) into the unification algorithm rather than treating their axiomatization within the general theorem proving process. Similar proposals were also made in the setting of Knuth-Bendix completion in term rewriting [30, 22]. As already pointed out by Plotkin, in the equational setting most general unifiers need not exist and their rôle is instead taken on by minimal complete sets of unifiers, i.e., sets of unifiers such that every unifier is an instance of a unifier in this set, and no distinct elements in the set are comparable w.r.t. the instantiation preorder. As instantiation preorder, Plotkin uses what we call the *restricted instantiation preorder*, i.e., $\sigma \leq_{E}^{X} \theta$ where E is the theory modulo which unification is considered and X is the set of variables occurring in the unification problem. This preorder requires the existence of a substitution λ such that $\lambda(\sigma(x)) \approx_E \theta(x)$ holds for all variables $x \in X$. He explains the use of equality modulo E ($\approx_E$) in this definition, but does not comment on the restriction to the variables of the unification problem.

Siekmann proposed to characterize equational theories according to the cardinality and existence of minimal complete sets of unifiers into the types unitary, finitary, infinitary, and zero. In the first overview paper on results in this direction [32], he uses the unrestricted instantiation preorder, but in later overview papers [35, 36, 37] he switches to the restricted one in the formal definition of unification types, like Plotkin without explanation. Due to potential applications of equational unification in resolution-based theorem proving and term rewriting, unification properties (among them the unification type) of frequently encountered equational axioms such as associativity, commutative, idempotency, distributivity and their combinations were extensively studied in the automated deduction and term rewriting communities in the 1980s and 1990s (see [21, 12, 13] for overviews). The obtained results were shown for the restricted setting.

The first concrete indication that "bad things" may happen if one uses the unrestricted instantiation preorder was given in [3], where it was shown that the theory ACUI of an associative, commutative, and idempotent binary function symbol f with a unit 0, which is unitary for elementary unification w.r.t. the restricted instantiation preorder [4], is not unitary w.r.t. the unrestricted one. This result was strengthened in [7], where it was demonstrated that also type finitary is not possible, but the exact unification type of ACUI (infinitary or zero) remained open. In our FSCD 2025 paper [5], we were able to settle this question by showing that the unrestricted unification type of ACUI is actually infinitary. We were also able to prove that the same is true for the theories AC and ACU. Quite surprisingly, it turned out that also "good things" may happen when switching from the restricted to the unrestricted instantiation preorder: we were also able to show that the unification type of the description logic $\mathcal{EL}$ actually improves from type zero [8] to infinitary. In addition to these results for specific theories/logics, we established some general results on the relationship between the two preorders, which among other things imply that for associativity A and for commutativity C the unification type does not depend on which of the two preorders is employed. The results shown in [5] are the ones printed in black in Table 1, where the cells containing a cross indicate combinations that are not possible. In the preprint [6], which is submitted for publication to a journal, we have determined the unrestricted unification type of quite a number of additional equational theories (see the entries printed in blue in Table 1). However, the main improvement achieved in [6] is that, unlike in [5], we proved these results not separately for each theory, but instead were able to establish results for whole classes of theories. The cells marked with "?" in Table 1 describe cases for which we have not yet found examples or a theorem that would preclude them.

The purpose of this abstract is two fold. First, we provide an overview of our previous results in [5, 6], which have so far not been presented at UNIF. Due to space constraints, we refer the reader to [6] for detailed definitions and full proofs. Second, we add in this paper three new entries to the results table where the unification type stays unitary when switching from the restricted to the unrestricted instantiation preorder (see the entries printed in red in Table 1). To this end, we show that the general result applied to $\emptyset$, C, I, and A in [6] can also be applied to these theories.

2 The results

In what follows, we introduce the equational theories contained in Table 1. With the exception of K, all the accompanying references refer to the *restricted* unification type of these theories w.r.t. elementary unification, i.e., where the unification problem may not contain additional free constants or function symbols. For K, the result was shown in the unrestricted setting, but it was stated that the same proof methods also apply to the restricted preorder.

- The symbol $\emptyset$ denotes the empty theory. Type unitary for $\emptyset$ was shown in [33, 25].
- A corresponds to the theory of an associative function symbol. Plotkin showed that A has unification type infinitary [31].

restricted \ unrestricted	unitary	finitary	infinitary	zero
unitary	$\emptyset$, B, AG, GAUSS	?	ACU, ACUI	?
finitary	$\times$	C, I	AC, ACI	?
infinitary	$\times$	$\times$	A, D	?
zero	$\times$	$\times$	$\mathcal{EL}$, AI, $\mathcal{FL}_0$, ACUh	K

Table 1: Overview on results about the restricted and unrestricted unification type for elementary unification.

- For the theory C of a commutative function symbol, type finitary was proved in [34].
- The theory I axiomatizes idempotency of a binary function symbol. The results in [26, 18] imply that I has unification type finitary for elementary unification.
- B stands for the theory of Boolean rings, for which type unitary was established in [16, 28].
- D is the equational theory of both-sided distributivity. Elementary unification type infinitary for D follows from the results shown in [39] and [24]. More precisely, in his PhD thesis [39], Szabo showed the upper bound "at most infinitary" for the restricted unification type of D. The corresponding lower bound "at least infinitary" was established in Section 5.4 of [24].
- The theory AC is obtained as the union of the theories A and C, i.e., $\mathsf{AC} = \mathsf{A} \cup \mathsf{C}$. The theories ACI and AI are obtained in a similar way. Type finitary for AC and ACI were respectively proved in [38] and [23]. For the theory AI of an associative and idempotent function symbol, type zero was shown in [1] for elementary unification.
- The equational theories ACU and ACUI are obtained by respectively extending AC and ACI with a unit. For elementary unification, type unitary for ACU is shown in [38] and for ACUI in [4].
- The equational theory ACUh extends ACU with a homomorphism. Type zero for ACUh was established in [2].
- AG corresponds to the theory of *Abelian groups*, whereas GAUSS is obtained by extending AG with a homomorphism h and the identity $x + h(h(x)) \approx 0$. For AG, unification type unitary was determined in [27] for elementary unification. In [29], Nutt showed that the same is the case for the theory GAUSS. Note that here we use the definition of GAUSS provided in [11].
- $\mathcal{EL}$ and $\mathcal{FL}_0$ are equational theories that can be used to axiomatize equivalence in the description logics $\mathcal{EL}$ and $\mathcal{FL}_0$. Type zero for $\mathcal{EL}$ was shown in [8] and for $\mathcal{FL}_0$ in [9].
- K axiomatizes equivalence in the modal logic K. It was shown in [20] by Jerábek that its unification type is zero w.r.t. the *unrestricted* instantiation preorder.

3 The Methods

The purpose of this section is to sketch some of the approaches used to prove the results on the unrestricted unification type depicted in Table 1.

The first observation is that, due to the fact that $\leq_E^V \subseteq \leq_E^{\mathrm{Var}(\Gamma)}$, the restricted unification type cannot be worse than the unrestricted one if the latter is unitary or finitary. The following theorem justifies the cells containing a cross in Table 1, which indicates that such a combination is not possible.

Theorem 1. *Let E be an equational theory. If E has unrestricted unification type unitary (finitary), then it has restricted unification type unitary (finitary or unitary).*

The next theorem pinpoints cases where the restricted and the unrestricted unification type coincide. For a given substitution σ, recall that $\mathrm{Dom}(\sigma)$ denotes the finite set of variables x such that $\sigma(x) \neq x$, and that $\mathrm{VRan}(\sigma)$ consists of the variables occurring in $\sigma(x)$ for some $x \in \mathrm{Dom}(\sigma)$.

Theorem 2 (Theorem 4.5 of [6])**.** *Let E be an equational theory, Γ an E-unification problem, and $\mathcal{S}$ a set of E-unifiers of Γ such that $\mathrm{VRan}(\sigma) \cup \mathrm{Dom}(\sigma) \subseteq \mathrm{Var}(\Gamma)$ holds for all $\sigma \in \mathcal{S}$ and $\mathcal{S}$ is complete w.r.t. the restricted or the unrestricted instantiation preorder. Then the restricted and the unrestricted E-unification types of Γ coincide.*

Basically, the theorem says that the unification type does not change if there is a complete set of unifiers (for the restricted or unrestricted setting) that does not introduce new variables, i.e., variables not occurring in the unification problem. The following corollary lists theories where this condition is satisfied.

Corollary 3. *For the theories $\emptyset$, C, A, I, K, B, AG, and GAUSS the restricted and the unrestricted unification types coincide.*

For the theories $\emptyset$, C, A, I, and K, this is shown in [6] (Corollary 4.6 and Corollary 4.7). For B, it is an easy consequence of the known algorithms for computing most general unifiers (see, e.g., the description of the algorithm based on Löwenheim's formula in [10]). For AG and GAUSS, note that both are so-called monoidal theories, for which unification can be reduced to solving linear equations over an associated semiring [29, 11]. For AG, this is the ring of integers $\mathbb{Z}$ and for GAUSS it is the ring of Gaussian integers $\mathbb{Z}[i]$. Both rings are principal ideal domains, which implies that the set of solutions of a system of linear equations with n variables has a generating set consisting of at most n elements (see, e.g., Theorem 3.7 in [19]). Since each of the elements of such a generating set corresponds to a variable in the most general unifier in the restricted setting, this implies that no new variables are needed, and thus Theorem 2 applies.

It remains to explain the results corresponding to the equational theories contained in the third column of Table 1 (with the exception of the theory A). We first showed the lower bound "at least infinitary" for the unrestricted unification type of these theories. For theories whose restricted unification type is at least infinitary, the contrapositive of Theorem 1 yields this lower bound also for the unrestricted type. This observation applies to the theories AI, D, $\mathcal{EL}$, $\mathcal{FL}_0$, and ACUh.

Corollary 4. *The unrestricted unification type of AI, D, $\mathcal{EL}$, $\mathcal{FL}_0$, and ACUh for elementary unification is at least infinitary.*

For the theories AC, ACI, ACU, and ACUI, the lower bound "at least infinitary" cannot be shown using the contrapositive of Theorem 1 since these theories are unitary or finitary in the restricted case. For these theories we had to prove the lower bounds directly. Basically, these proofs employ a sufficient criterion for "at least infinitary" in the unrestricted case that applies to regular theories and was already introduced in [7]. For each theory, one must show that this criterion applies, which basically means that one must find unification problems where complete sets of unifiers in the restricted case need to use new variables (see Section 5 in [6] for detailed proofs).

Proposition 5. *The unrestricted unification type of* AC*,* ACI*,* ACU*, and* ACUI *for elementary unification is at least infinitary.*

We also showed matching "at most infinitary" upper bounds for the theories in Corollary 4 and Proposition 5. To demonstrate that these theories cannot have unrestricted unification type zero, we proved that they are *Noetherian* w.r.t. $\leq_E^V$. That is, every strictly descending chain of substitutions $\sigma_1 >_E^V \sigma_2 >_E^V \sigma_3 >_E^V \cdots$ is finite. The notion of a Noetherian equational theory was introduced in [14] for the restricted preorder, and it was shown that it precludes restricted unification type zero. We showed in [6] that the same applies to the unrestricted case. To establish that the theories under consideration are Noetherian, we proceeded in two steps. First, we proved the following proposition (where $\sim_E^V$ is the equivalence relation induced by the preorder $\leq_E^V$).

Proposition 6 (Proposition 6.3 of [6])**.** *Let E be a regular theory and let σ, θ be substitutions such that $\sigma \leq_E^V \theta$. Then there is a substitution σ' such that $\sigma \sim_E^V \sigma'$ and $\mathrm{Dom}(\sigma') \cup \mathrm{VRan}(\sigma') \subseteq \mathrm{Dom}(\theta) \cup \mathrm{VRan}(\theta)$.*

This proposition allows us to reduce the search for a minimal unifier σ below a given unifier θ to substitutions that do not use more variables than θ. Based on this, we then were able to prove that *finite* theories [14] and regular, *locally finite* theories [15] are Noetherian w.r.t. the unrestricted instantiation preorder. This allowed us to establish the "at most infinitary" upper bounds for the finite theories AC and D and for the locally finite theories AI, ACI, and ACUI. The theory $\mathcal{EL}$ is regular, but not locally finite. However, a proof similar to the one employed for locally finite theories can be used to establish that $\mathcal{EL}$ is Noetherian. Finally, for the class of monoidal theories, we defined the subclass of "restrictive" monoidal theories and proved that the regular theories of this subclass are also Noetherian. Since ACU, ACUh, and $\mathcal{FL}_0$ are all members of this subclass, this implies that their unrestricted unification type cannot be zero. For precise definitions and full proofs, we refer the reader to Section 6 in [6].

Theorem 7. *The unrestricted unification type of* AI*,* D*,* AC*,* ACI*,* ACU*,* ACUI*,* ACUh*, $\mathcal{FL}_0$, and $\mathcal{EL}$ for elementary unification is infinitary.*

4 Future Work

The cells marked with "?" in Table 1 describe cases that are conceivable from the order-theoretic point of view, which only takes into account that the unrestricted preorder is a subset of the restricted one, but for which we have not yet found examples or a theorem that would preclude them. One topic for future research in this direction is to find either examples of equational theories that can be put in such a cell or to prove a theorem that says that such a cell must be empty. Some of our general methods only apply for regular theories, i.e., theories where the identities contain the same variables in the left- and right-hand sides. It would be interesting to see whether these methods can also be applied to non-regular theories. The theories AG, GAUSS, K, and B are not regular, but they could be handled using Theorem 2, which does not require regularity.

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