

False-name-proof Combinatorial Auction Protocol: Groves Mechanism with Submodular Approximation

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ABSTRACT

This paper develops a new combinatorial auction protocol called the Groves Mechanism with SubModular Approximation (GM-SMA). This protocol satisfies the following characteristics: (1) it is false-name-proof, (2) each winner is included in a Pareto efficient allocation, and (3) as long as a Pareto efficient allocation is achieved, the protocol is robust against the collusion of losers and the outcome is in the core. As far as the authors are aware, the GM-SMA is the first protocol that satisfies all three of these characteristics.

The basic ideas of the GM-SMA are as follows: (i) It is based on the VCG protocol, i.e., the payment of a winner in this protocol is identical to the payment in one instance of the Groves mechanism, which is a class of protocols that includes the VCG. (ii) When calculating the payment of a bidder, we approximate the valuations of other bidders by using a submodular valuation function (submodular approximation). Simulation results show that the GM-SMA achieves a better social surplus and seller's revenue than existing false-name-proof protocols, as long as the submodular approximation is close enough to the original valuations.

Categories and Subject Descriptors

I.2.11 [Artificial Intelligence]: Distributed Artificial Intelligence—*Multi-agent systems*

General Terms

Theory, Economics

Keywords

Combinatorial Auction, Strategy-Proof

1. INTRODUCTION

Internet auctions have become an integral part of Electronic Commerce and a promising field for applying AI and

agent technologies. Among various studies related to Internet auctions, those on combinatorial auctions have lately attracted considerable attention (an extensive survey is presented in [3]). Although conventional auctions sell a single item at a time, combinatorial auctions sell multiple items with interdependent values simultaneously and allow the bidders to bid on any combination of items. In a combinatorial auction, a bidder can express complementary or substitutable preferences over multiple bids. By taking into account complementary/substitutable preferences, we can increase the participants' utilities and seller's revenue.

One desirable characteristic of an auction protocol is that it is *strategy-proof*. A protocol is strategy-proof if, for each bidder, declaring his/her true valuation is a *dominant strategy*, i.e., an optimal strategy regardless of the actions of other bidders. In theory, the revelation principle states that in the design of an auction protocol, we can restrict our attention to strategy-proof protocols without loss of generality [8]. In other words, if a certain property (e.g., Pareto efficiency) can be achieved using some auction protocol in a dominant-strategy equilibrium, i.e., a combination of dominant strategies of bidders, then the property can also be achieved using a strategy-proof auction protocol.

Furthermore, a strategy-proof protocol is also practical for Internet auctions. For example, if we use the first-price sealed-bid auction, which is not strategy-proof, the bidding prices must be securely concealed until the auction is closed. On the other hand, if we use a strategy-proof protocol, knowing the bidding prices of other bidders is useless; consequently, such security issues become less critical.

The Vickrey-Clarke-Groves (VCG) protocol [2, 4, 9] is a strategy-proof protocol that can be applied to combinatorial auctions. The VCG protocol satisfies Pareto efficiency. Under several natural assumptions, the VCG protocol is the only strategy-proof, Pareto efficient protocol [6].

However, the VCG protocol has several limitations, including the problems described below.

Vulnerability to false-name bids: The authors have pointed out the possibility of a new type of fraud called false-name bids, which utilizes the anonymity available on the Internet [11, 12]. False-name bids are bids submitted under fictitious names, e.g., multiple e-mail addresses. Such a dishonest action is very difficult to detect, since identifying each participant on the Internet is virtually impossible.

We say a protocol is *false-name-proof* if, for each bidder,

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declaring his/her true valuations using a single identifier (although the bidder can use multiple identifiers) is a dominant strategy. The VCG is not false-name-proof [11, 12].

Vulnerability to loser collusion: As shown in [1], in the VCG protocol, there is a chance that some bidders, who would be losers if they bid their true valuations, become winners and increase their utility if they collude and adjust their bids.

The outcome is not in the core: As shown in [1], in the VCG protocol, there is a chance that the result of the auction is not in the core. This means that a seller has an incentive to deviate from the protocol and to sell goods to the bidders who are not winners in the VCG protocol.

In this paper, we introduce a new combinatorial auction protocol called the *Groves Mechanism with SubModular Approximation* (GM-SMA). The basic ideas of this protocol are as follows.

- This protocol is based on the VCG protocol. More precisely, the payment of a winner in this protocol is identical to the payment in one instance of the Groves mechanism [4], which is a class of protocols that includes the VCG¹.
- In this protocol, when calculating the payment of bidder i , we approximate the valuations of bidders except for i , by using a *submodular* valuation function (submodular approximation).

The GM-SMA satisfies the following characteristics.

- (1) The GM-SMA is false-name-proof.
- (2) In the GM-SMA, each winner is a bidder who is included in a Pareto efficient allocation².
- (3) In the GM-SMA, as long as the allocation is Pareto efficient, the protocol is robust against the collusion of the losers and the outcome is in the core.

As far as the authors are aware, the GM-SMA is the first protocol that satisfies all three of these characteristics.

The VCG protocol can always achieve a Pareto efficient allocation as long as there exists no false-name bid. However, it is not false-name-proof and cannot guarantee to achieve a Pareto efficient allocation when false-name bids are possible. In addition, even without false-name bids, the VCG protocol does not satisfy characteristic (3).

Furthermore, in existing false-name-proof protocols [10, 11], characteristic (2) is not satisfied, i.e., a bidder who is not included in any Pareto efficient allocation becomes a winner quite often. In such a case, the outcome is not in the core. Also, losers who are in a Pareto efficient allocation will have a strong incentive to collude and act as a single bidder.

In [1], an ascending protocol that utilizes a proxy is presented. In this protocol, a bidder cannot increase his/her

¹Strictly speaking, the GM-SMA is not an instance of the Groves mechanism, since it is not guaranteed to achieve a Pareto efficient allocation.

²Note that the converse is not true, i.e., a bidder who is included in a Pareto efficient allocation is not necessarily a winner. Thus, there is a chance that the GM-SMA could fail to achieve a Pareto efficient allocation. This is inevitable since the GM-SMA is false-name-proof [12].

utility by using multiple identifiers. Also, as long as each bidder bids straightforwardly, a Pareto efficient allocation is achieved and characteristic (3) is satisfied. However, unless the valuations of bidders are submodular, bidding straightforwardly is not a dominant strategy, i.e., a bidder may have an incentive to misreport his/her valuation.

The rest of this paper is organized as follows. First, we describe the model of combinatorial auctions (Section 2). Next, we describe a general framework for describing strategy-proof and false-name-proof protocols called the Price-Oriented, Rationing-Free (PORF) protocol [10] (Section 3). By describing a protocol as a PORF protocol, proving the protocol is strategy/false-name proof becomes much easier. Then, we describe the VCG protocol and its limitations (Section 4) as well as existing false-name-proof protocols and their limitations (Section 5). Furthermore, we describe the details of the GM-SMA (Section 6) and show its characteristics (Section 7). Finally, we evaluate the social surplus and the revenue of the seller in the GM-SMA through a simulation (Section 8).

2. MODEL

Assume there are a set of bidders $N = \{1, 2, \dots, n\}$ and a set of goods $M = \{1, 2, \dots, m\}$. Each bidder i has his/her preferences over $B \subseteq M$. Formally, we model this by supposing that bidder i privately observes a parameter, or signal, θ_i , which determines his/her preferences. We refer to θ_i as the *type* of bidder i . We assume θ_i is drawn from a set Θ . We also assume a *quasi-linear, private value* model with *no allocative externality*, defined as follows.

DEFINITION 1. *utility of a bidder*

The utility of bidder i , when i obtains a bundle, i.e., a subset of goods $B \subseteq M$ and pays $p_{B,i}$, is represented as $v(B, \theta_i) - p_{B,i}$.

We assume the valuation v is normalized by $v(\emptyset, \theta_i) = 0$. Furthermore, we assume *free disposal*, i.e., $v(B', \theta_i) \geq v(B, \theta_i)$ for all $B' \supseteq B$.

In a traditional definition [7], an auction protocol is (dominant strategy) *incentive compatible* (or *strategy-proof*) if declaring the true type/valuation is a dominant strategy for each bidder, i.e., an optimal strategy regardless of the actions of other bidders.

In this paper, we extend the traditional definition of incentive compatibility so that it can address false-name bid manipulations, i.e., we define that an auction protocol is (dominant strategy) incentive compatible if declaring the true type *by using a single identifier* is a dominant strategy for each bidder. To distinguish between the traditional and extended definitions of incentive compatibility, we refer to the traditional definition as *strategy-proof* and to the extended definition as *false-name-proof*.

As in the case of strategy-proof protocols, the revelation principle holds for false-name-proof protocols [12]. Thus, we can restrict our attention to false-name-proof protocols without loss of generality.

We say an auction protocol is *Pareto efficient* when the sum of all participants' utilities (including that of the auctioneer), i.e., the social surplus, is maximized in a dominant-strategy equilibrium. We have proved that there exists no false-name-proof protocol that satisfies Pareto efficiency [12]. Therefore, we need to sacrifice efficiency to some extent when false-name bids are possible.

3. PRICE-ORIENTED, RATIONING-FREE (PORF) PROTOCOL

In this section, we describe a general framework for describing strategy-proof protocols called Price-Oriented, Rationing-Free (PORF) protocol [10]. By describing a protocol as a PORF protocol, proving that the protocol is strategy-proof or false-name-proof becomes much easier.

A PORF Protocol is defined as follows.

DEFINITION 2. *PORF Protocol*

- Each bidder i declares his/her type $\tilde{\theta}_i$, which is not necessarily the true type θ_i .
- For each bidder i and for each bundle $B \subseteq M$, the price $p_{B,i}$ is defined. This price must be determined independently of i 's declared type $\tilde{\theta}_i$, while it can be dependent on declared types of other bidders.
- We assume $p_{\emptyset,i} = 0$ holds. Also, if $B \subseteq B'$, $p_{B,i} \leq p_{B',i}$ holds.
- For bidder i , a bundle B^* is allocated, where $B^* = \arg \max_{B \subseteq M} v(B, \tilde{\theta}_i) - p_{B,i}$. Bidder i pays $p_{B^*,i}$. If there exist multiple bundles that maximize i 's utility, one of these bundles is allocated.
- The result of the allocation satisfies allocation feasibility, i.e., for two bidders i, j and bundles allocated to these bidders B_i^* and B_j^* , $B_i^* \cap B_j^* = \emptyset$ holds.

It is straightforward to show that a PORF protocol is strategy-proof. The price of bidder i is determined independently of i 's declared type, and he/she can obtain the bundle that maximizes his/her utility independently of the allocations of other bidders, i.e., the protocol is rationing-free.

On the other hand, in a PORF protocol, the prices must be determined appropriately to satisfy allocation feasibility. The definition of a PORF protocol requires that if there exist multiple bundles that maximize i 's utility, then one of these bundles must be allocated, but it does not specify exactly which bundle should be allocated. Therefore, if there exist multiple choices, the auctioneer can adjust the allocation of multiple bidders in order to satisfy allocation feasibility.

In [10], it is shown that any strategy-proof protocol can be described as a PORF protocol, i.e., the following theorem holds.

THEOREM 1. *If a protocol is strategy-proof, then the protocol can be described as a PORF protocol.*

In this paper, we assume that a protocol satisfies the following condition.

DEFINITION 3. *weakly-anonymous pricing rule (WAP)*

For bidder i , the price of bundle B is given as a function of the types of other bidders, i.e., the price can be described as $p(B, \Theta_{N'})$, where N' is the set of bidders other than i , and $\Theta_{N'}$ is the set of types of bidders in N' .

The above condition requires that if two bidders face the same types of opponents, their prices must be identical for all bundles. The WAP condition is intuitively natural and virtually all well-known protocols, including the VCG, satisfy this condition.

For a PORF protocol that satisfies the WAP condition, we define the following additional condition.

DEFINITION 4. *no super-additive price increase (NSA)*
For any subset of bidders $S \subseteq N$ and $N' = N \setminus S$, and for $i \in S$, let us denote B_i as a bundle that maximizes i 's utility, then $\sum_{i \in S} p(B_i, \bigcup_{j \in S \setminus \{i\}} \{\theta_j\} \cup \Theta_{N'}) \geq p(\bigcup_{i \in S} B_i, \Theta_{N'})$.

An intuitive description of this condition is that the price of buying a combination of bundles (the right side of the inequality) must be smaller than or equal to the sum of the prices for buying these bundles separately (the left side).

In [10], it is shown that for a PORF protocol with the WAP, the NSA is a sufficient condition for a protocol to be false-name-proof, i.e., the following theorem holds.

THEOREM 2. *If a PORF protocol with the WAP satisfies the NSA condition, then the protocol is false-name-proof.*

4. VCG PROTOCOL

Next, we show how the VCG protocol can be described as a PORF protocol. Since a PORF protocol is strategy-proof, in the rest of this paper, we assume each bidder i declares his/her true type θ_i .

To simplify the protocol description, we introduce the following notation. For a set of goods B and a set of bidders S , where Θ_S is a set of types of bidders in S , we define $V^*(B, \Theta_S)$ as the sum of the valuations of S when B is allocated optimally among S .

To be precise, for a feasible allocation $g = (B_1, B_2, \dots)$, where $\bigcup_{i \in S} B_i \subseteq B$ and for all $i \neq i'$, $B_i \cap B_{i'} = \emptyset$, $V^*(B, \Theta_S)$ is defined as $\max_g \sum_{i \in S} v(B_i, \theta_i)$, where θ_i is the type of bidder i .

The price of bundle B for bidder i (VCG price) is defined as follows:

$$p_{B,i} = V^*(M, \Theta_{N \setminus \{i\}}) - V^*(M \setminus B, \Theta_{N \setminus \{i\}}).$$

This protocol is identical to the Vickrey-Clarke-Groves (VCG) mechanism [2, 4, 9], i.e., if B is allocated to i in a Pareto efficient allocation, then $p_{B,i}$ is equal to the payment in the VCG protocol and B maximizes i 's utility.

This protocol satisfies allocation feasibility since the following theorem holds.

THEOREM 3. *If B is allocated to i in a Pareto efficient allocation, then B maximizes i 's utility at the VCG price.*

Proof: If we assume this theorem does not hold, then there exists another bundle $B' \neq B$ that maximizes i 's utility. Thus, the following formula holds.

$$v(B', \theta_i) - p_{B',i} > v(B, \theta_i) - p_{B,i}$$

From this formula and the definition of $p_{B',i}, p_{B,i}$, we obtain:

$$v(B', \theta_i) + V^*(M \setminus B', \Theta_{N \setminus \{i\}}) > v(B, \theta_i) + V^*(M \setminus B, \Theta_{N \setminus \{i\}}).$$

This contradicts the assumption that allocating B to i is Pareto efficient. $\square$

Note that this theorem also holds when B is an empty set. From this theorem, if there exist multiple Pareto efficient allocations, the auctioneer can choose one of them arbitrarily, since every bidder has the highest preference for the bundle that is allocated in that allocation (which can be an empty bundle). Thus, allocation feasibility is satisfied.

Let us describe how this protocol works.

EXAMPLE 1. *Assume there are two goods 1 and 2, and three bidders, bidder 1, 2, and 3, whose types are θ_1, θ_2 , and θ_3 , respectively. The valuation for a bundle $v(B, \theta_i)$ is determined as follows.*

	{1}	{2}	{1, 2}
θ_1	6	0	6
θ_2	0	0	8
θ_3	0	5	5

Accordingly, the prices of these bundles for each bidder are given as follows.

	{1}	{2}	{1, 2}
bidder 1	3	8	8
bidder 2	6	5	11
bidder 3	8	2	8

As a result, bidder 1 obtains good 1 at price 3, and bidder 3 obtains good 2 at price 2.

Now, we can see that this result is not in the core. Let us assume another allocation where both goods are sold to bidder 2 at price 7. Then, both the seller and bidder 2 prefer this allocation to the outcome of the VCG protocol. Such a coalition, in this case, the coalition of the seller and bidder 2, is called a *blocking* coalition.

Let us consider another situation.

EXAMPLE 2. Assume there are two goods 1 and 2, and three bidders, bidder 1, 2, and 3, whose types are θ_1, θ_2 , and θ_3 , respectively. The valuation for a bundle $v(B, \theta_i)$ is determined as follows.

	{1}	{2}	{1, 2}
θ_1	4	0	4
θ_2	0	0	8
θ_3	0	3	3

Accordingly, the prices of these bundles for each bidder are given as follows.

	{1}	{2}	{1, 2}
bidder 1	5	8	8
bidder 2	4	3	7
bidder 3	8	4	8

As a result, bidder 2 obtains both goods at price 7.

Now, we can see that the VCG protocol is vulnerable to the collusion of losers. If bidder 1 and bidder 3 collude and increase their valuations, they can create a situation identical to Example 1. Then, bidder 1 obtains good 1 at price 3, and bidder 3 obtains good 2 at price 2. For each bidder, this price is less than his/her valuation, thus bidders can increase their utilities by engaging in collusion.

5. EXISTING FALSE-NAME-PROOF PROTOCOLS

In this section, we describe a false-name-proof protocol called the Max Minimal-Bundle (M-MB) protocol [10] as a representative false-name-proof protocol.

To simplify the protocol description, we introduce a concept called a *minimal bundle*.

DEFINITION 5. *minimal bundle*

Bundle B is called *minimal* for bidder i if for all $B' \subset B$ and $B' \neq B$, $v(B', \theta_i) < v(B, \theta_i)$ holds.

In the M-MB protocol, the price of bundle B for bidder i is defined as follows:

- $p_{B,i} = \max_{B_j \subseteq M, j \neq i} v(B_j, \theta_j)$, where $B \cap B_j \neq \emptyset$ and B_j is minimal for bidder j .

In short, the price of bundle B is equal to the highest valuation of a bundle, which is minimal and conflicting with bundle B .

It is clear that this protocol satisfies allocation feasibility [10]. Also, as shown in [10], this protocol is false-name-proof since it satisfies the NSA condition.

Let us describe how this protocol works. Let us consider a situation identical to Example 1. The prices of these bundles are given as follows.

	{1}	{2}	{1, 2}
bidder 1	8	8	8
bidder 2	6	5	6
bidder 3	8	8	8

As a result, bidder 2 obtains both goods at price 6.

As shown in this example, in the M-MB protocol, some goods can be allocated to a bidder who is not included in any Pareto efficient allocation. This is true for other existing false-name-proof protocols such as the LDS protocol [11]. In such a case, losers who are in a Pareto efficient allocation will have a strong incentive to collude. For example, if bidder 1 and bidder 3 collude and act as a single bidder whose valuation for the set is 11, then they can obtain both goods at price 8, which is smaller than the sum of their valuations.

6. GROVES MECHANISM WITH SUBMODULAR APPROXIMATION (GM-SMA)

In this section, we describe the details of the Groves Mechanism with SubModular Approximation (GM-SMA).

6.1 Protocol Description

To define the prices used in the GM-SMA, we introduce a concept called submodular approximation.

DEFINITION 6. *submodular approximation U^**

The function $U^*(B, \Theta_S)$ defined below is called a *submodular approximation* of V^* .

For a feasible allocation $g = (B_1, B_2, \dots)$, where $\bigcup_{i \in S} B_i \subseteq B$ and for all $i \neq i'$, $B_i \cap B_{i'} = \emptyset$, $U^*(B, \Theta_S)$ is defined as $\max_g \sum_{i \in S} v'(B_i, \theta_i)$. v' is a function that satisfies $v'(B, \theta_i) \geq v(B, \theta_i)$ for all i, B .

Furthermore, we assume v' is chosen so that U^* becomes submodular, i.e., for all $S \subseteq N, B', B'' \in M$, the following condition holds:

$$U^*(B', \Theta_S) + U^*(B'', \Theta_S) \geq U^*(B' \cup B'', \Theta_S) + U^*(B' \cap B'', \Theta_S).$$

From this definition, U^* satisfies the following conditions:

- For all $S \subset N, B \subset M$, $U^*(B, \Theta_S) \geq V^*(B, \Theta_S)$ holds.
- For all $S \subset N, B \subset M$, $U^*(M, \Theta_N) \geq U^*(B, \Theta_S) + U^*(M \setminus B, \Theta_{N \setminus S})$ holds.

The Groves Mechanism with SubModular Approximation (GM-SMA) is described as a PORF protocol in which prices are defined as follows.

- When $B = \emptyset$, $p_{B,i} = 0$.
- Otherwise, $p_{B,i} = U^*(M, \Theta_{N \setminus \{i\}}) - V^*(M \setminus B, \Theta_{N \setminus \{i\}})$.

Note that we use U^* only for the first term.

The PORF description of a protocol is very convenient for proving the theoretical properties of the protocol, but implementing it can be rather costly. Alternatively, the GM-SMA can be implemented as follows.

1. The auctioneer finds a Pareto efficient allocation. The auctioneer can choose an arbitrary one if there exist multiple Pareto efficient allocations.
2. When B is allocated to bidder i in that allocation, the auctioneer calculates the GM-SMA price $p_{B,i}$. If $v(B, \theta_i) \geq p_{B,i}$, then B is allocated to bidder i at price $p_{B,i}$. If $v(B, \theta_i) < p_{B,i}$, B is not allocated to any bidder.

This protocol is quite similar to the Groves mechanism [4], since the payment of a winner in this protocol is identical to the payment in one instance of the Groves mechanism. However, it is not an instance of the Groves mechanism because it does not guarantee to achieve a Pareto efficient allocation. This protocol can be considered as a modification of the Groves mechanism, in which a winner has an option not to obtain the goods if the payment becomes too high.

6.2 Examples

Here, we describe how the GM-SMA works in the situation of Example 1. In this example, we use $v'(\{1\}, \theta_2) = v'(\{2\}, \theta_2) = 8/2 = 4$, and we use $v'(\cdot, \cdot) = v(\cdot, \cdot)$ for other bundles and bidders. By choosing v' in this way, U^* satisfies the submodular condition.

The GM-SMA prices are given as follows.

	$\{1\}$	$\{2\}$	$\{1, 2\}$
bidder 1	4	9	9
bidder 2	6	5	11
bidder 3	10	4	10

As a result, bidder 1 obtains good 1 at price 4, and bidder 3 obtains good 2 at price 4.

This outcome is in the core. Since the revenue of the seller is 8, the seller does not have an incentive to deviate from the protocol with bidder 2 (who is willing to pay at most 8).

Next, we describe how the protocol works in the situation of Example 2. Here, v' is defined in a similar way to the previous example. The GM-SMA prices are given as follows.

	$\{1\}$	$\{2\}$	$\{1, 2\}$
bidder 1	5	8	8
bidder 2	4	3	7
bidder 3	8	4	8

As a result, bidder 2 obtains both goods at price 7. In this case, even if bidder 1 and bidder 3 collude and create a situation identical to Example 1, their utilities cannot be positive since each must pay 4 in that situation. In this case, the GM-SMA gives the same result as the VCG.

6.3 Submodular Approximation

In this subsection, we discuss how to choose v' so that U^* becomes submodular. It is known that if each bidder's valuation satisfies a condition called the *gross substitutes* condition³, then U^* becomes submodular [5].

DEFINITION 7. *gross substitutes*

Given a price vector $p = (p_1, \dots, p_m)$, we denote

$$D_i(p) = \{B \subset M : \forall C \subset M, v(B, \theta_i) - \sum_{j \in B} p_j \geq v(C, \theta_i) - \sum_{j \in C} p_j\}.$$

$D_i(p)$ represents the collection of bundles that maximizes the net utility of bidder i under price vector p . We say that the gross substitutes condition is satisfied, if for any two price vectors p and p' such that $p' \geq p$, $p'_j = p_j$, and $j \in B \in D_i(p)$, then there exists $B' \in D_i(p')$ such that $j \in B'$.

In short, the gross substitutes condition states that if good j is demanded with price vector p , it is still demanded if the price of j remains the same, while the prices of some other goods increase.

We describe two representative valuations that satisfy the gross substitutes condition.

DEFINITION 8. *unit-demand valuation*

Bidder i 's valuation is called unit-demand valuation if, for each good $j \in M$, a valuation v_{ij} is given and, for any bundle B , $v(B, \theta_i)$ is calculated as $\max_{j \in B} v_{ij}$.

In short, a bidder with a unit-demand valuation requires only one of M .

DEFINITION 9. *additive valuation*

Bidder i 's valuation is an additive valuation if, for each good $j \in M$, a valuation v_{ij} is given and, for any bundle B , $v(B, \theta_i)$ is calculated as $\sum_{j \in B} v_{ij}$.

In short, the valuation for a bundle of an additive valuation bidder is the sum of the valuations of each good.

We can obtain v' by approximating v by unit-demand valuation or additive valuation. For example, if we approximate the function v by additive valuation, we first calculate a valuation vector $\vec{v}_i = (v_{i1}, v_{i2}, \dots, v_{im})$ that satisfies the following conditions. Then, we define $v'(B, \theta_i)$ as $\sum_{j \in B} v_{ij}$.

- For each bundle B , $v(B, \theta_i) \leq \sum_{j \in B} v_{ij}$.
- $\vec{v}_i$ is minimal, i.e., the above condition is not satisfied if we decrease any v_{ij} .

So far, we have considered a method for constructing U^* by using a gross substitutes valuation. However, the gross substitutes condition is not a necessary condition for U^* to be submodular [5]. This means that there is a chance that we can construct U^* based on a weaker condition than the gross substitutes condition. In that case, we can expect to obtain a better approximation of v . We intend to examine such a method in our future works.

7. CHARACTERISTICS OF GM-SMA

In this section, we examine the characteristics of the GM-SMA.

³The fact that each v' is submodular is not sufficient to guarantee that U^* is submodular [5].

7.1 Allocation Feasibility

The following characteristic holds.

PROPOSITION 1. *The GM-SMA price of a bundle is larger than (or equal to) the VCG price. Also, for bidder i , the difference between the GM-SMA price and the VCG price is the same for every bundle except for an empty bundle, i.e., the difference is equal to $U^*(M, \Theta_{N \setminus \{i\}}) - V^*(M, \Theta_{N \setminus \{i\}})$.*

The proof is straightforward, since U^* is greater than or equal to V^* .

THEOREM 4. *The GM-SMA satisfies allocation feasibility.*

Proof: This is derived from the fact that the VCG protocol satisfies allocation feasibility. From Proposition 1, the GM-SMA prices are larger than the VCG prices by the same amount (except for an empty bundle). Thus, if a bidder is included in a Pareto efficient allocation, the bundle in the allocation maximizes the bidder's utility with the GM-SMA price, or the bidder prefers to obtain an empty bundle. Therefore, if there exist multiple Pareto efficient allocations, the auctioneer can choose one Pareto efficient allocation arbitrarily. Then, every bidder has the highest preference for the bundle that is allocated in that allocation, or an empty bundle. Thus, allocation feasibility is satisfied. $\square$

On the other hand, since a bidder who obtains some goods in the VCG might prefer obtaining an empty bundle, the GM-SMA cannot guarantee Pareto efficiency. This is inevitable since there exists no false-name-proof protocol that satisfies Pareto efficiency.

7.2 False-name-Proofness

In this subsection, we show that the GM-SMA is false-name-proof.

THEOREM 5. *The GM-SMA is false-name-proof.*

Proof: This can be proven by showing that the GM-SMA prices satisfy the NSA condition. We show when bidder i' obtains B' and bidder i'' obtains B'' , then $p_{B,i}$, i.e., the price for obtaining $B = B' \cup B''$ with a single identifier i , is less than (or equal to) $p_{B',i'} + p_{B'',i''}$. We can prove the case of more than two identifiers in a similar way.

From our definition of the GM-SMA prices, the following conditions hold. N' is a set of bidders except i', i'' .

$$p_{B',i'} = U^*(M, \Theta_{N' \cup \{i''\}}) - V^*(M \setminus B', \Theta_{N' \cup \{i''\}})$$

$$p_{B'',i''} = U^*(M, \Theta_{N' \cup \{i'\}}) - V^*(M \setminus B'', \Theta_{N' \cup \{i'\}})$$

$$p_{B,i} = U^*(M, \Theta_{N'}) - V^*(M \setminus B, \Theta_{N'})$$

From the fact that i' obtains B' and i'' obtains B'' , the following conditions hold.

$$V^*(M \setminus B', \Theta_{N' \cup \{i''\}}) = v(B'', \theta_{i''}) + V^*(M \setminus B, \Theta_{N'})$$

$$V^*(M \setminus B'', \Theta_{N' \cup \{i'\}}) = v(B', \theta_{i'}) + V^*(M \setminus B, \Theta_{N'})$$

Also, from the definition of U^* , the following conditions hold.

$$\begin{aligned} U^*(M, \Theta_{N' \cup \{i'\}}) &\geq U^*(B', \Theta_{\{i'\}}) + U^*(M \setminus B', \Theta_{N'}) \\ &\geq v(B', \theta_{i'}) + U^*(M \setminus B', \Theta_{N'}) \end{aligned}$$

$$\begin{aligned} U^*(M, \Theta_{N' \cup \{i''\}}) &\geq U^*(B'', \Theta_{\{i''\}}) + U^*(M \setminus B'', \Theta_{N'}) \\ &\geq v(B'', \theta_{i''}) + U^*(M \setminus B'', \Theta_{N'}) \end{aligned}$$

From above conditions, we obtain:

$$\begin{aligned} p_{B',i'} + p_{B'',i''} - p_{B,i} &= U^*(M, \Theta_{N' \cup \{i''\}}) - V^*(M \setminus B', \Theta_{N' \cup \{i''\}}) \\ &\quad + U^*(M, \Theta_{N' \cup \{i'\}}) - V^*(M \setminus B'', \Theta_{N' \cup \{i'\}}) \\ &\quad - [U^*(M, \Theta_{N'}) - V^*(M \setminus B, \Theta_{N'})] \\ &\geq v(B'', \theta_{i''}) + U^*(M \setminus B'', \Theta_{N'}) \\ &\quad - v(B'', \theta_{i''}) - V^*(M \setminus B, \Theta_{N'}) \\ &\quad + v(B', \theta_{i'}) + U^*(M \setminus B', \Theta_{N'}) \\ &\quad - v(B', \theta_{i'}) - V^*(M \setminus B, \Theta_{N'}) \\ &\quad - U^*(M, \Theta_{N'}) + V^*(M \setminus B, \Theta_{N'}) \\ &\geq U^*(M \setminus B', \Theta_{N'}) + U^*(M \setminus B'', \Theta_{N'}) \\ &\quad - U^*(M, \Theta_{N'}) - V^*(M \setminus B, \Theta_{N'}) \\ &\geq U^*(M \setminus B', \Theta_{N'}) + U^*(M \setminus B'', \Theta_{N'}) \\ &\quad - U^*(M, \Theta_{N'}) - U^*(M \setminus B, \Theta_{N'}) \end{aligned}$$

Since U^* is submodular, the following condition holds:

$$\begin{aligned} U^*(M \setminus B', \Theta_{N'}) + U^*(M \setminus B'', \Theta_{N'}) \\ \geq U^*(M, \Theta_{N'}) + U^*(M \setminus B, \Theta_{N'}). \end{aligned}$$

Therefore, the above formula is non-negative. Thus, $p_{B',i'} + p_{B'',i''} \geq p_{B,i}$ holds. $\square$

7.3 Robustness against Collusion of Losers

Next, we examine the effect of the collusion of losers.

DEFINITION 10. *strict loser*

We call a bidder who is not included in any Pareto efficient allocation a strict loser.

In the GM-SMA, a loser can be a strict loser or a non-strict loser. A non-strict loser is included in some Pareto efficient allocation but fails to obtain any good.

THEOREM 6. *In the GM-SMA, if strict losers collude and obtain some goods, the sum of their utilities becomes negative.*

Proof: Let us assume a set of strict losers S colludes and obtains B using a single identifier i .

From the fact that each bidder in S is a strict loser, the following formula holds. N' is a set of bidders other than S .

$$V^*(B, \Theta_S) + V^*(M \setminus B, \Theta_{N'}) < V^*(M, \Theta_{N'})$$

Also, the following condition holds for $p_{B,i}$.

$$\begin{aligned} p_{B,i} &= U^*(M, \Theta_{N'}) - V^*(M \setminus B, \Theta_{N'}) \\ &\geq V^*(M, \Theta_{N'}) - V^*(M \setminus B, \Theta_{N'}) \\ &> V^*(B, \Theta_S) \end{aligned}$$

Thus, the sum of the utilities of S becomes negative. Also, since the GM-SMA prices satisfy the NSA condition from Theorem 5, even if S uses multiple identifiers, the sum of the prices does not decrease. Therefore, the sum of the utilities of S remains negative even if S uses multiple identifiers. $\square$

On the other hand, the VCG prices do not satisfy the NSA condition. Therefore, as shown in Example 2, the sum of the losers' utilities can be positive if they collude and use multiple identifiers.

If a Pareto efficient allocation is achieved in the GM-SMA, all losers are strict losers. Thus, the following theorem holds.

THEOREM 7. *When a Pareto efficient allocation is achieved in the GM-SMA, if losers collude and obtain some goods, the sum of their utilities becomes negative.*

7.4 Core Outcome

Next, we examine whether the outcome of the GM-SMA is in the core. First, we show that the following lemma holds.

LEMMA 1. *If U^* is submodular for goods, then U^* is bidder-submodular, i.e., for any set of goods B and for any set of bidders Z, W , the following formula holds: $U^*(B, \Theta_Z) + U^*(B, \Theta_W) \geq U^*(B, \Theta_{Z \cup W}) + U^*(B, \Theta_{Z \cap W})$.*

The details of the proof are described in [12].

The following theorem holds.

THEOREM 8. *When the GM-SMA achieves a Pareto efficient allocation, then the outcome is in the core, i.e., there exists no blocking coalition.*

Proof: Let us assume that goods are allocated to a set of bidders $S_{ww} \cup S_{wl}$ in a Pareto efficient allocation, and bundle B_i is allocated to bidder i in $S_{ww} \cup S_{wl}$. Now, let us assume there exists a blocking coalition that consists of the seller and $S_{ww} \cup S_{lw}$, where $S_{wl} \cap S_{lw} = \emptyset$. In other words, in the blocking coalition, the auctioneer takes away the goods from S_{wl} and re-allocates goods among $S_{ww} \cup S_{lw}$. We assume B'_i is re-allocated to bidder i in $S_{ww} \cup S_{lw}$.

In the blocking coalition, the utility of the seller must be larger than the utility in the GM-SMA, i.e., $\sum_{i \in S_{ww} \cup S_{wl}} p_{B_i, i}$. To maximize the revenue of the seller, while each bidder in $S_{ww} \cup S_{lw}$ obtains the same utility as the GM-SMA, each bidder i in S_{lw} should pay $v(B'_i, \theta_i)$, and each bidder i in S_{ww} should pay $v(B'_i, \theta_i) - (v(B_i, \theta_i) - p_{B_i, i})$. This is because if a bidder pays more than this, the bidder will lose the incentive to join the coalition.

The revenue of the seller in the blocking coalition is given as follows. We represent $B_{S_{ww}} = \bigcup_{i \in S_{ww}} B_i$, i.e., a set of goods allocated to S_{ww} in the Pareto efficient allocation.

$$\begin{aligned} & \sum_{i \in S_{lw}} v(B'_i, \theta_i) + \sum_{i \in S_{ww}} [v(B'_i, \theta_i) - (v(B_i, \theta_i) - p_{B_i, i})] \\ &= V^*(M, \Theta_{S_{ww} \cup S_{lw}}) - V^*(B_{S_{ww}}, \Theta_{S_{ww}}) + \sum_{i \in S_{ww}} p_{B_i, i} \end{aligned}$$

By subtracting the revenue of the seller in the blocking coalition from the revenue of the seller in the GM-SMA, the remainder is given as follows:

$$\sum_{i \in S_{wl}} p_{B_i, i} + V^*(B_{S_{ww}}, \Theta_{S_{ww}}) - V^*(M, \Theta_{S_{ww} \cup S_{lw}}).$$

The following condition holds. We represent $B_{S_{wl}} = \bigcup_{i \in S_{wl}} B_i$, i.e., a set of goods allocated to S_{wl} in the Pareto efficient allocation.

$$\begin{aligned} & \sum_{i \in S_{wl}} p_{B_i, i} \\ &= \sum_{i \in S_{wl}} [U^*(M, \Theta_{N \setminus \{i\}}) - V^*(M \setminus B_i, \Theta_{N \setminus \{i\}})] \\ &= \sum_{i \in S_{wl}} [U^*(M, \Theta_{N \setminus \{i\}}) - V^*(B_{S_{ww}}, \Theta_{S_{ww}}) \\ & \quad - V^*(B_{S_{wl}} \setminus B_i, \Theta_{S_{wl} \setminus \{i\}})] \end{aligned} \quad (1)$$

From the fact that U^* is bidder-submodular, the following conditions hold.

For $i, j \in S_{wl}, i \neq j$,

$$\begin{aligned} & U^*(M, \Theta_{N \setminus \{i\}}) + U^*(M, \Theta_{N \setminus \{j\}}) \\ & \geq U^*(M, \Theta_N) + U^*(M, \Theta_{N \setminus \{i, j\}}). \end{aligned}$$

For $i, j, k \in S_{wl}, i \neq j, j \neq k, k \neq i$,

$$\begin{aligned} & U^*(M, \Theta_{N \setminus \{i\}}) + U^*(M, \Theta_{N \setminus \{j\}}) + U^*(M, \Theta_{N \setminus \{k\}}) \\ & \geq 2 \cdot U^*(M, \Theta_N) + U^*(M, \Theta_{N \setminus \{i, j, k\}}). \end{aligned}$$

By repeating this, we obtain:

$$\begin{aligned} & \sum_{i \in S_{wl}} U^*(M, \Theta_{N \setminus \{i\}}) \\ & \geq (|S_{wl}| - 1)U^*(M, \Theta_N) + U^*(M, \Theta_{N \setminus S_{wl}}). \end{aligned} \quad (2)$$

Also, the following formula holds:

$$\begin{aligned} & \sum_{i \in S_{wl}} V^*(B_{S_{wl}} \setminus B_i, \Theta_{S_{wl} \setminus \{i\}}) \\ & = (|S_{wl}| - 1)V^*(B_{S_{wl}}, \Theta_{S_{wl}}). \end{aligned} \quad (3)$$

In short, for each $i \in S_{wl}$, $v(B_i, \theta_i)$ is counted $|S_{wl}| - 1$ times in the above formula.

From formulae 1, 2, and 3, we obtain:

$$\begin{aligned} & \sum_{i \in S_{wl}} p_{B_i, i} + V^*(B_{S_{ww}}, \Theta_{S_{ww}}) - V^*(M, \Theta_{S_{ww} \cup S_{lw}}) \\ & \geq (|S_{wl}| - 1)U^*(M, \Theta_N) + U^*(M, \Theta_{S_{N \setminus S_{wl}}}) \\ & \quad - |S_{wl}|V^*(B_{S_{ww}}, \Theta_{S_{ww}}) - (|S_{wl}| - 1)V^*(B_{S_{wl}}, \Theta_{S_{wl}}) \\ & \quad + V^*(B_{S_{ww}}, \Theta_{S_{ww}}) - V^*(M, \Theta_{S_{ww} \cup S_{lw}}) \\ & \geq (|S_{wl}| - 1)U^*(M, \Theta_N) \\ & \quad - (|S_{wl}| - 1)(V^*(B_{S_{ww}}, \Theta_{S_{ww}}) + V^*(B_{S_{wl}}, \Theta_{S_{wl}})) \\ & \quad + U^*(M, \Theta_{S_{N \setminus S_{wl}}}) - V^*(M, \Theta_{S_{ww} \cup S_{lw}}) \\ & \geq 0. \end{aligned}$$

As a result, the revenue of the seller in this coalition does not exceed the revenue of the GM-SMA. Therefore, this coalition cannot be a blocking coalition. $\square$

8. EVALUATION

In this section, we evaluate the social surplus and the revenue of the seller in the GM-SMA through a simulation. As a representative of existing false-name-proof protocols, we use the M-MB protocol [10]. This is because using another false-name-proof protocol, i.e., the LDS protocol, would require us to tune various parameters to obtain a good social surplus [11].

We decide the valuation of bidder i by the following method.

- With probability r , where $0 \leq r \leq 1$, bidder i is a *single-minded* bidder, and with probability $1 - r$, bidder i is a unit-demand valuation bidder.
- If bidder i is a single-minded bidder, i wants to have bundle B , where $|B| = k$. B is chosen randomly, and the valuation for B is randomly chosen from $[0, 1000k]$. The gross utility of i is 0 if i fails to obtain any single good in B .
- If bidder i is a unit-demand valuation bidder, i wants to have any one of k goods. These goods are chosen randomly, and the valuation for the good is randomly chosen from $[0, 1000]$.

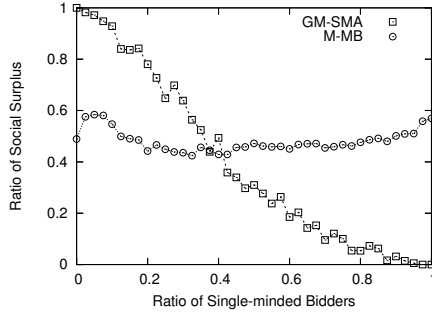


Figure 1: Ratio of Social Surplus

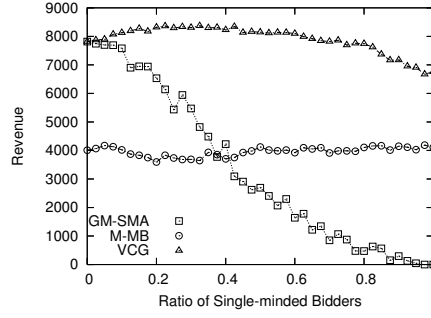


Figure 2: Revenue

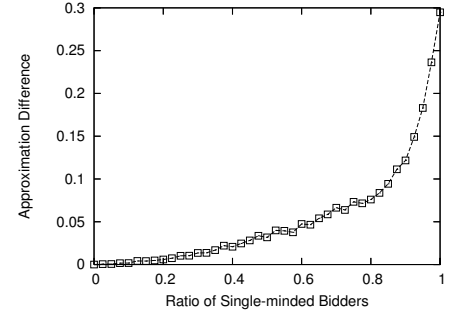


Figure 3: Approximation Difference

If bidder i is a unit-demand valuation bidder, we use $v'(\cdot, \cdot) = v(\cdot, \cdot)$. If bidder i is a single-minded bidder, we approximate v using an additive valuation, where $v_{ij} = v(B, \theta_i)/|B|$ for $j \in B$.

We set $N = 50$, $M = 10$, and $k = 4$, and generated 100 instances. Figure 1 shows the average social surplus of the GM-SMA and the M-MB Protocol by varying the ratio of single-minded bidders. If the ratio is zero, all bidders have unit-demand valuation and their valuations are submodular. Thus, the outcome of the GM-SMA is equivalent to that of the VCG. The social surplus is normalized by the social surplus of a Pareto efficient allocation. Figure 2 shows the revenue of the GM-SMA, M-MB, and VCG protocols. In the VCG protocol, we assume each bidder declares his/her true valuation without using false-name bids. We can see that when the ratio of single-minded bidders is less than 0.4, the GM-SMA outperforms the M-MB protocol both in terms of the efficiency and the revenue.

Also, to evaluate the accuracy of the submodular approximation, for each bidder, we calculate *approximation difference* defined as follows:

$$\frac{U^*(M, \Theta_{N \setminus \{i\}}) - V^*(M, \Theta_{N \setminus \{i\}})}{V^*(M, \Theta_{N \setminus \{i\}})}.$$

If this approximation difference were smaller, the submodular approximation would be more accurate. Figure 3 shows the average approximation difference. We can see that the efficiency and the revenue of the GM-SMA critically depend on the accuracy of the submodular approximation.

9. CONCLUSIONS

In this paper, we developed a new combinatorial auction protocol called the Groves Mechanism with SubModular Approximation (GM-SMA). The basic ideas of this protocol are as follows: (i) it is based on the VCG protocol, (ii) it utilizes submodular approximation, i.e., when calculating the payment of bidder i , we approximate the valuations of bidders other than i by using a submodular valuation function.

The GM-SMA satisfies the following characteristics: (1) it is false-name-proof, (2) each winner is a bidder who is included in a Pareto efficient allocation, (3) as long as the allocation is Pareto efficient, the protocol is robust against the collusion of losers and the outcome is in the core. The GM-SMA is the first protocol that satisfies all three of these characteristics.

Simulation results showed that the GM-SMA achieves a better social surplus and seller's revenue than an existing

false-name-proof protocol, as long as the submodular approximation is close enough to the original valuations.

The efficiency and the revenue of the GM-SMA critically depend on the accuracy of the submodular approximation. Our future works include investigating better approximation methods.

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