

Computationally Efficient and Revenue Optimized Auctioneer's Strategy for Expanding Auctions

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ABSTRACT

An expanding auction—a special type of an ascending-price open-cry auction—allows the auctioneer to dynamically add (identical) items to the single item offered initially. Expanding auctions are becoming an increasingly popular business-to-consumer auction mechanism. The auctioneer's revenue from an expanding auction depends, in large part, on its schedule for increasing the number of units offered. As we show, the naïve strategies commonly used for increasing the number of items offered in contemporary e-commerce implementations of expanding auctions are sub-optimal. In this study we provide a strategy that, given some assumptions about buyers' behaviors, maximizes the expected revenues of the auctioneer in an expanding auction. We model the auction process as a state graph in which nodes are auction states and edges are transitions. With this model, finding the optimal strategy is equivalent to solving a search problem on the state graph. We prove that the search problem to be solved, although seemingly exponentially complex, is actually linearly bounded. Based on this result, we introduce an informed decision strategy that optimizes the auctioneer's revenue.

Categories and Subject Descriptors

I.2.11 [Distributed Artificial Intelligence]: Intelligent agents and multiagent systems.

Keywords

Expanding auction, auctioneer's revenue.

1. INTRODUCTION

Auctions are a popular electronic commerce tool [7]. Multi-unit auctions are also in use, however they are not as widely deployed as single-unit auctions are. This is despite the availability of an array of theoretical multi-unit auction solutions [5, 8]. We believe that the reasons for the limited use of multi-unit auctions are practical limitations of existing solutions such as high computational costs, difficulty of optimizing auctioneer's revenues [9], attack vulnerability, etc. The expanding multi-unit auction (initially studied in [11], though in limited detail), which

is in use in several business-to-consumer (B2C) sites, is not an exception in this respect. Agents might be able to resolve some of these problems [10, 12], either on behalf of auctioneers or on behalf of buyers, however they should be provided with appropriate strategies for such decisions. The auctioneer's strategies for placing additional items up for sale in the expanding auction that are currently implemented in e-commerce sites do not optimize its revenues. In this study we overcome this limitation. We provide a novel strategy for the auctioneer to decide whether it should increase the number of items sold, or keep the number unchanged. The strategy is computationally feasible in real-world auctions, and it optimizes the auctioneers' expected revenues. This work is the first to provide such a strategy for an expanding auction.

Various mechanisms were developed to support the sale of multiple items [5, 7]. Some of these are sealed-bid auctions, e.g., Generalized Vickrey Auction (GVA) [3, 13]. GVA ensures that bidders expose their true valuation functions, which are defined over the possible allocations of the goods. GVA calculates the allocation that maximizes the sum of the bidders' utilities hence maximizing the auctioneer's revenue. The main disadvantage of this mechanism is that it is too complicated for practitioners, and in particular for consumers, to understand [2].

Ascending multi-unit auctions [2, 4] provide a reliable process of price discovery. This is important in auctions where a common value component is present, as is the case for commodities sold to consumers. An ascending bid multi-unit auction may be realized by enhancing the demand schedules approach. This method may be considered as a multi-round version of the multi-unit sealed bid mechanism [4]. Associated activity rules ensure that every bidder will contribute to price discovery as long as the auction process proceeds.

A better open format mechanism for allocating multiple units is the ascending clock mechanism [2]. Compared to the ascending bid multi-unit format this design has many advantages. Bidders have to submit only a single quantity in each round instead of a demand schedule, which makes the auction easier to implement and more comprehensible. Collusions are avoided by this mechanism, yet a uniform price is set in a similar manner to the uniform price auctions discussed earlier. This protocol yields revenues similar to the expected outcome with GVA. The main disadvantage here is that the number of units sold must be smaller than the number of units requested (as demand is greater than supply). Therefore this mechanism is more suitable to limited supply

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goods like communication spectrum or treasury bills, but not for commodities.

The expanding auction format is not well studied in terms of its theoretical properties. It is an open ascending price auction, and thus it has similarities to other such protocols. However, it has a simple, comprehensible user interface. Further, it is simple to implement. And as we show in this study, optimizing the auctioneer's expected revenues can be performed in linear time.

2. EXPANDING AUCTIONS

One of the reasons for multi-unit auctions' lack of popularity as B2C Internet auctions may be the likelihood of premature termination of the auction execution process. Premature termination may occur when the number of participants in the auction does not exceed the number of units offered (assuming each bidder desires one unit of the good). In single unit auctions such premature termination has a minor chance of occurring because the participation of a single bidder is usually unlikely. In multi-unit auctions the auctioneer may set a reservation price to prevent the loss expected from premature termination. However, this approach may deter bidders from the auction.

An emerging new auction type used in B2C e-commerce sites, which addresses the problem of premature auction termination, is the expanding multi-unit auction¹ (in short: the expanding auction). In an expanding auction, initially one item of a good is offered to the bidders. The bidders are free to raise their bids whereas the auctioneer is free to raise the number of units offered by one each time a new bid is received, with no upper limit on the number of units to be offered. The auction terminates when no more bids are placed. The winner list consists of the n highest bids offered where n is the number of units offered upon auction termination. The first bidder to enter the winner list is the first one to leave it. Similarly, with the Japanese auction [6] the auctioneer sets a bid rate used to gradually raise the price. Bidders are not allowed to raise their bids at a rate other than the one set by the auctioneer. It is assumed that each bidder desires one unit of the good and is risk neutral, therefore his/her utility function is given by $U_i = V_i - p$, where p is the price paid and V_i is bidder i 's valuation of the good.

The expanding auction increases the auctioneer's control over the execution process. In a typical multi-unit auction the auctioneer has no control over how the execution will proceed once the initial parameters have been set. In contrast, in the expanding auction, the number of units offered may be changed dynamically. This allows the auctioneer to terminate the execution by gradually raising the number of units offered, or to continue the process to a certain extent by preserving the number of units offered. The control granted enables the auctioneer to set the number of units offered according to the number of bidders participating in the auction, making it easier to avoid the premature termination phenomenon observed in typical multi-unit auction mechanisms.

¹ We found no reference in the literature for the mechanism described here as the 'Expanding multi-unit auction' mechanism.

The most important aspect in the implementation of an expanding auction is the decision under which conditions to raise the number of units offered and under which to preserve it unchanged. This decision has a significant effect on auction execution, and more importantly on the auctioneer's revenues.

A simple but naive strategy for addressing this issue, used by B2C sites, is the step function strategy. For example, a step function strategy is used by an auction site named OISale. (details are in the OISale statute [1]). The strategy is based on a step function set by the auctioneer, composed of <price level, number of units> pairs that determine the number of units to be offered when the current bid reaches a certain level. Figure 1 depicts the step function strategy. When the current bid reaches p_1 the auctioneer offers one more unit, raising the number of units offered to 2. When the price reaches p_2 the auctioneer offers an additional unit raising the number of units to 3, and so on.

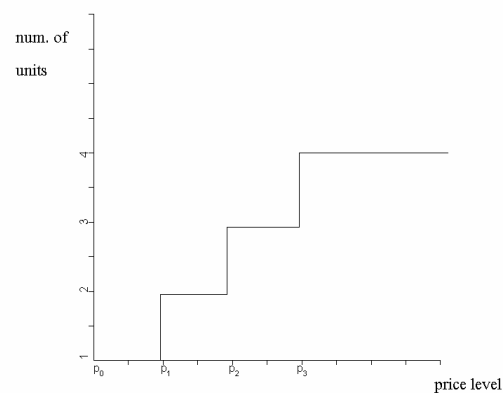


Figure 1. A step function policy

The step function strategy is simple to implement, however it is not trivial to configure its parameters in order to maximize the auctioneer's revenue. The same step function may yield higher revenues under certain conditions and lower revenues in others. This is illustrated in Example 1.

Example 1. Let (b_1, b_2, b_3) be 3 bidders with maximal bidding values (100, 1000, 2000), correspondingly. The bidders participate in an expanding auction in which the minimum bid rate is 10 and the step function is as given in Figure 1. Suppose that $p_1=100$; b_2 and b_3 may win the auction with bids equal to (100, 110) respectively. If $p_1=1000$, b_2 and b_3 may win the auction with bids equal to (1000, 1010), resulting in higher revenues obtained by the auctioneer compared to $p_1=100$. In contrast, when the bidding values are (100, 110, 120), $p_1=100$ results in higher revenues obtained by the auctioneer. With $p_1=100$, b_2 and b_3 will win the auction with bids equal to (100, 110), respectively, whereas with $p_1=1000$ only b_3 will win the good at a price of 120.

The goal of this study is to overcome the weakness of the naïve step function policy presented above. We devise a strategy that, by taking into account the expected maximal bidding values, outperforms the step function strategy. It is important to note that this study assumes the new strategy it offers will maintain its optimality as long as the behavior of the bidder population does not undergo a fundamental change. Clearly, in a population of rational bidders, this assumption

might not hold. Yet, typical human participants in B2C sites are not rational bidders. Therefore, the suggested new strategy should be valuable for contemporary expanding auction sites. Optimal bidder strategies and their effect on the suggested auctioneer's strategy are left for future research and are out of the scope of this paper.

3. THE INFORMED DECISION STRATEGY

We model the expanding auction by a state graph. The nodes of the graph represent the states of the auction at a given point in time, and the edges represent transitions between states. Each state denoted by S has the following properties:

- The price a bidder must pay to join the winner list, denoted by l_d , where d denotes the bid sequence. For instance, l_2 denotes the price a bidder must pay to join the winner list after 2 bids have already been submitted and accepted by the auctioneer. Note that all states that may be formed after the auctioneer has received exactly d bids share the same price level l_d .
- The number of units the auctioneer is willing to sell at price l_d , denoted by n_s .
- A list B_s of maximal bidding values² (MBV) of the bidders currently participating in the auction. B_s contains MBVs of bidders which are either in the winner list, or willing to pay more than l_d and may therefore join the winner list at a later stage.
- A list of MBVs of the bidders in the winner list, denoted by B_{sw} . Clearly, $B_{sw} \subseteq B_s$.

Two operators will define the transition between states. The operators are defining the auctioneer's decision on *how to proceed with the auction process* given a particular state. The *raise()* operator corresponds to the auctioneer's decision to raise the number of units by one whereas the *preserve()* operator corresponds to his/her decision to keep the number of units offered unchanged. The operators are activated on states and produce new states.

The winner list is a queue that consists of bids placed by bidders. The size of this queue is equal to the number of units offered by the auctioneer. Once a bidder enters the winner list he/she is positioned at the end of the list. The first bidder to leave the list is the first bidder that entered the list (FIFO). The transition between states may affect the size of the queue, that is, the size is increased by one once the auctioneer decides to raise the number of units offered, otherwise the size remains unchanged.

Example 2. This example illustrates the state tree representing possible executions of an expanding auction process (cf. Figure 2). It has the following properties:

- The initial number of units offered by the auctioneer denoted by n_0 is 1.
- The reservation price denoted by l_0 is 90.

² The maximal bidding value of a bidder is the maximal bid he/she is willing to offer during the auction. This value may differ from the private value, depending on his/her strategy.

- The minimum bid increment denoted by Δ is 10.
- The initial list of the MBVs of the participating bidders denoted by B_0 is (100,120,130).

To simplify the illustration, in this example (and only in this example) it is assumed that the auctioneer first receives bids originating from bidders that have lower MBVs than other participating bidders have. This means that if multiple bidders submit equal bids at the same price level l_d the auctioneer will accept the bid originating from the bidder with the lowest MBV among the MBVs of the submitting bidders. The assumption ensures a single representation of the state graph illustrated in Figure 2. It will not be assumed further that this assumption holds.

In Figure 2, a state S_i is represented as follows: $S_i=(b_1, b_2, b_3 | n_i)$. In this representation, b_i refers to the MBV of bidder i , and n_i is the number of items offered by the auctioneer. Within the list of MBVs, some may be emphasized in bold-face font. The emphasized MBVs correspond to the bidders which are in the current winner list. Note that since not all bidders participate in the auction at all states, in some states the MBV list includes only a subset of bidders.

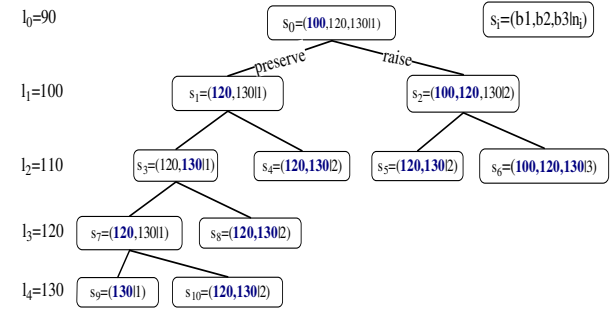


Figure 2. A state graph representing the dynamics of an auction process

The auction in Figure 2 starts with bidder b_1 obtaining the unit offered at 90\$. In case that the auctioneer decides to raise the number of units offered, bidder b_2 joins the winner list by offering 100\$ for the second unit. A transition from S_2 to its *raise* descendent S_6 results in b_3 joining the winner list with a bid of 110\$. More interesting is the transition from S_2 to its *preserve* descendent, which results in the removal of b_1 from the winner list to be replaced by bidder b_3 . Bidder b_1 also leaves the auction since he/she is not willing to pay more than 100\$ for the good. The auction halts when all bidders in a given state are also in the winner list and therefore do not have the incentive to bid. These states are the terminating states ($S_4, S_5, S_6, S_8, S_9, S_{10}$).

It is important to observe that given the MBVs of bidders entering the auction we can infer the most beneficial strategy by forming and inspecting the state tree. In this example, raising the number of units offered repeatedly leads the process to state S_6 in which the auctioneer gains $100+100+90=300$, thus outperforming all other terminating states.

In developing a decision strategy that is based on the state tree described, we face two fundamental problems. Firstly, the MBVs of bidders entering the auction are unknown. Secondly,

the search for the most beneficial strategy may be expensive in terms of complexity of the search problem to be solved.

To address the first problem, we assume that for each bidder b_i participating in the auction, a probability distribution function f_i of his/her maximal bidding value is given or can be learned from historical biddings. The probability distribution function f_i depicts the price bidder i is willing to pay for the good compared to its catalog price. The comparison to the catalog price is expressed in percentage points thus normalizing the data collected across different types of goods.

Using the statistical data gathered, a strategy that takes into account the bidding profile of the bidders participating in the auction can be devised. The informed decision strategy (in short, IDS) we propose is such a strategy. It is a simulation-based strategy that, given the current state of the auction, attempts to determine the auctioneer's action that would yield the best results. It does this by generating MBVs vectors of bidders participating in an auction and the current state of the auction. To find what the auctioneer's best choice for proceeding the auction is, a search problem that takes into account all possible scenarios has to be solved for each MBV vector generated. A solution should determine whether the auctioneer should increase the number of units sold by one, or preserve it unchanged. The knowledge applied to the simulation quantifies the value of each choice, thus improving the chance of making a good decision.

Given the auctioneer's reservation price l_0 and the maximal bidding value of the bidders participating l_{max} , we can establish a bound on the number of bid levels. The worst case is arrived at when only one unit is offered during the lifetime of the auction. Hence, the number of bid levels is bounded by $(l_{max} - l_0)/\Delta$. Since the branching factor of the tree is 2, the number of states in the state tree is bounded by $O(2^{(l_{max}-l_0)/\Delta})$. Therefore, computing the complete state tree is too expensive in terms of the time it takes for the auctioneer to respond to bid offers submitted by the participating bidders. The responsiveness of the auction process will be downgraded significantly, causing bidders to leave the auction. In Section 4 we show that the complexity of the search problem can be reduced to linear time.

Provided that bidders' MBVs can be estimated and the complexity of the search problem can be reduced to linear time, the IDS may be implemented in real-world Internet auctions. We presume that an auctioneer implementing the IDS would like to control the amount of time allocated to computing the action needed to maximize his/her revenues. This will enable the auctioneer to guarantee an acceptable responsiveness level for new bids received.

The algorithm presented below describes the implementation of the IDS within the auction execution process. The time frame allocated by the auctioneer for computing the optimal strategy is denoted by t_p , and the time in which a new bid that raises the price of the good sold is received is denoted by t_0 :

1. $t_0 \leftarrow \text{now}$, $nRaise \leftarrow 0$, $nPreserve \leftarrow 0$
2. Generate an MBV vector v_{sample} based on the probability distribution functions of participating bidders.

3. Using the IDS compute the optimal action to be taken based on vector v_{sample} . If raising the number of units sold by one is the best course of action then $nRaise \leftarrow nRaise + 1$, else $nPreserve \leftarrow nPreserve + 1$.
4. If $\text{now} < t_0 + t_p$ go to step 2.
5. If $nRaise > nPreserve$ raise the number of units sold by one, if $nRaise < nPreserve$ preserve the number of units sold. Otherwise choose one of the operators randomly.

Detailed explanations of step 3 above are in the proceeding sections.

4. CALCULATING THE STRATEGY IN LINEAR TIME

Given a state S and the required statistical data for deriving the MBVs of the bidders currently participating in an auction, it is possible to compute the optimal set of actions the auctioneer has to perform to maximize his/her revenue. As shown above, the complexity of this analysis is bounded by $O(2^{(l_{max}-l_0)/\Delta})$.

This complexity is infeasible for real-world applications of auction sites. In this section we show that the complexity of the search problem is actually linear in the maximal number of bid levels of the auction. We do this by utilizing the knowledge we have with regards to the properties of the problem domain we address. This knowledge enables us to prune a substantial part of the states in the graph. We show that the search problem can be bounded by $(l_{max} - l_0)/\Delta$, thus proving that the analysis can be completed in a linearly bounded computation time. This result enables the implementation of the IDS in real-world Internet auctions.

The proof starts by introducing a set of invariants that capture some basic properties of the state tree. Using these invariants, we initially prove that the number of states to be analyzed can be reduced to $O(((l_{max} - l_0)/\Delta)^2)$, thus providing a polynomial time bounded analysis procedure. Based on the first proof, we later prove that the number of states that have to be analyzed is linearly bounded by $O((l_{max} - l_0)/\Delta)$.

Invariant 1. Given a state S , a descendent state will be produced if there exists a bidder $b^* \in B_S$ which is not in the current winner list, and his/her MBV satisfies $p_{b^*} \geq l_d + \Delta$. When this condition holds, state S will have 2 descendent states; otherwise it will have no descendent states. Formally: $b^* = \exists b \in B_S \mid b \notin B_{sw}, p_b \geq l_d + \Delta$

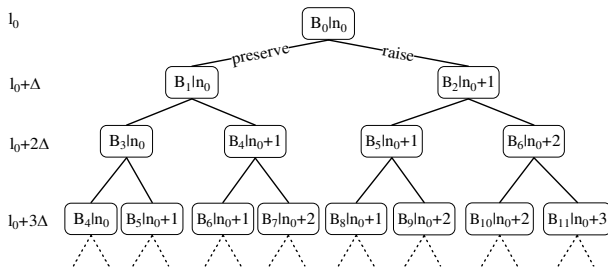
Invariant 2. Given a state S , we denote the bidders' list that would result from the auctioneer's decision to raise the number of units offered by $B_{raise(s)}$. The list $B_{raise(s)}$ consists of the bidders belonging to S 's winner list and an additional bidder b^* that joins S 's winner list (see Invariant 1). Bidders that do not participate in S 's winner list, but their MBVs satisfy $p_b > l_d + \Delta$, are included in the descendent state bidder list although they do not participate in its winner list. Formally:

$$B_{raise(s)} = \{(b \in B_S \mid (b \in B_{sw}) \vee (p_b > l_d + \Delta) \vee (b = b^*))\}$$

Invariant 3. Given a state S , we denote the bidders' list that would result from the auctioneer's decision to preserve the number of units offered by $B_{preserve(s)}$. When traversing from S to its preserve descendent state, a bidder $b' \in B_{sw}$ will be removed from S 's winner list, substituted by another bidder b^*

$$B_{preserve(s)} = \{(b \in B_S \mid (b \in B_{sw}, b \neq b') \vee (p_b > l_d + \Delta) \vee b^*)\}$$

Figure 3 illustrates a formal description of the state graph formed as a result of computing an auction execution. Each node S_i in the graph consists of its bidder list denoted by B_i . The initial number of units offered by the auctioneer is denoted by n_0 and the reservation price set initially by the auctioneer is denoted by I_0 . The auctioneer's decision to raise the number of units is reflected in the graph by traversing to the right. The descendent state created is characterized by an additional unit offered to the participating bidders and a price level that is higher by Δ . In case that the auctioneer preserves the number of units the graph branches to the left forming a state that is characterized by the same number of units offered and a price level that is higher by Δ . Note that the state tree formed is not necessarily a complete tree but is shown as such to cover cases in which a complete tree is formed.



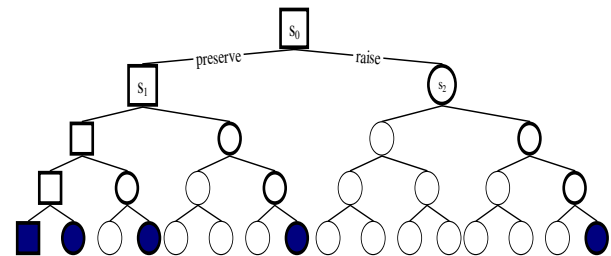
For a given state S_i we define its preserve branch denoted by $preserve_branch(S_i)$ as the set of states formed by iteratively traversing left starting at state S_i until we reach a terminating state. The following algorithm defines (and constructs) $preserve_branch(S_i)$:

- In a similar manner we define $raise_branch(S_i)$. $raise_branch(S_i)$ is formed by iteratively traversing right, starting at state S_i to its raise descendents. $raise_branch(S_i)$ is constructed by the algorithm:

3. $raise_branch(S_i) \leftarrow raise_branch(S_i) \cup \{raise(S)\}$
4. $S \leftarrow \{raise(S)\}$
5. go to step 2

Theorem 1. Let T_0 be a state tree of an expanding auction whose root is S_0 . Let S_{T_0} be the set of terminating states of T_0 . Let $S_{T_0}^*$ be the subset of terminating states reached by traversing on *raise_branch*(S_i) for each state S_i in *preserve_branch*(S_0). We define $S_{T_0}^*$ formally as follows. Let $P_{T_0} = \text{preserve_branch}(S_0)$ and $R_{T_0} = \bigcup_{s \in P_{T_0}} \text{raise_branch}(s)$ then $S_{T_0}^* = \{s \in R_{T_0} \cup P_{T_0} \mid \deg(s) = 0\}$. This theorem presents two claims: (1) $S_{T_0} = S_{T_0}^*$. That is, $S_{T_0}^*$ covers all the terminating states in the state tree produced by the auction execution. (2) The number of states to be visited for computing $S_{T_0}^*$ is bounded by $O(((l_{\max} - l_0)/\Delta)^2)$.

From Theorem 1 it follows that one needs to search only $O(((l_{\max} - l_0)/\Delta)^2)$ nodes to find the best auctioneer's action. For space reasons, the proof of Claim 1 is omitted. The definitions introduced in Theorem 1 and the approach to proving it are illustrated in Figure 4.



Theorem 2. Let S_1, S_2 be two states such that $S_2 = \text{preserve}(S_1)$. Let S_{1r}, S_{2r} be the terminating states reached by traversing $\text{raise_branch}(S_1)$ and $\text{raise_branch}(S_2)$, respectively. Let $\text{Rev}(S_{1r})$ and $\text{Rev}(S_{2r})$ be the revenues collected by the auctioneer at states S_{1r} and S_{2r} , respectively. Given $\text{Rev}(S_{1r})$, $\text{Rev}(S_{2r})$ can be computed as following:

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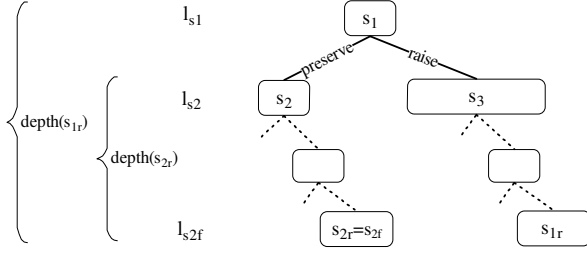


Figure 5. The auctioneer's revenue in case that $P_b < (depth(S_{1r})+1) \cdot \Delta + l_{s1}$

This theorem presents three claims. Claim (1) states that, in the case of $P_b < (depth(S_{1r})+1) \cdot \Delta + l_{s1}$, when traversing from S_1 to S_2 the auctioneer loses the revenue received from bidder b 's participation in the winner list at state S_1 . Therefore the auctioneer's revenue at state S_{2r} is smaller by P_b than its potential revenue at state S_{1r} .

Claim (2) states that, in the case of $P_b \geq (depth(S_{1r})+1) \cdot \Delta + l_{s1}$, bidder b submits an additional bid at state S_{2f} and thus increases the potential revenue at state S_{2r} by $(depth(S_{1r})+1) \cdot \Delta$ compared to the revenue obtained at state S_{1r} .

Claim (3) states that the number of states produced while traversing from S_2 to S_{2r} equals $depth(S_{1r})$ in case $P_b \geq (depth(S_{1r})+1) \cdot \Delta + l_{s1}$ or equals $depth(S_{1r})-1$ otherwise.

Proof: Traversing $raise_branch(S_1)$ to a terminating state we discover $depth(S_{1r})$ new states. These states were produced as a result of $depth(S_{1r})$ bids received by the auctioneer. The same bids are received by the auctioneer while traversing from S_1 to S_2 and then along $raise_branch(S_1)$. Observing state S_2 and $raise(S_1)$ it is evident that $B_{s2} \cup \{b\} = B_{raise(s1)}$ since while traversing from S_1 to S_2 bidder b , which leaves the winner list, might leave the bidder list as well. Assuming that $B_{s2} = B_{raise(s1)}$ we can conclude that since these states share the same price level l_{s2} , all $depth(S_{1r})$ bids received by the auctioneer while traversing $raise_branch(S_1)$ will produce new states while traversing from S_1 to S_2 and then along $raise_branch(S_2)$. Moreover, knowing that none of these bids belong to bidder b , which participates in S_1 winner list and therefore does not have the incentive to send bids to the auctioneer, we conclude that even if $B_{s2} = B_{raise(s1)} \setminus b$, all $depth(S_{1r})$ bids will produce new states while traversing from S_1 to S_2 and then along $raise_branch(S_2)$ until we reach state S_{2f} . Observing states S_{1r} , S_{2f} shown Figures 5 and 6 we conclude that they both share the same price level since they are both at a distance of $depth(S_{1r})$ from S_1 , and that $B_{s2f} \cup \{b\} = B_{s1r}$, since b has left the winner list while traversing from S_1 to S_2 , and all bids that were accepted by the auctioneer while traversing from S_1 to S_{1r} were also accepted while traversing from S_1 to S_{2f} . It is important to observe that in case that $b \chi B_{s2f}$, b is the only bidder in state S_{2f} that is willing to raise the price level. Otherwise, there exists another bidder b' that is willing to do so; that is, $P_{b'} \geq l_{s2f} + \Delta$, and therefore the same bidder b' should participate in S_{1r} leading us to the conclusion that S_{1r} is not a terminating state.

In order for bidder b to participate in S_{2f} , the condition $P_b \geq depth(S_{1r}) \cdot \Delta + l_{s1}$ must hold true. Further, bidder b will be willing to raise the price level if $P_b \geq (depth(S_{1r})+1) \cdot \Delta + l_{s1}$; this results in a new state in which all bidders participate in the

winner list, and therefore it is a terminating state. That is, we reached S_{2r} (see Figure 6). The revenue obtained in this case equals $Rev(S_{1r}) + (depth(S_{1r}) + |B_{s1w}|) \cdot \Delta$, since bidder b pays $l_{s1} + (depth(S_{1r}) + 1) \cdot \Delta$ instead of $l_{s1} - \Delta \cdot (|B_{s1w}| - 1)$ in state S_{1r} . The number of states produced while traversing from S_2 to S_{2r} consists of $depth(S_{1r})-1$ states produced while traversing from S_2 to S_{2f} and an additional state produced in this case, therefore $depth(S_{1r}) = depth(S_{2r})$.

If the condition above does not hold, bidder b will not participate in S_{2f} while other bidders participate in the winner list. Therefore, we reached the terminating state S_{2r} . Since bidder b does not participate in the winner list the auctioneer's revenue decreases by $l_{s1} - \Delta \cdot (|B_{s1w}| - 1)$ compared to S_{1r} revenue. The number of states produced while traversing from S_2 to S_{2r} consists of $depth(S_{1r})-1$ states produced while traversing from S_2 to S_{2f} , therefore $depth(S_{1r})-1 = depth(S_{2r})$. ■

For space limitations, the proof of the theorem is presented only in part. Theorem 2 provides a better understanding of the auction execution process. Based on this understanding, we can refine, and precisely define, step 3 of the algorithm presented in Section 3. That step computes the optimal action to be taken based on the current state of the auction denoted by S_1 and the MBVs of the participating bidders given by vector v_{sample} .

Theorem 3. The number of states to be visited for computing IDS is bounded by $O((l_{max} - l_0)/\Delta)$. The proof is expressed by the algorithm presented below. ■

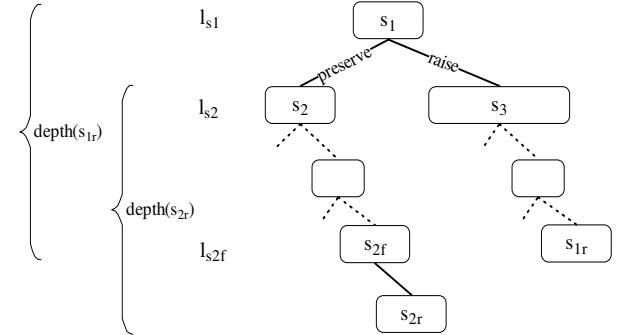


Figure 6. The auctioneer's revenues in case that $P_b \geq (depth(S_{1r})+1) \cdot \Delta + l_{s1}$

Figure 7 illustrates the details of step 3 of the algorithm. Starting from the current state of the auction, denoted by S_1 , compute $depth(S_{1r})$ and $rev(S_{1r})$. This is done by computing each state produced along S_1 raise branch until S_{1r} is reached. Then, compute $depth(y_r)$ and $rev(y_r)$, where y_r is the terminating state reached by traversing from S_1 to its preserve descendent and then traversing along its descendent raise branch. The computation of $depth(y_r)$ and $rev(y_r)$ is immediate (as shown in Theorem 2). In case that the revenue expected by reaching y_r is higher than the one expected by reaching x_r , traversing to $y = preserve_desc(x)^3$ is preferable. That is, it is better to preserve the number of units offered. Otherwise, unless y is a terminating state, it is necessary to check whether y can lead to a better revenue than $rev(x_r)$ could. This means

³ $preserve(x)$ appears as $preserve_desc(x)$ in Figure 7.

that it is necessary to compute the optimal strategy as if y is the current state ($x \leftarrow y$). Since we already know the value of $depth(y_r)$ and $rev(y_r)$, there is no need to perform the first step of the algorithm, which is time-consuming. The number of states computed along the execution of the algorithm is composed of $(l_{max}-l_0)/\Delta$ states computed at the initial phase of the algorithm and $(l_{max}-l_0)/\Delta$ states computed until reaching a terminating state. Summing up the number of states we compute during the algorithm we find out that this number is linear bounded by $O((l_{max}-l_0)/\Delta)$. This result proves Theorem 3.

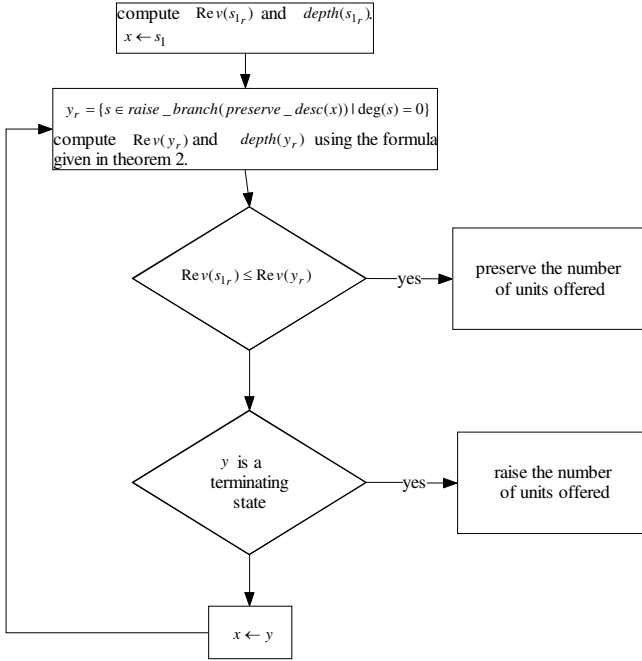


Figure 7. Illustration of step 3 of the informed decision strategy algorithm.

5. EXPERIMENTAL EVALUATION

The theoretical model presented above was tested experimentally to assess its added value compared to other techniques for running auction sites in the context of conducting expanding auctions.

Our simulation captured the dynamics of the expanding auction mechanism discussed above. Two strategies have been tested with such auctions:

- The informed decision strategy we propose.
- A post optimal decision strategy – this strategy is equivalent to the IDS except that here instead of sampling a MBV vector based on statistical knowledge gathered, the strategy considers the real MBV generated for each bidder participating in the auction. This strategy is superior compared to the IDS and serves as a measure of evaluating the quality of the solution offered by the IDS.

Both strategies have been tested in two types of environments: a homogenous environment in which bidders participating are sharing the same probability distribution function, and;

heterogeneous environment in which the probability distribution function of each bidder is distinct.

For each setting, the parameters of the experiment were set to allow producing statistically significant results. For example, for an experiment in which the bidders' MBV distribution has a low variance to expectation ratio, 1000 trials were sufficient to arrive at statistical significance. Testing the same feature in the case of a high variance vs. expectation ratio required 4000 trials to produce stable results.

We have conducted a series of experiments in which all bidders originated from a homogenous distribution function; the variance/expectation ratio of the distribution function was set to 0.25. The number of bidders participating in the auction was set to 100. The results show that the IDS gains 70-80 percent of the post-optimal strategy in terms of revenue. Increasing the variance/expectation ratio shows degradation in the results compared to the post-optimal strategy. This is expected as the uncertainty of the sample MBV vectors produced increases.

The experiments with homogeneous distribution functions served as a reference. We have also performed experiments where we mixed homogenous and heterogeneous bidders. The results obtained when conducting experiments in mixed environments were highly dependent on the variance/expectation ratio, the portion of bidder types participating in the auction and the bidder selection distribution function. Interestingly, for experiments where the mutual variance/expectation ratio was set to 0.25, the portion of bidder types was 50% of each, and the bidder selection distribution function was uniform, the results can be computed from results of other experiments. In particular, one can average the results of similar experiments performed on fully homogenous and fully heterogeneous bidder populations.

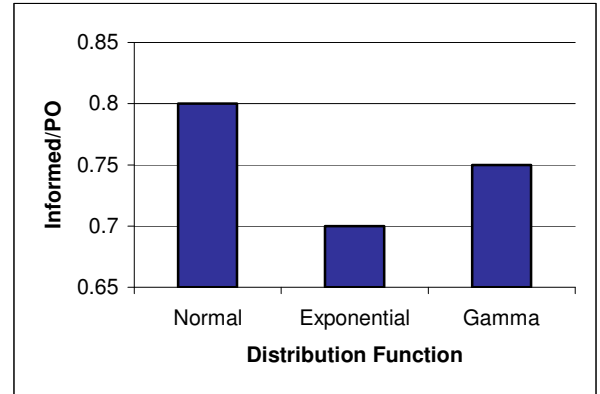


Figure 8. Informed Decision Strategy vs. Post-optimal strategy (variance/expectation ratio=0.25).

In a set of experiments, using settings as stated above, we have performed experiments in which normally distributed bidders, exponentially distributed bidders and Gamma distributed bidders participate in the same auction. The expected revenue arrived at was 0.7-0.8 of the post-optimal results (cf. Figure 8).

Note that we did not perform experiments that compare our strategy to the naïve step strategy currently used in web implementations of the expanding auction. Such experiments could have further exemplified the advantage of our strategy. However, the reason for not performing such experiments is that, as we have shown by example, the step function can lead to results far from optimal. In contrast, because our strategy selects the best possible action given the knowledge available, it would always outperform the step strategy.

6. CONCLUSION

The expanding auction mechanism discussed in this paper is different from other mechanisms, such as the ascending clock mechanism or the GVA mechanism, in the sense that it does not require that the auctioneer set the number of units he/she is willing sell before the auction starts. This property makes the expanding multi-unit auction mechanism suitable for B2C commodities sales, where the auctioneer might prefer flexibility in the number of units offered to the participants.

The study performed here introduces a practical solution to the management of expanding multi-unit auctions. The informed decision strategy proposed was proven, theoretically and empirically, to address computation and time constraints that must be met by real life auctions. It was shown that the optimal strategy for searching the problem space—seemingly exponentially bounded—can be solved in linear time. The naïve step function strategy used in B2C expanding auction sites provides a sub-optimal mechanism to establish the price levels in which the auctioneer should raise the number of units offered. The revenues obtained by our new strategy outperform this naïve strategy.

Empirical results have shown that under diverse probability distribution functions the informed decision strategy we introduced produces significant gains. The auctioneer's expected revenues constitute 70%-80% of the post-optimal revenues. This is an excellent result given the probabilistic nature of the information used by our strategy. Future research needs to examine the effect of rational bidder strategies on our result. Regardless, IDS is very useful by itself, as bidders in contemporary dynamic sales sites hardly implement rational bidding.

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