

# A Robust Combinatorial Auction Mechanism against Shill Bidders

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## ABSTRACT

This paper presents a method for discovering and detecting shill bids in combinatorial auctions. The Vickrey-Clarke-Groves Mechanism is one of the most important combinatorial auctions because it can satisfy the strategy-proof property, individual rationality, and Pareto efficiency, that is, it is the only mechanism that simultaneously satisfies these properties. As Yokoo et al. pointed out, false-name bids and shill bids pose an emerging problem for auctions, since on the Internet it is easy to establish different e-mail addresses and accounts for auction sites. Yokoo et al. proved that VCG cannot satisfy the false-name-proof property, and they also proved that no auction protocol can satisfy all three of the above properties and the false-name proof property simultaneously. Their approach concentrates on designing a new mechanism that has desirable properties, but this is quite complicated. As a new approach against shill-bids, in this paper, we design a mechanism that utilizes VCG and an algorithm for finding potential shill bids. Our mechanism is quite simple compared with Yokoo's approaches [11][12][13]. Our mechanism can judge whether there might be a shill bid from the results of the VCG procedure. We prove a theorem stating that shill bidders cannot increase their utilities unless all shill bidders win in the auction. Based on this theorem, our proposed mechanism compares the agents' utilities in a conventional auction with those in an auction where a shill bidder does not join in the auction. When these agents' utilities are different between the above cases, such agents might be shill bidders. Then, our mechanism allocates items to the shill bidders as a group from the set of items obtained through successful bids by the agent in the conventional auction. This process prevents shill bidders from increasing unfair profits. Furthermore, even though shill bidders par-

ticipate in the auction, the seller's profit does not decrease using our proposed method. Thus, our mechanism detects shill bids when it only detects the possibility of shill bids. Our proposed method has the following three key advantages. First, we propose a method to detect shill bidders by comparison between bidders utilities. Our method is superior than existing complex mechanisms in the point of view of generalization and wide-use, because our auction mechanism employs only VCG. Second, even though there are shill bidders in an auction, incentive compatibility property is preserved using our mechanism. Finally, the schemer, in our mechanism, does never have incentive to make shill bidders. The schemer's utility does not increase in our mechanism even though a schemer make shill bidders. Namely, not to make shill bidders is dominant strategy for the schemer.

## Categories and Subject Descriptors

I.2.11 [Multiagent Systems]: Design, Theory, and Electronic Commerce

## Keywords

Computational Mechanism Design, Combinatorial Auctions, Shill Bids, Vickrey-Clarke-Groves Mechanism, Incentive Compatibility and Pareto Efficiency.

## 1. INTRODUCTION

This paper presents a method for detecting shill bids and for the allocation of items in combinatorial auctions. Auction theory has received much attention from computer scientists and economic scientists in recent years [7]. One reason for this interest is the fact that Internet auctions such as Yahoo Auction and eBay have developed very quickly and widely. Also, auctions in B2B trades are increasing rapidly. Moreover, there is growing interest in using auction mechanisms to solve distributed resource allocation problems in the field of AI and multi-agent systems. Combinatorial auctions have been studied very widely as one of most important auction formats. In a combinatorial auction, bidders can make bids on multiple bundles of different items[9].

The Vickrey-Clarke-Groves (VCG) Mechanism is one of the combinatorial auction mechanisms that are strategy-proof, i.e., the dominant strategy is for bidders to declare their true evaluation value for a good, and its allocation is Pareto efficient. Many scientists in the field of auction

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theory have focused on VCG because of its strategy-proof property. The details of VCG are described in Section 2.

As Yokoo et al. pointed out, false-name bids and shill bids are an emerging problem for auctions, since on the Internet it is easy to establish different e-mail addresses and accounts for auction sites [13]. Bidders who make false-name bids and shill bids can benefit if the auction is not robust against false-name bids. A method to prevent false-name bids and shill bids is a major issue that auction theorists need to resolve.

Yokoo et al. proved that VCG cannot satisfy the false-name-proof property [13]. Moreover, they proved that there is no auction protocol that can satisfy all of the three major properties, i.e., false-name-proof property, Pareto efficiency, and strategy-proof property. They have developed several other complicated auction protocols that can satisfy at least the false-name-proof property. Their approach is to find alternative auction protocols and mechanisms. However, satisfying the false-name proof property prevents them from developing Pareto efficient and strategy-proof protocols.

On the contrary, in this paper we propose a new approach to detecting shill bidders as well as an allocation method in combinatorial auctions. Even though there exist shill bidders, our method can employ the VCG mechanism without using complicated mechanisms such as the protocol proposed by Yokoo et al. Our mechanism can determine whether there might be a shill bid from the results of the procedure of a VCG mechanism.

The mechanism proposed in this paper consists mainly of the following two steps. First, our mechanism judges the possibility of shill bidders based on the result of an auction. Auctions where some bidders bid their valuations as zero are attempted, and other bidders' utilities are calculated in this situation. The utility of each bidder is compared between a conventional VCG and the altered outcome. When there are bidders whose utilities change with the situation, our mechanism judges that such bidders might be shill bidders. Then, items are allocated to successful bidders. However, in this step, the following method of allocation is used, the items are allocated to bidders who are not shill bidders. On the other hand, some items are allocated as a set to a group of shill bidders and bidders who might be shill bidders, for a merged valuation function. This prevents shill bidders from making unfair profits and decreasing total revenue for sellers. Namely, in our approach, we build a mechanism to find shill bids in order to avoid them in VCG. This differs from the approach of Yokoo et al., which builds mechanisms to avoid shill bids. In Yokoo's approach, they have been trying to design a different mechanism that can satisfy desired properties (e.g. false-name proofness). However, the resulting mechanisms have been complicated due to the difficulty of this approach. On the other hand, our mechanism detects shill bidders and regards them as a single bidder. In our mechanism, VCG is employed without any complicated modifications. This gives our mechanism more generic usefulness since VCG is a well known protocol compared with Yokoo's proposed protocols.

A shill bid is defined as two or more bids created by a single person, who can unfairly gain a benefit from creating such bids. The straightforward way to find shill bids is to find a bidder whose utility becomes negative when his/her bids and those of another bidder are merged. The merging method is described in Section 3. However, this straightfor-

ward method requires an exponential amount of computing power, since we need to check all possible combinations of bidders. Thus, we use a theorem involving the characteristics of shill bids and propose a new method for detecting shill bidders. The theorem shows that a group of shill bidders cannot increase their utilities when a shill bidder cannot be a successful bidder. Using this method, the computational costs are reduced dramatically compared with the merge investment method. Comparing bidders utilities between the normal VCG and the above method, the group of shill bidders is detected. Our mechanism allows a set of items to be allocated to a shill bidders group for merged investments. In addition to detecting shill bidders, the VCG's properties are preserved. Further, a seller's revenue never decreases, even though shill bidders bid in the auction.

Our proposed method has the following properties and advantages. First, we propose a method to detect shill bidders by comparison between bidders utilities. Our method is superior than existing complex mechanisms in the point of view of generalization and wide-use, because our auction mechanism employs only VCG. Second, even though there are shill bidders in an auction, incentive compatibility property is preserved using our mechanism but efficiency is not preserved in some cases. Finally, the schemer, in our mechanism, does never have incentive to make shill bidders. The schemer's utility does not increase in our mechanism even though a schemer make shill bidders. Namely, not to make shill bidders is dominant strategy for the schemer.

The rest of this paper consists of the following six parts. In Section 2, we give preliminaries, explaining several terms and concepts of auctions. In Section 3, we explain shill bids and show the characteristics of shill bidding in VCG. In Section 4, the method of detecting shill bidders and allocating items used in this paper is introduced. In Section 5, discussions are presented to show the properties of our mechanism. Finally, we give our concluding remarks.

## 2. PRELIMINARIES

### 2.1 Model

Here, we describe the model and definitions needed for the present work. The auction participants of trading consist of a seller and bidders. The seller prepares multiple items for sale, and bidders place bids for bundles they want to purchase.

- In an auction, we define that a set of bidders/agents is  $N = \{1, 2, \dots, i, \dots, n\}$  and a set of items is  $A = \{a_1, a_2, \dots, a_k, \dots, a_m\}$ .
- $v_i^{a_k}$  is bidder  $i$ 's value, at which the  $i$ th bidder bids for the  $k$ th item ( $1 \leq i \leq n, 1 \leq k \leq m$ ).
- $v_i(B_i^{a_k, a_l})$  is bidder  $i$ 's value, at which the  $i$ th bidder bids for the bundle including the  $k$ th and  $l$ th items ( $1 \leq i \leq n, 1 \leq k, l \leq m$ ). This notation is used when a bidder evaluates more than two items.
- $p_i^{a_k}$  is the payment when agent  $i$  can purchase an item  $a_k$ . When bidder  $i$  purchases the set of bundles of items, the payment is shown as  $p_i(B_i^{a_k, a_l})$ .
- $G_i$  is the bundle of items allocated to agent  $i$ .

- The set of choices is  $G = \{(G_1, \dots, G_n) : G_i \cap G_j = \emptyset, G_i \subseteq G_j\}$ .

**ASSUMPTION 1 (QUASI-LINEAR UTILITY).** *Agent  $i$ 's utility  $u_i$  is defined as the difference between the value  $v_i$  of the allocated good and the monetary transfer  $p_i$  for the allocated good.  $u_i = v_i - p_i$ . Such a utility is called a quasi-linear utility.*

**ASSUMPTION 2 (PRIVATE VALUE).** *In this paper, we employ the assumption that the characteristics of items are private value for all bidders. Each agent's type is independent of the types of the other bidders.*

**ASSUMPTION 3 (SINGLE-MINDED VALUATION).** *We assume single-minded valuation, that is, bidders never bid as substitute values for each item.*

Namely, in this paper, when the number of items in a bundle increases, the total value for the bundle decreases. This means that free disposal is assumed.

Each bidder  $i$  has preferences for the subset  $G_i \subseteq G$  of goods. Formally, each bidder has type  $\theta_i$ , that is, in a type's set  $\Theta$ . Based on the type, we show that the bidder's utility is  $v_i(G_i, \theta_i) - p_i^{G_i}$  when the bidder purchases item  $G_i$  for  $p_i^{G_i}$ . Note that  $(v_i(G_i, \theta_i))$  is bidder  $i$ 's value for bundle  $G_i \subseteq G$ .

## 2.2 VCG: Vickrey-Clarke-Groves Mechanism

The Vickrey-Clarke-Groves mechanism [2][4][10] has some key properties, that is, it's strategy-proof and Pareto efficient if there exists no false-name bid. We say an auction protocol is Pareto efficient when the sum of all participants' utilities (including that of the auctioneer), i.e., the social surplus, is maximized. If the number of goods is one, in a Pareto efficient allocation, the good is awarded to the bidder having the highest evaluation value corresponding the quality of the good.

In the VCG, first each agent tells his/her value  $v_i(G_i, \theta_i)$  to the seller. We omit the "type" notation and simply write  $v_i(G_i, \theta_i) = v_i(G_i)$ . The efficient allocation is calculated as an allocation to maximize the total value:

$$G^* = \arg \max_{G=(G_1, \dots, G_n)} \sum_{i \in N} v_i(G_i).$$

The auctioneer informs the payment amount to the bidders. Agent  $i$ 's payment  $p_i$  is defined as follows.

$$p_i = \sum_{j \neq i} v_j(G_{\sim i}^*) - \sum_{j \neq i} v_j(G^*).$$

Here,  $G_{\sim i}^*$  is the allocation that maximizes the sum of all agents' values other than that of agent  $i$ . This is the allocation in which the total value is maximum when all agents, other than agent  $i$ , bid their values:

$$G_{\sim i}^* = \arg \max_{G \setminus G_i} \sum_{N-i} v_j(G_j).$$

## 3. SHILL BIDS

### 3.1 Weakness against shill bids

In auction research, some papers have reported the influence of false name bids in combinatorial auctions, such as

VCG [13]. These are called "shill bids." Here, we show an example of shill bids.

Assume there are two bidders and two items. Each agent bids for a bundle, that is,  $\{a_1, a_2, (a_1, a_2)\}$ .

Agent 1's value  $v_1(B_1^{a_1, a_2}) : \{\$6, \$6, \$12\}$

Agent 2's value  $v_2(B_2^{a_1, a_2}) : \{\$0, \$0, \$8\}$

In this case, both items are allocated to agent 1 for 8 dollars. Agent 1's utility is calculated as  $12 - 8 = 4$ .

If agent 1 creates a false agent 3, his/her utility increases.

Agent 1's value  $v_1(B_1^{a_1, a_2}) : \{\$6, \$0, \$6\}$

Agent 2's value  $v_2(B_2^{a_1, a_2}) : \{\$0, \$0, \$8\}$

Agent 3's value  $v_3(B_3^{a_1, a_2}) : \{\$0, \$6, \$6\}$

Agent 1 can purchase item  $a_1$  and agent 3 can buy item  $a_2$ . Each agent's payment amount is  $8 - 6 = 2$ , and each agent's utility is calculated as  $6 - 2 = 4$ . Namely, agent 1's utility is 8 dollars (because agent 3 is the same as agent 1).

Yokoo et al. examined the effect of false-name bids on combinatorial auction protocols [13]. False-name bids are bids submitted by a single bidder using multiple identifiers such as multiple e-mail addresses. They showed a formal model of combinatorial auction protocols in which false-name bids are possible. The obtained results can be summarized as follows: (1) the Vickrey-Clarke-Groves (VCG) mechanism, which is strategy-proof and Pareto efficient when there exists no false-name bid, is not false-name proof; (2) there exists no false-name-proof combinatorial auction protocol that satisfies Pareto efficiency; (3) one sufficient condition where the VCG mechanism is false-name-proof is identified, i.e., the concavity of a surplus function over bidders.

### 3.2 Schemer's utility

In this paper, a "schemer" is an auction participant who contrives shill-bids and creates "shill bidders". When shill bidders are not successful bidders, the schemer's utility never increases. Namely, the schemer cannot increase his/her utility by using a "decoy" bidder. The schemer's utility does not increase unless all shill bidders and the schemer become successful bidders. The following theorem shows the characteristics of shill-bids.

**THEOREM 1 (SHILL-BIDDERS MUST BE WINNERS).** *An agent who creates shill bidders cannot increase his/her utility unless a shill bidder wins.*

[Proof] We prove that bidder agent  $i$ 's utility  $u_i$  does not increase when the agent's shill bidder does not win in an auction. When bidder  $i$  does not create any shill bidders, bidder  $i$ 's payment  $p_i$  can be illustrated as  $p_i = \sum_{j \neq i} v_j(G_{\sim i}^*) - \sum_{j \neq i} v_j(G^*)$ . When the bidder agent creates shill bidders, bidder  $i$ 's payment  $p'_i$  is  $p'_i = \sum_{j \neq i} v_j(G'_{\sim i}) - \sum_{j \neq i} v_j(G'^*)$ .

Here, we show  $p'_i \geq p_i$ , assuming that bidder  $i$ 's shill bidders do not win in the auction. Also, the set of allocations in the auction does not change. Namely,  $G' = G \not\subseteq s$  holds. The difference between  $p'_i$  and  $p_i$  is shown as follows.  $p'_i - p_i = \sum_{j \neq i} v_j(G'_{\sim i}) - \sum_{j \neq i} v_j(G_{\sim i}) - (\sum_{j \neq i} v_j(G'^*) - \sum_{j \neq i} v_j(G^*)) = \sum_{j \neq i} v_j(G'_{\sim i}) - \sum_{j \neq i} v_j(G_{\sim i})$ .

The objective function of the winner determination problem solved over a strictly larger set of bids (that is, one with additional shill bids) cannot decrease. By adding bids the winner determination value never goes down. Thus, shill bidders' utilities do not increase when the shill bidder does not win.  $\square$

### 3.3 Characteristics of shill bids

Shill bids are characterized by the fact that a successful shill bidder's payment amount is decided based on the other shill bidders and the schemer. Namely, the schemers get unfair profits from the shill bidders' bids because the shill bidders' valuations are used in calculating the schemer's payment amount.

Here, we give the following two lemmas characterized by shill bidding.

**LEMMA 1 (SHILL BIDDER'S UTILITY).** *A shill bidder's utility cannot be independent of the values bid by other shill bidders.*

[Proof] Essentially, the lemma can be proven based on the definition of payment in the VCG mechanism. We give an example of this situation. Using the example of the section 2, we consider the situation where bidder 3's valuation changes. The shill bidders group includes bidders 1 and 3. Each bidder bids his/her value as follows.

$$\begin{aligned} v_1(B_1^{a_1, a_2}) &: \{\$6, \$0, \$6\} \\ v_2(B_2^{a_1, a_2}) &: \{\$0, \$0, \$8\} \\ v_3(B_3^{a_1, a_2}) &: \{\$0, \$4, \$4\} \end{aligned}$$

Bidder 1's payment amount can be calculated as  $8 - 4 = 4$  for item 1 and bidder 3's payment amount is  $8 - 6 = 2$  for item 3. Thus, the schemer's (bidder 1's) utility can be computed as \$6. In section 2, the schemer's utility is \$8. On the other hand, in this situation, the schemer's utility has a different value. Namely, when shill bidders' valuations for items change, the bidders' payoff in the same shill-bid group changes. A shill bidder's payment is not independent of the other shill bidders' or the schemer's value.  $\square$

**LEMMA 2 (SHILL BIDDER'S PAYMENT).** *Each shill bidder's payment amount is calculated based on the valuations of the other shill bidders in the shill-bid group.*

[Proof] This lemma can be proven easily based on the definition of VCG. In VCG, a successful bidder's payment is shown as  $p_i = \sum_{j \neq i} v_j(G_{\sim i}^*) - \sum_{j \neq i} v_j(G^*)$ . The  $\sum_{j \neq i} v_j(G_{\sim i}^*)$  in the right side of the equation means the sum of the winners' total valuations except for bidder  $i$ 's valuation. This first member of the equation includes all other shill bidders' values. This means that each shill bidder's payment amount is calculated based on the other shill bidders' valuations in the same shill-bid group, as described above. Thus, a shill bid can be characterized by its payment amount being interdependent with other shill bidders' valuations.  $\square$

In VCG, bidder  $i$ 's payment amount is based on the bidders' values in the bidder set  $N \setminus i$  in equation as shown in section 2.2. On the another hand, bidder  $i$ 's valuation is used in calculating the other shill bidders' payment amounts. Let us consider the situation where a schemer does not create shill bidders. In this situation, the schemer's payment increases compared with the situation where there are shill bidders, since there is no shill bidder who can influence to the schemer's payment. Intuitively, we can detect shill bidders by merging investments among winners. We can determine the possibility of shill bidders by comparing the utilities between a calculation using an individual agent's value and one using the merged values. Figure 1 shows a conception of the merged value method. Winners' utilities under normal VCG are reduced than the utilities under the merged investment. Thus, the merged investment method is one of appropriate methods for detecting shill bidders.

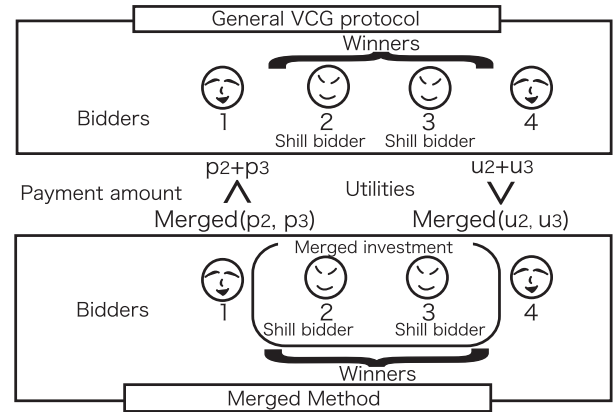


Figure 1: Merged Investment

## 4. DETECTING SHILL BIDDERS AND ALLOCATING ITEMS

### 4.1 Merged investment

Shill bidders are detected based on a comparison between bidders' utilities. Concretely, first, each bidder's value is merged with other bidders' values. An auction is conducted under the condition of merged investment. The merged bidder's utility is computed in this situation and compared with the sum of each bidder's utility in general VCG. If the bidder's utility is reduced in the merged case, such a bidder may be guilty of shill bidding. Thus, we construct an artificial situation where a shill bidder cannot influence the other shill bidders through collusion, and then we can detect the shill bidders in VCG. The next section describes our method of detecting shill bidders in detail.

A feature of shill bids is that the scheming agent can increase his/her utility when he/she bids the values divided into multiple bids. We show a straightforward method to determine the possibility of shill bidding by using the winners' values in a combinatorial auction. Shill bidders are created by separating the bids of an original agent. Namely, the original agent can increase utility by dividing his/her bidding actions. When the original bidder and his/her shill bidders are winners of an auction, we can determine how much the agent increases the utility by comparing the utilities of divided bidding with those of merged investment bidding. Intuitively, we apply a winner-based algorithm, which is based on a comparison of utilities between bidding in real auctions and bidding based on our method. Below, we describe our proposed algorithm in detail.

**Input:** Bids on bundles from each participant.

**Output:** True if there is a potential shill bid.  
False if there is no potential shill bid.

**Function** Detect a Shill bid

**begin**

Determine winners and calculate payments based on VCG.

Create a power set  $S$  for a set of participants.

**for each**  $s \in S$

Merge participants' values in  $s$

by merge function  $f(s)$ .  
Determining winners and calculating payments  
based on merged values by VCG.  
 $u_{f(s)} :=$  the utility of  $s$  after merging.  
 $u_{sum_s} :=$  sum of the utilities in  $s$  before merging.  
**if**  $u_{f(s)} < u_{sum_s}$   
    **return** True  
**else**  
    **return** False  
**end.**

In the above method, the number of merged investments is calculated as  $2^n - n - 1$  when the number of winner is  $n$ , namely the computational cost is  $O(2^n)$ . Therefore, a huge computational cost is required to judge whether there are shill bidders in the auction. Consequently, we propose a new method for detecting shill bidders. Based on [Theorem 1], a schemer's utility decreases if a shill bidder does not win in the auction, since the shill-bid scheme fails. Assume that there are shill bidders  $i$  and  $j$ . Bidder  $i$ 's utility decreases if bidder  $j$  does not join the auction because shill bidder  $j$ 's valuation is used in calculating  $i$ 's payment. Using this method, the number of computations is just  $n - 1$  if all bidders win in the auction.

## 4.2 Method of detecting shill bidders

Based on [Theorem 1], shill bidders cannot increase their utilities unless their confederates win the auction. Then, how can we detect shill bidders by using this feature? We tentatively try each auction where winners are excluded, one by one. Figure 2 shows a conception of our new proposed method. If there are shill bidders, such bidders' utilities are reduced by the above procedure through comparison between a normal auction and the trial auctions. Thus, shill bidders are detected by comparing the differences in their utilities.

Here, we give an outline of our proposed mechanism. First, an allocation is calculated and successful bidders' utilities are computed in VCG. Second, bidders' utilities are calculated after attempting auctions in which each winner is excluded one by one. Third, each bidder's utility is compared between the first step and the second step. The bidders whose utilities change are classified into a shill-bidder group. Then, the auction is retried among innocent bidders and the shill-bidder group. The bundle is allocated as a set to the shill-bidder group. The reason why we allocate items as a set in this method is that a shill-bidder group sometimes includes false-shill bidders. False-shill bidders, who bid unexpectedly in the manner of shill bidding, are innocent. In this paper, we focus on preventing shill bidding and fraud. We permit the mechanism to allocate items to the shill-bidder group to cope with shill bidding, but we never employ the traditional auction method in which items are allocated to each bidder. Thus, our mechanism cannot only detect shill bidders but also preserve the advantages of VCG.

Here, we concretely explain the procedure of our mechanism.

**[Algorithm] (Step 1)** Assume that  $n$  bidders join the auction and that the winners are determined based on VCG. Items are allocated to the successful bidders in a combination such that the total of all bidders' valuations is maximum.  $G^* = \arg \max_{G=(G_1, \dots, G_n)} \sum_{i \in N} v_i(G_i)$ . The successful bidder  $i$ 's payment amount  $p_i$  is calculated based on

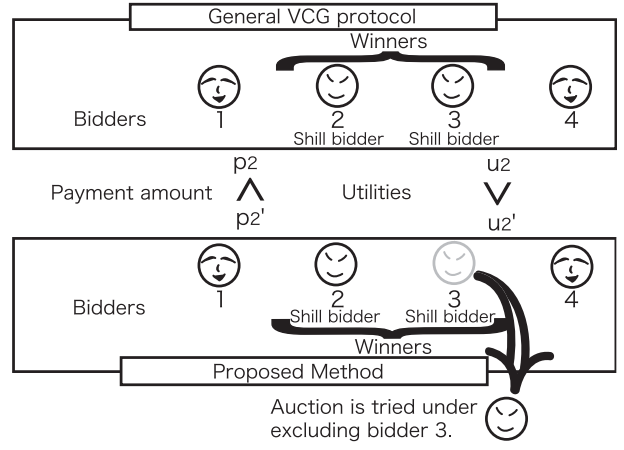


Figure 2: Proposed Method

the following equation.  $p_i = \sum_{i \neq j} v_j(G_{\sim i}^*) - \sum_{i \neq j} v_j(G^*)$ .  $G_{\sim i}^*$  is the combination where all bidders' valuations are maximized except for bidder  $i$ 's valuation. This allocation is defined as follows.  $G_{\sim i}^* = \arg \max_{G \setminus G_i} \sum_{N-i} v_j(G_j)$ .

**(Step 2)** Winners' utilities are calculated. We simply define that a set of successful bidders is shown as  $W = \{1, 2, \dots, \iota, \dots, \theta\}$ . Bidder  $\iota$ 's surplus is  $u_\iota$ .

**(Step 3)** An auction in which a bidder  $\theta$  does not participate is tried. In this auction, the allocation is described as  $G_{\sim \theta} = \arg \max_{G=(G_1, \dots, G_n)} \sum_{N-\theta} v_i(G_i)$ . The set of winners is  $W = \{1, 2, \dots, \iota, \dots, \theta - 1\}$ . In this allocation, each bidder's utility is compared with his/her utility at the allocation in **(Step 1)**.

**(Step 4)** The difference in the winners utilities,  $u_\iota - u_\iota^{\sim \theta}$ , is calculated.  $u_\iota^{\sim \theta}$  is winner  $\iota$ 's utility in an auction that bidder  $\theta$  does not join. A set of winners  $\{w_{\sim \iota}\} \in W$  is detected under  $u_\iota - u_\iota^{\sim \theta} > 0$ .

**(Step 5)** In **(Step 4)**, a winners group  $(\{w_{\sim \iota} \setminus \theta\} \in W) = (\{w_{\sim \theta} \setminus \iota\} \in W)$ , whose utilities are reduced, is established. Items are allocated to the winners except for bidders in the above winners group (shill bidders), that is,  $W \setminus w$ .

**(Step 6)** Winners  $w$ 's values are merged. The payment amount is calculated in the merged investment. For this payment, items are allocated as a set to the shill bidders.

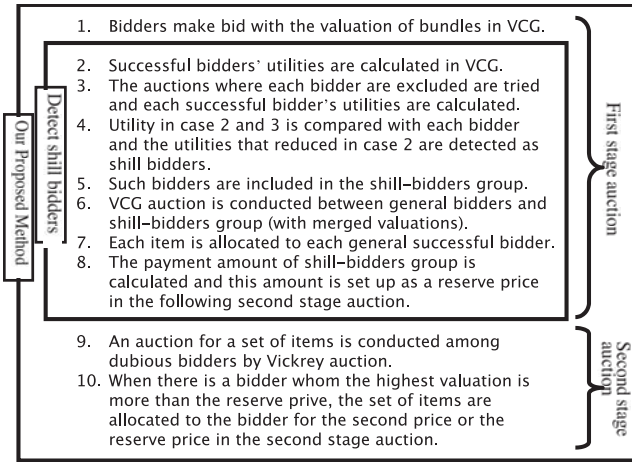
## 4.3 Example

In this section, an example of our proposed mechanism is shown in the case where five bidders participate in an auction for two items. In this situation, assume that there are a schemer and a shill bidder created by the schemer. Bidders bid their valuations for each item and bundle as follows.

$$\begin{aligned} v_1(B_1^{a_1, a_2}) &: \{\$8, \$7, \$15\} \\ v_2(B_2^{a_1, a_2}) &: \{\$7, \$8, \$15\} \\ v_3(B_3^{a_1, a_2}) &: \{\$12, \$0, \$12\} \\ v_4(B_4^{a_1, a_2}) &: \{\$0, \$12, \$12\} \\ v_5(B_5^{a_1, a_2}) &: \{\$0, \$0, \$20\} \end{aligned}$$

In this round, the successful bidders are 3 and 4. Each bidder's payment amount is 8. Namely,  $u_3$  and  $u_4$  are both 4 dollars.

Here, an auction where bidder 3 does not participate is tried based on our proposed mechanism to detect shill bidders. In this case, the winners are bidder 5 or bidders 1 and



**Figure 3: Proposed mechanism**

4. Bidder 4's payment amount is 12 dollars.  $u_4$  is 0. Bidder 4's utility is reduced compared with the normal auction.

Similarly, there is the same result when bidder 4 is excluded from a set of bidders. In this case, the winners are bidder 5 or bidders 2 and 3. Bidder 3's payment amount is 12 dollars.  $u_3$  is 0. Bidder 3's utility is reduced compared with the normal auction. Bidders 3's and 4's utilities decrease after the integration of valuations.

In an allocation, bidders 3's and 4's valuations are merged by a certain merge function  $F$ . For example, we assume that  $v_{f(i,i+1,\dots,j)}$  is the following maximum function among agents' values for each item. Merged value  $v_{f(i,i+1,\dots,j)}$  is shown as  $(\max_{i,i+1,\dots,j}\{v_i^{a_1}\}, \max_{i,i+1,\dots,j}\{v_i^{a_2}\}, \dots, \max_{i,i+1,\dots,j}\{v_i(B_i^{a_1,\dots,a_m})\})$  for agent  $i$ 's value  $(v_i^{a_1}, v_i^{a_2}, \dots, v_i(B_i^{a_1,\dots,a_m}))$  in the maximum selection method. In this example, the merged valuation  $v_{F(3,4)}$  for bidders 3 and 4 is  $\{12, 12, 24\}$ . Using this merged valuation, the auction is retried. The integrated bidders group's payment amount  $p_{F(3,4)}$  is calculated as  $20 - 0 = 20$ . Both items are allocated to the merged bidders for \$20 as a set.

#### 4.4 Robust auction mechanism against shill bidders

Figure 3 shows an outline of our proposed mechanism. The mechanism is distinguished between two stages. In the first stage, VCG auction is conducted. and shill bidders are detected as shown in the above subsections. Items are allocated to general bidders who are no possibility of shill bidder. After that, the remained set of items are allocated to shill-bidders group. Here, reserve price is calculated as a result of first stage auction. In the second stage, each dubious bidder as a shill agent bids to a set of items like a Vickrey auction. A bidder who bids the highest valuation makes a successful bid when the valuation is more than the reserve price.

In our mechanism, there are two situation concerned with efficiency. When the set of items is allocated successfully to the bidder for the bidder's reserve value or less, the allocation is Pareto efficient. On the other hand, the false-name proof and strategy-proof property is preserved based on this method.

Here, we give the concrete allocation method. The method

employs like the single item auction that is, each bidder bids only one bid value to the set of items. The set of items is allocated based on the second highest value. Thus, the method is essentially same as the Vickrey auction.

We assume that there are bidders  $w = \{i'_1, \dots, i'_l, \dots, i'_n\}$  in shill bidders group. Bidders' valuations are shown as  $\{v_{i'_1}, \dots, v_{i'_l}, \dots, v_{i'_n}\}$ . Bidders' payments included in the shill bidders group is shown as  $\{p_{i'_1}, \dots, p_{i'_l}, \dots, p_{i'_n}\}$ . Bidders' utilities is shown as  $\{u_{i'_1}, \dots, u_{i'_l}, \dots, u_{i'_n}\}$ . We decide a reserve price based on the price (such as the end of subsection 4.3) allocated to the shill bidders group at the first stage. The set of items is shown as  $A' \subset A$ .

**[Algorithm] (Step 1)** Bidders who are included in the shill bidders group bid their valuations  $v_{i'_l}$  for the set of items  $A'$ .

**(Step 2)** We assume simply higher order of bidders valuations is shown as  $\{v_{i'_1} > v_{i'_2} > \dots > v_{i'_l} > \dots > v_{i'_n}\}$ . If the bidder's valuation  $v_{i'_2}$  is not more than the reserve price, items are not allocated to the winner. Otherwise, **(Step 3)** is employed.

**(Step 3)** If the bidder's valuation  $v_{i'_2}$  is more than the reserve price, the winner of this auction is decided based on the second highest price which is bid by a bidder in the shill bidders group. Namely, bidder  $i'_1$ 's payment amount  $p_{i'_1}$  is  $v_{i'_2}$ .

## 5. DISCUSSION

### 5.1 Properties of protocol

Here, we give a discussion on the properties and validity of our mechanism. Our mechanism preserves the desirable advantages of VCG, that is, Pareto efficiency, individual rationality, and incentive compatibility. First, Pareto efficiency is proven self-evidently because items are allocated in a combination where participants' total utilities are maximum in VCG. The individual rationality property can be shown simply by the definition of VCG. Here, we give the following theorem to prove the incentive compatibility property of our mechanism.

**THEOREM 2 (STRATEGY-PROOF PROPERTY).** *Incentive compatibility is preserved in our mechanism.*

[Proof] In literature [13], Yokoo et al. proposed a framework for mechanism design called PORF protocol, that is, designed mechanism is satisfied incentive compatibility property. Mechanisms based on PORF's characteristics are incentive compatible mechanism and strategy-proof mechanism. Our proposed mechanism has the PORF's feature as shown in the following explanations.

First, we give some definitions for explanations. Bidder  $i$  is a  $i$ th bidder who participates in an auction. Shill bidders (including innocent bidders in the shill bidders group) are shown as  $i'$ . Shill bidders group is shown as  $w$ , that is,  $i'$  is included in  $w$ . In our proposed method, all bidders' payments and shill bidders' payments included in shill bidders group are not depend on their own bid valuations. Thus, our mechanism is satisfied the strategy-proof property.

Second, the bidders  $i$ 's payment amount is calculated based on VCG. On another hand, the payment amount for shill bidder  $i'$  is decided based on the reserve price  $r(= p_w)$  in the second stage. The reservation value  $r$  is independent with the valuations included in the shill bidders group.

Finally, an allocation feasibility for bidder  $i$  is reserved due to same as VCG. On the other hand, the allocation for shill bidder  $i'$  is reserved because items are allocated to a bidder who is in the shill bidders group.

As shown the above issues, our proposed mechanism is satisfied the PORF framework. Even though a bidder's valuation who is not in the shill bidders group can give effects to shill bidders group's payment amount, the payment amount is not reduced by the valuation.  $\square$

In our proposed method, a schemer can not increase their utilities if he/she makes shill bidders compared with the situation where the schemer does not make shill bidders. This situation is same as normal VCG auction because shill bidders' values are merged. Considering situations in the real auctions, to make shill bidders puts some costs for the schemer. Thus, if the schemer does not make shill bidders, the schemer's strategy becomes dominant strategy.

When the set of items is allocated successfully, the allocation is efficient. On the other, if the bidder's valuation  $v_{i'2}$  is not more than the reserve price, the allocation is not efficient because items are remained unsold. The efficiency is related the number of bidders in shill-bidders group and the rate of false-shill bidders in the group. The efficiency is shown as a decreasing function with the rate of false-shill bidders in the group. Here, we give a proposition that a schemer never has an incentive to make shill bidders in our mechanism. Schemer's dominant strategy is not to make shill bidders.

**ASSUMPTION 4 (STRATEGY SPACE).** *We assume the following two strategies, that is, a schemer makes shill bidders and does not make shill bidders.*

**PROPOSITION 1 (DOMINANT STRATEGY).** *Schemer's dominant strategy is not to make shill bidders in our mechanism.*

[Proof] After detecting shill bidders, second-price auction is conducted based on bidders' valuations and reserve price for the set of items. In this auction, if there is a winner, his/her payment amount is the reserve price  $r'$  or more than the price because the second-price should be more than the reserve price. Namely, the following holds.  $p_{i'1} = v_{i'2} \geq r'$ . The reserve price is essentially same as the payment amount in which bidders do not bid by shill bidding.

First, we consider a situation where shill bidders group is organized by only bidders made by the schemer. If the schemer does not make shill bidders, he/she can purchase items for  $r'$ . On the other hand, If the schemer make shill bidders, he/she purchase items for  $r'$  or more than the reserve value. Even though a shill bidder can make successful bid and his/her valuation is  $v_{i'1}$ , his/her utility  $u_{i'1}$  is calculated as  $v_{i'1} - v_{i'2}$ . On the other hand, when the schemer does not make shill bidders, his/her utility  $u_{i'1}$  is calculated as  $v_{i'1} - r'$ . Clearly, the following holds.  $v_{i'1} - v_{i'2} < v_{i'1} - r'$ . Thus, the schemer, in this case, does never have incentive to make shill bidders.

Second, we consider a situation where shill bidders group is organized by innocent bidders (false-shill bidders) and shill bidders made by the schemer. In this situation, there are following two cases, (1) the case in which the schemer wins the second auction and (2) the case in which the schemer loses the second auction.  $p_{i'1} = v_{i'2} \geq r'$  holds concerned with the schemer's payment amount. The schemer's utility  $u_{i'1}$  is calculated as  $v_{i'1} - v_{i'2}$ . On the other hand,

when the schemer does not make shill bidders, his/her utility  $u_{i'1}$  is calculated as  $v_{i'1} - r'$ . Clearly, the following holds.  $v_{i'1} - v_{i'2} < v_{i'1} - r'$ . In the latter case, the schemer's utility is 0 because he/she can not purchase items. If the schemer does not make shill bidders, he/she can make a successful bid in the first stage auction(VCG). Clearly, the schemer's utility is  $v_{i'1} - r'(> 0)$ .

Thus, the schemer, in this case, does never have incentive to make shill bidders. In any case, the schemer's utility does not increase in our mechanism. Namely, not to make shill bidders is dominant strategy for the schemer.  $\square$

## 5.2 Multiple schemes

Our mechanism can cope with multiple schemers in an auction. If there are multiple schemers, our mechanism detects multiple shill-bidder groups created by the schemers.

In our mechanism, some bidders may casually submit bid values that seem like unnatural shill bidders' valuations. Such bidders are included in the shill-bidder group. However, the goal of our protocol is to detect and prevent shill bidding. To solve this problem, our mechanism allocates items to a shill-bidder group as a set of items. When the shill-bidder group consists of all shill bidders, the result of the mechanism is the same as an auction in which the schemer does not commit unfair practices. Even if the shill-bidder group includes an innocent bidder, we can provide many types of methods of allocation and calculation of payment amount. In the shill-bidder group, items are allocated based on the method in which each bidder's payment does not depend on his/her valuation.

## 6. RELATED WORK

Milgrom analyzed the shill-biddable feature in VCG [7]. Bidders in VCG can profitably use shill bidders, intentionally increasing competition in order to generate a lower price. Thus, the Vickrey auction provides opportunities and incentives for collusion among low-value, losing bidders. This feature is a result of the monotonic increase problem. However, this work does not provide a method of detecting shill bidding in combinatorial auctions.

Yokoo et al. reported the effect of false-name bids in combinatorial auctions [13]. To solve the problem, Yokoo, Sakurai and Matsubara proposed novel auction protocols that are robust against false-name bids [11]. One such protocol is called the Leveled Division Set (LDS) protocol, which is a modification of the VCG. It utilizes the reservation prices of auctioned goods for making decisions on whether to sell goods in a bundle or separately. Furthermore, they also proposed an Iterative Reducing (IR) protocol that is robust against false-name bids in multi-unit auctions [12]. The IR protocol is easier to use than the LDS, since the combination of bundles is automatically determined in a flexible manner according to the declared values of agents. They concentrated on designing mechanisms that could perform as alternatives to VCG. Due to our fundamentally different purpose, we do not simply adopt off-the-shelf methods for mechanism design.

Ausubel and Milgrom referred and characterized shill bidding [1]. They showed a theorem in which the substitute condition is sufficient for shill bidding and loser collusion to be unprofitable. Further, if the set of bidder values is otherwise sufficient inclusive, the condition is also necessary for the theorem. However, in our proposed method, we do not

only show the features of shill bidding, but we also give a concrete mechanism for detecting shill bidding.

Some researchers [3][5][6][8][9] have proposed methods for computing and calculating the efficient solution in combinatorial auctions. These analyses contributed to the pursuit of a computational algorithm for winner determination in combinatorial auctions, but they did not deal with shill bidding and thus are fundamentally different approaches from our work. However, some of these algorithms can be incorporated in our work. Combinatorial auctions have a computationally hard problem in which the number of combinations increases when the number of participants/items increases in an auction, since agents can bid their values as a set of bundled items.

Sandholm [8] proposed a fast winner determination algorithm for combinatorial auctions. Sandholm et al. [9] also showed how different features of a combinatorial market affect the complexity of determining the winners. These works studied auctions, reverse auctions, and exchanges, with one or multiple units of each item, with and without free disposal. We theoretically analyzed the complexity of finding a feasible, approximate, or efficient solution.

Fujishima et al. proposed two algorithms to mitigate the computational complexity of combinatorial auctions [3]. Their Combinatorial Auction Structured Search (CASS) algorithm determines efficient allocations very quickly and also provides good "any-time" performance. Their second algorithm, called Virtual Simultaneous Auction (VSA), is based on a simulation technique. CASS considers fewer partial allocations than the brute force method because it structures the search space to avoid considering allocations containing conflicting bids. It also caches the results of partial searches and prunes the search tree. On the other hand, VSA generates a virtual simultaneous auction from the bids submitted in a real combinatorial auction and then carries out simulation in the virtual auction to find a good allocation of goods for the real auction. In our work, to determine efficient allocations quickly in each VCG, we employ the CASS method. However, Fujishima's paper does not focus on shill bids.

Leyton-Brown et al. proposed an algorithm for computing the optimal winning bids in a multiple-unit combinatorial auction [6]. This paper describes the general problem in which each good may have multiple units and each bid specifies an unrestricted number of units desired for each good. The paper proves the correctness of our branch-and-bound algorithm based on a dynamic programming procedure. Lehmann et al. proposed a greedy optimization method for computing solutions of combinatorial auctions [5]. However, the VCG payment scheme does not provide for a truth-revealing mechanism. Therefore, they introduced another scheme that guarantees truthfulness for a restricted class of auction participants.

## 7. CONCLUSIONS

This paper proposed a method for detecting shill bids in combinatorial auctions. Our mechanism can judge whether there might be a shill bid from the results of the VCG procedure. We proved a theorem stating that shill bidders cannot increase their utilities unless all shill bidders win in the auction. Based on this theorem, our proposed mechanism compares the agents' utilities in a conventional auction with those in an auction where a shill bidder does not join in the

auction. This process prevents shill bidders from increasing unfair profits.

Our mechanism has the three advantages. (1) Our mechanism is more superior than existing complex mechanisms in the point of view of generalization, because our mechanism employs only VCG. (2) Even though shill bidders exist in an auction, incentive compatibility property is preserved using our mechanism. (3) The schemer does never have incentive to make shill bidders in our mechanism. Namely, not to make shill bidders is dominant strategy for the schemer.

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