

A Memetic Algorithm for Efficient Solution of 2D and 3D Shape Matching Problems

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Abstract. Shape representation and the associated morphing (repair) operators play a vital role in any shape optimization exercise. This paper presents a novel and an efficient methodology for morphing via smart repair of control points, wherein a repaired sequence of control points are generated. The repaired set of control points are then used to define the curve or the surface using a B-spline representation, while the control points themselves are optimized using a memetic algorithm. While the authors have already proposed the approach for 2D shape matching, this paper extends the approach to deal with 3D shape matching problems. Two 2D and one 3D examples have been presented to illustrate the performance of the proposed approach.

Keywords: shape representation, design optimization, shape matching.

1 Introduction

Shape representation and optimization is a key element in any product design process. Shape representation schemes are required for the generation of shapes which in turn facilitates the design of functional articles. For example, aerofoils, converging-diverging nozzles, ship-hulls, medical prosthesis, structural elements, and many other functional articles require shape generation and shape modification to achieve the desired performance. The performance parameters may vary from an application to another depending on the intended use of the object. In a shape optimization exercise, the shapes are generally modified using an optimization algorithm. Efficiency of the optimization process and flexibility of shape representation scheme are the two major issues requiring serious attention. A lack of flexibility in shape representation will limit the evolution of various shapes whilst an inefficient optimization algorithm will require evaluation of numerous shapes prior to its convergence, both of which are not desirable. Several methods for shape representation have been proposed over the years such as implicit polynomials [9, 16], cubic splines [26, 28], Bezier curves [26, 31] and B-splines [5, 25]. A B-spline representation has been used in the present study wherein the control points of the curve or surface are identified through an optimization algorithm. In a B-spline representation, a small number of control points are able to represent a wide variety of complex geometries and a change in the position of the control points are known to affect the shape locally. The B-spline basis also allows the order of the basis function and hence the degree of the resulting curve to be changed without changing the number

of defining polygon vertices [26]. Its strong convex-hull property, variation diminishing and affine invariance properties are also attractive in the realm of shape optimization.

Evolutionary design optimization has been an active area of research in the last two decades [1–3, 8, 12, 13, 15]. Several applications span across a range of domains such as aerodynamic design [27], hydrodynamic design [3], structural design [13, 19, 21], and electromagnetic component design [2, 17, 32]. Shape matching is often a pre-cursor to a shape optimization exercise, where the efficiencies of the underlying shape representation scheme and the performance of the optimization algorithm is investigated in depth. An efficient optimization algorithm coupled with a flexible and effective shape representation scheme is the key to a successful approach that requires limited number of design evaluations to arrive at the optimum shape [14].

A number of approaches have also been reported in literature that attempts to solve 2D and 3D shape matching problems [4, 18, 20, 29, 30, 34–37]. A genetic algorithm with multi-parent recombination using latin square crossover operator was adopted in [35], whereas an individual mutative evolution strategy (iES) with covariance matrix adaptation evolution strategy (CMA) was used over a B-spline representation in [36]. An effective genetic algorithm with an enhanced geometry representation scheme was also presented by Wang in [34] and an adaptive constraint concept was used to direct search in regions of interest. A mixed integer quadratic approach was suggested using 3D shape constraints to guide shape matching [30] whereas, an unconstrained quadratic optimization problem formulation was proposed instead of handling a large set of complicated SOCP constraints by Zhu in [37] for 3D deformable surface tracking. Three different morphing methods were introduced and employed within a standard evolution strategy in [20]. A method for the automatic construction of control volumes was proposed in [18] based on the concept of evolvability as a potential capacity of representations. In [4], an incremental abstraction of control points was embodied into evolution strategies with a modified 1/5 rule. It is important to highlight that the computational cost involved in the above studies are still significant and often restricted bounds were used to limit the search space.

In summary, population based optimization algorithms have had limited success for 2D shape matching problems and their performance for 3D shape matching problems were far from satisfactory. Such approaches in principle allowed the connectivity and location of control points to evolve within the course of optimization. Such an attempt is likely to yield a number of impractical shapes with crossings which in turn results in a significant computational cost. In this paper, we alleviate the problem via repair, wherein the set of control points are repaired and sorted. While the sequence of control points are generated via repair, their locations are evolved using the memetic algorithm.

Rest of the paper is organized as follows. Aspects of shape matching metrics and the proposed method are discussed in Section 2 and 3 respectively. The results are reported in Section 4. Finally Section 5 concludes the paper.

2 Shape Matching Metrics

Shape matching usually refers to transforming a shape, and measuring its resemblance with a target shape using some similarity measures. In the present study, we assume

that the length, width and height of the box enclosing the target shape is known along with its location of centroid. Two commonly used shape matching metrics include Euclidean distance and Hausdorff distance. For two finite point sets $A = \{a_1, a_2, \dots, a_m\}$ and $B = \{b_1, b_2, \dots, b_m\}$ (where, m indicates the total number of points in a particular point set), the distance is defined as $L_j(A, B) = (\sum_{i=1}^m |a_i - b_i|^j)^{\frac{1}{j}}$. For $j = 2$, this yields the Euclidean distance L_2 [24]. Hausdorff distance is a classical non correspondence-based shape matching metric [10, 11, 33]. The Hausdorff distance between A and B is defined as

$$H(A, B) = \max \{h(A, B), h(B, A)\} \quad (1)$$

where,

$$h(A, B) = \max_{a \in A} \min_{b \in B} \|a - b\| \quad (2)$$

and $\|\cdot\|$ is the underlying norm on the points of A and B , usually Euclidean distance. $h(A, B)$ in effect ranks each point of A based on its distance to the nearest point of B and then uses the largest ranked of such point as the distance (the most mismatched point of A) [10]. The Hausdorff distance $H(A, B)$ is the maximum of $h(A, B)$ and $h(B, A)$.

3 Proposed Method

In the proposed method, the number of control points required to represent the shape, the dimensions of the box enclosing the shape and the centroid of the shape are assumed to be known. The variables of the optimization problem are the x , y and z coordinates of the control points, the range of which are the same as the dimensions of the enclosing box. Every solution generated through the process of initialization or recombination is repaired, wherein the sequence of the control points are changed to obtain a non-intersecting control polygon net while maintaining the specific values of its coordinates. The concept is illustrated using Fig. 1a, 1b and 1c, where the original randomly generated control polygon net and the convex-hull are presented alongside its repaired form. The repaired solution (ordered set of vertices in the control polygon net) is used to create the shape using B-splines and compared with the target shape based on the metrics described above. For the 3D shape optimization problem, the shape is assumed to be disintegrated to several 2D shape optimization problems (cross-sections of 3D shape) termed as stations, wherein the set of control points representing the 2D shape in each station are repaired.

3.1 Initialization and Repair Strategy

The number of control points (N) used to describe a shape is a user defined parameter. Since the limits of x , y and z coordinates i.e. the space enclosing the target shape is known, a random solution is created and repaired during the phase of initialization of 2D shape. Firstly, a convex-hull is generated using a set of control points (Fig. 1b). Thereafter, the points (lying inside the convex-hull) nearest to their adjacent edges are inserted to generate the non-intersecting control polygon net (Fig. 1c). In order to avoid

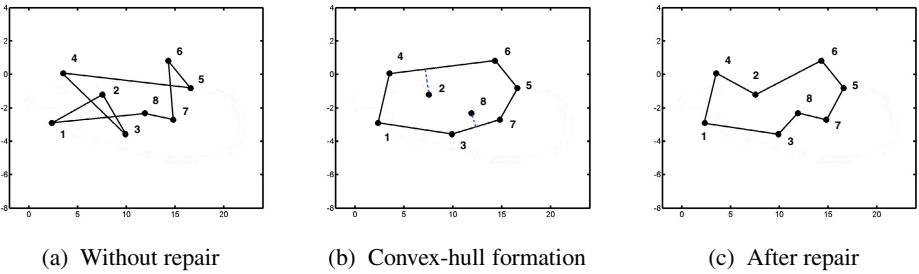


Fig. 1. Effect of repair scheme on the position of initial control points

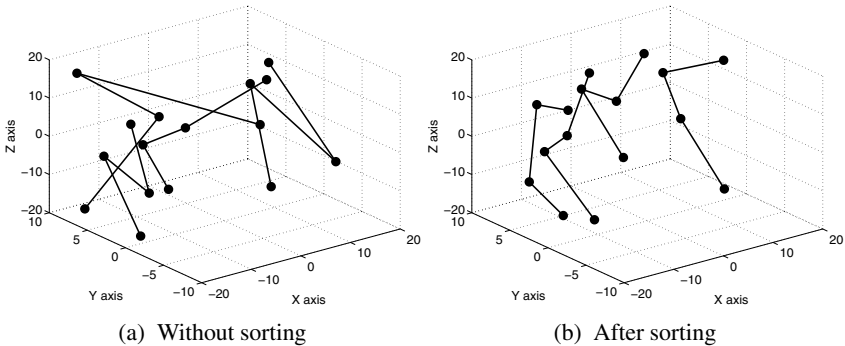


Fig. 2. Effect of sorting on the position of control points

entangling or crossings of control points between two neighbouring stations, every set of x coordinates of control points spanning the shape in x direction is sorted in ascending order during the optimization process. The effect of sorting on the position of control points is illustrated in Figure 2a and Figure 2b.

3.2 Matching Error

Since the centroid of the target shape is known, the shape generated using the repaired solution (reordered set of control points) is shifted such that its centroid matches the centroid of the target shape. The maximum of Euclidean and Hausdorff distance is computed using the generated shape and the target shape.

3.3 Optimization Algorithm

Evolution: The pseudo-code of the proposed optimization is presented in Algorithm 1. In the proposed algorithm, two different evolution strategies are used for generating the offspring population. These are:

- 1. *EA-like evolution* – This includes simulated binary (SBX) crossover and polynomial mutation [7].

Algorithm 1. Proposed Algorithm

Require: Population size (S), Number of generations (S_g), Crossover and mutation parameters, Target shape (s_T)

- 1: Initialize (pop_1) {Assumed number of control points}
- 2: Repair (pop_1) {Create the sequence of control points}
- 3: $CS_{pop_1} = \text{Centroid_Shift}(pop_1, s_T)$ {Shift all pop_1 solutions such that the centroids of those solutions match the centroid of the target shape}
- 4: $\text{Matching_Error}_{pop_1} = \text{Max}(E_dist, H_dist)$ {For all pop_1 evaluate the maximum of Euclidean and Hausdorff distances as matching error between target and generated shape}
- 5: **for** $i = 2$ to S_g **do**
- 6: **if** $\text{rand}(0,1) \leq 0.5$ **then**
- 7: $childpop_i = \text{Evolve_EA}(pop_{i-1})$
- 8: **else**
- 9: $childpop_i = \text{Evolve_DE}(pop_{i-1})$
- 10: **end if**
- 11: $childpop_i = \text{Repair}(childpop_i)$
- 12: $CS_{childpop_i} = \text{Centroid_Shift}(childpop_i, s_T)$
- 13: Evaluate ($childpop_i$)
- 14: $R = \text{Rank}(pop_{i-1} + childpop_i)$
- 15: $pop_i = \text{Reduce}(R)$ {Selecting best S solutions for the next generation}
- 16: $\mathbf{x}_{\text{best}} \leftarrow \text{Local_search}(\mathbf{x})$ { $\mathbf{x}_{\text{best}}$ is the best solution found using local search from $\mathbf{x}$ }
- 17: Replace worst solution in pop_i with $\mathbf{x}_{\text{best}}$
- 18: $pop_i = \text{Rank}(pop_i)$ {Rank the solutions again in pop_i }
- 19: **end for**

2. *DE-like evolution* – This includes the DE exponential crossover and mutation, as described in [6, 23].

Ranking and Reduction: Since the test problems studied in the paper are formulated as single-objective, unconstrained minimization problems, the ranking is done by sorting the objective values in ascending order. The best N solutions from the (parent+child) population form the population for the next generation.

Local Search: At each generation, in addition to generation of new solutions using recombination and mutation, a local search is used for further improvement. Sequential quadratic programming (SQP) [22] is used for the local search in the present study. After performing the local search, the worst solution in the population is replaced by the best solution found from the local search.

4 Results

Two 2D shape and one 3D shape examples have been used to test the performance of the proposed approach. The parameters used for the algorithm are the same for each shape matching exercise and listed in Table 1.

4.1 2D Shape Example

From the two 2D examples, the target shapes i.e the damselfly-wing data contains 516 points whereas the stingray data contains 740 points. A population size of 40 and a maximum of 5,000 function evaluations are set for both the examples. For both the problems, we have assumed that the shapes can be represented using 16 control points which render to a 32 variable optimization problem. The results after 5,000 function evaluations for both examples are shown in Table 2.

4.2 3D Shape Example

A flower vase shape (or a converging-diverging nozzle) is considered for the 3D case. The target flower vase data consists of 676 points. A population size 400 and a maximum of 100,000 function evaluations are set for this problem. The flower vase shape has been assumed to be represented using 10X8 control polygon net (a total 80 control points) which translates to a 240 (3X80) variable optimization problem. The results after 100,000 function evaluations for the flower vase shape example are reported in Table 2.

For all the problems the best and worst values are reported as the minimum and maximum errors across 20 runs, respectively. The median value reported is the average of 10th and 11th values in the sorted list of matching errors obtained across 20 runs. The small values of standard deviations of all the shape examples reflect the consistency in terms of convergence of the algorithm. Various states of evolution of generated shape towards the target or original shape are shown in Fig. 4, 5 and 6. The average computational time for repairing a solution is around 0.0014 seconds (averaged over 1000 solutions) whereas the average computational cost of computing shape matching error is 0.0053 seconds (averaged over 1000 instances) for a problem involving 16 control points. In case of 3D flower vase problem, the average computational cost of

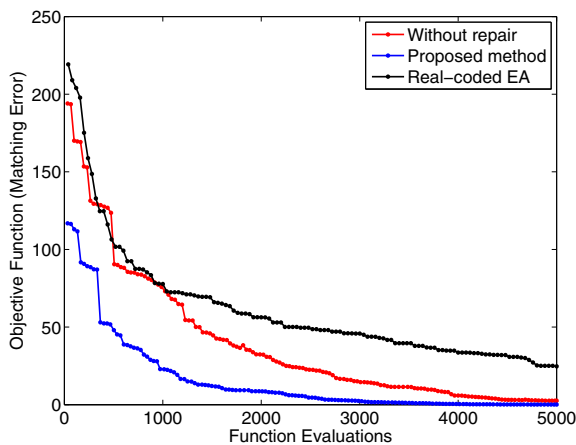


Fig. 3. Progress plot of the best shape of stingray for single objective matching error minimization

Table 1. Parameters used for the proposed algorithm

Parameter	Value
Population size	40/400
Max. function evaluations	5000/100000
Crossover probability (for both SBX/exponential)	1.0
Crossover index	10
Mutation probability	0.05
Mutation index (polynomial)	20
Scale factor F (for DE mutation)	0.9

Table 2. Results with repair and centroid shift using hybrid evolutionary algorithm

Shapes	Error	Best	Worst	Mean	Median	Std.
Damselfly-wing	Max(Eucli,HD)	1.4e-05	3.2e-04	8.3e-05	6.1e-05	8.1e-05
Stingray	Max(Eucli,HD)	0.067	1.863	0.706	0.589	0.518
Flower vase	Max(Eucli,HD)	0.848	1.553	1.244	1.264	0.166

Table 3. Results without repair and centroid shift using hybrid evolutionary algorithm

Shapes	Error	Best	Worst	Mean	Median	Std.
Damselfly-wing	Max(Eucli,HD)	4.6e-03	7.6e-03	5.1e-03	4.8e-03	7.1e-04
Stingray	Max(Eucli,HD)	2.578	11.668	4.947	4.590	2.176
Flower vase	Max(Eucli,HD)	2.367	3.817	3.091	3.081	0.333

Table 4. Results without repair and centroid shift using real-coded evolutionary algorithm

Shapes	Error	Best	Worst	Mean	Median	Std.
Damselfly-wing	Max(Eucli,HD)	3.026	6.958	5.132	5.202	1.210
Stingray	Max(Eucli,HD)	24.665	50.710	40.771	42.092	6.677
Flower vase	Max(Eucli,HD)	44.694	51.002	48.324	48.043	1.627

computing its error is 0.0820 seconds and the average computational cost of repairing a solution is 0.0066 seconds. All the computations are performed on a computer with the following configuration.

- Processor - Intel Core2Duo E8400 @ 3.00GHz, 2.99GHz.
- RAM - 4GB.
- Operating System - Microsoft Windows XP .

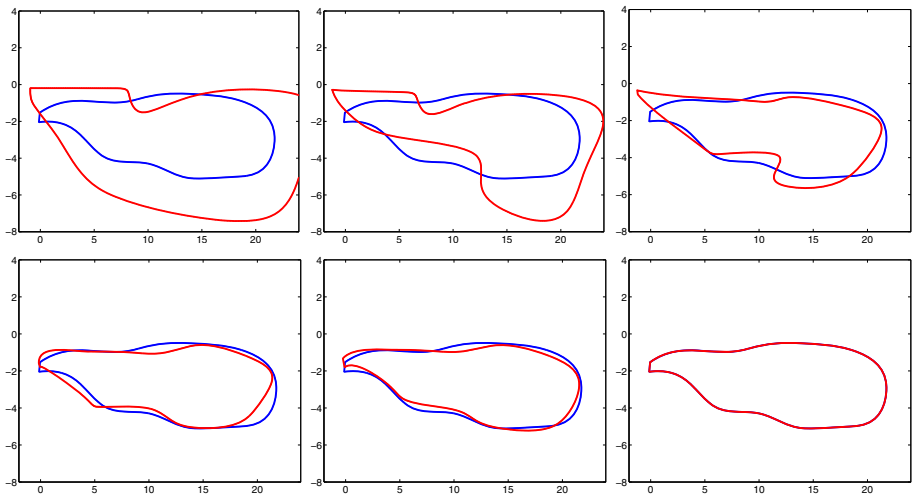


Fig. 4. Evolutions of generated damselfly-wing (red) towards the target damselfly-wing (blue) for matching

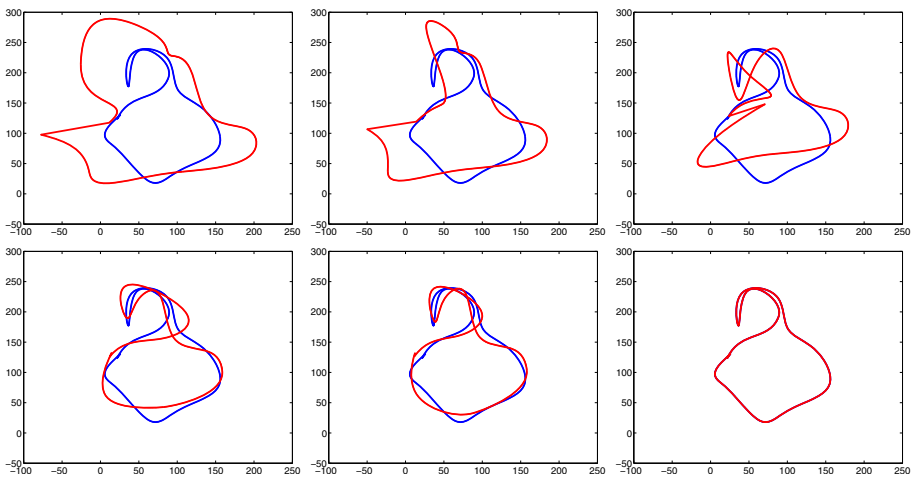


Fig. 5. Evolutions of generated stingray (red) towards the target stingray (blue) for matching

In an attempt to observe the performance of the repair scheme and the centroid shift mechanism, the same exercise was conducted with the same parameter setting without the reordering of the control points (without repair) and centroid shift. The performance statistics are presented in Table 3. It is clear from Table 2 and Table 3, that a significant improvement in performance has been achieved through the use of repair and centroid shift strategy. For the sake of comparison, results of using a real-coded evolutionary

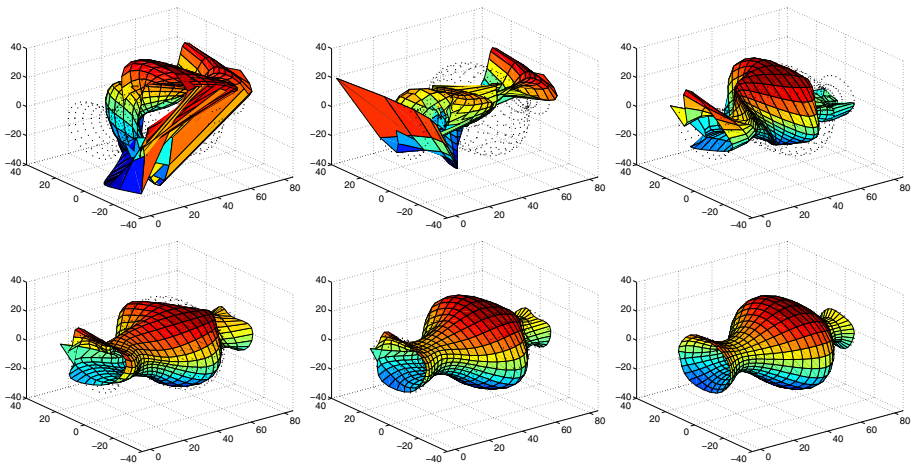


Fig. 6. Evolutions of generated flower vase (evolving surface) towards the target flower vase (point cloud) for matching

algorithm without repair and centroid shift are also presented in Table 4. The parameter setting for the runs are the same as listed in Table 1. Progress plot of the best shape of stingray shape example is presented in Figure 3, where the results for the three different cases are compared. It is interesting to observe that the best results of the proposed algorithm for the two shape examples are more than 100 times better than the results of the real-coded evolutionary algorithm.

5 Conclusion

In the present paper, an efficient method is introduced that is capable of solving 2D and 3D shape matching problems. The control points of the underlying B-spline representation are optimized using a memetic algorithm, while the presence of repair or morphing operation ensures generation of valid shapes. The memetic algorithm is embedded with multiple recombination strategies (EA and DE) and a local search to improve its efficiency. Two commonly used measures (Euclidean and Hausdorff distance) have been considered as the measure of similarity between the generated and the target shape. The proposed method has been tested using two 2D examples of varying complexity, each of which was solved to desired accuracy within 5,000 evaluations, while for the complex 3D example, 100,000 evaluations were required. The performance of the repair strategy has been studied for both 2D and 3D examples to highlight the benefits. Such a capability is the first step towards a cutting edge shape optimization approach, where one is interested to uncover novel shapes with extreme performance characteristics.

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