

A Novel D-S Theory Based AHP Decision Apparatus under Subjective Factor Disturbances

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Abstract. In real life, sometimes Multi-Criteria Decision Making (MCDM) problems are dealt with inevitably under the disturbance of subjective factors such as intuition, feeling and emotion. However, the existing work cannot handle the issue well. As a result, it is difficult for them to predict and give some decision support for MCDM problem of this kind. Thus, to address the issue, this paper constructs a novel Analytic Hierarchy Process (AHP) approach based on Dempster-Shafer (D-S) theory. More specifically, our model can: (i) handle the subjective factor disturbances by distinguish complete (there not exists any hidden subjective factors impact the decision) and incomplete (the converse of complete) criteria; (ii) differentiate incomplete and complete relative ranking of the groups of decision alternatives over a criterion; and (iii) handle the ambiguous criteria evaluations of the groups of decision alternatives. Moreover, we give two methods to reduce the subjective factor disturbances. Finally, we illustrated our approach with a real-life problem.

1 Introduction

In real-life, often a decision is influenced by different hidden subjective factors, such as intuition, feeling, emotion of the decision maker. For example, an investor wants to pick up a house from three options that an agency introduces to him. When he enters a house, if he feels very uncomfortable suddenly, although the agency tells him how nice this house is at the reasonable price, the investor may still hesitate to make a deal. Another example is that a man wants to buy a second-hand car. Although the car dealer offers a good price and shows many excellent aspects of his car, if the man feels that the dealer cannot be trusted, he would not buy. Apparently, in both two Multi-Criteria Decision Making (MCDM) problems, the final choice of the decision makers has been affected by their feelings or intuition. Therefore, subjective factor disturbances actually plays a vital role in real-life MCDMs [7]. Especially, when we need predict what will happen if subjective factors will indeed work, the factors have to be reflected in the decision model that is used to predict this case.

Actually, decision making problems with the effects of subjective factors have been an active area of research that obtain more and more attentions from researchers [1]. Some focus on eliciting the influence of subjective factors for the

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decision making by psychological investigations [10,4,21] and experiments in behavioral economics [8,17,6]. All of these researches show that subjective factors such as emotions and intuition are, in manifold ways, involved in process of decision making. However, all of these researches do not give a formal treatment to this kind of decision problems, which is the main contribution of our model proposed in this paper. Another line of research incorporating subjective factor disturbances into models of decision making can be seen in the development of prospect theory [9], regret theory [13], dual-process models [18], smooth ambiguity model [11], and so on. However, all of these models focus on the decision making in single criterion, which means that they do not account the effect of subjective factors as the criteria weights in MCDM problems. Moreover, because of the subjective factor disturbances, it is difficult even impossible to elicit accurate evaluations for each decision alternative regarding each criterion in MCDM problem [12]. As a result, to handle hidden subjective factors in MCDM problem has to base on some uncertainties theory in MCDM. There are some important methods have been developed to deal with uncertainty in MCDM problems. For example, fuzzy sets approach [25] is used to model and solve fuzzy MCDM problems [16,22]. Beynon et al. [3] extend the AHP model to a DS/AHP method that introduce the basic concepts of D-S theory based AHP model to deal with the incomplete pairwise compare and the range of levels of uncertainty in MCDM problem. Nevertheless, all of these models do not consider the effects of subjective factors for the finally decision. Also, these MCDM approaches are essentially based on traditional evaluation methods, which cannot well handle uncertainty of different kinds of ambiguity, such as missing, imprecise, and uncertain evaluations of multi-criteria.

In order to address these issues, we propose model extend Analytic Hierarchy Process (AHP), which is one of the most well-known MCDM techniques. Also our extension is based on D-S theory, which is one primary theory to handle the ambiguity information. So, this paper actually proposes a novel method that incorporates AHP method and D-S theory to handle MCDM problems under subjective factor disturbances. First, as the subjective factor disturbances will impact the choices of a decision maker and change the criteria weights in the given MCDM problems, we identify two situations of criteria by considering whether or not there exists subjective factor disturbances. Therefore, we construct two comparison rules. (i) When the importance ordering of all criteria have been revealed (i.e., when subjective factor disturbances do not exist), the decision maker will apply the complete relative ranking rule. (ii) When the importance ordering of all criteria are incomplete (i.e., when subjective factor disturbances do exist), the decision maker will apply incomplete relative ranking rule. Then, our new model employs the rules to set the preference degree of decision alternatives. After combining the criteria weights and preference degree of decision alternatives, our method can find the optimal decision alternatives.

Actually, compared with the DS/AHP method that is also based on the D-S theory, our model can solve some problems of it while maintaining its advantage. (i) The DS/AHP method cannot explain why the criterion weights established

by a pairwise comparison matrix is still constructed as in AHP, but not by identifying a especial group of focal elements (set of single element sets) as favorable from the frame of discernment (of all criteria), but our model can apply the same rules to set the criteria weight and the relative ranking of groups of decision alternatives against the each criterion. (ii) The DS/AHP method does not offer an approach to obtain the optimal decision alternative based on the final mass function of each decision alternative, but our model does. Moreover, we extend the DS/AHP method to deal with subjective factor disturbances and ambiguity information.

This paper advances the state of art in the field of MCDM in the following aspects. (i) Our model can deal with the influence of subjective factors of the decision maker, and discerning two types of the criteria: revealed and hidden ones. (ii) While remaining the advantages of the DS/AHP model, our model can handle the complete and incomplete relative ranking of all choices over a criterion. (iii) Our model can incorporate the ambiguity aversion principle of minimax regret [14,15,23,24] to handle three types ambiguity evaluations that are mentioned in [14,15]. And (iv) according to our model, we give two methods to reduce the influence of subjective factor disturbances to make decision more rationally.

The rest of this paper is organised as follows. Section 2 recaps D-S theory and the decision rule under ambiguity. Section 3 models the subjective factor disturbances. Section 4 discerns the relative ranking of groups of decision alternatives on each criterion and discusses how to combine the criteria's weights and the decision alternatives' relative ranking. Section 5 shows some properties of our method. Section 6 illustrates our model by a real-life problem. Finally, Section 7 conclude the paper with future work.

2 Preliminaries

This section recaps D-S theory [19] and the decision rule under ambiguity [14,15,23,24].

2.1 Basics of D-S Theory

Definition 1. *Let an exhaustive set of mutually disjoint atomic Θ be a frame of discernment.*

- (i) *Function $m : 2^\Theta \rightarrow [0, 1]$ is called a basic probability assignment or a mass function if $m(\phi) = 0$ and $\sum_{A \subseteq \Theta} m(A) = 1$.*
- (ii) *Function $Bel : 2^\Theta \rightarrow [0, 1]$, defined as follows, is a belief function over Θ :*

$$Bel(A) = \sum_{B \subseteq A} m(B). \quad (1)$$

If $m(B) > 0$, then B is said to be a focal element of Bel .

(iii) Function $Pl : 2^\Theta \rightarrow [0, 1]$, defined as follows, is called a plausibility function over Θ :

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B). \quad (2)$$

Moreover, D-S theory also provides a rule to combine different mass functions:

Definition 2 (Dempster combination rule). Let m_1 and m_2 be two basic probability assignment over the discernment frame Θ . Then function $m_{12} = m_1 \oplus m_2$ is given by:

$$m_{12}(\{x\}) = \begin{cases} 0 & \text{if } x = \emptyset \\ \frac{\sum_{A_i \cap B_j = x} m_1(A_i)m_2(B_j)}{1 - \sum_{A_i \cap B_j = \emptyset} m_1(A_i)m_2(B_j)} & \text{if } x \neq \emptyset \end{cases} \quad (3)$$

The following is the concept of a mass function's ambiguity degree, which is a normalised version of the generalised Hartley measure for nonspecificity [5].

Definition 3. Let m be a mass function over discernment frame Θ , and $|A|$ be the cardinality of set A . Then the ambiguity degree of mass function m , denoted as δ , is given by

$$\delta = \frac{\sum_{A \subseteq \Theta} m(A) \log_2 |A|}{\log_2 |\Theta|}. \quad (4)$$

The point-valued expected utility can be extended to the interval one [20]:

Definition 4. For choice a specified by mass function m over Θ , its expected utility interval is $EUI(a) = [\underline{E}(a), \overline{E}(a)]$, where

$$\underline{E}(a) = \sum_{A \subseteq \Theta} \min(A) m(A), \quad (5)$$

$$\overline{E}(a) = \sum_{A \subseteq \Theta} \max(A) m(A). \quad (6)$$

In the above definition, if each $A \subseteq \Theta$ has only one element, $m(A)$ degenerates to probability and formulas (5) and (6) degenerate to the point-valued expected utility. In other words, the interval-valued expected utility is caused by $m(A) > 0$ where $|A| > 2$ (i.e., ambiguity).

2.2 Ambiguity Aversion Principle of Minimax Regret

In [14,15,23,24], we propose the following decision rule:

Definition 5 (The ambiguity aversion principle of minimax regret). Let m be an initial mass function over a set of utilities $\Theta = \{x_1, \dots, x_n\}$, $EUI(x) = [\underline{E}(x), \overline{E}(x)]$ be the expected utility interval of choice x and $\delta(b)$ be the ambiguity

degree of choice b . Then the ambiguity-avoiding maximum regret of the choice a against choice b is defined as

$$\mathfrak{R}_a^b = \varepsilon(b) - \underline{E}(a), \quad (7)$$

where $\varepsilon(b) = \underline{E}(b) + (1 - \delta(b))(\overline{E}(b) - \underline{E}(b))$, called the ambiguity-avoided upper expected utility of choice b . For any two choices a and b , the strict preference ordering $\succ$ is defined as follow:

$$a \succ b \Leftrightarrow \mathfrak{R}_a^b < \mathfrak{R}_b^a. \quad (8)$$

In [23], we prove that the binary relation $\succ$ defined in the above is a preference relation (asymmetric, acyclic and transitive) and so with the binary relation $\sim$ ($x \sim y$ if $x \not\succ y$ and $y \not\succ x$), we can compare any choices properly.

Also, in [14,15], we give a point-valued preference degree over a group of identified decision alternatives with the expected utility interval, which can induce a preference ordering equivalent to that in Definition 5. That is the following definition:

Definition 6. Let $EUI(A, c) = [\underline{E}(A, c), \overline{E}(A, c)]$ be an interval-valued expected utility of the group of decision alternatives A against criterion c , and $\delta(A, c)$ be the ambiguous degree of $EUI(A, c)$, then the preference degree of A is given by

$$\rho_c(A) = \frac{2\underline{E}(A, c) + (1 - \delta(A, c))(\overline{E}(A, c) - \underline{E}(A, c))}{2}. \quad (9)$$

Also, in [14,15], we present the following method to obtain the optimal decision alternative by the idea similar to that of the ambiguous aversion principle of minimax regret.

Definition 7. Let $m(\{a_i\})$ is a mass function over $\Theta = \{a_1, a_2, \dots, a_n\}$, where each a_i is a decision alternative, which induces belief function $Bel(\{a_i\})$ and plausibility function $Pl(\{a_i\})$, then the degree of preference of a_1 over a_2 , denoted by $P_{a_2}^{a_1} \in [0, 1]$, is given by:

$$P_{a_2}^{a_1} = (Bel(\{a_1\}) + \mu(\{a_1\})) - (Bel(\{a_2\}) + \mu(\{a_2\})), \quad (10)$$

where $\mu(\{a_i\}) = (1 - \delta_U(a_i))(Pl(\{a_i\}) - Bel(\{a_i\}))$ ($i = 1, 2$) and $\delta_U(a_i) = \frac{\sum_{\{a_i\} \cap B \neq \emptyset} m(B) \log_2 |B|}{\log_2 |\Theta|}$.

Thus, the preference relation between two decision alternatives can be defined as follows:

Definition 8. The preference relation between two decision alternatives a and b is defined as follows: (i) $a \succ b$ if $P_b^a > 0$; (ii) $a \prec b$ if $P_b^a < 0$; and (iii) $a \sim b$ if $P_b^a = 0$.

The binary relation in Definition 8 is proved indeed a preference ordering [14,15].

3 To Model Subjective Factor Disturbances

This section will handle the issue of subjective factor disturbances in MCDM, which has two characteristics. (i) The subjective factors will change the criteria weights in a given problem. For example, although the apples quality is high and the price is reasonable, which should be the main concerns of a buyer, the buyer might still not buy if he feels that the salesman cannot be trusted. In this case, apparently the intuition of the decision maker weakens the importance of the revealed criteria (i.e., quality and price) in the final decision. (ii) The decision maker cannot tell how important the subjective factors are in affecting his decision. To reflect these two characteristics, we take the subjective factors as a hidden criterion, denoted as T with an imprecise value of weight. Therefore, we can distinguish two situations for criteria: (i) complete criteria, i.e., the decision maker is sure that he has considered all relevant criteria; and (ii) incomplete criteria. i.e., there is a hidden criterion T , which is caused by some subjective factors that might disturb his decision. Formally, we have:

Definition 9. Let discernment frame $\Theta \subseteq \{c_1, \dots, c_n\}$, which is the set of all the criteria of an MCDM problem. Then the criteria are said to be complete if $\Theta = \{c_1, \dots, c_n\}$, while the criteria are said to be incomplete if $\Theta \supset \{c_1, \dots, c_n\}$.

In the situation of incomplete criteria, as the decision maker is uncertain about the weight of the hidden criterion T , the decision maker has no evidence about the weight of it. According to D-S theory, a belief degree should be assigned to the whole discernment frame. On the other hand, the complete criteria mean that the decision maker has the evidence to support the relative ranking of all criteria. So, we need not to assign any belief degree to the discernment frame to express the ignorance of the decision maker.

From the previous discussion, we can distinguish complete and incomplete criteria. First, in the case of incomplete criteria, when the relative ranking of some criteria weights are hidden, the decision maker adopts a frame of discernment to express his ignorance for the final relative ranking. These groups of decision alternatives and the discernment frame constitute a specific matrix. Through comparing a group of decision alternative with discernment frame Θ , the decision maker can express his preference ordering of the given criteria. So, our method satisfies the following theorem and corollary of [2]:

Theorem 1. For $(d+1)$ by $(d+1)$ matrix

$$A_{d+1} = \begin{pmatrix} 1 & 0 & \cdots & 0 & a_1 \\ 0 & 1 & \ddots & \vdots & a_2 \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & 1 & a_d \\ 1/a_1 & 1/a_2 & \cdots & 1/a_d & 1 \end{pmatrix}, \quad d = 1, 2, \dots, \quad (11)$$

where $a_1, \dots, a_d$ are positive real values, the largest eigenvalue defined λ_{d+1}^L associated with matrix A_{d+1} is $\lambda_{d+1}^L = 1 + \sqrt{d}$.

Corollary 1 (Incomplete Relative Ranking Rule (IRRR)). *For $(d + 1)$ by $(d + 1)$ matrix A_{d+1} given in Theorem 1, the normalised values within the eigenvector defined by $(x_1, \dots, x_{d+1})$ associated with the largest eigenvalue of the matrix A_{d+1} are as follow:*

$$x_j = \frac{a_j}{\sum_{i=1}^d a_i + \sqrt{d}}, \quad (12)$$

$$x_{d+1} = \frac{\sqrt{d}}{\sum_{i=1}^d a_i + \sqrt{d}}, \quad (j = 1, 2, \dots, d). \quad (13)$$

In this situation, based on the belief interval obtained by Corollary 1, by the similar idea of Definitions 6 and 7, we can obtain the point-value weight for each criterion under the condition of hidden criteria disturbance:

$$\omega_i = m(i) + \frac{(1 - m(\Theta))(m(\Theta))}{2d}, \quad (14)$$

$$\omega_T = 1 - \sum_{i=1}^d \omega_i = \frac{m(\Theta)(1 + m(\Theta))}{2}. \quad (15)$$

Now we turn to the situation of complete criteria. In this situation, as the decision maker actually has sufficient evidence for ranking all given criteria, he can rank the criteria without hesitation. As a result, he needs not to use the discernment frame to express his ignorance. Using the idea behind DS/AHP, we can introduce a new measurement: in the complete criteria, any relative ranking made on given criteria is compared with empty set $\emptyset$, which means no elements can be chosen by the decision maker, he must accept even turn out to be worst or least important criterion. In this aspect, we consider that empty set $\emptyset$ reflects the least important criterion. As the DS/AHP method [3] constructed, by Theorem 1, Corollary 1, and Definition 1, we can obtain:

Corollary 2 (Complete Relative Ranking Rule (CRRR)). *For the complete criteria, let s_i be one of d focal elements of a criterion with scale values a_i , then the normalised value for the relative ranking value of the decision alternatives over the criterion is:*

$$m(s_i) = \frac{a_i}{\sum_{j=1}^d a_j}, \quad (i = 1, 2, \dots, d). \quad (16)$$

Proof. By Definition 1, we have $m(\emptyset) = 0$ and $\sum_{A \subseteq \Theta} m(A) = 1$. And thus we have:

$$m(s_i) = \frac{\frac{a_i}{\sum_{j=1}^d a_j + \sqrt{d}}}{\sum_{A \subseteq \Theta} m(A)} = \frac{\frac{a_i}{\sum_{j=1}^d a_j + \sqrt{d}}}{1 - \frac{\sqrt{d}}{\sum_{j=1}^d a_j + \sqrt{d}}} = \frac{a_i}{\sum_{j=1}^d a_j}, \quad i = 1, 2, \dots, d. \quad \square$$

It is straightforward that the weight of each criterion is the percentage of its importance value from the total importance value of all revealed criteria.

4 Discern the Relative Ranking Value on an Individual Criterion

Similar to Definition 9, we can also distinguish two situations for the relative ranking of the groups of decision alternatives with respect to a given criterion:

Definition 10. Let $\Theta = \{a_1, \dots, a_n\}$ be the discernment frame of all decision alternatives, and $A_1, \dots, A_m$ ($A_i \cap A_j = \emptyset$ if $i \neq j$) be the groups of decision alternatives with respect to a given criterion that we can assign a preference scale value. Then the relative ranking of the groups of decision alternatives over a criterion is said to be complete if $A_1 \cup \dots \cup A_m = \Theta$; and it is said to be incomplete if $A_1 \cup \dots \cup A_m \subset \Theta$.

Actually, we can also use the CRRR and IRRR rules to solve these two cases. When the relative ranking of the decision alternatives against a criterion is incomplete, then the relative ranking of some decision alternatives against the given criterion is completely ignorant. As a result, a belief degree should be assigned to the discernment frame. On the other hand, the relative ranking of the decision alternatives regarding a criterion is complete, which means that the decision maker has the evidence to support the relative ranking of all decision alternatives over the criterion. As a result, we need not to assign any support belief degree to the discernment frame to express the ignorance of the decision maker.

Formally, by CRRR and IRRR, the relative ranking value of the decision maker regarding a criterion can be counted by formulas (17) and (18). Notice here without loss of generality, this paper uses 1 – 6 for discriminating levels of favourability, meaning *equal*, *strong*, *very strong*, *really strong*, *extremely strong*, and *absolutely strong*, respectively.

When the relative ranking is complete, it is

$$v(a_i) = m(s_i), \quad (17)$$

where $m(s_i)$ is given by formula (16). When the relative ranking is incomplete, it is

$$v_t(s_i) = x_j, \quad v(\Theta) = x_{d+1}, \quad (18)$$

where x_j and x_{d+1} is given by formulas (12) and (13). As a result, our model can solve the complete evaluation ignorance problem by introducing two methods of pairwise comparison.

After obtaining the criteria weights and the relative ranking value of the decision alternative of each criterion, the next step is to combine these two values and apply Dempster combination rule (i.e., formula (3)) to obtain the overall mass function for each decision alternative. In the DS/AHP method, Beynon [3] suggests that the weight of each criterion is incorporated into their respective knowledge matrix. So, the preference scale value of each decision alternative is multiplied by the respective weight of the given criterion.

However, if we adopt the DS/AHP method in the case that the relative ranking of the groups of decision alternatives is complete, the relative ranking value of

the groups of decision alternatives regarding a given criterion will not change no matter which weight we assign to the criterion. For this reason, we propose a new method to combine criteria weights and the relative ranking value of groups of decision alternative. Formally, we can set the mass function for the focal elements of each criterion as follows:

$$m(x) = \omega_i v(x), \quad m(\Theta) = 1 - \omega_i + v(\Theta)\omega_i, \quad (19)$$

where $v(x)$ and $v(\Theta)$ is the relative ranking value of groups of decision alternatives against criterion i , Θ is the frame of discernment of all decision alternatives, and ω_i is the weight of criterion i . This method means that because of the influence of the weight for a given criterion, the result (the relative ranking value) obtained by the criterion is on the level of ω available for the final ranking. The other level of $(1 - \omega)$ is unknown because we do not have evidence about the relative ranking of the decision alternatives regarding other criteria if we just consider the preference scales of the criterion.

Finally, the decision maker can obtain the mass function of each criterion by formula (19) and then use Dempster combination rule (i.e., formula (3)) to aggregate the overall mass function. After that, the decision maker can apply Definition 8 to set the preference ordering over the belief intervals and finally set the optimal option.

Moreover, in [14,15], we have pointed out that there are five types of evaluation over the groups of decision alternatives regarding each criterion, i.e., point-valued, risk lottery, missing, interval-value, and ambiguous lottery ones. Actually, in our ambiguity aversion AHP method, when the preference degree is point-valued, we can apply the complete relative ranking rule (CRRR) and incomplete relative ranking rule (IRRR) to obtain the relative rank value (the mass function for each criterion). As a result, we can apply formulas (5) and (6) and Definition 6 to obtain point-valued preference degree for these five types of evaluation, and then handle the ambiguity information directly.

5 How to Reduce the Disturbance

In real-life, sometimes it is necessary to reduce the effects of subjective factor disturbances in decision making. For example, in business, although a manager hates some suppliers, he should put the interest of the company in the first place and consider the deals more rationally or less emotionally. So, we need consider how to reduce the effect of subjective factor disturbances according to our model. This section will discuss this issue. First, we will give a formal definition for the effect of this kind before showing the properties.

Definition 11. *For a MCDM problem with subjective factor disturbances with the relative ranking of already known criterion is a_i , suppose the overall mass function of each group of decision alternatives is $m_1(X_t)$ in the condition of complete criteria, and the the overall mass function of each group of decision alternatives $m_2(X_t)$ in the condition of incomplete criteria then the effect of subjective factors T is*

$$f_t = \sum_{j=1}^{2^\Theta} (m_1(X_j) - m_2(X_j))^2. \quad (20)$$

This definition means that the effect of subjective factor disturbances are determined by the sum of difference of two sets of overall mass function about each group of decision alternatives, one of which consider the subjective factor disturbances and another is not. There are two reasons for this definition: (i) The effect of subjective factor disturbances is based on its impact for the final decision choice. For example, the feeling of uncomfortable for a house might change the final choice of decision maker. (ii) By Definitions 7 and 8, we can find that the preference ordering of the decision alternatives is determined by the value of $Bel(\{a_i\}) + \mu(\{a_i\})$. Moreover, because the ambiguity degree of the belief interval and $Bel(\{a_i\}) + \mu(\{a_i\})$ can be obtained by the overall mass function, we can just consider the change of overall mass function that considered the subjective factor disturbance or not to know the effect of subjective factors disturbances.

Now, we present a lemma as follows:

Lemma 1. *Suppose the mass function of the criteria's relative ranking is $m(i) = s$, $m(\Theta) = t$, where Θ is the discernment frame of each revealed criterion i and hidden criterion T . Then the higher the value of t , the higher the value of ω_T , vice versa.*

Proof. In this case, by formula (15), we can obtain the point-valued weight of each criterion:

$$\omega_i = s + \frac{(1-t)(t)}{2d}, \quad \omega_T = \frac{(1+t)(t)}{2}.$$

Since $0 \leq t \leq 1$, the higher the value of t , the higher the value of ω_T , *vice versa*. $\square$

Moreover, because the higher value of criteria weight of ω_T , the more effects the subjective factor disturbances will cause for the final decision, then by Lemma 1, we can derive:

Theorem 2. *The larger scale value (favourable degree) the decision maker uses as the basic for discriminating level of favourability, the less effects subjective factor disturbances will cause for the final decision, ceteris paribus.*

Proof. By formulas (15) and IPOR (Corollary 2), we have $\omega = 1 - \sum_{j=1}^d \omega_i = \frac{t(t+1)}{2}$, $t = \frac{\sqrt{d}}{\sum_{j=1}^d a_j + \sqrt{d}}$, where $t = m(\Theta)$. As a results, we can easily find that when the number of the revealed criteria does not change, and the decision maker uses the larger scale value (favourable degree) for the basic for discriminating level of favourability, which means that the value of $\sum_{j=1}^d a_j$ is higher, we find that the new weight of hidden criteria T : ω'_T is less than the old one. By Lemma 1, we can obtain the theorem directly. $\square$

This theorem means that to reduce the influence of subjective factor disturbances in an MCDM problem, the decision maker has to use large enough unit-scale to discriminate level of favourability on the criterion. This is consistent with our intuition that the more important (favourability level) the decision maker thinks the revealed criteria are, the more confident for him to insist on his choice. For example, the buyer might easily change his mind for buying some daily necessities, but in financial industry, it is hard for a CEO to change his mind, for example, when considering the feasibility of an equity financing project.

Theorem 3. *When the total preference degree of revealed criteria satisfies $\sum_{j=1}^d a_j < 2d + 1$, where d is the number of revealed criteria, the more criteria in consideration, and the less effects subjective factor disturbances cause upon the final decision, *ceteris paribus*.*

Proof. Let the number of the revealed criteria in an MCDM problem be d , T be the hidden criterion with weight ω_T . Now we add a new criterion that favourable degree compared with Θ is r (≥ 1). In order to prove that subjective factor disturbances will cause less affects for final decision, by Lemma 1 and Corollary 2, we have (noticing $\sum_{j=1}^d a_j = k$):

$$\begin{aligned} \frac{\sqrt{d}}{k + \sqrt{d}} &> \frac{\sqrt{d+1}}{\sum_{j=1}^{d+1} a_j + \sqrt{d+1}} \Leftrightarrow \sqrt{d}(k+1 + \sqrt{d+1}) > \sqrt{d+1}(k + \sqrt{d}) \quad (\text{when } r \geq 1) \\ &\Leftrightarrow (\sqrt{d}k + \sqrt{d})^2 > (\sqrt{d+1}k)^2 \\ &\Leftrightarrow 2d + 1 > k > 0 \quad \square \end{aligned}$$

By Theorems 2 and 3, we find that according to our model, there are two ways for reducing the effects of subjective factor disturbances: (i) to make a larger scale value to capture the importance degree of the revealed criteria; and (ii) when the scale value is small enough ($\sum_{j=1}^d a_j < 2d + 1$), to consider more relevant criteria to make decision. This result satisfies our intuition well: the decision maker will be not hesitated for subjective factors when making the decision if he has firmness of will or considers the problem very rationally.

6 The Problem of House Investment

This section illustrates our model by a real life house investment problem, which decision tree is shown in Fig. 1. Suppose a decision maker wants to buy a house for investment. There are four different alternatives A , B , C , and D and two criteria. Moreover, when he comes to house A , suppose he feels very uncomfortable suddenly.

To set the preference ordering for the decision alternatives over all the criteria, by the ambiguity aversion AHP method, first we use a 6-unit scale to discriminate the preference degrees of the decision alternatives and obtain the favourable matrix for criteria *price*, *quality* and *hidden* (see Table 1). From Fig. 1, we can see that for criterion *price*, three groups of decision alternatives have been identified and the union of these set of decision alternatives is the same as the discernment

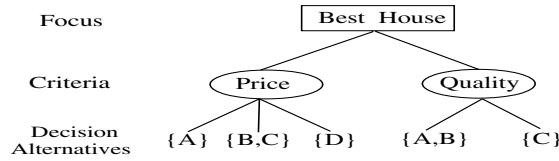


Fig. 1. Decision tree for the example of house investment

Table 1. Initial favourable matrix for price and quality criterion

Price	{A}	{B, C}	{D}	$\emptyset$	Quality	{A, B}	{C}	Θ	T	{A}	{B, C, D}	$\emptyset$
{A}	1	0	0	6	{A, B}	1	0	3	{A}	1	0	1
{B, C}	0	1	0	3	{C}	0	1	4	{B, C, D}	0	1	4
{D}	0	0	1	1	Θ	1/3	1/4	1	$\emptyset$	1	1/4	1
$\emptyset$	1/6	1/3	1	1								

frame. So, by Definition 9, the relative ranking of decision alternatives over the price criterion is complete, which means that the distinct groups of decision alternative are compared with the empty set (this set stands for the worst choice according to criterion *price*). For criterion *quality*, the union of two identified groups of decision alternatives are real subsets of the discernment frame, which means that the relative ranking of decision alternatives over criterion *quality* are incomplete. So, the decision maker compares the groups of decision alternatives with the discernment frame to express his ignorance in the preference degrees over criterion *quality*. Finally, for hidden criteria T , the decision maker feels very uncomfortable with house A , meaning that all other decision alternatives are must better than A . In order to distinguish the situation of no subjective feeling from that of an uncomfortable feeling, we set other decision alternative in higher level, but the preference relation amongst these decision alternatives is uncertain. As the decision maker knows all situations of his feelings, the relative ranking of decision alternatives over the hidden criterion is complete, meaning that the distinct groups of decision alternatives are compared with the empty set. This is exactly the meaning of Table 1. Now the decision maker can obtain the relative ranking value of each criterion by formulas (17) and (18) as follows: (i) for criterion *price*: $v(\{A\}) = 0.6$, $v(\{B,C\}) = 0.3$, $v(\{D\}) = 0.1$; (ii) for criterion *quality*: $v(\{A,B\}) = 0.357$, $v(\{C\}) = 0.475$, $v(\Theta) = 0.168$; and (iii) for the hidden criterion: $v(\{A\}) = 0.2$, $v(\{B, C, D\}) = 0.8$.

After confirming the favourable matrix, the decision maker will set the criteria weights. In this example, the decision maker feels very uncomfortable for house A , which means that this kind of subjective factor disturbed his final decision. As a result, the decision maker thinks that his criteria weight may change by the disturbance of hidden criterion T , which stands for his feeling about the houses. After using a 6-unit scale to discriminate importance degrees of the criteria, by the importance degrees of the criteria that is compared with the discernment frame, we can obtain the favourable matrix as follow:

$$\begin{pmatrix} \text{Criteria} & \text{Price} & \text{Quality} & \Theta \\ \text{Price} & 1 & 0 & 3 \\ \text{Quality} & 0 & 1 & 1 \\ \Theta & 1/3 & 1 & 1 \end{pmatrix} \quad (21)$$

By formulas (12) and (13), we can obtain: $m(p) = 0.554$, $m(q) = 0.185$, $m(\Theta) = 0.261$. Therefore, by formulas (14) and (15), the point-valued weight for each criterion is: $\omega_p = 0.602$, $\omega_q = 0.233$, $\omega_T = 0.165$. Then, form the relative ranking value of each criterion and the criteria weight, by formula (19), we can obtain the basic probability assignment in the D-S theory, which stands for the priority value over each criterion as follows: $m_p(\{A\}) = 0.361$, $m_p(\{B, C\}) = 0.181$, $m_p(\{D\}) = 0.06$, $m_p(\Theta) = 0.398$; $m_q(\{A, B\}) = 0.083$, $m_q(\{C\}) = 0.111$, $m_q(\Theta) = 0.806$; $m_T(\{A\}) = 0.033$, $m_T(\{B, C, D\}) = 0.132$, $m_T(\Theta) = 0.835$.

By Dempster combination rule (i.e., formula (3)), we can aggregate the overall mass function for all criteria as follows: $m_h(\{A\}) = 0.322$, $m_h(\{B\}) = 0.021$, $m_h(\{C\}) = 0.07$, $m_h(\{D\}) = 0.053$, $m_h(\{A, B\}) = 0.031$, $m_h(\{B, C\}) = 0.158$, $m_h(\{B, C, D\}) = 0.047$, $m_h(\Theta) = 0.298$.

Thus, form the above mass function, by formulas (1) and (2) we have:

$$\begin{aligned} [Bel(A), Pl(A)] &= [0.322, 0.651], [Bel(B), Pl(B)] = [0.021, 0.555], \\ [Bel(C), Pl(C)] &= [0.07, 0.573], [Bel(D), Pl(D)] = [0.053, 0.398]. \end{aligned}$$

Hence, form the belief interval and overall mass function, by Definitions 7, we can obtain

$$\begin{aligned} P_B^A &= 0.524, P_C^A = 0.435, P_D^A = 0.535, P_A^B = -0.524, P_C^B = -0.089, P_D^B = 0.011, \\ P_A^C &= -0.435, P_B^C = 0.089, P_D^C = 0.1, P_A^D = -0.535, P_B^D = -0.011, P_C^D = -0.1. \end{aligned}$$

Moreover, by Definition 8, we can set the preference ordering of the house investment problem as $A \succ C \succ B \succ D$. So, the decision alternative A is the optimal one over all the criteria.

7 Summary

This paper proposes a new MCDM method. (i) It can deal with the situation that the optimal option of decision maker is disturbed by some subjective factors (e.g., intuition, feeling, and emotion). (ii) It can differentiate two types of criteria that can consider the effects of subjective factors in MCDM problem. Moreover, our method offers two different rules (i.e., CRRR and IRRR) to deal with two criteria situations that distinguish by the existence of hidden criterion. (iii) It shows that actually the relative ranking values have two types that can be distinguished by the completeness of the information obtained. These two types can also be solved by CRRR and IRRR. And (iv) we identified two ways for reducing the effects of subjective factor disturbances to ensure the decision making is more rational. However, we still need to do some psychological experiments to check and improve our decision model in the future work.

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