

A Common Computational Framework for Semiring-based Argumentation Systems^{1,2}

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Abstract. We suggest semirings as a mean to parametrically represent “weighted” Argumentation frameworks (AF): different kinds of preference levels related to arguments, e.g. a score representing a “fuzziness”, a “cost” or a probability level of each argument, can be represented by choosing different semirings. The novel idea is to provide a common computational and quantitative framework where attacks (and/or supports) have an associated weight and, consequently, also the computation of the classical Dung’s semantics has an associated weight representing how much inconsistency we tolerate in the solution. The proposed semiring-based AF is then casted into a Soft Constraint Satisfaction Problem. Adding suitable sets of constraints permits to characterize the solution of the soft CSP as the desired Dung semantics. This allows for the application of the several solution techniques developed for Soft CSPs to AFs.

1 Introduction

Argumentation [11, 15] is based on the exchange and the evaluation of interacting arguments which may represent information of various kinds, especially beliefs or goals. Argumentation can be used for modeling some aspects of reasoning, decision making, and dialogue [9]. When an agent has conflicting beliefs (viewed as arguments), a set of plausible consequences can be derived through argumentation from the most acceptable arguments for the agent.

Many theoretical and practical developments build on Dung’s seminal theory of argumentation. A *Dung Argumentation Framework* (AF) is a directed graph consisting of a set of arguments and a binary conflict based *attack relation* among them. The sets of arguments to be considered are then defined under different semantics.

Constraint programming is a powerful paradigm for solving combinatorial search problems that draws on a wide range of techniques from artificial intelligence, computer science, databases, programming languages, and operations research [17]. The idea of the semiring-based formalism [6, 3] was to further extend the classical constraint notion by adding the concept of a structure representing the levels of satisfiability of the constraints. Such a structure is similar to a semiring (see Sec. 3). Problems defined according to the semiring-based framework are called *Soft Constraint Satisfaction Problems* (SCSPs). There already exist many efficient techniques, as constraint propagation [17], to solve such complex problems.

In this paper we show that different weighted AFs based on fuzziness, probability or a preference in general [15, 2], can be modeled

and solved with the same soft constraint framework by only changing the related semiring in order to optimize the different criteria. Also classical (crisp) AFs can be represented inside the soft framework by adopting the *Boolean* semiring. As far as we know, a common and powerful computational framework lacks in the literature of AFs. We consider the AFs where the attack (or support) relationship is associated with a more informative value representing the strength or the cost of the attack: suppose a judge must decide between the arguments of two parties, and often no conclusive demonstration of the rightness of one side is possible. The arguments will not have equal value for the judge and the case will be decided by preferring one argument over the other [15].

In this paper we revise the classical definitions provided by Dung, as the definition of AF and the Dung extensions (see Sec. 2), in order to solve these frameworks. Since attacks are associated with weights, the final solution comes with a global (in)consistency amount which characterizes how much inconsistency we are prepared to tolerate: given an inconsistency threshold α , we would be prepared to disregard attacks up to a total cost of α . The consistency amount of the solution is found by aggregating the attack weights together. This is particularly useful when the considered problem is over-constrained in case no classical (Dung’s) admissible extension can be found over the interaction graph. This consistency amount (or *budget*, as defined in [12]) represents how much we have to pay if we include some attack relationships that are not allowed in the classical Dung’s vision, but that we consider in order to obtain a minimum violating-cost solution: clearly, we prefer solutions obtained with a higher consistency budget.

Within the same framework we can also describe and solve two or more weighted AFs at the same time (with the cartesian product of semirings [6]).

The paper is structured as follows: in Sec. 2 we report the theory behind Argumentation and Sec. 3 summarizes the background about soft constraints. Section 4 redefines the framework provided by Dung [11] by expressing weights as semiring values and by adding the notion of α -consistency to the classical definitions. Then, in Sec. 5 we propose a mapping from weighted AFs to SCSPs. At last, in Sec. 6 we present the related work and Sec. 7 concludes the paper.

2 Dung Argumentation

Dung proposed an abstract framework for argumentation in which he focuses on the definition of the status of arguments [11]. For that purpose, it can be assumed that a set of arguments is given, as well as the different conflicts among them.

Definition 1 [11] *An Argumentation Framework (AF) is a pair $\langle \mathcal{A}_{rgs}, R \rangle$ of a set $\mathcal{A}_{rgs}$ of arguments and a binary relation R on*

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$\mathcal{A}_{rgs}$ called the attack relation. $\forall a_i, a_j \in \mathcal{A}_{rgs}, a_i R a_j$ means that a_i attacks a_j . An AF may be represented by a directed graph (the interaction graph) whose nodes are arguments and edges represent the attack relation. A set of arguments $\mathcal{B}$ attacks an argument a if a is attacked by an argument of $\mathcal{B}$. A set of arguments $\mathcal{B}$ attacks a set of arguments $\mathcal{C}$ if there is an argument $b \in \mathcal{B}$ which attacks an argument $c \in \mathcal{C}$.



Figure 1. A classical Argumentation Framework; e.g. b attacks c .

In Fig. 1 we show an example of AF represented as an *interaction graph*: the nodes represent the arguments and the directed arrow from b to c represents the attack of b towards c , that is $b R c$. Dung [11] gave several semantics of acceptability. These various semantics produce none, one or several acceptable sets of arguments, called extensions. The stable semantics is only defined via the notion of attacks:

Definition 2 [11] A set $\mathcal{B} \subseteq \mathcal{A}_{rgs}$ is *conflict-free* iff it does not exist two arguments a and b in $\mathcal{B}$ such that a attacks b . A *conflict-free* set $\mathcal{B} \subseteq \mathcal{A}_{rgs}$ is a *stable extension* iff for each argument which is not in $\mathcal{B}$, there exists an argument in $\mathcal{B}$ that attacks it.

The other semantics for acceptability rely upon the concept of defense. An admissible set of arguments according to Dung must be a conflict-free set which defends all its elements. Formally:

Definition 3 [11] An argument b is *defended* by a set $\mathcal{B} \subseteq \mathcal{A}_{rgs}$ (or $\mathcal{B}$ defends b) iff for any argument $a \in \mathcal{A}_{rgs}$, if a attacks b then $\mathcal{B}$ attacks a . A *conflict-free* set $\mathcal{B} \subseteq \mathcal{A}_{rgs}$ is *admissible* iff each argument in $\mathcal{B}$ is defended by $\mathcal{B}$.

Besides the stable semantics, three semantics refining admissibility have been introduced by Dung [11]:

Definition 4 [11] A *preferred extension* is a maximal (w.r.t. set inclusion) admissible subset of $\mathcal{A}_{rgs}$. An *admissible* $\mathcal{B} \subseteq \mathcal{A}_{rgs}$ is a *complete extension* iff each argument which is defended by $\mathcal{B}$ is in $\mathcal{B}$. The least (w.r.t. set inclusion) complete extension is the *grounded extension*.

3 Semirings and Soft Constraints

A c -semiring [6, 3] S (or simply semiring in the following) is a tuple $\langle A, +, \times, 0, 1 \rangle$ where A is a set with two special elements $0, 1 \in A$ (respectively the bottom and top elements of A) and with two operations $+$ and $\times$ that satisfy certain properties: $+$ is defined over (possibly infinite) sets of elements of A and is commutative, associative, idempotent, it is closed and 0 is its unit element and 1 is its absorbing element; $\times$ is closed, associative, commutative, distributes over $+$, 1 is its unit element, and 0 is its absorbing element (for the exhaustive definition, please refer to [6]). The $+$ operation defines a partial order $\leq_S$ over A such that $a \leq_S b$ iff $a + b = b$; we say that $a \leq_S b$ if b represents a value *better* than a . Other properties related to the two operations are that $+$ and $\times$ are monotone on $\leq_S$, 0 is its min and 1 its max, $\langle A, \leq_S \rangle$ is a complete lattice and $+$ is its lub.

A *soft constraint* [6, 3] may be seen as a constraint where each instantiation of its variables has an associated preference. Given $S = \langle A, +, \times, 0, 1 \rangle$ and an ordered set of variables V over a finite domain D , a soft constraint is a function which, given an assignment $\eta : V \rightarrow D$ of the variables, returns a value of the semiring. Using this notation $\mathcal{C} = \eta \rightarrow A$ is the set of all possible constraints that can be built starting from S , D and V . Any function in $\mathcal{C}$ depends on the assignment of only a finite subset of V . For instance, a binary constraint $c_{x,y}$ over variables x and y , is a function $c_{x,y} : V \rightarrow D \rightarrow A$, but it depends only on the assignment of variables $\{x, y\} \subseteq V$ (the *support*, or *scope*, of the constraint). Note that $c\eta[v := d_1]$ means $c\eta'$ where η' is η modified with the assignment $v := d_1$. Notice that $c\eta$ is the application of a constraint function $c : V \rightarrow D \rightarrow A$ to a function $\eta : V \rightarrow D$; what we obtain is a semiring value $c\eta = a$. $\bar{0}$ and $\bar{1}$ represent the constraint functions associating 0 and 1 to all assignments of domain values; the $\bar{a}$ function always returns the value a .

Given the set $\mathcal{C}$, the combination function $\otimes : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$ is defined as $(c_1 \otimes c_2)\eta = c_1\eta \times c_2\eta$ [6, 3]. The $\otimes$ builds a new constraint which associates with each tuple of domain values for such variables a semiring element which is obtained by multiplying the elements associated by the original constraints to the appropriate sub-tuples. Given a constraint $c \in \mathcal{C}$ and a variable $v \in V$, the *projection* [6, 3, 5] of c over $V - \{v\}$, written $c \downarrow_{(V \setminus \{v\})}$ is the constraint c' such that $c'\eta = \sum_{d \in D} c\eta[v := d]$. Informally, projecting means eliminating some variables from the support.

A SCSP [3] defined as $P = \langle C \rangle$ where C is the set of constraints. The *best level of consistency* notion defined as $blevel(P) = Sol(P) \downarrow_{\emptyset}$, where $Sol(P) = \bigotimes C$ [3]. A problem P is α -consistent if $blevel(P) = \alpha$ [3]; P is instead simply “consistent” iff there exists $\alpha >_S 0$ such that P is α -consistent [3]. P is inconsistent if it is not consistent.

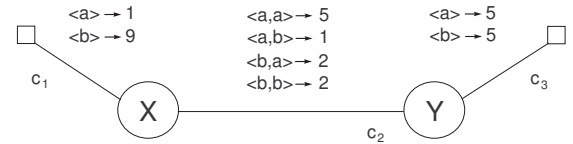


Figure 2. A soft CSP based on a Weighted semiring.

A SCSP Example. Figure 2 shows a weighted SCSP as a graph: the used semiring is the *Weighted* semiring, i.e. $\langle \mathbb{R}^+, \min, \hat{+}, \infty, 0 \rangle$ ($\hat{+}$ is the arithmetic plus operation). Variables and constraints are represented respectively by nodes and by arcs (unary for c_1 and c_3 , and binary for c_2), and semiring values are written to the right of each tuple. $D = \{a, b\}$. The solution of the CSP in Fig. 2 associates a semiring element to every domain value of variables X and Y by combining all the constraints together, i.e. $Sol(P) = \bigotimes C$. For instance, for the tuple $\langle a, a \rangle$ (that is, $X = Y = a$), we have to compute the sum of 1 (which is the value assigned to $X = a$ in constraint c_1), 5 (which is the value assigned to $\langle X = a, Y = a \rangle$ in c_2) and 5 (which is the value for $Y = a$ in c_3). Hence, the resulting value for this tuple is 11. For the other tuples, $\langle a, b \rangle \rightarrow 7$, $\langle b, a \rangle \rightarrow 16$ and $\langle b, b \rangle \rightarrow 16$. The *blevel* for the example in Fig. 2 is 7, related to the solution $X = a, Y = b$.

4 Expressing Weighted Argumentation Frameworks with Semirings

There have been a number of proposals for extending Dungs framework in order to allow for more sophisticated modeling and analysis of conflicting information. A common theme among some of these proposals is the observation that not all arguments are equal, and that the relative strength of the arguments needs to be taken into account somehow [15, 2, 1, 12, 14]. Weighted argumentation systems extend Dung-style abstract argumentation systems by adding numeric weights to every edge in the attack graph, intuitively corresponding to the strength of the attack, or equivalently, how reluctant we would be to disregard it.

In literature, we can find preferences directly associated with arguments [15] or, more frequently, with attacks [2, 1, 12, 14]. In this work we consider a weight as associated with the attack relationship, modeling its “strength”. For example, in Fig. 3, we represent a weighted interaction graph with three contradictory weather forecasts announced by *BBC*, *CNN* and *Fox*:

- a:** Today will be dry in London since BBC forecast sunshine
- b:** Today will be wet in London since CNN forecast rain
- c:** Today will be dry in London since Fox forecast sunshine

Therefore, $\mathcal{A}_{rgs} = \{a, b, c\}$, $a R b$ and $c R b$. In Fig. 3 each of these two attack relationships is associated with a fuzzy weight (in $[0, 1]$) representing the strength of the attack: since BBC forecast is more reliable than Fox forecast, a attacks b with more strength, i.e. 0.5, than c attacks b , i.e. 0.9 (0 represents the strongest possible attack and 1 the weakest one).



Figure 3. A fuzzy Argumentation Framework with fuzzy scores modeling the attack strength.

An argument can be seen as a chain of possible events that makes the hypothesis true. The credibility of a hypothesis can then be measured by the total probability that it is supported by arguments. The proper semiring to solve this problem consists in the *Probabilistic semiring* [3]: $\langle [0..1], \max, \hat{\times}, 0, 1 \rangle$, where the arithmetic multiplication (i.e. $\hat{\times}$) is used to compose the probability values together (assuming that the probabilities being composed are independent). Weights can be interpreted as subjective beliefs [12]. For example, a weight of $w \in (0, 1]$ on the attack of argument a_1 on argument a_2 might be understood as the belief that (a decision-maker considers) a_2 is false when a_1 is true. This belief could be modeled using probability [12].

The Fuzzy Argumentation [18] approach enriches the expressive power of the classical argumentation model by allowing to represent the relative strength of the attack relationships between arguments, as well as the degree to which arguments are accepted. In this case, the *Fuzzy semiring* $\langle [0..1], \min, \max, 0, 1 \rangle$ can be used, as for the example in Fig. 3.

In addition, the *Weighted semiring* $\langle \mathbb{R}^+, \min, \hat{+}, 0, 1 \rangle$, where $\hat{+}$ is the arithmetic plus ($0 = \infty$ and $1 = 0$), can model the (e.g. money) cost of the attack: for example, the number of votes in support of the attack [12]. Other possible interpretations of the weights are provided

in [12], as the chance to rank the relative strength of attacks: the single absolute weight is not important.

With the *Boolean semiring* $\langle \{true, false\}, \vee, \wedge, false, true \rangle$ we can cast the classic AFs originally defined by Dung [11] in the same semiring-based framework ($0 = false, 1 = true$).

Even if, as anticipated before, this paper deals with weights on attacks, it is still possible to aggregate these values and to obtain a problem with weights on arguments. Considering the example in Fig. 3, nodes not being attacked receive a fuzzy value of 1 (i.e. a and c), while for the others, the arithmetic mean (for example) of the weights of the attacks threatening them: for the *CNN rain* node, $(0.9 + 0.5) \setminus 2 = 0.7$ as in Fig. 4. Similarly, it may be possible to create the inverse mapping by shifting weights from arguments to attacks: considering Fig. 4, *BBC Sunshine* node attacks *CNN rain* node with a weight $(1 - 0.7) = 0.3$, that is the difference between the weights of the arguments (for example).



Figure 4. The weighted AF in Fig. 3 obtained by composing the weights of the attacks on the argument itself.

The following definition rephrases the notion of AF into *semiring-based AF*, i.e. an AF_S :

Definition 5 (semiring-based AF) A *semiring-based Argumentation Framework* (AF_S) is a quadruple $\langle \mathcal{A}_{rgs}, R, W, S \rangle$, where S is a semiring $\langle A, +, \times, 0, 1 \rangle$, $\mathcal{A}_{rgs}$ is a set of arguments, R the attack binary relation on $\mathcal{A}_{rgs}$, and $W : \mathcal{A}_{rgs} \times \mathcal{A}_{rgs} \rightarrow A$ a binary function called the *weight function*. Given $a, b \in \mathcal{A}_{rgs}$, $\forall (a, b) \in R$, $W(a, b) = s$ means that a attacks b with a strength level $s \in A$.

In Fig. 5 we provide an example of a weighted interaction graph describing the AF_S defined by $\mathcal{A}_{rgs} = \{a, b, c, d, e\}$, $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$ with $W(a, b) = 7$, $W(c, b) = 8$, $W(c, d) = 9$, $W(d, c) = 8$, $W(d, e) = 5$, $W(e, e) = 6$ and $S = \langle \mathbb{R}^+, \min, \hat{+}, \infty, 0 \rangle$ (i.e. the *Weighted semiring*). Therefore, each attack function is associated with a semiring value that represents the “strength” of the attack between two arguments. Still considering weather forecasts, the five arguments in Fig. 5 can be interpreted in the following way: **a:** *Sunshine*, **b:** *Rain*, **c:** *Sunshine and Hot*, **d:** *Cold and Mild Breeze* and **e:** *Strong Winds and no Waves* (which attacks itself because a contradiction).

We can consider the weights in Fig. 5 as user’s votes supporting the associated attack, collected on an on-line weather forecast forum. A semiring value equal to the top element of the semiring **1** represents no-attack relationships, which are not represented in Fig. 5 to have a light notation. As a consequence of this, the bottom element of the semiring, i.e. **0**, represents the strongest attack. Notice that the fuzzy preferences in [18] are considered with the inverse ordering w.r.t. ours: in [18] **1** represents the strongest attack and **0** the lowest one. Just to show some examples, by considering the weights in Fig. 3 in a *Fuzzy semirings*, if we have more attackers on a single argument the strongest attack rules over the others, e.g. $\min(0.9, 0.5) = 0.5$; by instead considering Fig. 5 and the *Weighted semiring*, the strength values of the attacks are summed together: e.g. a and c attacks b with $7 \hat{+} 8 = 15$. In Def. 6 we define the strength of

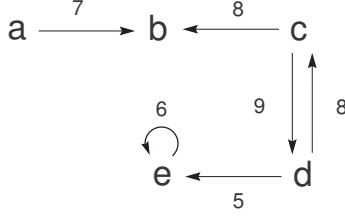


Figure 5. An example of a weighted interaction graph.

attack for sets of arguments that attacks an argument or another set of arguments ($\prod$ consists in applying the $\times$ operator of the semiring):

Definition 6 (attacks for sets of arguments) A set of arguments $\mathcal{B}$ attacks an argument a with a weight of k , that is $W(\mathcal{B}, a) = k$, if $\prod_{b \in \mathcal{B}} W(b, a) = k$. A set of arguments $\mathcal{B}$ attacks a set of arguments $\mathcal{C}$ with a weight of k , that is $W(\mathcal{B}, \mathcal{C}) = k$, if $\prod_{b \in \mathcal{B}, c \in \mathcal{C}} W(b, c) = k$.

In Def. 7 we redefine the notion of conflict-free set: conflicts can be now part of the solution until a cost threshold α is met, and not worse (we now have α -conflict-free solutions).

Definition 7 (α -conflict-free and α -stable extensions) A subset of the arguments $\mathcal{B} \subseteq \mathcal{A}$ is α -conflict-free for AF_S iff $W(\mathcal{B}, \mathcal{B}) \geq_S \alpha^5$. An α -conflict-free set $\mathcal{B}$ is an α -stable extension iff for each argument $c \notin \mathcal{B}$, $W(\mathcal{B}, c) <_S 1$, i.e. c is attacked by at least an element of $\mathcal{B}$.

To clarify the ideas w.r.t. the AF_S Fig. 5, while the set $\{a, b, c\}$ is not conflict-free in the crisp version of that problem because it includes the attacks between a and b and between c and b , it is instead 15-conflict-free because $W(a, b) \times_S W(c, b) = 15$.

Proposition 1 If an extension is α_1 -conflict-free, then the same extension is also α_2 -conflict-free if $\alpha_1 <_S \alpha_2$.

For instance, in Weighted semirings a 3-conflict-free extension is also 4-conflict-free.

Proposition 2 If an extension is conflict-free in the classical Dung semantics, then it is also 1-conflict-free in the weighted semantics.

Definition 7 proposes also the Dung's stable extension revisited in the semiring-based version. For example, considering the problem in Fig. 5 as unweighted (i.e. the classical Dung AF), the set $\mathcal{B} = \{a, d\}$ corresponds to the only stable extension. This set is also a 0-stable extension since there are no conflicts in $\mathcal{B}$ and b, c, e are attacked by an element in $\mathcal{B}$. As in Sec. 3 with the classical Dung's semantics, the other weighted extensions rely upon the concept of defense:

Definition 8 (defense) An argument b is defended by a set $\mathcal{B} \subseteq \mathcal{A}_{rgs}$ (or $\mathcal{B}$ defends b) iff $\forall a_i \in \mathcal{A}$ we have that $W(a_i, b) >_S W(\mathcal{B}, a_i)$.

Considering again the set $\mathcal{B} = \{a, d\}$, it α -defends d because $W(c, d) >_S W(a, b) \times W(d, c)$, i.e. $(9 >_S 15)$. This definition reminds the notion of collective defeat presented in [19, 16]. An α -admissible set of arguments must be an α -conflict-free set which α -defends all its elements. Formally:

Definition 9 (α -admissible extension) An α -conflict-free set $\mathcal{B} \subseteq \mathcal{A}_{rgs}$ is α -admissible iff each argument in $\mathcal{B}$ is defended by $\mathcal{B}$.

Not considering weights in Fig. 5, the admissible sets are: $\{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}$. Even in the AF_S in Fig. 5, these four extensions are 1-admissible since they are 1-conflict-free and defend themselves. Notice that a 1-conflict-free extension means that the set is conflict-free with every semiring. Moreover, for example, $\{a, b, c\}$ is 13-admissible because it is 13-conflict-free and c defends himself from d , since $W(d, c) >_S W(c, d)$ (i.e. $8 >_S 9$)⁶.

Besides the α -stable semantics, three semantics refining α -admissibility can be introduced:

Definition 10 (α -admissible, α -preferred, α -complete and α -grounded extensions) An α -preferred extension is a maximal (w.r.t. set inclusion) α -admissible subset of $\mathcal{A}_{rgs}$. An α -admissible $\mathcal{B} \subseteq \mathcal{A}_{rgs}$ is an α -complete extension iff each argument which is defended by $\mathcal{B}$ is in $\mathcal{B}$. The least (w.r.t. set inclusion) α -complete extension is the α -grounded extension.

Notice that now, if we can disregard consistency at will, we can always take the whole $\mathcal{A}_{rgs}$ set as an admissible and then preferred extension: $\{a, b, c, d, e\}$ in Fig. 5 the 43-admissible extension of course maximal, i.e. it is also preferred.

In Def. 11 we redefine also the semi-stable semantics as proposed in [8]. According to [8], given $a \in \mathcal{A}_{rgs}$ and $\mathcal{B} \subseteq \mathcal{A}_{rgs}$ we define a_α^+ as $\{c \mid W(a, c) <_S \alpha\}$ and $\mathcal{B}_\alpha^+$ as $\{c \mid W(\mathcal{B}, c) <_S \alpha\}$.

Definition 11 (semi- α -stable extension) Let $AF_S = \langle \mathcal{A}_{rgs}, R, W, S \rangle$ be a semiring-based AF and $\mathcal{B} \subseteq \mathcal{A}_{rgs}$. $\mathcal{B}$ is called an α -semi-stable extension iff $\mathcal{B}$ is α -complete and $\mathcal{B} \cup \mathcal{B}_\alpha^+$ is maximal.

Some classical properties among the extensions [8, 11] still hold:

Theorem 1 (extensions properties) For a given α ,

1. every α -preferred extension is also α -complete.
2. an α -grounded extension is contained in every α -preferred one.
3. every α -stable extension is also α -semi-stable.
4. every α -semi-stable extension is also α -preferred.
5. the 1-grounded extension is unique.

Proof Sketch. Following the line in [8]. 1) if $\mathcal{B}$ is the maximal set such that each argument in $\mathcal{B}$ is defended by $\mathcal{B}$, then each argument which is defended by $\mathcal{B}$ is in $\mathcal{B}$ (i.e. $\mathcal{B}$ is α -complete). 2) $\mathcal{B}$ is a superset of the α -grounded extension, i.e. the minimal α -complete one. Trivially, 3) α -stable implies α -semi-stable and 4) α -semi-stable implies α -preferred. 5) By computing the grounded extension as in [11] we always obtain a single 1-grounded extension. $\square$

5 Mapping from an AF_S to a SCSP.

In this section we propose a mapping from AF (and AF_S) to (S)CSPs: in this way it is possible to find all the α -extensions described in Sec. 4 as solution of the corresponding (S)CSP.

Given an $AF_S = \langle \mathcal{A}_{rgs}, R, W, S \rangle$, we define a variable for each argument $a_i \in \mathcal{A}_{rgs}$, i.e. $V = \{a_1, a_2, \dots, a_n\}$ and each of these argument can be taken or not as an element of one of the α -extensions, i.e. the domain of each variable is $D = \{1, 0\}$ (1 when the element is considered in the dung extension, 0 when it is not

⁵ in case of a partially ordered semiring $\geq_S$ is replaced by $\leq_S$

⁶ In the Weighted semiring, $>_S$ is equivalent to $<$, in the Probabilistic and Fuzzy ones, $>_S \equiv >$.

present). In the following explanation, notice that b attacks a means that b is a father of a in the interaction graph, and c attacks b and b attacks a means that c is a grandfather of a . To compute the different α -extensions we need to define specific sets of constraints:

1. **Conflict-free constraints.** Since we want to find the conflict-free sets, if $W(a_i, a_j) = s$ is in the graph we need assign a s consistency budget to the solution that includes both a_i and a_j in the considered α -conflict free extension: $c_{a_i, a_j}(a_i = 1, a_j = 1) = s$. For the other possible assignment of the variables ($(a = 0, b = 1)(a = 1, b = 0)$ and $(a = 0, b = 0)$), $c_{a_i, a_j} = 1$, since these assignments are permitted with no cost also in the classical semantics: in these cases we are choosing only one argument between the two (or none of the two) and thus, we have no α conflict.
2. **Admissible constraints.** For the admissibility, we need that, if a son argument a_i has a father node a_f but a_i has no grandfather node a_g , then we must avoid to take a_i in the extension because it is attacked and cannot be defended by any ancestor: this can be expressed with a binary constraint, $c_{a_f, a_i}(a_f = 0, a_i = 1) = 0$ and equal to 1 for the other assignments of a_f and a_i . Moreover, we need to add a $k + 1$ -ary constraint $c_{a_i, a_{g_1}, \dots, a_{g_k}}(a_i = 1, a_{g_1} = X_1, \dots, a_{g_k} = X_k)$, where $X_i, i = 1, \dots, k$ can be equal to 0 or 1 (i.e. each grandfather can be in the admissible set or not), which is equal to 0 if

$$\prod_{g_i \text{ for } i=1..k, \text{ s.t. } X_i=1} W(a_{g_i}, a_f) >_S W(a_f, a_i)$$

or equal to 1 if

$$\prod_{g_i \text{ for } i=1..k, \text{ s.t. } X_i=1} W(a_{g_i}, a_f) \leq_S W(a_f, a_i)$$

In words, the constraint has a value of 0 if the composition of the attacks of the grandfathers (taken in the set) towards the a_f father of a_i is better than the attack of a_f towards a_i . The same $k + 1$ -ary constraint is equal to 1 otherwise.

3. **Complete constraints.** To compute a complete extension $\mathcal{B}$, we need that each argument a_i which is defended by $\mathcal{B}$ is in $\mathcal{B}$ (see Sec. 2). This is already enforced with the admissible constraints, except for the nodes without a father. For these a_i nodes we need to add the constraint $c_{a_i}(a_i = 0) = 0$.
4. **Stable constraints.** If we have a son node a_i with multiple fathers $a_{f1}, a_{f2}, \dots, a_{fk}$, we need to add the constraint $c_{a_i, a_{f1}, \dots, a_{fk}}(a_i = 0, a_{f1} = 0, \dots, a_{fk} = 0) = 0$. In words, if a node is not taken in the extension (i.e. $a_i = 0$), then it must be attacked by at least one of the taken nodes, that is at least a father of a_i needs to be taken in the stable extension (that is, $a_{fj} = 1$). Moreover, if a node a_i has no father in the graph, it has to be included in the stable extension (notice a_i cannot be attacked by nodes inside the extension, since he has no father). The corresponding unary constraint is $c_{a_i}(a_i = 0) = 0$.

Notice that if we consider 1-conflict-free sets, what we obtain is exactly classical conflict free sets as proposed in Def. 1 by Dung. With Th. 2 we can find all the α -extensions by solving the corresponding SCSP obtained with the mapping:

Theorem 2 (solution equivalence) *Given an $AF_S = \langle \mathcal{A}_{rgs}, R, W, S \rangle$ and the corresponding SCSP P , the α consistent solutions of P corresponds to all the:*

- α -admissible extensions by using conflict-free and admissible constraints.

- α -complete extensions by using conflict-free, admissible and complete constraints.
- α -stable extensions by using conflict-free and stable constraints.

Corollary 1 (Dung's extensions equivalence) *By using the Boolean semiring the solutions of the CSP respectively correspond to all the classical admissible, complete and stable extensions.*

Moreover, to find the α -preferred extension we simply need to find all the maximal (w.r.t. set inclusion) α -admissible extensions of $\mathcal{A}_{rgs}$ s.t. $Sol(P) \Downarrow_{\emptyset} = \alpha$, that is to find all the α -admissible sets, keeping those with $Sol(P) \Downarrow_{\emptyset} = \alpha$ (if they exist) and at last returning the maximal ones. Similar considerations hold for the α -grounded extension, except for the last step in which we choose the minimal sets (w.r.t. set inclusion). Clearly, these maximal/minimal extensions are parametric w.r.t. the chosen α . Moreover, with the Fuzzy semiring we can immediately flatten some weights to the 0 bottom value:

Proposition 3 (fuzzy cut) *In a fuzzy AF_S , that is using $S = \langle [0..1], \min, \max, 0, 1 \rangle$, in order to find all the α -extensions it is possible to preprocess the problem by assigning 0 to all the attack relationships s.t. $\forall a, b \in \mathcal{A}_{rgs}, W(a, b) < \alpha$ [6].*

Notice that the presented soft constraint framework can be easily used to solve argumentation problems with additional constraints, as proposed in [10] only for boolean constraints. We can find further requirements on the sets of arguments which are expected as extensions, like *i*) “extensions must contain argument a when they contain b ” or *ii*) “extensions must not contain one of c or d when they contain a but do not contain b ”. All these additional constraints can be associated with a cost as well as the other ones, which represents the penalty to be paid: considering *ii*), $c(x_a = 1, x_b = 1) = 3$ means that we can however take a and b by paying 3 more in the inconsistency score.

At last, notice that the cartesian product of two semirings is still a semiring [6, 3], and this can be fruitfully used to describe multi-criteria constraint satisfaction and optimization problems by considering the efficient frontier as proposed in [4].

6 Related Work and Comparison

The framework proposed in this paper is able to solve the weighted proposals reported below (including the classical Dung framework [11]), both from the qualitative and (main novelty) quantitative point of view. It is only necessary to change the semiring in order to deal with the different kinds of weights.

In [18], the authors have developed the notion of fuzzy unification and incorporated it into a novel fuzzy argumentation framework for extended logic programming: the attacks are associated to a fuzzy strength value, i.e. a V -attack. As well, a V -argument A is V -acceptable w.r.t. the set $\mathcal{A}_{rgs}$ of V -arguments if each argument V -attacked A is V -attacked by an argument in $\mathcal{A}_{rgs}$.

In [2], AFs have been extended to *Value Based Argumentation Frameworks (VAF)* where V is a generic nonempty set of values and Val is a function mapping from elements of $\mathcal{A}_{rgs}$ to elements of V .

The work in [1] concerns the acceptability of arguments in preference-based AFs. Preferences are represented with a preordering relationships (partial or total) that resembles the ordering defined by the $+$ operator of semirings (see Sec. 3).

Probabilistic Argumentation [13]. This theory is an alternative approach for non-monotonic reasoning under uncertainty. It allows to judge open questions (hypotheses) about the unknown or future

world in the light of the given knowledge. From a qualitative point of view, the problem is to derive arguments in favor and against the hypothesis of interest.

In [12] the authors define the notion of consistency budget. The background motivations are very close to the ones behind our paper, but the authors do not directly redefine all the Dung's extensions according to the α -consistency budget, and do not provide a computational framework where to solve these weighted problems. In addition they formally show that some problems considering α thresholds are difficult, providing a strong motivations for using a constraint-based solving tool.

To compare these works with our solution, we need to say that in [12, 2] the authors redefine only the concept of conflict-free sets by tolerating a certain amount of attacks inside the set. What we do is instead a complete revision of the Dung's semantics (also admissible, complete and stable ones), by introducing also the notion of the weight of attack between a set and a single argument (i.e. in Def. 6) used for example in the new concept of defense (see Def. 8)

In [7] the authors solved AFs with weights on arguments instead of attacks. As already said, preferences on arguments are used in many works in literature in [15, 2]. Notice that we consider this kind of weights also by adding *Preference constraints*: the unary weight function $W(a_i) = s$ ($s \in A$) of an AF_S can be modeled with the unary constraints $c_{a_i}(a_i = 1) = s$, otherwise, when a_i is assigned to 0, the argument is not taken in the considered extension and so its cost must not be computed.

Notice that in [15, 2, 1] the preference among arguments is given in a qualitative way, that is argument a is better than argument b , which is better than argument c ; in this paper we study the problem from a quantitative point of view, with scores associated with arguments. We suggest the algebraic semiring structure (see Sec. 3) as a mean to parametrically represent and solve all the "weighted" AFs presented in literature (see Sec. 6), i.e. to represent the scores.

In [16] the authors define the "collective defeat", where we have a set B of nodes each of which is defeated by other members of the set and none of which is defeated by undefeated nodes outside the set. Compared to Def. 8, we do not require that each member in B is defeated by other members of B and the defeat relationship is weighted.

7 Conclusions

In literature we can find range of proposals that extend Dung's proposal [11] for abstract argumentation by incorporating preferences or weights on attacks between arguments (or on arguments themselves). This paper offers a single general framework, based on soft constraints, for considering this range of proposals. By having a uniform framework, it may be possible to see more clearly the relationships between different proposals. In the future, it may also offer the possibility to identify new results concerning classes of these proposals. We propose a cross-fertilization between AFs and SCSPs in order to use all the theory and solution techniques advanced for constraint programming [17] which are well-suited to solve combinatorial problems. This can be fruitfully exploited for solving hard problems related to weighted argumentation [12].

We have enhanced the definitions of Dung's classical extensions with an α consistency budget, which allows to solve over-constrained AFs by tolerating a certain amount of inconsistency. We have proposed a mapping to SCSPs in order to find all the α extensions: soft constraints can be used to model the mechanisms behind argumentation, when classical (i.e. no weights), weighted, fuzzy or probabilistic

problems must be solved. We have proposed an unifying computational framework where by only parametrically changing the semiring in the framework, we can answer to different AFs.

In the future, we want to extend the implementation presented in [7] to consider also weights on attacks, as presented in this paper, instead of only on arguments, as done in [7]. As a distinct goal, we want to cluster arguments according to their (for example) coherence, still using soft constraints as the framework to obtain the solution. This can be useful to check the discrepancies/likeness during a negotiation process, inside different interviews to the same political candidate or during discussions in general. As an example, "We do not want immigrants with the right to vote" is clearly closer to "Immigration must be stopped", than to "We need a multicultural and open society in order to enrich the life of everyone and boost our economy", and should belong to the same cluster.

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