

Preference-Based Argumentation Framework with Varied-Preference Intensity

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Abstract. Recently, Dung’s argumentation has been extended in order to consider the strength of the defeat relation, i.e., to quantify the degree to which an argument defeats another one. We construct an argumentation framework with varied-strength defeats from a preference-based argumentation framework with an intensity degree in the preference relation. We also consider the case when the preference over the arguments is constructed from a valued logic.

1 INTRODUCTION

Argumentation is a model for reasoning about an inconsistent knowledge. Dung has proposed an abstract argumentation framework that is composed of a set of arguments and a binary defeat relation between the arguments [5]. This framework has been instantiated to take into account the importance of the arguments, yielding to various preference-based argumentation frameworks [9, 2, 3].

Recently, Dung’s argumentation framework has been extended to consider defeat relations with varied strengths [7, 8, 6] (see Section 2). In this paper we instantiate this framework by taking into account the intensity of preference over arguments (see Section 3). More precisely, the intensity in the preference is propagated to the defeat relation: the larger the preference between two arguments, the larger the defeat. We also investigate the case where this preference relation is computed from weights associated to arguments (see Section 4). From an analysis of this situation, we derive some constraints on the construction of the preference relation from the weights.

2 ARGUMENTATION THEORY

An argumentation framework (AF) is a tuple $\langle \mathcal{A}, \rightarrow \rangle$ where $\mathcal{A}$ is a finite set (of arguments) and $\rightarrow$ is a binary (defeat) relation defined on $\mathcal{A} \times \mathcal{A}$ [5]. Two basic properties are necessary to define the acceptable arguments (also called extensions): the conflict freeness and the defense of an argument by a set of arguments ($A \subseteq \mathcal{A}$ defends $a \in \mathcal{A}$ if $\forall b \in \mathcal{A}$ such that $b \rightarrow a$, $\exists c \in A$ such that $c \rightarrow b$).

A preference-based argumentation framework is a 3-tuple $\langle \mathcal{A}, \rightsquigarrow, \succ \rangle$ where $\mathcal{A}$ is a set of arguments, $\rightsquigarrow$ is a binary relation defined on $\mathcal{A} \times \mathcal{A}$, called attack relation, and $\succ$ is a complete or partial order on $\mathcal{A} \times \mathcal{A}$, called preference relation [1]. A preference-based argumentation framework $\langle \mathcal{A}, \rightsquigarrow, \succ \rangle$ represents $\langle \mathcal{A}, \rightarrow \rangle$ if $\forall a, b \in \mathcal{A}$, we have $a \rightarrow b$ iff $(a \rightsquigarrow b \text{ and } \text{not}(b \succ a))$.

An argumentation framework with varied-strength defeats (AFV) is a 3-tuple $\langle \mathcal{A}, \rightarrow, VDef \rangle$ where $\langle \mathcal{A}, \rightarrow \rangle$ is an AF and $VDef$ is a function defined from $\rightarrow$ to $[0, 1]$ [6]. Extensions are also defined from the conflict freeness and the notion of defense. Intuitively, when $b \rightarrow a$ and $c \rightarrow b$, the strength of defeats should play a role in the definition of the defense since c is considered as a “serious” defender of a if the defeat of c on b is at least as strong as the defeat of b on a . The set $A \subseteq \mathcal{A}$ defends $a \in \mathcal{A}$ w.r.t. $\langle \mathcal{A}, \rightarrow, VDef \rangle$ iff for all $b \in \mathcal{A}$ such that $b \rightarrow a$ and $VDef(b, a) > 0$, there exists $c \in A$ with [7] (under the name of “strong” and “normal” defense in [7])

$$c \rightarrow b \text{ and } VDef(c, b) \geq VDef(b, a).$$

Regarding the notion of conflict-freeness, one can either use the standard definition for an AF [7] or the concept of α -conflict-freeness [6].

3 VALUED PREFERENCE-BASED AF

A valued preference relation on $\mathcal{A}$ is a function $P : \mathcal{A} \times \mathcal{A} \rightarrow [0, 1]$. $P(a, b)$ is the degree of credibility of the statement “ a is strictly preferred to b ”. The preference relation over arguments may serve to evaluate how strong a defeat relation is in preference-based argumentation framework.

We instantiate AFV with a preference-based argumentation framework where preferences have varied intensity. A *valued preference-based argumentation framework* is a 3-tuple $\langle \mathcal{A}, \rightsquigarrow, P \rangle$ where $\mathcal{A}$ is the set of arguments, $\rightsquigarrow$ is a binary attack relation defined on $\mathcal{A} \times \mathcal{A}$ and P is valued preference relation on $\mathcal{A}$. A valued preference-based argumentation framework $\langle \mathcal{A}, \rightsquigarrow, P \rangle$ represents an AFV $\langle \mathcal{A}, \rightarrow, VDef \rangle$ iff $\rightsquigarrow = \rightarrow$ and

$$VDef(a, b) = 1 - P(b, a).$$

Example 1 Let $\langle \mathcal{A}, \rightsquigarrow, \succ \rangle$ be a preference-based argumentation framework where $\mathcal{A} = \{a_1, a_2, a_3, a_4\}$, $a_1 \rightsquigarrow a_2$, $a_2 \rightsquigarrow a_1$, $a_1 \rightsquigarrow a_4$, $a_4 \rightsquigarrow a_1$, $a_2 \rightsquigarrow a_3$, $a_3 \rightsquigarrow a_2$, $a_3 \rightsquigarrow a_4$, $a_4 \rightsquigarrow a_3$, $a_2 \succ a_1$ and $a_4 \succ a_3$. $\langle \mathcal{A}, \rightsquigarrow, \succ \rangle$ represents Dung’s AF $\langle \mathcal{A}, \rightarrow \rangle$ with $a_2 \rightarrow a_1$, $a_1 \rightarrow a_4$, $a_4 \rightarrow a_1$, $a_2 \rightarrow a_3$, $a_3 \rightarrow a_2$, $a_4 \rightarrow a_3$ [4]. There are two stable extensions $A = \{a_1, a_3\}$ and $B = \{a_2, a_4\}$. The authors of [4] have noticed that A should not be considered as a stable extension as each argument in A is less preferred to at least one argument in B (as $a_2 \succ a_1$ and $a_4 \succ a_3$). Therefore B may be considered the only stable extension.

The problem raised in this example can be solved if one is able to differentiate between strict preference and incomparability. This is possible with our approach, with the following valued preference relation $\hat{P}_\succ$ defined from $\succ$: $\hat{P}_\succ(a, b) = 1$ and $\hat{P}_\succ(b, a) = 0$ if $a \succ b$, and $\hat{P}_\succ(a, b) = \frac{1}{2}$ otherwise. Then we have $VDef(a_4, a_3) = 1$,

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$VDef(a_3, a_2) = \frac{1}{2}$, $VDef(a_2, a_3) = \frac{1}{2}$, $VDef(a_2, a_1) = 1$, $VDef(a_1, a_4) = \frac{1}{2}$, and $VDef(a_4, a_1) = \frac{1}{2}$. The other values of $VDef$ vanish. Now, $A = \{a_1, a_3\}$ is no more an admissible extension since the defeat of $a_2 \in B = \{a_2, a_4\}$ on $a_1 \in A$ is stronger than the defense that A can give. Hence there remains only one stable extension namely B . The problem raised by this example on the stable extension is thus solved by the introduction of the strength of defeat relations.

It is worth noticing that an argumentation framework with varied-strength defeats represented by a valued preference-based argumentation framework is general and can also recover Dung's argumentation framework represented by a preference-based AF.

4 ARGUMENTS VALUATION

In this section we study the case where the preference relation is derived from weights associated to arguments. Let w be a function from $\mathcal{A}$ to $[0, 1]$, where $w(a)$ is the weight associated to the argument a . This weight is generally computed from the weights associated to knowledge from which arguments are built [2].

Given w , the preference relation in preference-based argumentation framework $\langle \mathcal{A}, \rightsquigarrow, \succ^w \rangle$ is defined in the following way [1]:

$$\forall a, b \in \mathcal{A}, a \succ^w b \text{ iff } w(a) > w(b).$$

Note that a valued preference relation P^w cannot uniquely be constructed from w . Here are two examples (for all $a, b \in \mathcal{A}$):

$$P_1^w(a, b) = \begin{cases} 1 & \text{if } w(a) > w(b) \\ w(a) - w(b) + 1 & \text{if } w(a) \leq w(b) \end{cases}$$

$$P_2^w(a, b) = \begin{cases} 0 & \text{if } w(a) < w(b) \\ w(a) - w(b) & \text{if } w(a) \geq w(b) \end{cases}$$

We compare the defense in both frameworks $\langle \mathcal{A}, \rightsquigarrow, \succ^w \rangle$ and $\langle \mathcal{A}, \rightsquigarrow, P_k^w \rangle$. Let $\langle \mathcal{A}, \multimap_w \rangle$ (resp. $\langle \mathcal{A}, \multimap_w, VDef_k \rangle$) be Dung's AF (resp. AFV) represented by $\langle \mathcal{A}, \rightsquigarrow, \succ^w \rangle$ (resp. $\langle \mathcal{A}, \rightsquigarrow, P_k^w \rangle$).

Let us first consider P_1^w . We obtain the same situations of defense w.r.t. $\langle \mathcal{A}, \rightsquigarrow, \succ^w \rangle$ and $\langle \mathcal{A}, \rightsquigarrow, P_1^w \rangle$, except when $w(a) < w(b) < w(c)$ and $w(b) - w(a) > w(c) - w(b)$. Indeed, c defends a w.r.t. $\langle \mathcal{A}, \rightsquigarrow, \succ^w \rangle$ (since $b \multimap_w a$ and $c \multimap_w b$) but not w.r.t. $\langle \mathcal{A}, \rightsquigarrow, P_1^w \rangle$ (since $VDef_1(c, b) < VDef_1(b, a)$). However, the intuition of the Boolean case ($\succ^w$) is valid: c is stronger than both b and a , and, because of that, c deserves to defend a against the attack of b (even if c is just slightly stronger than b). Consequently, we conclude that the expression P_1^w is not suitable.

In order to determine which expressions of P are suitable, we assume that the strict preference relation can be written from w as

$$\forall a, b \in \mathcal{A} \quad P^w(a, b) = p(w(a), w(b)) \quad (1)$$

where $p : [0, 1]^2 \rightarrow [0, 1]$. Function p shall be continuous, non-decreasing in the first argument and non-increasing in the second argument. Moreover, we have the boundary conditions:

$$p(0, 1) = 0 \text{ and } p(1, 0) = 1.$$

We obtain $VDef(a, b) = 1 - p(w(b), w(a))$. The situation $p(t, t)$ for $t \in [0, 1]$ corresponds to two arguments a and b having the same weight t . The degree of preference of a over b shall not depend on t . Hence, for symmetry reasons, we assume the following condition:

$$\forall t, v \in [0, 1] \quad p(t, t) = p(v, v). \quad (2)$$

We assume that the function p is fixed and does not depend on $\mathcal{A}$ and w . The function p is supposed to satisfy all previous requirements.

The condition raised in the study of P_1^w can be formalized in the following way.

Unrestricted positive defense (UPD): Let $\mathcal{A}$ be a set of arguments, and w be a function from $\mathcal{A}$ to $[0, 1]$. Let $\langle \mathcal{A}, \rightsquigarrow, P^w \rangle$ be a valued preference-based argumentation framework, where P^w is given by (1), representing an AFV $\langle \mathcal{A}, \multimap, VDef \rangle$. Let $A \subseteq \mathcal{A}$, $a, b \in \mathcal{A}$ and $c \in A$. If $c \rightsquigarrow b$, $b \rightsquigarrow a$ and $w(c) \geq w(b) \geq w(a)$ then c defends a against b w.r.t. $\langle \mathcal{A}, \multimap, VDef \rangle$.

Proposition 1 Under UPD, $p(t, v) = 0$ whenever $t \leq v$.

From Proposition 1, there is no way the statement " a is strictly preferred to b " is validated when $w(a) < w(b)$. One is sure about the credibility of this assertion only when $w(a)$ is significantly larger than $w(b)$. Therefore P_1^w is ruled out and P_2^w is suitable.

Considering P_2^w , the case where $w(c) < w(b) < w(a)$ is of particular interest. It consists in two sub-cases. When $w(b) - w(c) > w(a) - w(b)$ (i.e. $w(c) \ll w(b) < w(a)$), c does not defend a w.r.t. $\langle \mathcal{A}, \rightsquigarrow, P_2^w \rangle$ (since $VDef_2(c, b) < VDef_2(b, a)$). Since c , that is supposed to defend a , is much weaker than a and b , it is indeed reasonable that the defense of a by c fails in this case. When $w(b) - w(c) < w(a) - w(b)$ (i.e. $w(c) < w(b) \ll w(a)$), c now defends a w.r.t. $\langle \mathcal{A}, \rightsquigarrow, P_2^w \rangle$ (since $VDef_2(c, b) > VDef_2(b, a)$). Since the weight of c is not too far from that of b compared to a , one may admit that c is sufficiently strong to defend a against b .

5 CONCLUSION

In this paper we investigated a way to capture the strength of a defeat relation from the intensity of preferences over arguments: the larger the preference between two arguments, the larger the defeat. We developed a valued preference-based argumentation framework that represents an AFV. When the valued preference relation is constructed from a weight function w defined on the arguments, we showed, from a property called **UPD** that, for every a, b , a is clearly not strictly preferred to b w.r.t. the valued preference relation if $w(a) \leq w(b)$.

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