

On Admissibility in Timed Abstract Argumentation Frameworks

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Abstract. In this work, we define Timed Abstract Argumentation Frameworks, a novel argumentation formalism where arguments are only valid for consideration in a given period of time, which is defined for every individual argument. Thus, the attainability of attacks and defenses is related to time, and the outcome of the framework may vary accordingly. We define a timed, interval-based notion of grounded extension as a skeptical semantics.

1 Introduction

The study of pure semantic notions in a given scenario of arguments and their relationships requires some level of abstraction. Abstract argumentation systems [4, 6, 1, 2] are formalisms for argumentation where some components remain unspecified, being the structure of an argument the main abstraction. The emphasis is put on the semantic notion of finding the set of accepted arguments. Most of these systems are based on the single abstract concept of *attack* represented as an abstract relation, and extensions are defined as sets of possibly accepted arguments. The simplest abstract framework is defined by Dung in [4]. It only includes a set of abstract arguments and a binary relation of attack between arguments. Several semantic notions were defined and the Dung's argument extensions became the foundation of further research.

In this paper we present a formal model of arguments that are relevant only for a certain period of time, called here *timed arguments*, and its semantic consequences. A timed argument refers to information that is somehow dependant on time. This kind of argument has a limited influence in the system, given by the temporal context in which these arguments are taken into account. As related work in combining time and argumentation, in [5] a calculus for representing temporal knowledge is proposed, and defined in terms of propositional logic. This calculus is then considered with respect to argumentation, where an argument is defined in the standard way: an argument is a pair constituted by a minimally consistent subset of a database entailing its conclusion. This work is thus related to [3]. In contrast, here we maintain our development at the abstract level in an effort to capture intuitions related with the dynamic interplay of arguments as they become available and cease to be so.

In the following sections we propose an abstract argumentation framework equipped with a special kind of timed arguments, which are relevant only for an interval of time. We formalize the notion of defense between timed arguments and we present a semantics based on acceptability [4].

2 Timed Argumentation Framework

The consideration of time restrictions for arguments is formalized through an *availability function*, which defines a temporal interval for each argument in the framework. This interval states the period of time in which an argument is available for consideration in the argumentation scenario. The formal definition of our timed abstract argumentation framework follows.

Definition 1 A *timed abstract argumentation framework (TAF)* is a 3-tuple $\langle Args, Atts, Av \rangle$ where $Args$ is a set of arguments, $Atts$ is a binary relation defined over $Args$ and Av is the *availability function for timed arguments*, defined as $Av : Args \rightarrow [a, b]$ with $a, b \in \mathbb{Z} \cup \{-\infty, \infty\}$.

As stated before, the availability of arguments is tied to a temporal restriction. Thus, an attack to an argument may be *attainable* under certain conditions: if the availability of both the attacker and the attacked argument eventually overlaps. *Attainable attacks* are attacks that will eventually occur in some period of time.

Definition 2 Let $\Phi = \langle Args, Atts, Av \rangle$ be a TAF, and let $\mathcal{A}, \mathcal{B} \subseteq Args$ such that $(\mathcal{B}, \mathcal{A}) \in Atts$. The attack $(\mathcal{B}, \mathcal{A})$ is said to be *attainable* if $Av(\mathcal{A}) \cap Av(\mathcal{B}) \neq \emptyset$. The attack is said to be *attainable* in $Av(\mathcal{A}) \cap Av(\mathcal{B})$.

The set of all attainable attacks in the framework Φ is denoted $AttAtts_\Phi$. It is also possible to define the attainability of attacks at a particular timed intervals, as shown next.

Definition 3 Let $\Phi = \langle Args, Atts, Av \rangle$ be a TAF, and let $\mathcal{A}, \mathcal{B} \subseteq Args$ such that $(\mathcal{B}, \mathcal{A}) \in Atts$. The attack $(\mathcal{B}, \mathcal{A})$ is said to be *attainable at I* if: $(Av(\mathcal{A}) \cap Av(\mathcal{B})) \cap I \neq \emptyset$.

The set of attainable attacks of Φ at interval I is denoted $AttAtts_\Phi^I$.

3 Acceptability on Timed Argumentation

As attacks may occur only on a period of time (that in which the participants are available), argument defense is also occasional. The classical definition of acceptability requires an adaptation to a timed context. It is easy to analyze a defense over a single moment or time point, since attacks are either available or not, and thus an argument may be defended or not. However, when we expand the analysis over a wider temporal domain the situation is more complex, since defenses may occur sporadically. An argument should be considered

defended if it has defenders during its entire availability period. For example, an argument $\mathcal{A}$ may be defended by $\mathcal{X}$ in the first half of its time interval, and later by an argument $\mathcal{Y}$ in the second half. Although $\mathcal{X}$ is not capable of providing a full defense, argument $\mathcal{A}$ is defended while $\mathcal{A}$ is available. In order to define argument defense in timed abstract argumentation, we need to define when an argument is *threatened* by other arguments. An argument $\mathcal{A}$ is said to be a *threatened argument* if there is at least one argument $\mathcal{B}$, such that $(\mathcal{B}, \mathcal{A}) \in \text{AttAtts}_\Phi$.

Definition 4 Let $\Phi = \langle \text{Args}, \text{Atts}, \text{Av} \rangle$ be a TAF, and let $\mathcal{A}, \mathcal{B} \in \text{Args}$ such that $(\mathcal{B}, \mathcal{A}) \in \text{AttAtts}_\Phi$. The threat interval of $\mathcal{B}$ to $\mathcal{A}$, denoted $\tau_{\mathcal{B}}^{\mathcal{A}}$, is defined as $\text{Av}(\mathcal{A}) \cap \text{Av}(\mathcal{B})$.

The threat interval is formed by timepoints in common and it is the period of time in which an argument attacks another. In our framework, a defense occurs when an argument $\mathcal{X}$ is threatened by another argument $\mathcal{Y}$, which in turn is threatened by a third argument $\mathcal{Z}$. The first one is defended by the last one. However, this requirement is not a sufficient condition. Time restrictions must be taken into account: the defender $\mathcal{Z}$ must attack $\mathcal{Y}$ while this argument attacks $\mathcal{X}$.

Definition 5 Let $\Phi = \langle \text{Args}, \text{Atts}, \text{Av} \rangle$ be a TAF, and let $\mathcal{A}, \mathcal{B}, \mathcal{C} \in \text{Args}$ such that $(\mathcal{B}, \mathcal{A}) \in \text{AttAtts}_\Phi$ and $(\mathcal{C}, \mathcal{B}) \in \text{AttAtts}_\Phi$. The interval in which $\mathcal{C}$ defends $\mathcal{A}$ against $\mathcal{B}$, denoted $\delta_{\mathcal{C}\mathcal{B}}^{\mathcal{A}}$, is defined as (i) $\delta_{\mathcal{C}\mathcal{B}}^{\mathcal{A}} = \text{Av}(\mathcal{C}) \cap \tau_{\mathcal{B}}^{\mathcal{A}}$ if this intersection is non-empty, or (ii) $\delta_{\mathcal{C}\mathcal{B}}^{\mathcal{A}} = \emptyset$ otherwise.

An argument $\mathcal{C}$ is said to be a *defender* of another argument $\mathcal{A}$ if the provided defense interval is non-empty. Note that if the defense does not cover the $\mathcal{B}$ -threat interval then the defense task is not fully achieved, since there are moments of time where the threat of $\mathcal{B}$ still prevails, yet $\mathcal{C}$ is still considered a defender.

Example 1 Consider the TAF $\langle \text{Args}, \text{Atts}, \text{Av} \rangle$, where $\text{Args} = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$, $\text{Atts} = \{(\mathcal{B}, \mathcal{A}), (\mathcal{C}, \mathcal{B})\}$, and $\text{Av}(\mathcal{A}) = [5, 20]$, $\text{Av}(\mathcal{B}) = [5, 15]$, $\text{Av}(\mathcal{C}) = (-\infty, 7]$. All the attacks in Atts are attainable. The threat interval $\tau_{\mathcal{B}}^{\mathcal{A}} = [5, 15] = \text{Av}(\mathcal{A}) \cap \text{Av}(\mathcal{B})$. Argument $\mathcal{C}$ is a possible defender for $\mathcal{A}$ against $\mathcal{B}$, the defense interval $\delta_{\mathcal{C}\mathcal{B}}^{\mathcal{A}} = [5, 7]$. Notice that $\mathcal{C}$ does not provide full defense. If $\text{Av}(\mathcal{C}) = [0, 20]$ then $\mathcal{C}$ becomes a full defender since $\delta_{\mathcal{C}\mathcal{B}}^{\mathcal{A}} = \tau_{\mathcal{B}}^{\mathcal{A}}$.

Given an argument $\mathcal{A}$ and a set of arguments S , we are interested in the interval of defense of every defender included in S .

Definition 6 Let $\Phi = \langle \text{Args}, \text{Atts}, \text{Av} \rangle$ be a TAF, and let $\mathcal{A}, \mathcal{B} \in \text{Args}$ such that $(\mathcal{B}, \mathcal{A}) \in \text{AttAtts}_\Phi$, and let $S \subseteq \text{Args}$. The set of defense intervals of $\mathcal{A}$ against $\mathcal{B}$ in S , denoted as $\Delta_{\mathcal{B}}^{\mathcal{A}}(S)$, is defined as: $\Delta_{\mathcal{B}}^{\mathcal{A}}(S) = \{\delta_{\mathcal{X}\mathcal{B}}^{\mathcal{A}} : \mathcal{X} \in S, \mathcal{X} \text{ is a defender of } \mathcal{A} \text{ against } \mathcal{B}\}$.

Acceptability and admissibility must be slightly adapted to a timed context. A set of arguments S defends an argument $\mathcal{A}$ whenever S provides enough intervals of defense against every attacker of $\mathcal{A}$.

Definition 7 Let $\Phi = \langle \text{Args}, \text{Atts}, \text{Av} \rangle$ be a TAF, and let $\mathcal{A} \in \text{Args}$, $S \subseteq \text{Args}$. Argument $\mathcal{A}$ is acceptable with respect to S if $\forall \mathcal{X} \in \text{Args}$ such that $(\mathcal{X}, \mathcal{A}) \in \text{AttAtts}_\Phi$, S defends $\mathcal{A}$ from $\mathcal{X}$.

The analysis of admissibility, which leads to the grounded extension, can be restricted to a particular interval of time. Here, the required defense of an argument is only relevant in such an interval.

Definition 8 Let S be a set of intervals, and let I be an interval of time. The set S is said to be *t-connected* on I , denoted $S \rightsquigarrow I$, if:

- i. $\exists X \in S$ such that $I^- \in X$ and $I^+ \in X$, or
 - ii. $\exists Y \in S$ such that $I^+ \in Y$ and $(S \ominus Y^+) \rightsquigarrow [I^-, (Y^- - 1)]$
- Where $S \ominus i = \{I : I \in S \wedge i \notin I\}$ for some set of intervals S and some moment of time i .

Definition 9 Let $\Phi = \langle \text{Args}, \text{Atts}, \text{Av} \rangle$ be a TAF and I an interval. Let $\mathcal{A}, \mathcal{B} \in \text{Args}$ such that $(\mathcal{B}, \mathcal{A}) \in \text{AttAtts}_\Phi$ and let $S \subseteq \text{Args}$. The set S defends $\mathcal{A}$ from $\mathcal{B}$ in I if $\Delta_{\mathcal{B}}^{\mathcal{A}}(S)$ is *t-connected* on $I \cap \tau_{\mathcal{B}}^{\mathcal{A}}$.

Some arguments may not be available in several periods of time. When the analysis is restricted to a particular interval the availability of arguments plays a major role. It only makes sense to take into account arguments that are available in the studied interval, even in a single time point. The adapted acceptability notion follows.

Definition 10 Let $\Phi = \langle \text{Args}, \text{Atts}, \text{Av} \rangle$ be a TAF, and let $\mathcal{A} \in \text{Args}$, $S \subseteq \text{Args}$ and I an interval. Argument $\mathcal{A}$ is acceptable with respect to S in I if $\text{Av}(\mathcal{A}) \cap I \neq \emptyset$ and $\forall \mathcal{X} \in \text{Args}$ such that $(\mathcal{X}, \mathcal{A}) \in \text{AttAtts}_\Phi$ then S defends $\mathcal{A}$ from $\mathcal{X}$ in I .

If an argument $\mathcal{A}$ is acceptable with respect to a set S in I , if it is available at I and there exists a set of defenders $S_1 \subseteq S$, such that $\Delta_{\mathcal{X}}^{\mathcal{A}}(S_1)$ covers the threat interval in I , for each attacker $\mathcal{X}$ of $\mathcal{A}$. Note that an argument may not be acceptable w.r.t any set in its entire availability interval, and yet it may be acceptable w.r.t. some set in a particular interval. The interval-based acceptability function is defined as $F_\Phi^I = \{\mathcal{A} : \mathcal{A} \text{ is acceptable with respect to } S \text{ in } I\}$.

The iterative application of the acceptability function leads to a time-restricted notion of grounded extension.

Definition 11 Let $\Phi = \langle \text{Args}, \text{Atts}, \text{Av} \rangle$ be a TAF and I an interval. The timed grounded extension in I of Φ , noted as tGE_Φ^I , is the least fix point of function F_Φ^I .

The set tGE_Φ^I contains arguments that can be defended to the grounds while they are available in I .

4 Conclusions and future work

In this work we proposed a novel abstract argumentation framework where arguments are only valid for consideration in a given period of time. Thus, the attainability of attacks and defenses is related to time. We introduced the notion of timed arguments and we formally defined time-related defense conditions. The classical grounded extension is adapted to this context. We are studying the definition of timed frameworks with dense time and arguments with intermittent availability. We are also interested in the treatment of cycles of timed arguments and its semantic consequences.

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