

Balancing Optimality and Robustness in Resource Allocation Problems

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Abstract. We formally define a flexible robustness mechanism for resource allocation problems encoded as combinatorial auctions based on repairable solutions. Our notion of robustness is focused on resources that become unavailable once an allocation has been found. We extend previous works on encoding auctions to weighted Max-SAT and supermodels for the boolean satisfiability framework (SAT) with a mechanism that adds flexibility to the robust solutions.

1 INTRODUCTION

Robustness is a desirable quality in real world resource allocation problems that have to deal with uncertainty. The changes (breakages) in the environment that we consider are resources that become unavailable once a solution to the resource allocation problem has been found and announced to the participants. Robustness for CA has already been addressed regarding bid withdrawal in [6]. But, as far as we know, the problem of resource unavailability has not yet taken into account. In this paper we introduce a new mechanism for finding robust solutions to auctions with a bounded number of allowed breakages (a) in resource availability, a bounded number of repairs (b) in bid assignment, and a minimum guaranteed profit (β) for the auctioneer. We call this a (a,b,β) -super solution.

2 RELATED WORK

Our approach is based in the encoding of auctions as weighted Max-SAT formulas [7], so we have taken the existing notion of robustness for plain SAT as a basis [4]. This approach introduces the notion of *supermodel*, where the original formula is reformulated into a new one that has a model if and only if the original has a supermodel. The concept of (a,b) -supermodel for propositional logic has been generalized to that of (a,b) -super solution in the context of constraint programming (CP) in [5]. These works are close to ours but they are only focused in finding a solution that can be repaired. They do not consider the cost of the repair (but only the total number of variables changed). In our work the cost is also an important aspect that needs to be minimized.

There are also some works on robustness for Combinatorial Auctions, however the only work that deals with repairable solutions is the one of [6], although it addresses the problem of bid withdrawal (i.e. a bidder that withdraws a winning bid), while we consider the problem of resource unavailability.

3 ROBUST AUCTIONS AS PARTIAL WEIGHTED MAX-SAT PROBLEMS

A partial weighted Max-SAT formula has the form $F = C \wedge W$, where C denotes the set of mandatory constraints, and W denotes the set of weighted, non-mandatory constraints. In the standard modeling (non-robust) of CA [7], a set of boolean variables encode each bid $b_1, \dots, b_n$ to indicate whether it is winner or loser. Hence, the mandatory clauses (C) include the bid incompatibilities, such that for each pair of incompatible bids i, j ($i \neq j$), i.e. such that $S_i \cap S_j \neq \emptyset$, we state $\neg b_i \vee \neg b_j$ since it is not possible to assign both bids as winners. The non-mandatory clauses (W) are the prices (weights) of the bids, so we add a weighted unit clause (b_i, p_i) indicating that if the bid i is not part of the solution, then there is a loss of revenue of p_i .

We extend this encoding adding another set of boolean variables representing resource availability $g_1, \dots, g_m$ and add to the mandatory clauses (C), $b_j \Rightarrow g_{i_1} \wedge \dots \wedge g_{i_{n_j}}$ to indicate that whenever bid j is accepted, then all of its goods must be available. Robustness with repairable solutions is defined based on three parameters: a , b and β , where a is the number of breakages that the solution can absorb, b is the maximum size of a repair (number of variables changed), and β is the minimum benefit of the solution (including its possible repairs).

In order to deal with the benefit constraints we need to move from SAT to SMT (Satisfiability Modulo Theories), incorporating the theory of linear integer arithmetic (LIA). First, we add to the mandatory constraints (C) the restriction of the minimum benefit. Therefore we state $\sum_{j \in 1..|B|} b_j \cdot w_j \geq \beta$.

Finally, in order to turn this into a robust CA, we add an additional set of formulas with the aim of assuring that for each possible breakage of size at most a from the breakage set G (resources), there exists a repair of size at most b from the repair set B (bids) with a benefit greater or equal to β . Therefore, we add a new set of constraints, one for each possible breakage $S \subseteq G, |S| \leq a$, that is composed of a set of restrictions $C_{\bar{S}}$ analogous to C but with the variables of the breakage (S) flipped (negated), and using new variables for the bids (b'_{i_S}). Each set of restrictions (representing a breakage) cannot change (repair) the value of more than b variables. Therefore, in order to do limit the number of changes we define new variables d_i to compute them, such that $(d_{i_S} = 0 \Leftrightarrow b_i = b'_{i_S}) \wedge (d_{i_S} = 1 \Leftrightarrow b_i \neq b'_{i_S})$.

At the end, the partial weighted Max-SMT formula encoding a robust CA is the following formula:

$$C \wedge W \wedge \bigwedge_{S \in S_1^{a+}} \left(C_{\bar{S}} \wedge \sum_i d_{i_S} \leq b \right) \quad (1)$$

where S_1^{a+} denotes the set of all non-empty subsets of a set S with size at most a . The correctness of this encoding is proved in [1].

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3.1 Flexible robustness

The previous encoding forces the solutions to be (a, b, β) repairable, which is very strict. In order to add the possibility of allowing some of the breakages to be left unrepaired we slightly modify it. We add a mechanism to count the number of repairable breakages, so that we can later maximize it (i.e. maximize the reparability of the solution). Thus, we create a new boolean variable R_S for each possible breakage S , that will be true if and only if the breakage is repairable. We associate a weight p_S to each variable R_S , and state the weighted clause (R_S, p_S) for each S . The satisfaction of these new weighted clauses will be maximized together with the weights of the bids.

The weights' values of these variables allow us to express different robustness notions as for example optimal solutions as robust as possible (OR), robust solutions as optimal as possible (RO), or trade-off solutions. The difference between them is in the weights that are set for the bids' prices and for the robustness clauses. For the OR the weights of the robustness clauses must be much lower than the bids' prices, and inversely for the RO. For the hybrid setting we introduce a new parameter α , that defines the trade-off between optimality and robustness, so that $\alpha = 0$ is equivalent to OR and $\alpha = 1$ to RO.

4 EXPERIMENTATION

For the experimentation we have used CATS (Combinatorial Auction Test Suite) [3] with the L7 distribution. We analyzed the effect of some of the parameters of the (a, b, β) -super solutions, as well as the number of bids participating in the CA. The parameter a was set to 1, and the number of goods being sold was set to 15. We varied the number of bids in $\{15, 20, 25, 30\}$, the number of repairs b in $\{1, 2, 3, 4, 5\}$ and the value β , was set according to various percentages of the optimal revenue when robustness is not considered, concretely $\{50\%, 60\%, 70\%, 80\%, 90\%\}$. For each case we generated 50 instances, solved each without robustness to get the optimal revenue, and then the robust version to compare with. All the instances were solved using Yices [2] with a timeout of 1000 seconds. The experiments were performed with an Intel Core 2 Duo with 1.86GHz and 2GB of RAM, running under Windows XP SP2.

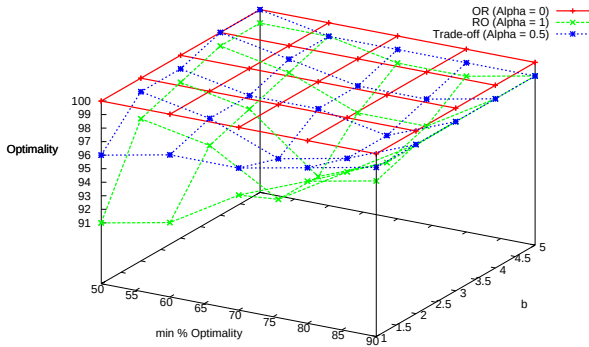


Figure 1. Optimality varying b and β .

Figures 1 and 2 graphically show the results with 15 bids regarding optimality and robustness respectively. The X axis represents the minimum benefit percentage of the solution (and its repairs). The Y axis represents the size of the repair b . We observe that when the

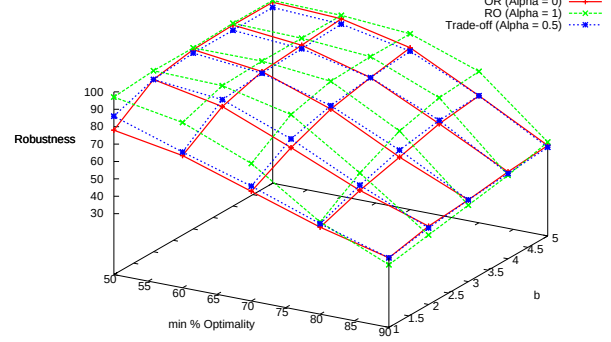


Figure 2. Robustness varying b and β .

value of β decreases, the value of robustness increases, since the problem is less restricted and it is easier to find robust solutions. Also, when the size of the repair (b) increases, the value of robustness is also increased, since more repairs are allowed. We can also see that when the value of α increases the percentage of optimality decreases (since the solutions are more robust, and thus less optimal), and conversely the robustness increases. The differences between the graphs are higher in the robustness graph than in the optimality, with the RO graph obtaining 20 percentual points more robustness than OR in the instances with lower values of α . These results show that the optimality of the solutions when looking for robust solutions is not affected in the same level as the robustness when searching for optimal solutions.

5 CONCLUSIONS

We presented a formal definition for resource unavailability-based robustness for combinatorial auctions. Our notion of robustness balances the number of allowed repairs when a break occurs and the loss of revenue for the auctioneer, by defining (a, b, β) super-solutions, in a similar way as supermodels for SAT. The encoding has been performed in partial weighted Max-SMT.

We analyzed the relation between robustness and optimality and proposed a mechanism to define the trade-off between robustness and optimality of the solutions with a parameter α . In the experimentation section we see that the loss of optimality when searching robust solutions is lower than the loss of robustness when looking for optimal solutions.

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