

Algorithms and Complexity

Paul E. Dunne
Dept. of Computer Science
University of Liverpool
United Kingdom

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Introduction I

1. The models and semantics described provide *analytic methods* for a formal *computational theory* of argument.
2. These do not, however, discuss how such methods can be *realised in practice* nor what *costs* may be incurred in doing so (and the extent to which costs are *unavoidable*).
3. These are topics dealt with in the study of algorithmics and computational complexity.

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Introduction II

- What sort of computational tasks might we wish to solve in practice?
Verify that a subset of arguments meets the criteria of some semantics.
Decide if an argument is a member of *some/all* such subsets.
Decide if *any* (or *any non-empty*) suitable subset exists.
Construct one/all such subsets.
Decide if *two different* semantics *coincide*.

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Introduction III

- These arise both in AFs and ABFs.
- For *deductive systems* natural questions are
 - Is (Φ, α) an argument?
 - Is (Φ, α) *warranted*?
- Algorithmic and complexity questions have been studied in all three models.

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Overview

- Formal definition of problems.
- Classical semantics in AFs (and ABFs)
- Novel semantics – semi-stable & ideal
- Parametric case – resolution-based.
- Special case efficient algorithms.
- Augmented AFs (VAF, EAF, WAS).

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“Canonical” Decision Problems

- $VER_{\sigma}(<X, A>, S)$ – does $\sigma(S)$ hold?
- $CA_{\sigma}(<X, A>, x)$ – $\exists S : x \in S$ and $\sigma(S)$?
- $SA_{\sigma}(<X, A>, x)$ – $\forall S \sigma(S) \Rightarrow (x \in S)$?
- $EX_{\sigma}(<X, A>)$ – $\exists S : \sigma(S)$?
- $NE_{\sigma}(<X, A>)$ – $\exists S : \sigma(S)$ and $S \neq \emptyset$?
- $COIN_{\sigma, \tau}(<X, A>)$ – $\forall S \sigma(S) \Leftrightarrow \tau(S)$?

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Grounded, Admissible, Preferred, Stable

- The status of *all* the canonical decision questions w.r.t. "standard" semantics is completely resolved for (finite) AFs.
- GR: efficient methods for all problems (using iterative algorithm of Part 3 earlier).
- ADM: VER easy; EX and SA trivial; CA and NE NP-complete.
- PR: EX trivial; CA and NE NP-complete; VER coNP-complete; SA Π_2 -complete.
- ST: VER easy; CA, EX NP-complete; SA coNP-complete.
- COIN (ST=PR): Π_2 -complete.
- From Dung(1995), Dimopoulos & Torres (1996); Dunne & Bench-Capon (2002).

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Proving "negative" results

- "Almost all" of the known hardness results use variations on a translation from CNF formulae to AFs, first given in Dimopoulos & Torres (1996).
- Similar constructions underpin most results obtained for novel semantics and augmented frameworks.

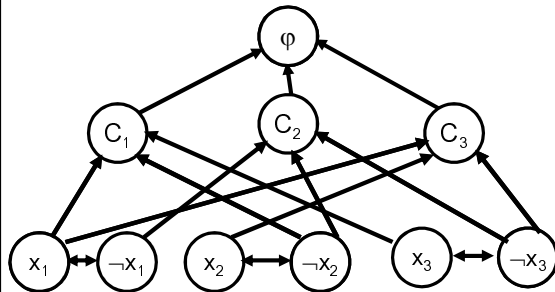
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The standard translation

- Represent a 3-CNF formula, φ , with clauses $\{C_1, C_2, \dots, C_k\}$ defined by literals (y_{i1}, y_{i2}, y_{i3}) as an AF with the argument labelled φ , in an admissible set if and only the 3-CNF formula, φ , is satisfiable.

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$(x_1, \neg x_2, \neg x_3), (\neg x_1, \neg x_2, x_3), (x_1, x_2, \neg x_3)$



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Applications (in addition to CA)

- Forms the basis of lower bound proofs in
 - Existence of stable extension
 - Verification of preferred extension
 - Non-emptiness (ADM and PR)
- Variation gives lower bound on
 - SA (preferred extensions)
 - Coincidence of PR and ST

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Assumption-based frameworks

- For standard semantics, a similarly complete picture has been obtained for various settings of ABFs.
- The constructions are presented in work of Dimopoulos, Nebel & Toni (2002).
- The key element is in relating bounds to the complexity of checking “*derivability*” within the supporting ABF theory $\langle L, R \rangle$.

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Novel semantics

- Despite the computational problems arising in standard semantics, these do not feature as a motivation for proposals such as semi-stable or ideal semantics.
- It turns out that computationally these, to some extent, raise greater difficulties.

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Semi-stable semantics

- EX: trivial (in *finite* AFs).
- VER: coNP-complete
- CA, SA: Σ_2 -complete, Π_2 -complete.
- NE: coNP-complete
- CA and SA recent (optimal) improvements by Dvorak & Woltran (IPL, April 2010) to lower bounds of Dunne & Caminada (JELIA, 2008).
- Again all obtained by variant of standard translation from CNFs to AFs.
- No treatment in ABFs yet undertaken (but results for LP settings of ABFs follow from AF case; DL & AEL open problems).

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Ideal Semantics

- Similar complete picture (in both AF and ABF settings) given in Dunne (AIJ, 173:15539-1591, 2009)
- Recall that CA=SA, i.e. unique status semantics.
- Lower bound constructions again use form of standard translation, but require some *non-trivial structural machinery* from computational complexity.
- Pro: CA, VER, NE "easier" than SA in preferred semantics. (AF and ABF).
- Pro: also construction of ideal extension easier.
- Con: "intermediate" hardness between NP/coNP and Σ_2 -complete/ Π_2 -complete. ("qualified" Θ_2 -complete)

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Resolution-based semantics.

- Main case of interest is *grounded* version – single to multiple status. But,
- VER & NE *remain* efficiently decidable.
- CA (now non-trivial) NP-complete.
- Some restricted cases allow *exact counting* of the *number of extensions*.
- Results from (Baroni, Dunne, Giacomin, IJCAI, 2009; COMMA 2010)

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Special case algorithms I

- The computational difficulty of many key questions motivates looking for *special classes* of AF where these are “*easier*”.
- Dung (1995) already establishes AFs without *cycles* as one such class.
- Coste-Marquis, Devred & Marquis (2005) give *symmetric* AFs as another.
- Dunne (2007) shows *bipartite* AFs are a third such class.

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Special case algorithms II

- Much recent activity has focused on so-called *Fixed Parameter Tractability* (FPT) measures.
- In particular, AFs with *bounded treewidth* via *Courcelle's Thm.* have *linear time methods* for properties expressible by a *suitable second-order formula*.
- Both Dunne (2007) and Dvorak and Woltran (2010) present such formulae for a number of decision questions in both standard and novel approaches.

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Special case algorithms III

- Some “natural” graph-theoretic forms, make no difference:

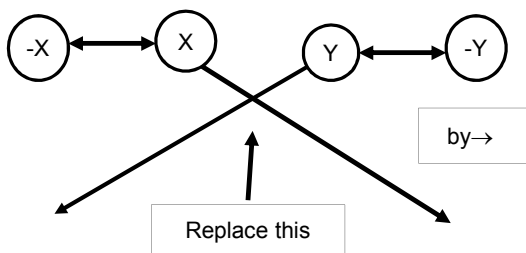
Planarity

Limiting number of *attacks/attackers* to 2
k-partite AFs ($k \geq 3$)

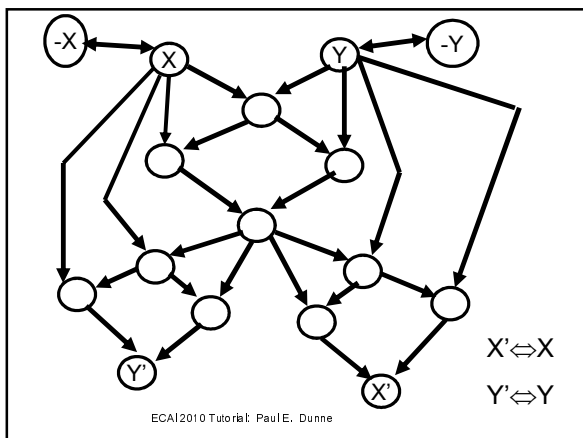
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Planar Crossovers

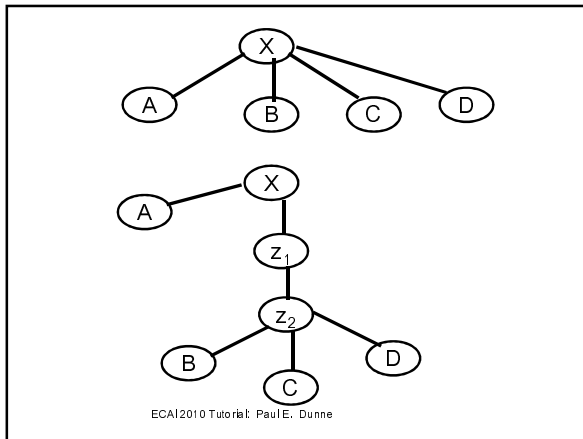
In standard translation



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Augmented Frameworks - VAFs

- Recall the two main questions are
SBA (Subjective Acceptance)
OBA (Objective Acceptance)
- These ask if an argument is in the grounded extension of the AF induced by *at least one/every* audience.
- That is, the AFs formed by removing failing attacks.

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Algorithms for SBA & OBA

- For k a “*small*” number of values (e.g. $k \leq 8$) one method is simply to run through all $k!$ specific audiences.
- This, however, is clearly infeasible for “*larger*” sizes, e.g. $k \geq 20$.
- There is some evidence, however, that such brute-force approaches *may be unavoidable*, even when the VAF *structure is limited*.

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SBA & OBA

- A straightforward twist on the standard translation shows SBA is NP-complete and OBA coNP-complete in general.
- In contrast to AFs, however, this status remains unchanged even when $\langle X, A \rangle$ is a *binary tree in which no value occurs more than three times*. (Dunne, AIJ, 171:701-729, 2007)
- Recently, Kim, Ordyniak and Szeider have shown similar *negative results* for VAFs in which *no value occurs more than twice or at most one attack involves identical values*. (COMMA 2010).
- These *disprove* conjectures of Dunne (2007).

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Positive results

- Until very recently, only *one* tractable general class of VAF was known: *symmetric frameworks*.
- Recently found: *bipartite* AFs with *no value occurring more than twice* and; all AFs whose “*extended graph structure*” has bounded treewidth. (Kim et al, COMMA 2010)
- VAFs whose “*value graph*” is a tree with maximum degree d , at most m branching vertices and $d \times m$ constant. (Dunne, COMMA 2010)

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EAFs

- Reinstatement sets complicate concept of “ x is EAF-acceptable w.r.t. S ”.
- Thus status of testing EAF admissibility and computation of grounded extension is unclear.
- In fact, deciding if an argument is EAF acceptable w.r.t. S can be done efficiently.
- This leads to efficient methods for construction of grounded extension, etc.
- Methods are presented in Dunne, Modgil, & Bench-Capon (ECAI 2010, Session 1E).

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Problematic Aspects I

- Recall that EAF semantics can lead to “pathological” behaviour compared to AFs.
- There may be EAF preferred extensions in which some elements of the EAF grounded extension do not occur.
- There may be EAF stable extensions that are not EAF preferred.
- How difficult is it to detect such behaviour?

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Problematic Aspects II

- Deciding if all EAF preferred extensions contain EAF grounded extension (GS):
coNP-hard
upper bound Π_2
- Deciding if all EAF stable extensions are EAF preferred (SC): coNP-complete.
- Also from Dunne, Modgil, & Bench-Capon (ECAI 2010, Session 1E).

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Weighted Argument Systems I

- The canonical questions are now qualified by an additional parameter describing the *inconsistency budget*.
- For example, WCA (*weighted CA*) w.r.t. grounded extensions asks:
“Given $\langle X, A, w \rangle, x, K$ is there a subset B of A whose *total weight* is *at most* K and for which x is in the grounded extension of $\langle X, A \setminus B \rangle$?”

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Weighted Argument Systems II

- For the case of *grounded semantics*, WCA is NP-complete in general)
- even if $\langle X, A \rangle$ is *acyclic* (similar to VAF)
- even if $\langle X, A \rangle$ is *symmetric*. (to date, the *only known example* of hard CA problem for $\langle X, A \rangle$ symmetric).
- Computing the minimum budget required to accept an argument is as *hard as computing the minimum length TSP solution* but *can* be done *efficiently* when $\langle X, A \rangle$ is both *acyclic* and *bipartite*.
- These are given in (extended version of) (Dunne et al., AAMAS, 2009).

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Summary

- Algorithmic and complexity issues are central to the effective realisation of the semantic models of acceptability in AFs.
- While properties of most AF based semantics are now well understood, augmented frameworks still offer a number of challenging questions.
- The “negative” complexity results should not be seen as the “final word” but rather as motivating alternative directions: e.g. special case behaviour; average-case methods, FPT approaches. While some initial study has been carried out (special case and FPT) others have yet to be seriously investigated (average-case).

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