

Building a Fuzzy Multi-objective Portfolio Selection model with Distinct Risk Measurements

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Abstract—Based on portfolio selection theory, this study proposes an improved fuzzy multi-objective model that can evaluate the invest risk exactly and increase the probability of obtaining the expected return. In building the model, fuzzy Value-at-Risk (VaR) is used to evaluate the exact future risk, in term of loss. The VaR can directly reflect the greatest loss of a selection case under a given confidence level. On the other hand, variance is utilized to make the selection more stable. This model can provide investors with more significant information in decision-making. To better solve this model, an improved particle swarm optimization algorithm is designed to mitigate the conventional local convergence problem. Finally, the proposed model and algorithm are exemplified by some numerical examples. Experiment results show that the model and algorithm are effective in solving the multi-objective portfolio selection problem.

Index Terms—Fuzzy multi-objective portfolio selection model, fuzzy variable, fuzzy Value-at-Risk, fuzzy simulation, improved particle swarm optimization.

I. INTRODUCTION

In finance, portfolio selection theory plays a pivotal role in decision-making for portfolio investors. As an early adopter, Markowitz [1] proposed a significant portfolio selection theory in 1952. He used the single period variance of returns as a risk measure to develop techniques for portfolio optimization. From then on, various researches have been carried on.

Security returns in conventional models are always determined by precise historical data. However, such data are not always available. On the other hand, securities are always affected by infinitive factors. It is impossible to simulate the influence of all factors well. Therefore, in today's stock markets, it is not reasonable to evaluate future security returns only based on the historical data. To make the forecasting more accuracy, the expert's opinions from related fields need to be considered. In prediction, to handle the uncertain factors which affect the security return and the imprecise expert's opinions, it is in reason to treat security returns as variables with imprecise distributions, i.e., fuzzy variables.

Based on fuzzy set theory, recently, various fuzzy portfolio selection models are proposed [2-9]. Among these fuzzy portfolio selection models, the most important distinction could be the risk measure method. Different techniques provide different criteria to select a portfolio. Huang [8] employed the variance of fuzzy variables as a risk measurement. As

an extension of conventional variance theory, the variance of fuzzy variables can reflect an average deviation level of a selection case to the expected value in fuzzy environment. Huang [3] also developed a mean-entropy model. As a physics concept, entropy was utilized here to reduce the uncertainty of a portfolio and make the prediction easily. Li *et al.* [4] proposed a mean-variance-skewness portfolio selection model. Being combined with variance, the skewness was taken for a new risk measurement which can evaluate an average deviation level from the expected value with asymmetry fuzzy returns. Recently, Wang *et al.* [9] introduced fuzzy Value-at-Risk (VaR) to this field. The VaR can directly reflect the loss of one selection case under given confidence level. It was used as a novel risk measurement. Different from previous methods which try to make the portfolio stable, the VaR can tell investors the exact future risk in term of loss.

Nevertheless, the above [2-9] are all single-objective models based on Markowitz's principle. That is, the return and risk are compensated by each other. Some decision makers do not want to set a fixed level to return or the risk as the constraints. They want to choose the investment scheme which can maximize the profit and minimize the potential risk together. Therefore, a single optimized portfolio cannot satisfy all investors. On the other hand, sometimes portfolio optimization models are required to consider more criteria than the standard mean-variance model. Therefore, to provide comprehensive selection information for investors, recently multi-objective portfolio selections models (MOPSM) were built. From the literatures, some studies on MOPSM have been investigated. Jana *et al.* [10] proposed a multi-objective portfolio based model and added another entropy objective function to generate a well diversified asset portfolio within optimal asset allocation. In this research, a fuzzy programming technique is utilized as the solution of the multi-objective non-linear programming model. Steuer *et al.* [11] took some additional arguments of portfolio selection theory into consideration, such as: dividends, liquidity, social responsibility, amount of short selling, and so forth. This study is significant for non-standard investors to make decisions. Zhang *et al.* [12] presented a fuzzy multi-objective mean-variance-skewness model with transaction costs. The skewness of portfolio return is considered as a critical factor

in selecting a portfolio. In order to solve the above model, a hybrid intelligent algorithm was designed by integrating simulated annealing algorithm, relevance vector machine and fuzzy simulation.

In this study, based on the property of fuzzy Value-at-Risk, we propose a mean-variance-VaR multi-objective portfolio selection model (VaR-MOPSM).

The remainder of this paper is organized as follows: Section 2 introduces some basic information on fuzzy variables. We also explain the concepts and calculations of the variance and fuzzy VaR. Section 3 illustrates the motivations behind our research. In Section 4, we briefly review previous MOPSM and build the VaR-MOPSM. The model can be solved by an improved particle swarm optimization (IPSO) introduced in Section 5. In Section 6, we apply the proposed algorithm to solve a numerical example. In order to demonstrate the effectiveness of the IPSO, we also compare it with existing methods in this Section. Finally Section 7 summarizes our conclusions.

II. PRELIMINARIES

To make a better prediction of security returns, fuzzy set theory is employed to portfolio selection problem. In this section, we briefly review some features on fuzzy variables. The fuzzy variable is a fundamental mathematical tool to describe fuzzy uncertainty, and there are mainly three types of fuzzy variables frequently used: triangular, trapezoidal and normal distributions. Before introducing the variance and fuzzy VaR, it is necessary to review some basic knowledge on the credibility theory of fuzzy variables.

Suppose ξ is a fuzzy variable whose membership function is μ_ξ , and r is a real number. Then, the possibility, necessity and credibility of event $\xi \leq r$ are expressed, respectively, as follows:

$$\text{Pos}\{\xi \leq r\} = \sup_{t \leq r} \mu_\xi(t), \quad (1)$$

$$\text{Nec}\{\xi \leq r\} = 1 - \sup_{t > r} \mu_\xi(t), \quad (2)$$

$$\text{Cr}\{\xi \leq r\} = \frac{1}{2} [\text{Pos}\{\xi \leq r\} + \text{Nec}\{\xi \leq r\}]. \quad (3)$$

The credibility measure is formed on the basis of the possibility and necessity measures, and in the simplest case, it is taken as their average. The credibility measure is a self-dual set function [13], *i.e.* $\text{Cr}\{\xi \leq r\} = 1 - \text{Cr}\{\xi > r\}$. Equation (3) also results in the following:

$$\text{Cr}\{\xi \leq r\} = \frac{1}{2} \left[\sup_{t \leq r} \mu_\xi(t) + 1 - \sup_{t > r} \mu_\xi(t) \right]. \quad (4)$$

For further information on fuzzy variables, one may refer to [13], [14] and [15].

A. Expected value

The most general definition of expected value of fuzzy variable was given by Liu and Liu [13].

Let ξ be a fuzzy variable. Then the expected value of ξ is defined by:

$$E[\xi] = \int_0^{+\infty} \text{Cr}\{\xi \geq r\} dr - \int_{-\infty}^0 \text{Cr}\{\xi \leq r\} dr. \quad (5)$$

B. Variance

The variance of a fuzzy variable provides a measure of the spread of the distribution around its expected value. A small value of variance indicates that the fuzzy variable is tightly concentrated around its expected value; and a large value of variance indicates that the fuzzy variable has a wide spread around its expected value. Therefore, the variance can be used to represent the level of risk in a fuzzy system.

If the fuzzy variable has a finite expected value, according to Liu and Liu [13], its variance is defined as:

$$V[\xi] = E[(\xi - e)^2] \quad (6)$$

C. Value-at-Risk

The Value-at-Risk (VaR) of an investment is the likelihood of the greatest loss with a given confidence level [17], [18]. Following Wang *et al.* [16], if $\tilde{L}_t$ denotes the fuzzy power demand of period t , then, in a fuzzy environment, the VaR of L_t with a confidence of $(1 - \beta)$ can be written as:

$$\text{VaR}_{1-\beta} = \sup\{\lambda | \text{Cr}(\tilde{L}_t \geq \lambda) \geq \beta\}, \quad (7)$$

where $\beta \in (0, 1)$.

Equation (7) tells us that the greatest value of $\tilde{L}_t$ under confidence level $(1 - \beta)$ is the largest λ .

Recently, the measure of Value-at-Risk has been applied to many engineering problems to build fuzzy optimization models [19], [20], [21], [22].

III. MOTIVATIONS

As we mentioned in Section I, the most important distinction of the portfolio selection models could be the risk measure methods. Investor probability obtains quite different optimal results with different methods. In Table I below, we list the risk measurements in detail which have been applied to fuzzy portfolio selection problem.

The risk measurements listed in Table I can be sorted into two types. The first type includes mean-risk, mean-variance, mean-semivariance, entropy and mean-variance-skewness which find the optimal solutions by minimizing the risk measure functions, thus maximizing the stability of a portfolio. However, these evaluation methods do not focus on the exact risk of future loss, which is significant to investors. The second type is fuzzy VaR. Using fuzzy VaR as the risk measurement, investors can assign different confidence levels and then use the model to gain how much return they can

TABLE I
THE COMPARISONS OF THE FUZZY VaR AND EXISTING RISK MEASUREMENTS

Methods Name	Risk Measure Function	Purpose
Mean-risk [23]	$\text{Cr}\{b - (x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n) \geq r\} \leq \alpha(r)$	Describe the credibility values of the bad events, which should be lower than the given tolerance levels
Mean-variance [2]	$V[x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n] \leq r$	Reflects an average deviation level from the expected value, which is limited by the given value
Mean-semivariance [5]	$H[x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n] \leq r$	Measures only the lower deviation from the expected return
Entropy optimization [3]	$SV[x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n] \leq r$	Minimize the entropy can reduce the uncertainty of a portfolio and make the prediction easily
Mean-variance-skewness [4]	$V[x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n] \leq \alpha$	Reflects an average deviation level from the expected value with asymmetry fuzzy returns
	$S[x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n] \geq \beta$	
Fuzzy Value-at-Risk [9]	$\text{VaR}_{1-\beta} = \sup\{\lambda \text{Cr}(\mathcal{L} \geq \lambda) \geq \beta\}$	Directly reflect the loss of one selection case under given confidence level

expect and how much future loss they may suffer in the worst case scenario. Nevertheless, the VaR cannot enhance the stability of a portfolio which may not make the selection case trusted.

To the best of our knowledge, the conventional multi-objective portfolio selection models were built based on the first type of risk measurements. That is to say, all of the existing MOPSMs only focus on enhancing the stability of a selection case from different aspects, but the exact risk in term of loss is not considered.

In this study, we proposed a MOPSM based on both types risk measure methods: variance is utilized to evaluate the portfolio stability while fuzzy VaR is applied to measure the exact future risk. Therefore, the obtained selection result could be more helpful in decision making.

IV. MODELING

In this Section, we briefly review the existing multi-objective portfolio selection models and then provide our model. Some notations are given as follows:

x_i : Proportion invested on security i , $i = 1, 2, \dots, n$.

ξ_i : The fuzzy return of security i .

E : The expected return of a selection case.

V : The variance of a selection case.

S : The skewness of a selection case.

$(1 - \beta)$: The confidence level of Value-at-Risk.

$\text{VaR}_{1-\beta}$: The VaR of a selection case under confidence level

$(1 - \beta)$.

Cr : The credibility functions of fuzzy variables.

L : The loss functions of selection cases, defined as $L =$

$$-(x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n).$$

In the conventional MOPSM, the objective considered was to maximize the investment return and minimize several risk measure functions at the same time. Thus, it can be summarized as:

$$\begin{cases} \max E[x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n] \\ \min V[x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n] \\ \min -S[x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n] \\ \text{Subject to} \\ x_1 + x_2 + \dots + x_n = 1; \\ x_i \geq 0, i = 1, 2, \dots, n. \end{cases} \quad (8)$$

If we quantify the skewness function of model (8) by risk measurements mentioned in Table I, some other existing fuzzy portfolio selection models can be obtained.

In this study, we retain the Markowitz mean-variance selection principle; however, the fuzzy VaR is considered as another risk measure method. In this case, the proposed VaR-MOPSM can be described as:

$$\begin{cases} \max E[x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n] \\ \min V[x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n] \\ \min \text{VaR}_{1-\beta} = \sup\{\lambda | \text{Cr}(\mathcal{L} \geq \lambda) \geq \beta\} \\ \text{Subject to} \\ x_1 + x_2 + \dots + x_n = 1; \\ x_i \geq 0, i = 1, 2, \dots, n. \end{cases} \quad (9)$$

V. SOLUTION

The VaR-MOPSM is a complex nonlinear optimization problem which cannot be solved by linear programming methods or simplex algorithms. Therefore, in this section, we provide a fuzzy simulation based multi-objective PSO algorithm to get the solutions.

A. Fuzzy simulation

In Section II, we have introduced the calculation of expected value, fuzzy variance and VaR of single fuzzy variable. However, we are required to deal with a series of different distributed fuzzy variables, such as $\xi = (\xi_1, \xi_2, \dots, \xi_n)$. It is impossible to solve the problem as $V[x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n]$ or $\text{VaR}_{1-\beta} = \sup\{\lambda | \text{Cr}(\xi \geq \lambda) \geq \beta\}$ from the above definitions. The fuzzy simulation developed by Liu and Liu [13] plays a pivotal role in solving the above problem. Employing this technique, the theoretical calculation of fuzzy VaR can be briefly summarized as follows:

Suppose $\xi_i = (\xi_1, \xi_2, \dots, \xi_n)$ is a continuous fuzzy variables with support $\prod_{i=1}^n [a_i^L, a_i^U]$, where a_i^L and a_i^U are the lower and upper supports of ξ_i . We first divide each fuzzy variable into l parts and then approximate the membership function μ_{ξ_i} by a sequence of discrete fuzzy vectors ζ_i^r which can be constructed as:

$$\zeta_i^r = a_i^L + \frac{r}{l}(a_i^U - a_i^L), \quad (10)$$

where, l and r are integers, and $0 \leq r \leq l$.

Second, the loss variable $\mathcal{L} = -(x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n)$ can be simulated by the fuzzy vectors ζ_i^r .

Third, on the basis of the approximate membership function of $\mathcal{L}$, the credibility function $\text{Cr}\{\mathcal{L} \geq \lambda\}$ can be calculated by equation (4).

Finally, we can obtain an approximate values of variance and fuzzy VaR by equation (6) and (7).

Therefore, the larger the value of l , the more exact the result will be, but the longer the computation time will be. We can conclude that the value of l should be set specially to different optimization problems. For more about this methods, you may refer to [4], [16] and [28].

B. Improved particle swarm optimization algorithm

The PSO algorithm was initially proposed by Kennedy [24] in 1995. It has been widely used and demonstrated its effectiveness in a variety of fields; refer to [9], [25], [26] and [27] for details.

1) *Improving conventional PSO to mitigate the local convergence problem:* It is well known that the PSO algorithm can find the optimal solution in fewer iterations than other algorithms, but may suffer from the local convergence problem. To overcome the above problem and better solve the multi-objective model, we improve the original PSO algorithm by the escape speed (ES) and particle restart position (PRP).

ES: If the optimal solution cannot be updated after a fixed number of iterations, then it will be considered as having fallen into local convergence. Then, each particle's speed will be

modified by the escape speed. If the escape speed is sufficiently large, all the particles can jump out of their current loops and restart searching in different areas.

PRP: After the particle speed has been modified by the escape speed, the particles will be distributed randomly. For most of the selection problems, there should be a constraint on the search space; for example, in our problem, each position value belongs to the range (0,1). Then, the particle restart position is generated by the edge value of the search space.

2) The evaluation method of the multi-objective problem:

To solve the multi-objective model, the following researches have been done:

Find the ideal Pareto solution. Pareto solutions are those for which improvement in one object can only occur with the worsening of at least one other objective. If we optimize each object (expected value or variance or VaR) without considering others, the ideal Pareto solution can be created. Such processes can be summarized as follows:

Scenario I: the expected return is considered as the unique object, then, the proposed model (9) degenerate to the following:

$$\begin{cases} \max E[x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n] \\ \text{Subject to} \\ x_1 + x_2 + \dots + x_n = 1; \\ x_i \geq 0, i = 1, 2, \dots, n. \end{cases} \quad (11)$$

Scenario II: the variance is considered as the unique object, then, model (9) is revised to:

$$\begin{cases} \min V[x_1\xi_1 + x_2\xi_2 + \dots + x_n\xi_n] \\ \text{Subject to} \\ x_1 + x_2 + \dots + x_n = 1; \\ x_i \geq 0, i = 1, 2, \dots, n. \end{cases} \quad (12)$$

Scenario III: the fuzzy VaR is considered as the unique objective, then,

$$\begin{cases} \min \text{VaR}_{1-\beta} = \sup\{\lambda | \text{Cr}(\mathcal{L} \geq \lambda) \geq \beta\} \\ \text{Subject to} \\ x_1 + x_2 + \dots + x_n = 1; \\ x_i \geq 0, i = 1, 2, \dots, n. \end{cases} \quad (13)$$

The revised models (11), (12), (13) are single object optimization problem which can be easily solved by simplex methods. Supposedly, these models results in $E_{\max}$, $V_{\min}$ and $\text{VaR}_{\min}$, then, the ideal Pareto solution is considered as $(E_{\max}, V_{\min}, \text{VaR}_{\min})$.

In model (7), if one selection case i produces the result of (E_i, V_i, VaR_i) , the fitness value (FV) of portfolio i is considered as the distance between the ideal Pareto solution and optimal solution i , which can be obtained by the following equation (14).

$$\text{FV}(i) = \sqrt{(E_{\max} - E_i)^2 + (V_{\min} - V_i)^2 + (\text{VaR}_{\min} - \text{VaR}_i)^2} \quad (14)$$

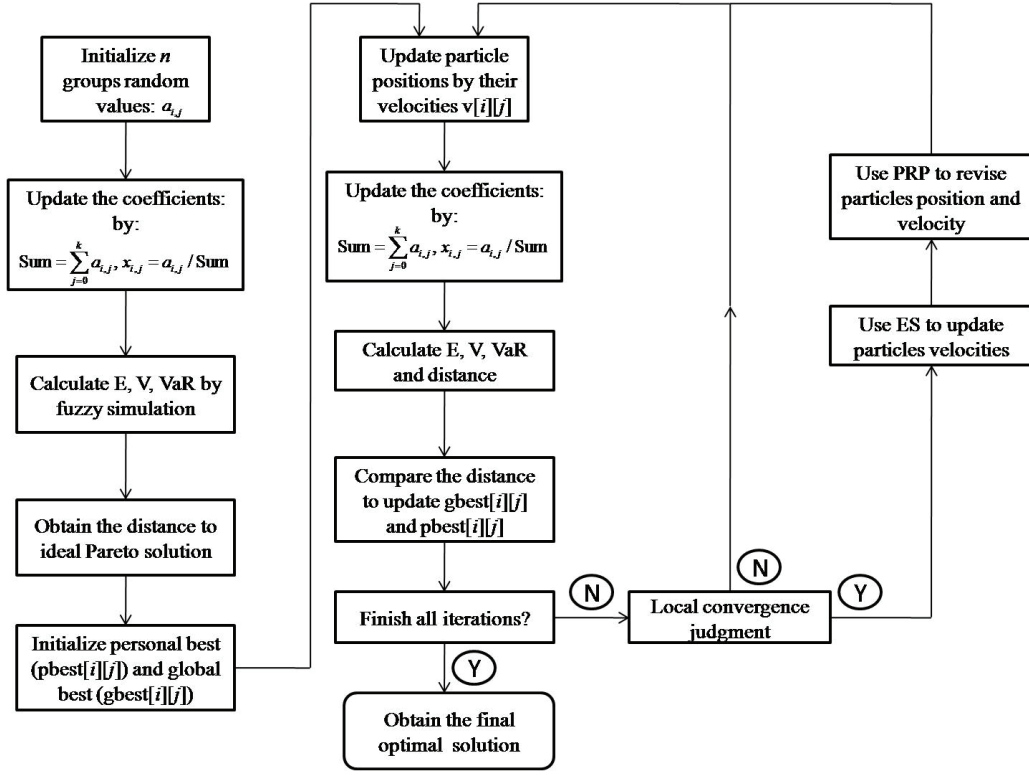


Fig. 1. The flow chart of IPSO

Therefore, the smaller the distance is, the better the optimal solution will be.

We use Figure 1 above to illustrate the IPSO.

VI. NUMERICAL EXAMPLE

In this section, the proposed model and algorithm are exemplified by the following numerical example.

Example: Suppose that there are ten securities whose returns are independent triangular or normally distributed fuzzy variables, as given in Table II. The confidence level of fuzzy Value-at-Risk is set as 0.9, that is $\beta = 0.1$.

TABLE II
SECURITY RETURNS

Security No.	Return	Security No.	Return
1	(-0.8, 0.4, 2.0)	6	N(0.9, 1.7 ²)
2	(-0.7, 0.6, 1.8)	7	N(0.6, 1.0 ²)
3	(-1.1, 0.6, 2.8)	8	N(0.8, 1.4 ²)
4	(-1.6, 0.9, 3.2)	9	N(0.7, 1.1 ²)
5	(-1.4, 0.8, 3.9)	10	N(1.1, 1.9 ²)

Then, apply these data to model (9), we get the following selection problem.

$$\begin{cases} \max E[x_1\xi_1 + x_2\xi_2 + \dots + x_{10}\xi_{10}] \\ \min V[x_1\xi_1 + x_2\xi_2 + \dots + x_{10}\xi_{10}] \\ \min \text{VaR}_{0.9} = \sup\{\lambda | \text{Cr}(\mathcal{L} \geq \lambda) \geq 0.1\} \\ \text{Subject to} \\ x_1 + x_2 + \dots + x_{10} = 1; \\ x_i \geq 0, i = 1, 2, \dots, 10. \end{cases} \quad (15)$$

Now, we use the IPSO algorithm to get the optimal solutions. All of the following experiments were implemented on a personal computer: Dell E8500 3.16 GHz CPU with RAM 2.96 GB.

A. The performance of IPSO

We first applied IPSO to solve model (15). After 100 iterations, the optimal fitness value (FV) is 0.146, and the capital should be allocated as Figure 2 shows, the numerical value of the selection is (0.00; 0.47; 0.00; 0.00; 0.52; 0.00; 0.00; 0.01; 0.00; 0.00).

From the numerical data provided in this Example, we may find security 2 is overlapped by security 3. However, from the selection result, only security 2 is considered as an invest proportion. The reason is: The VaR and variance of security 2 is smaller than those of security 3, when compute the distance to ideal Pareto solution, security 2 performs better.

B. Comparisons with other algorithms

Some existing approaches have been proposed to solve fuzzy portfolio optimization problems, such as genetic algorithms (GA) [3], [4] and simulated annealing algorithm

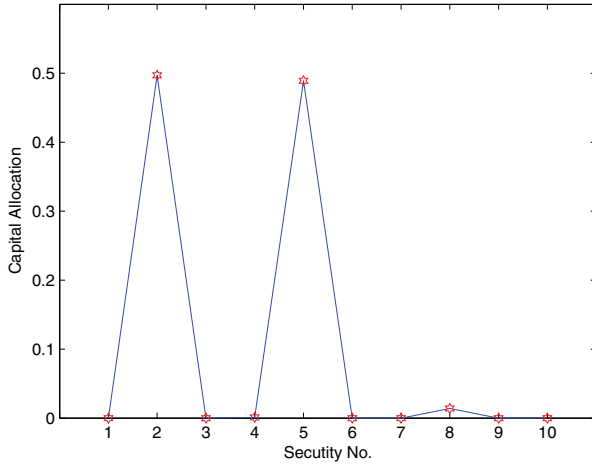


Fig. 2. Optimal solution of IPSO (Capital allocations)

(SAA) [7]. In this study, we first revised the fitness functions of GA and SAA to solve the multi-objective portfolio selection problem. Then, the proposed IPSO is compared with other approaches from two aspects: the optimal solutions and runtime cost.

1) *Solutions comparisons:* The IPSO, GA and SAA are applied to solve the above numerical example. For GA, the crossover and mutation probability is set as 0.8 and 0.05, the Chromosome number is 20. The temperature reduction coefficient of SAA is selected as 0.984. The following Figure 3 shows the optimal process of each method during the 100 times iterations.

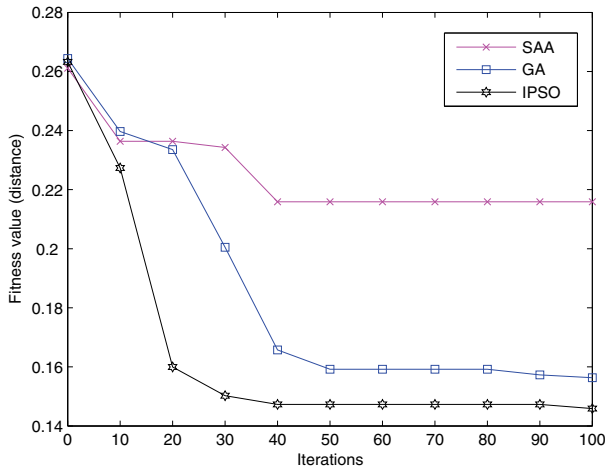


Fig. 3. Optimal solutions of different approaches

From the Figure 3, we can conclude that: the proposed IPSO can mitigate the local convergence problem very well and find better solutions than other methods.

2) *Runtime comparisons:* The runtime costs (50, 100, 200 iterations) of each algorithm are depicted in Figure 4.

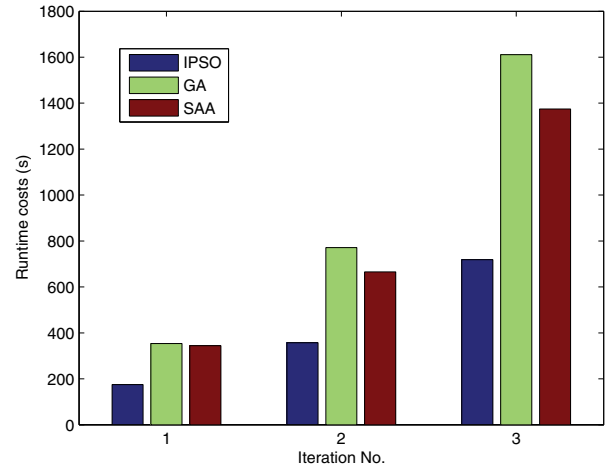


Fig. 4. Optimal solutions of different approaches

From the comparisons results in Figures 3 and 4, we can conclude that the proposed IPSO has better performance than GA and SA for our VaR-MOPSM.

C. Discussion on the optimal results of VaR-MOPSM with previous MOPSMs

Here, we provide more explanations on the effectiveness of VaR-MOPSM by linguistic analysis.

Because the risk measurements of the existing portfolio selection models are quite different, each method produces varying criteria to select a portfolio, so comparisons among the selection results of different models may not be very meaningful. However, it is important to note that, investors clearly understand the purposes of the first type risk measurements (provided in Section III), such as mean-variance, mean-semivariance or entropy; minimize these values can improve the selection. However, they cannot identify the potential risk included in these numerical values. For instance, in the conventional MOPSM, if the risk level r is modified from 1.6 to 1.7, then the investors will obtain a selection case with higher expected return and risk. However, it is quite difficult for them to determine how much higher the future risk becomes, so they cannot balance the higher expected return with the higher risk. As a result, investors cannot clearly determine the parameters and evaluate the results of these models. Based on the above analysis, the existing MOPSMs are not sensitive in measuring the future risk. In our VaR-MOPSM, on the other hand, investors can determine the confidence levels as they wish, and any change in these values will be directly reflected by the greatest loss obtained from the fuzzy VaR. Therefore, from this perspective, the multi-objective portfolio selection model with the fuzzy VaR can provide more effective information for investors and help them make decisions more easily.

VII. CONCLUSION

In this paper, we built a novel multi-objective fuzzy portfolio selection model (VaR-MOPSM) which takes the expected value, Value-at-Risk and variance as the criteria to evaluate a selection case. Applying this model, investors can well balance the future return with the potential loss, and therefore make more proper decisions. The model is formed as a non-linear complicated multi-objective optimization problem, which cannot be solved by linear programming methods or simplex algorithms. To solve the VaR-MOPSM, a fuzzy simulation based improved particle swarm optimization algorithm (IPSO) was designed as the solution. The proposed model and algorithm were exemplified by some numerical experiments. From the experiments' results and discussion on the optimal results of VaR-MOPSM with previous approaches, we concluded that: 1. the VaR-MOPSM is more helpful for general investors in decision-making; 2. the IPSO can mitigate the local convergence very well; 3. IPSO is more effective in solving the multi-objective than other existing approaches. The VaR-MOPSM model proposed in this paper is also applicable to various investment or engineering problems.

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