

Direct Adaptive Fuzzy Control for Nonaffine Nonlinear Systems with Unknown Control Direction

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Abstract—This paper deals with direct adaptive fuzzy control for a class of uncertain nonaffine nonlinear systems with unknown control direction. Within this scheme, a fuzzy system is used to generate directly the control input signal without dynamic system estimation. The adjustable parameters in the used fuzzy system are updated using a gradient descent adaptation algorithm that incorporates a Nussbaum-type function to deal with the unknown control direction. The stability of the closed-loop system is performed using a Lyapunov approach. Simulation results are provided to verify the effectiveness of the proposed design.

Keywords- fuzzy control, adaptive control, nonaffine nonlinear systems, Nussbaum function.

I. INTRODUCTION

In the past years, based on the universal approximation property of fuzzy systems, adaptive fuzzy control of uncertain nonlinear systems has been intensively conducted and some developments have been achieved in the literature [1, 2]. The stability study in such adaptive methods is performed by using the Lyapunov design approach. Most of the proposed adaptive fuzzy control methods can be roughly classified into two types: the direct adaptive fuzzy control approach where fuzzy systems are used to approximate unknown ideal controllers [1-4], and the indirect adaptive fuzzy control approach where fuzzy systems are used to identify the plant dynamics [1-3, 5]. In [1-5], adaptive fuzzy control design for the affine-in-control nonlinear systems is presented. Adaptive fuzzy control design for nonaffine systems is discussed in [6-9].

In the direct schemes [1-4, 6, 8, 9], the parameter adaptation laws used are designed based on a Lyapunov approach, where an error signal between the desired output and the actual output is used to update the adjustable parameters. However, since in the direct adaptive control approach, the purpose is to approximate an unknown ideal controller by using a fuzzy controller, so it is more convenient to try to adjust the free parameters of the fuzzy controller by using directly the error between the ideal controller and the fuzzy controller [4, 7]. In [7], a direct adaptive fuzzy controller was designed for a class of nonaffine systems where the gradient descent method was used to design the parameter adaptation law in order to minimize the error between an ideal controller and the fuzzy controller.

However, in [7], the control direction is assumed known a priori, i.e., the sign of the control gain (the control gain is the partial derivative over control variable for nonaffine systems) is assumed known for the designer. Without this assumption, the adaptive control design becomes much more difficult, because in this case, one cannot decide the direction along which the control operates and/or the direction of the search of controller parameters. In the adaptive control literature, the unknown control direction problem is mainly solved by using the Nussbaum-type function [10, 11], and it has been successfully applied for robust control design [12, 13] adaptive control design for nonlinear systems [14-17], and adaptive fuzzy control for nonaffine systems [18, 19]. In the literature, other approaches solved the unknown control direction problem by using: the correction vector method for first order nonlinear systems [20]; a switching scheme based on a monitoring function [21]; a hysteresis-type function for indirect adaptive fuzzy control [22, 23]; or by estimating unknown parameters involved in the control gain [24].

In this work we present a direct adaptive fuzzy control scheme for a class of uncertain nonaffine nonlinear systems with unknown control direction. In the design; instead of using fuzzy systems to approximate the unknown nonlinearities in the system, a fuzzy system is used to approximate an unknown desired controller that can achieve control objectives, and a Nussbaum function is introduced in the parameter adaptation law to solve the problem of unknown control direction. The parameter adaptation law is designed to minimize the error between the ideal controller and the fuzzy controller by using the gradient descent method. Using the Lyapunov approach, it is proved that the proposed adaptive fuzzy controller can guarantee the boundedness of all the signals of the closed-loop system and the asymptotic convergence of the tracking error. In fact, this paper presents an extension of our previous work [7] to the case of unknown control direction.

The remainder of the paper is organized as follows. The problem formulation and preliminaries are given in section II. The proposed direct adaptive control scheme is presented in section III with its adaptive law and stability analysis. In section IV, the proposed adaptive control algorithm is used to control a simple nonaffine nonlinear system.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Problem Formulation

Consider the class of single-input single-output (SISO) nonlinear systems modeled by

$$\begin{cases} \dot{x}_i = x_{i+1}, i=1, \dots, n-1 \\ \dot{x}_n = f(\mathbf{x}, u) \\ y = x_1 \end{cases} \quad (1)$$

or, equivalently

$$y^{(n)} = f(\mathbf{x}, u) \quad (2)$$

where $\mathbf{x} = [x_1, \dots, x_n]^T \in \mathbb{R}^n$, is the state vector of the system which is assumed available for measurement, $u \in \mathbb{R}$ is the scalar control input, $y \in \mathbb{R}$ is the scalar system output, and $f(\mathbf{x}, u)$ is an unknown smooth nonlinear function.

In respect of the dynamic system (1), the following assumption will be made:

Assumption 1: The function $g(\mathbf{x}, u) = \frac{\partial f(\mathbf{x}, u)}{\partial u}$ is nonzero and bounded. Moreover, its sign, i.e. control direction, is unknown.

The objective is to design an adaptive fuzzy controller for system (1) such that the system output $y(t)$ follows a desired trajectory $y_d(t)$ while all signals in the closed-loop system remain bounded.

Regarding the development of the control law, the following assumption should also be made:

Assumption 2: The desired trajectory $y_d(t)$ and its first n derivatives are smooth and bounded.

Now, let us define the tracking error as

$$e = y_d - y \quad (3)$$

and let s be a sliding surface defined as

$$s = \left(\frac{d}{dt} + \lambda \right)^{n-1} e, \quad \lambda > 0 \quad (4)$$

From (4), $s(t) = 0$ represents a linear differential equation whose solution implies that the tracking error $e(t)$ converges to zero with a time constant $(n-1)/\lambda$ [22]. Thus, the control objective becomes the design of a controller to keep $s(t)$ at zero.

The time derivative of s is given by

$$\dot{s} = v_0 - f(\mathbf{x}, u) \quad (5)$$

where

$$v_0 = y_d^{(n)} + \beta_{n-1} e^{(n-1)} + \dots + \beta_1 \dot{e} \quad (6)$$

$$\text{with } \beta_j = \frac{(n-1)!}{(n-j)!(j-1)!} \lambda^{n-j}, \quad j = 1, \dots, n-1.$$

By adding and subtracting $k_1 s + k_2 \tanh(s/\varepsilon)$ to the right-hand side of (6), one obtains

$$\dot{s} = -k_1 s - k_2 \tanh(s/\varepsilon) - (f(\mathbf{x}, u) - v) \quad (7)$$

where k_1 and k_2 are positive design parameters, and $v = v_0 + k_1 s + k_2 \tanh(s/\varepsilon)$.

From Assumption 1 and the fact that the signal v , does not explicitly depend upon the control input u , the partial derivative of $f(\mathbf{x}, u) - v$ with respect to the input u satisfies

$$\frac{\partial (f(\mathbf{x}, u) - v)}{\partial u} = \frac{\partial f(\mathbf{x}, u)}{\partial u} \neq 0 \quad (8)$$

Based on the implicit function theorem [5, 13], we know that the nonlinear algebraic equation $f(\mathbf{x}, u) - v = 0$ is locally solvable for the input u for each $\mathbf{x}$ and v . Therefore, there exists some ideal controller $u^*(\mathbf{x}, v)$ satisfying the following equality for all $\mathbf{x}$ and v

$$f(\mathbf{x}, u^*(\mathbf{x}, v)) - v = 0 \quad (9)$$

If the control input u is chosen as the ideal control law u^* , i.e., $u = u^*(\mathbf{x}, v)$, the closed-loop error dynamic (7) becomes

$$\dot{s} = -k_1 s - k_2 \tanh(s/\varepsilon) \quad (10)$$

From which one can conclude that $s(t) \rightarrow 0$ as $t \rightarrow \infty$ and, therefore, $e(t)$ and all its time derivatives up to $n-1$ converge to zero [25].

However, the implicit function theory only guarantees the existence of the ideal controller u^* for system (1), and does not prescribe a technique for constructing it even if the dynamics of the system are well-known. In the next section, a fuzzy system will be used to construct adaptively this unknown ideal implicit controller and Nussbaum technique will be used to cope with the unknown control direction.

B. Fuzzy Systems

We use in this work the zero-order Takagi-Sugeno fuzzy system that performs a mapping from an input vector $\mathbf{z} = [z_1, \dots, z_m]^T \in \Omega_z \subset \mathbb{R}^m$ to a scalar output variable $y_f \in \mathbb{R}$, where $\Omega_z = \Omega_{z_1} \times \dots \times \Omega_{z_m}$ and $\Omega_{z_i} \subset \mathbb{R}$. If we define M_i fuzzy sets F_i^j , $j = 1, \dots, M_i$, for each input z_i , then the fuzzy

system will be characterized by a set of if-then rules of the form [1, 2]

$$\begin{aligned} R^k : & \text{If } z_1 \text{ is } G_1^k \text{ and } \dots \text{ and } z_m \text{ is } G_m^k \\ & \text{Then } y_f \text{ is } y_f^k \quad (k=1, \dots, N) \end{aligned} \quad (11)$$

where $G_i^k \in \{F_i^1, \dots, F_i^{M_i}\}$, $i=1, \dots, n$, y_f^k is the crisp output of the k -th rule, and N is the total number of rules.

By using the singleton fuzzifier and the product inference engine, the final output of the fuzzy system is given by [1, 2]

$$y_f(z) = \frac{\sum_{k=1}^N \mu_k(z) y_f^k}{\sum_{k=1}^N \mu_k(z)} \quad (12)$$

where

$$\mu_k(z) = \prod_{i=1}^m \mu_{G_i^k}(z_i), \text{ with } \mu_{G_i^k} \in \{\mu_{F_i^1}, \dots, \mu_{F_i^{M_i}}\},$$

with $\mu_{F_i^j}(x_i)$ is the membership function of the fuzzy set F_i^j .

By introducing the concept of fuzzy basis functions [1], the output given by (10) can be rewritten in the following compact form

$$y_f(z) = w^T(z) \theta \quad (13)$$

where $\theta = [y_f^1, \dots, y_f^N]^T$ is a vector grouping all consequent parameters, and $w(z) = [w_1(z), \dots, w_N(z)]^T$ is a set of fuzzy basis functions defined as

$$w_k(z) = \frac{\mu_k(z)}{\sum_{j=1}^N \mu_j(z)}, \quad k=1, \dots, N. \quad (14)$$

The fuzzy system (11) is assumed to be well-defined so that $\sum_{j=1}^N \mu_j(z) \neq 0$ for all $z \in \Omega_z$.

It has been proved in [1] that fuzzy systems in the form of (12) with Gaussian membership functions can approximate continuous functions over a compact set to an arbitrary degree of accuracy provided that enough number of rules are considered.

C. Nussbaum Functions

In order to deal with the unknown control direction (sign of $g(x, u)$), the Nussbaum gain technique is used in this paper. The Nussbaum function technique was originally proposed in [10] and has been effectively used in controller design in solving the difficulty of unknown control directions [11-17]. A function $N(\tau)$ is called a Nussbaum-type function if it has the following properties [10]

$$\limsup_{\nu \rightarrow +\infty} \left(\frac{1}{\nu} \int_0^\nu N(\tau) d\tau \right) = +\infty \quad (15)$$

$$\liminf_{\nu \rightarrow +\infty} \left(\frac{1}{\nu} \int_0^\nu N(\tau) d\tau \right) = -\infty \quad (16)$$

For example, the continuous functions $e^{\tau^2} \cos(\tau)$ and $\tau^2 \cos(\tau)$ are Nussbaum-type functions. In this paper, the even Nussbaum function $\tau^2 \cos\left(\frac{\pi}{2} \tau\right)$ is used.

The following Lemma regarding the property of Nussbaum functions is used in the controller design and theorem proof of next section.

Lemma 1: Let $V(\cdot)$ and $\tau(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \geq 0$, $\forall t \in [0, t_f)$, $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds: $V(t) \leq c_0 + \int_0^t (g(\zeta) N(\tau(\zeta)) + c_1) \dot{\tau}(\zeta) d\zeta$ for $\forall t \in [0, t_f)$, where $g(\cdot)$ is a piecewise continuous time function which takes values in the unknown closed interval $I = [\underline{g}, \bar{g}]$ with $0 \notin I$, c_1 any positive number and c_0 is a constant, then $V(t)$, $\tau(t)$ and $\int_0^t (g(\zeta) N(\tau(\zeta)) + c_1) \dot{\tau}(\zeta) d\zeta$ are bounded on $[0, t_f)$.

Proof: For the proof, please refer to [12].

III. ADAPTIVE FUZZY CONTROLLER DESIGN

In the previous section, based on the implicit function theory, it has been shown that there exists an ideal control law $u^*(x, v)$ that can achieve control objectives. Since the implicit function theory only ensures the existence of this ideal controller and does not provide a way or a procedure for building it. In this section, we propose to use a fuzzy system to adaptively build this unknown ideal implicit controller.

To develop the control law, we assume that the ideal controller $u^*(x, v)$ can be approximated by a fuzzy system in the form of (11) as follows

$$u^*(z) = w^T(z) \theta^* \quad (17)$$

where z is the input vector of the fuzzy system, $w(z)$ is a fuzzy basis function vector fixed by the designer, and θ^* is a unknown bounded time-varying parameter vector with bounded time derivative.

Since the ideal parameter vector θ^* is unknown, let us use its estimate θ instead to form the adaptive control

$$u(z) = w^T(z) \theta \quad (18)$$

To design an adaptation law for the free parameters θ , if the control direction (sign of $g(\mathbf{x}, u)$) is known, one can consider the minimization problem of the cost function [7]

$$J(\theta) = \frac{1}{2}(u^* - u)^2 = \frac{1}{2}(u^* - \mathbf{w}^T(\mathbf{z})\theta)^2$$

By using the gradient descent method, the following parameters update law can be obtained [7]

$$\dot{\theta} = \gamma \text{sgn}(g(\mathbf{x}, u)) \mathbf{w}^T(\mathbf{z})(\dot{s} + k_1 s + k_2 \tanh(s/\varepsilon))$$

with $\gamma > 0$. However, since the control direction (sign of $g(\mathbf{x}, u)$) is unknown, we have to use a Nussbaum-type function instead for designing the parameter adaptation law as the following

$$\dot{\theta} = \gamma N(\tau) \mathbf{w}^T(\mathbf{z})(\dot{s} + k_1 s + k_2 \tanh(s/\varepsilon)) \quad (19)$$

$$\dot{\tau} = \eta (\dot{s} + k_1 s + k_2 \tanh(s/\varepsilon))^2 \quad (20)$$

$$N(\tau) = \tau^2 \cos\left(\frac{\pi}{2}\tau\right) \quad (21)$$

where γ and η are strictly positive constants.

Theorem 1: For nonlinear system (1), under Assumptions 1 and 2 and Lemma 1, the control law (18) with adaptation laws (19)–(21) guarantee the boundedness of all the signals in the closed-loop system and the asymptotic convergence of the tracking error.

Proof: By invoking the mean value theorem, there exists a constant $\bar{\lambda}$ with $0 < \bar{\lambda} < 1$, such that the nonlinear function $f(\mathbf{x}, u)$ can be expressed around u^* as follows

$$f(\mathbf{x}, u) = f(\mathbf{x}, u^*) + g(\mathbf{x}, \bar{u})(u - u^*) \quad (22)$$

where $\bar{u} = \bar{\lambda} u + (1 - \bar{\lambda})u^*$.

By substituting (22) into (7), we get

$$\dot{s} = -k_1 s - k_2 \tanh(s/\varepsilon) + g(\mathbf{x}, \bar{u})(u^* - u) \quad (23)$$

With (17) and (18), one can rewrite (23) as

$$\dot{s} = -k_1 s - k_2 \tanh(s/\varepsilon) + g(\mathbf{x}, \bar{u}) \mathbf{w}^T(\mathbf{z}) \tilde{\theta} \quad (24)$$

where $\tilde{\theta} = \theta^* - \theta$ is the parameter error vector.

Now, consider the following positive definite function

$$V = \frac{1}{2}s^2 + \frac{1}{2\gamma} \tilde{\theta}^T \tilde{\theta} \quad (25)$$

The time derivative of V is

$$\dot{V} = s\dot{s} - \frac{1}{\gamma} \tilde{\theta}^T \dot{\tilde{\theta}} + \frac{1}{\gamma} \tilde{\theta}^T \dot{\theta}^* \quad (26)$$

Substituting (19) and (24) into (26), and using the equality $\mathbf{w}^T(\mathbf{z}) \tilde{\theta} = \tilde{\theta}^T \mathbf{w}(\mathbf{z})$, it follows that

$$\begin{aligned} \dot{V} = & -k_1 s^2 - k_2 s \tanh(s/\varepsilon) + s g(\mathbf{x}, \bar{u}) \mathbf{w}^T(\mathbf{z}) \tilde{\theta} + \frac{1}{\gamma} \tilde{\theta}^T \dot{\theta}^* \\ & - g(\mathbf{x}, \bar{u}) N(\tau) \mathbf{w}^T(\mathbf{z}) \tilde{\theta} (\dot{s} + k_1 s + k_2 \tanh(s/\varepsilon)) \end{aligned} \quad (27)$$

From (24) one can obtain

$$\dot{s} + k_1 s + k_2 \tanh(s/\varepsilon) = g(\mathbf{x}, \bar{u}) \mathbf{w}^T(\mathbf{z}) \tilde{\theta} \quad (28)$$

Thus, (26) can be rewritten as

$$\begin{aligned} \dot{V} = & -k_1 s^2 - k_2 s \tanh\left(\frac{s}{\varepsilon}\right) - \frac{N(\tau)}{g(\mathbf{x}, \bar{u})} \left(\dot{s} + k_1 s + k_2 \tanh\left(\frac{s}{\varepsilon}\right)\right)^2 \\ & + s g(\mathbf{x}, \bar{u}) \mathbf{w}^T(\mathbf{z}) \tilde{\theta} + \frac{1}{\gamma} \tilde{\theta}^T \dot{\theta}^* \end{aligned} \quad (29)$$

Using (20) and the inequality

$$\begin{aligned} s g(\mathbf{x}, \bar{u}) \mathbf{w}^T(\mathbf{z}) \tilde{\theta} & \leq \frac{1}{4\eta} s^2 + \eta (g(\mathbf{x}, \bar{u}) \mathbf{w}^T(\mathbf{z}) \tilde{\theta})^2 \\ & \leq \frac{1}{4\eta} s^2 + \eta (\dot{s} + k_1 s + k_2 \tanh(s/\varepsilon))^2 \end{aligned} \quad (30)$$

Equation (29) can be bounded as

$$\dot{V} \leq -\left(k_1 - \frac{1}{4\eta}\right) s^2 - k_2 s \tanh\left(\frac{s}{\varepsilon}\right) + \dot{\tau} - \frac{N(\tau)}{g(\mathbf{x}, \bar{u})} \dot{\tau} + \frac{1}{\gamma} \tilde{\theta}^T \dot{\theta}^* \quad (31)$$

Now, we suppose that the following inequality holds

$$\begin{aligned} \|\tilde{\theta}^T \dot{\theta}^*\| & \leq \kappa_1 s^2 + \kappa_2 (g(\mathbf{x}, \bar{u}) \mathbf{w}^T(\mathbf{z}) \tilde{\theta})^2 + \delta^2 \\ & \leq \kappa_1 s^2 + \kappa_2 (\dot{s} + k_1 s + k_2 \tanh(s/\varepsilon))^2 + \delta^2 \\ & \leq \kappa_1 s^2 + \frac{\kappa_2}{\eta} \dot{\tau} + \delta^2 \end{aligned} \quad (32)$$

where κ_1 and κ_2 are positive constants, and δ is a bounded square integrable function.

With (32), (33) can be upper bounded by

$$\begin{aligned} \dot{V} \leq & -\left(k_1 - \frac{1}{4\eta} - \frac{\kappa_1}{\gamma}\right) s^2 - k_2 s \tanh\left(\frac{s}{\varepsilon}\right) \\ & + (h(\mathbf{x}) N(\tau) + c_1) \dot{\tau} + \frac{1}{\gamma} \delta^2 \end{aligned} \quad (33)$$

with $c_1 = 1 + \frac{\kappa_2}{\gamma\eta}$ and $h(\mathbf{x}, \bar{u}) = -1/(\eta g(\mathbf{x}, \bar{u}))$.

Assuming that $k_1 > 1/\left(\frac{1}{4\eta} + \frac{\kappa_1}{\gamma}\right)$, then by integrating (33) over $[0, t]$, one can obtain the following inequality

$$V(t) \leq c_0 + \int_0^t (h(\zeta)N(\tau(\zeta)) + c_1) \tau(\zeta) d\zeta \quad (34)$$

where $c_0 = V(0) + \frac{1}{\gamma} \int_0^\infty \delta^2(t) dt$.

According to Assumption 1, we can consider $h(\cdot)$ as a time-varying parameter which takes values in the interval $I_h = \left[-(\eta \underline{g})^{-1}, -(\eta \bar{g})^{-1}\right]$ with $0 \notin I_h$. Thus, Using Lemma 1, we can conclude from (33) the boundedness of the signals $V(t)$, $N(t)$ and $\int_0^t (h(\zeta)N(\zeta) + c_1) \tau(\zeta) d\zeta$ for $t \in [0, t_f]$. According to [13], since no finite-time escape phenomenon may happen, then $t_f = \infty$. Therefore, $s(t)$, $\hat{\theta}(t)$, $\mathbf{x}(t)$, and $u(t)$ are bounded. As an intermediate result, $s(t)$ is square integrable and $\dot{s}(t)$ is bounded. Moreover, by invoking Barbalat's lemma, we can conclude the asymptotic convergence of $s(t)$, which further implies the asymptotic convergence of the tracking error $e(t)$ and its first $(n-1)$ derivatives.

Remark 1: In this paper, since the signal $\dot{s}(t)$ in the adaptive laws (19) and (20) is not available for measurement, the following approximation is used $\dot{s}(t) \approx (s(t) - s(t - \Delta t))/\Delta t$ where Δt is a small positive constant. Notice that because of the integral structure of adaptive laws (19) and (20), the noise effect will be considerably reduced.

IV. SIMULATION RESULTS

To demonstrate the effectiveness of the above proposed adaptive controller, the following second-order nonlinear nonaffine system is used for simulation [7]

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_1^2 + 0.15u + 0.1(1 + x_2^2)u + \sin(0.1u) \end{cases} \quad (35)$$

The control objective is to force the system output $y(t) = x_1(t)$ to track the desired trajectory $y_d(t) = \sin(t) + \cos(0.5t)$.

Within this simulation, the unknown ideal controller is approximated by a fuzzy system in the form of (15). The input variables of the fuzzy system are chosen as $\mathbf{z} = [x_1, x_2]^T$. For each variable z_i , $i = 1, 2$, five Gaussian membership functions are used with centers $-1, -0.5, 0, 0.5$ and 1 , and a variance equal to 4 .

In this simulation, the system initial conditions are $\mathbf{x}(0) = [0.1, 0]^T$, the adjustable parameters θ are initialized with zero values, and the design parameters are $\lambda = 1$, $k_1 = 0.5$, $k_2 = 0.5$, $\gamma = 0.1$, $\eta = 0.01$ and $\varepsilon_0 = 0.01$.

Simulation results for the case of an unknown control direction (where $\text{sgn}(g(\mathbf{x}, u)) = 1$) are shown through Figs. 1 to 4. We can see that actual trajectories converge to the desired ones and that the control input signal is bounded.

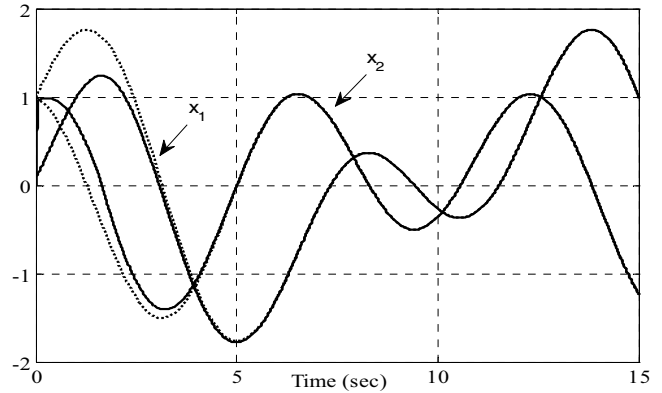


Fig. 1. Actual system trajectories (solid lines) and desired trajectories (dotted lines)

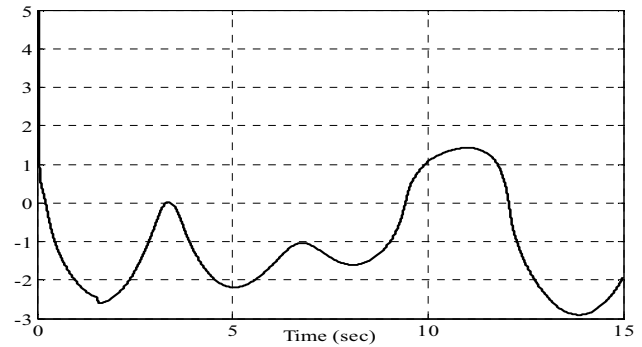


Fig. 2. Control input signal

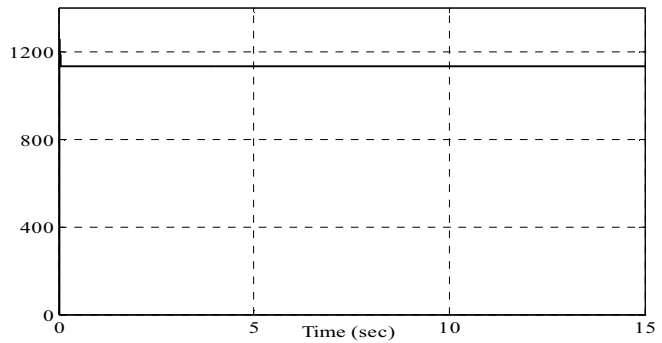


Fig. 3. Nussbaum function $N(\tau)$

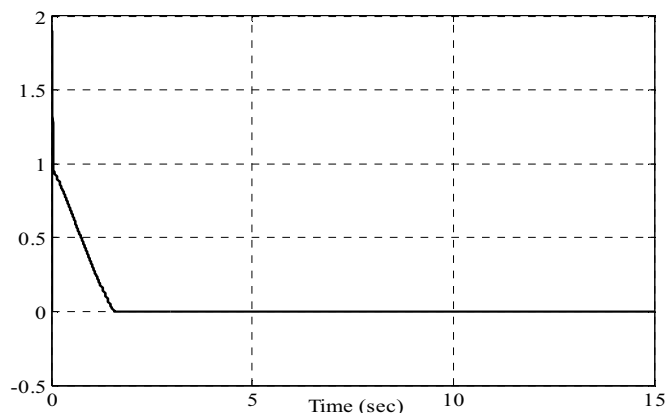


Fig. 4. Sliding surface $s(t)$

V. CONCLUSION

This paper proposed a stable direct adaptive fuzzy control scheme for a class of uncertain SISO nonaffine nonlinear systems with unknown control direction. The scheme consists of an adaptive fuzzy controller with its adaptive law. Within this scheme, a fuzzy system is used to construct adaptively an unknown ideal implicit controller, and its adjustable parameters are updated, by using the gradient descent method, in order to minimize the error between the unknown controller and the fuzzy controller. The parameters adaptation law incorporates a Nussbaum function in order to cope with the unknown control direction problem. Using the Lyapunov synthesis method, all signals in the closed-loop control system have been theoretically shown to be bounded and the tracking error has been shown to be asymptotically stable. Simulation results have verified the effectiveness of the proposed adaptive control approach.

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