

Permutation Flow Shop Scheduling: Fuzzy Particle Swarm Optimization Approach

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Abstract—A fuzzy particle swarm optimization (PSO) for the minimization of makespan in permutation flow shop scheduling problem is presented in this paper. In the proposed fuzzy PSO, the inertia weight of PSO and the control parameter of the cross-mutated operation are determined by a set of fuzzy rules. To escape the local optimum, cross-mutated operation is introduced. In order to make PSO suitable for solving permutation flow shop scheduling problem, a roulette wheel mechanism is proposed to convert the continuous position values of particles to job permutations. Meanwhile, a swap-based local search for scheduling problem is designed for the local exploration on a discrete job permutation space. Flow shop benchmark functions are employed to evaluate the performance of the fuzzy PSO for flow shop scheduling problems and the results indicate that the algorithm performs better compared with existing hybrid PSO algorithms.

Index Terms—Flow shop Scheduling, Fuzzy logic, Particle swarm optimization.

I. INTRODUCTION

Permutation flow shop scheduling problem is a NP hard problem, existing mathematical methods, i.e., integer programming, branch-and-prune method, dynamic programming, etc, are only applicable to small-scale problem and they are difficult to find the global solution of the problem. Thus, various global algorithms such as genetic algorithm (GA) [1], differential evolution (DE) [2] and ant colony optimization (ACO) [3] [4] are proposed. Recently, a new global searching algorithm namely particle swarm optimization (PSO) is widely used to different industrial areas [5]–[8]. Especially, it is effectively to tackle the flow shop scheduling problems [9]–[14]. Comparing with other population based stochastic optimization methods, such as evolutionary algorithms, PSO has comparable or even superior search performance for many hard optimization problems with faster and more stable convergence rates [15], [16]. However, observations reveal that PSO converges sharply in the early stage of the searching process, but saturates or even terminates in the later stage. It behaves like the traditional local searching methods that trap in local optima.

Recently, different hybrid PSO methods have been proposed to overcome the drawback of trapping in local optima. The hybrid PSO has been first proposed in 1998 [17], in which a

standard selection mechanism is integrated with PSO. A new hybrid gradient descent PSO (HGPSO), which is integrated with gradient information to achieve faster convergence without getting trapped in local minima is proposed [18]. However, the computational demand of HGPSO is increased by the process of the gradient descent. In addition, it is poor to handle the multimodal problem that contain many local minima. In [19], a hybrid PSO algorithm named HGAPSO is proposed, which incorporates GA's evolutionary operations of crossover, mutation and reproduction. In the paper [5], a hybrid PSO named HPSOM is proposed, in which a constant mutating space is used in mutation. In both HGAPSO and HPSOM, the solution space can be explored by performing mutation operations on particles along the search, and premature convergence is more likely to be avoided. However, the mutating space is kept unchanged all the time throughout the search, and the space for the permutation of particles in PSO is also fixed. It can be improved by varying the mutating space along the search. A hybrid PSO with wavelet mutation operation (HPSOWM) is proposed [8], which the mutating space is varying by applying wavelet theory. The solution quality and solution reliability (standard deviation upon many trials) are improved.

In order to make PSO be suitable to solve the combinational (scheduling) problem, a converting process from the continuous position values of particles to job permutations is used. Conventionally, the Smallest Position Value (SPV) rule [11]–[13] is actually a mapping from an N -dimensional continuous space to a sequence with N numbers. The corresponding sequence is represented by the order of the vector. As we know, the positions of the particles in PSO are updated that causes velocity and evolution. In some case, the positions of the particles are updated, the corresponding sequence is still unchanged by using SPV rule [14]. This kind of inefficiency of the continuous space will result in the decrease in convergence rate. To overcome this drawback, a sequence-order system based on the roulette wheel mechanism is proposed in this paper. In this mechanism, the probability representation is instead of the position value which is represented by SPV rule.

In [20], an improved version of PSO with an inertia weight factor is introduced. The aim of the inertia weight provides a balance between the global exploration and local exploitation and it is governed by a linear characteristic function. However, not all the optimization problems are linear. Thus, in this paper, a hybrid fuzzy logic-based PSO with cross-mutated operation is proposed. This proposed approach can be divided into two main parts. First, an adaptive inertia weight is proposed, which incorporates with the fuzzy inference system. Fuzzy logic is good in representing some expert knowledge and experience in some linguistic rule which can be easily understood by the human being. Using fuzzy inference to determine the inertia weight of PSO, such that, the characteristic of function of inertia weight becomes nonlinear. It will be shown that the nonlinear characteristic of the inertia weight performs more better solution quality in this paper. Second, a new operation namely cross-mutated (CM) operation is introduced. The CM operation effectively solves the drawback of PSO, which makes PSO easier to trap in the local optima. In CM operation, the control parameter is determined by fuzzy rules and the operation becomes adaptive. Additionally, a swap-based local search method is designed for the local exploration on a discrete job permutation space. This method is used to fine tune the sequence of the flow shop scheduling problem. In this paper, a suite of benchmark problems for permutation flow shop scheduling problem are used to illustrate the performance of the proposed approach. The resulting adaptive inertia weight and CM operation aid the proposed fuzzy logic-based PSO with cross-mutated operation to perform better solution quality compared with improved PSO (IPSO) [20] and other hybrid PSO methods [5], [8], [18], [19].

This paper is organized as follows. Section 2 presents the formulation of flow shop scheduling problem. The implementation of the flow shop scheduling problem with hybrid PSO will be discussed in Section 3. In this section, the solution representation, hybrid fuzzy-based PSO with cross-mutated operation, and the swap local search method are discussed. In Section 4, benchmark problems will be given to evaluate the performance of the proposed method. Finally, a conclusion will be drawn in Section 5.

II. PERMUTATION FLOW SHOP SCHEDULING PROBLEM FORMULATION

The formulation of the permutation flow shop scheduling problem is described in this section. Suppose there is a set of n jobs to be processed in a set of m machines in the same order. The processing time $p_{j,k}$ for job j ($j = 1, 2, \dots, n$) on machine k ($k = 1, 2, \dots, m$) is given. The objective is to find a sequence for the processing of jobs in the machines to minimize the total completion time or makespan (C_{max}). There are several assumptions on this scheduling problem [21]: 1) Each job j can be processed at most on one machine k at the same time; 2) Each machine m can process only one job j at a time; 3) The processing of a job j on a machine k cannot be interrupted; 4) All jobs are independent and are available for processing at time 0; 5) The setup times of the jobs on machines are

negligible; 6) The machines are continuously available; and 7) If the next machine on the sequence needed by a job is not available, the job can wait and join the queue at that machine.

Let $\pi = J_1, J_2, \dots, J_n$ denote a permutation of all jobs, and $C(J_j, k)$ denotes the completion time of job J_j on the machine, then the completion time $C(J_j, k)$ can be calculated as follows:

$$C(J_1, 1) = p_{J_1, 1} \quad (1)$$

$$C(J_j, 1) = C(J_{j-1}, 1) + p_{J_j, 1}, j = 2, \dots, n \quad (2)$$

$$C(J_1, k) = C(J_1, k-1) + p_{J_1, k}, k = 2, \dots, m \quad (3)$$

$$C(J_j, k) = \max\{C(J_{j-1}, k), C(J_j, k-1)\} + p_{J_j, k}, \\ j = 2, \dots, n, k = 2, \dots, m \quad (4)$$

$$C_{max}(\pi) = C(J_n, m). \quad (5)$$

So, the flow shop problem with the makespan criterion is to find a permutation π^* in the set of all permutations Π such that,

$$C_{max}(\pi^*) \leq C(J_n, m), \forall \pi \in \Pi \quad (6)$$

III. THE HYBRID FUZZY-BASED PSO FOR FLOW SHOP SCHEDULING

A. Solution representation: roulette wheel mechanism

As mentioned in the introduction section, the existing solution representation method, smallest position value (SPV) rule has a drawback which is to decrease the convergence rate and performance. In this section, a sequence-order system based on the *roulette wheel mechanism* is proposed. In this method, the value of the continuous position of particles represents the probability of the higher ranking. In other words, the position of the higher value of the element of particles will have a higher chance to be select to job sequence list firstly. In this section, the process of the methodology is discussed.

The process of the solution representation with roulette wheel mechanism is described as follows. The roulette wheel mechanism is shown in Fig. 1.

- (1) Given a continuous position (element) of the particle x_j , $j = 1, 2, \dots, n$.
- (2) Assign a probability q_j to the x_j such that: $q_j = \frac{x_j}{\sum_{j=1}^n x_j}$.

Note that $\sum q_j$ should equal to 1.

- (3) Evaluate a cumulative probability $\hat{q}_j$ for each x_j such as: $\hat{q}_j = \sum_{i=1}^j q_i$.
- (4) Generate a set of floating-point numbers $d_h \in [0 \ 1]$ randomly, $h = 1, 2, \dots, \eta$; η is the number of generated floating-point number. These floating point numbers should be sorted with ascending order. Normally, the

number of generated floating-point number η equals to $20 \times n$.

- (5) Count the total number of the floating-point numbers for each j are in the range of $[\hat{q}_{j-1} \quad \hat{q}_j]$ such as: $S_j = \text{count}(\hat{q}_{j-1} \leq d_h \leq \hat{q}_j)$, where $\text{count}()$ is a function to count how many statement is met the condition. Note that $\hat{q}_0 = 0$.
- (6) Convert the continuous position of S_j to the sequence of job by using largest position value rule. For a given continuous position like $[23, 12, 41, 6]$, the corresponding sequence is represented by the order of the value. For example, in decent order, 41 is the largest value, so the first of the sequence should be 3; the second largest value is 2, so the second job of sequence should be 1. Thus, we can obtain the corresponding job sequence π is $[3, 1, 2, 4]$.

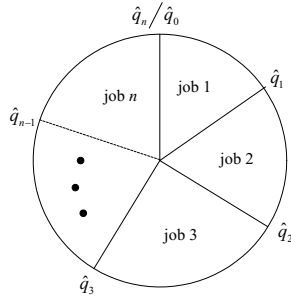


Fig. 1. The roulette wheel mechanism

B. Fuzzy logic-based PSO with cross-mutated operation

Particle swarm optimization (PSO) is a stochastic optimization method which developed in 1995 [15]. It models the processes of the sociological behavior associated with bird flocking and fish schooling. It uses a number of particle that constitute a swarm. Each particle traverses the search space looking for the global optimum. In recently, an improved PSO (IPSO) is proposed [20] where constriction and inertia weight factors are introduced and the searching ability is improved compared with standard PSO [15]. Based on the IPSO, an improved fuzzy logic-based particle swarm optimization with cross-mutated operation namely FPSOCM is proposed to tackle the permutation flow shop scheduling problem and shown in Fig. 2.

From Fig.2, $X(t)$ is denoted as a swarm at the t -th iteration. Each particle $\mathbf{x}^i(t) \in X(t)$ contains n elements $x_j^i(t) \in \mathbf{x}^i(t)$ at the t -th iteration, where $i = 1, 2, \dots, \gamma$ and $j = 1, 2, \dots, n$; γ denotes the number of particles in the swarm and n is the dimension of a particle. First, the particles of the swarm are initialized and then evaluated by a defined cost (objective) function. The evaluation cost value is represented by $f(\mathbf{x}^i(t))$. In the same time, current generation number t is initially set

at 0. The objective of PSO is to minimize the cost values of particles iteratively. The swarm evolves from iteration $t+1$ by repeating the processes as shown in Fig.2.

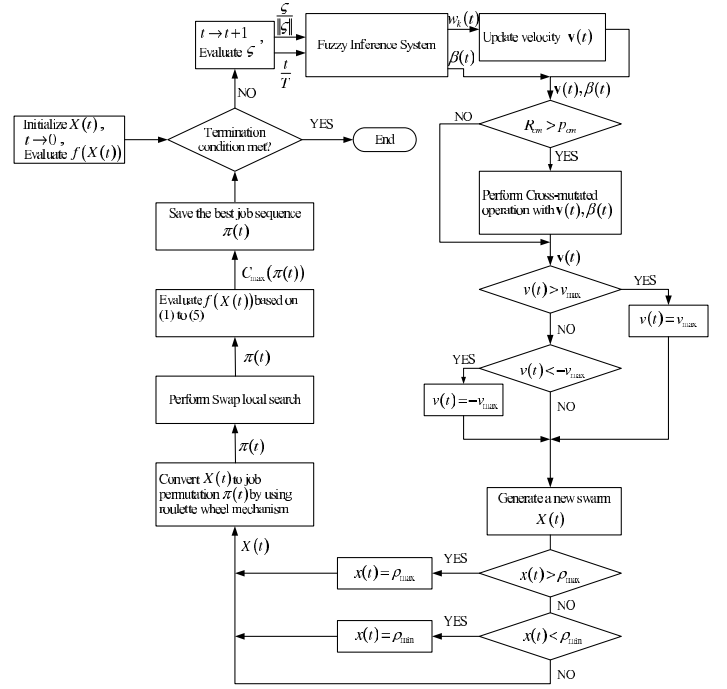


Fig. 2. The process diagram of FPSOCM for flow shop scheduling

In PSO, there has one important term and called velocity. The velocity is corresponding to the flight speed in the search space. The velocity $v_j^i(t)$ and the position $x_j^i(t)$ of the j -th element of the p -th particle at the t -th generation can be calculated using the following formulae:

$$v_j^i(t) = k \cdot \{ \omega_k(t) \cdot v_j^i(t-1) + \varphi_1 \cdot r_1 \cdot (p_j^i - x_j^i(t-1)) + \varphi_2 \cdot r_2 \cdot (g_j - x_j^i(t-1)) \} \quad (7)$$

$$x_j^i(t) = x_j^i(t-1) + v_j^i(t) \quad (8)$$

where $p^i = [p_1^i \quad p_2^i \quad \dots \quad p_n^i]$ and $g = [g_1 \quad g_2 \quad \dots \quad g_n]$; the best position of a particle i is represented as p^i ; the position of the best particle among all the particles is represented as g ; r_1 and r_2 return a uniform random number in the range of $[0,1]$, where ω_k is an inertia weight factor to provide a balance between the global and local explorations; φ_1 and φ_2 are acceleration constants; k is a constriction factor derived from the stability analysis of equation (7) to ensure the system to be converged but not prematurely [20]. Mathematically, k is a function of φ_1 and φ_2 as reflected in the following equation:

$$k = \frac{2}{|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}|} \quad (9)$$

where $\varphi = \varphi_1 + \varphi_2$ and $\varphi > 4$.

In this proposed fuzzy PSO, there are two main contributions that will enhance the performance of searching compared to other PSO methods. Firstly, an adaptive inertia weight $\omega_k(t)$ is proposed to improve the solution quality of searching. Secondly, cross-mutated (CM) operation is given to solve the drawback of IPSO that it is easier to trap into local minima. From Fig.2, we can see that the adaptive inertia weight $\omega_k(t)$ and the control parameter $\beta(t)$ of CM operation are determined by a fuzzy inference system.

1) *Adaptive inertia weight*: In IPSO [20], the inertia weight is used to provide a balance between the global exploration and local exploitation. A linear inertia weight is adopted in IPSO. In IPSO, when the value of t/T is smaller which implies that it is doing global exploration. Similarly, higher value of t/T implies that it is doing fine-tuning (local exploitation). We can see that the characteristic of this function is linear. However, some of the optimization problems are nonlinear. Thus, an adaptive (nonlinear) inertia weight is proposed to enhance the performance of the searching. In this paper, an adaptive inertia weight $\omega_k(t)$ is proposed, which is incorporated with a fuzzy inference system. In a fuzzy inference system, there are two inputs and two outputs. One of the output is adaptive inertia weight $\omega_k(t)$, the other one is control parameter $\beta(t)$ of CM (It will discussed in later). The inputs of the fuzzy system are $\varsigma(t)/\|\varsigma(t)\|$ and t/T . $\varsigma(t)/\|\varsigma(t)\|$ is a normalized standard deviation of cost value among all the particles. A larger value of $\varsigma(t)/\|\varsigma(t)\|$ implies that the variance between each cost value of particle $f(X^p(t))$ is larger. Conversely, when the value of $\varsigma(t)/\|\varsigma(t)\|$ is smaller, that means the variance between each cost value of particle $f(X^i(t))$ is smaller. In other words, the location of each particles are closer. The formulation of $\varsigma(t)/\|\varsigma(t)\|$ is defined as follows:

$$\varsigma(t) = \sqrt{\frac{1}{\gamma} \sum_i^{\gamma} (f(x^i(t)) - \bar{f}(x^i(t)))^2} \quad (10)$$

where

$$\bar{f}(x^i(t)) = \frac{1}{\gamma} \sum_{i=1}^{\gamma} f(x^i(t)) \quad (11)$$

$\|\cdot\|$ denotes the l_2 vector norm.

The adaptive inertia weight $\omega_k(t)$ is governed by the following fuzzy rules:

Rule j : IF $\varsigma(t)/\|\varsigma(t)\|$ is N_1^j , AND t/T is N_2^j , THEN $\omega_k(t) = \sigma_j$,

$$j = 1, 2, \dots, \varepsilon \quad (12)$$

where N_1^j and N_2^j are fuzzy terms of rule j , ε denotes the number of rules, $\sigma_j \in [\omega_{min}, \omega_{max}]$ is the singleton to be determined. In this paper, ω_{min} and ω_{max} are set at 0.1 and 1.1 respectively [8]. The final value of $\omega_k(t)$ is given by:

$$\omega_k(t) = \sum_{j=1}^{\varepsilon} m_j(t) \sigma_j \quad (13)$$

where

$$m_j(t) = \frac{\mu_{N_1^j}(\varsigma(t)/\|\varsigma(t)\|) \times \mu_{N_2^j}(t/T)}{\sum_{j=1}^{\varepsilon} (\mu_{N_1^j}(\varsigma(t)/\|\varsigma(t)\|) \times \mu_{N_2^j}(t/T))} \quad (14)$$

$\mu_{N_1^j}(\varsigma(t)/\|\varsigma(t)\|)$ and $\mu_{N_2^j}(t/T)$ are the membership function corresponding to N_1^j and N_2^j respectively.

In the proposed algorithm, there are 3 membership functions for each input and shown in Fig. 3. The 3 fuzzy terms are namely L (Low), M (Middle), and H (High). Based on the characteristic of $\varsigma(t)/\|\varsigma(t)\|$ and t/T , there are 9 linguistic IF-THEN fuzzy rules and the fuzzy rule table for determine $\omega_k(t)$ is shown in Fig. 4 (a). The rationale of the selected fuzzy rules for determine $\omega_k(t)$ is described as follows: the value of $\omega_k(t)$ is determined by fuzzy inputs $\varsigma(t)/\|\varsigma(t)\|$ and t/T . The value of t/T represents the iteration stage (smaller value of t/T represents the searching process in the early stage, a larger value of t/T represents the searching process in the later stage). The value of $\omega_k(t)$ should be higher as the value of t/T is smaller (in early stage) implies that a larger value of velocity of particle element is given for global searching. Similarly, a larger value of t/T implies that a higher value of velocity of particle element for local searching. Because of it, the values of $\omega_k(t)$ at t/T is “L”, are larger than t/T is set at “M”. For the same reason, the values of $\omega_k(t)$ at t/T is “M” are larger than that as t/T is “H”. The value of $\omega_k(t)$ should be smaller as t/T increase in order to reduce the value of velocity of particle element for fine-tuning.

As mentioned before, $\varsigma(t)/\|\varsigma(t)\|$ is a normalized standard deviation of cost value among all the particles. A larger value of $\varsigma(t)/\|\varsigma(t)\|$ implies that the variance between each cost value of particle $f(X^i(t))$ is larger. In other words, the location of each particle is far away. At t/T is “L”, which the searching process is in early stage and when $\varsigma(t)/\|\varsigma(t)\|$ is “H”, that implied the location of each particle is far away in early stage. Thus, a larger value of $\omega_k(t)$ should be given for global exploration. In a special case, when $\varsigma(t)/\|\varsigma(t)\|$ is “L”, that implies the location of each particle is closed in early stage.

We also set $\omega_k(t)$ to a larger value, because there has a high chance that the solutions are trap into local optima (it is affected from the smaller value of $\varsigma(t)/\|\varsigma(t)\|$). Therefore, a larger value of $\omega_k(t)$ is given to force the particle escape the local optima. As the value of $\varsigma(t)/\|\varsigma(t)\|$ is “M”, we set the value of $\omega_k(t)$ is slight smaller than that $\varsigma(t)/\|\varsigma(t)\|$ is “L” and $\varsigma(t)/\|\varsigma(t)\|$ is “H” in early stage (t/T is “L”). When t/T is “M”, which the searching process is in middle stage. The rationale of the suggested value of $\omega_k(t)$ is similar to rules that when t/T is “L”. However, there is one difference that when $\varsigma(t)/\|\varsigma(t)\|$ is “L”, the value of $\omega_k(t)$ is smaller than $\varsigma(t)/\|\varsigma(t)\|$ is “H”. It is because the optimal solution may be found in middle stage where a smaller value of $\omega_k(t)$ is given. At t/T is “H”, which the searching process is in later stage. In this stage, the searching process is undergo fine-tuning process (local exploitation) to find the optimal solution. Because of it, when the value of $\varsigma(t)/\|\varsigma(t)\|$ is “L” that implied the position

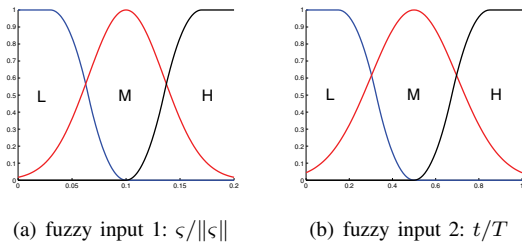


Fig. 3. Membership functions a) (x-axis: $\varsigma(t)/\|\varsigma(t)\|$, y-axis: $\mu_{N_1}(\varsigma(t)/\|\varsigma(t)\|)$) and b) (x-axis: t/T , y-axis: $\mu_{N_2}(t/T)$)

of most particles are closed and near the best solution. Thus, a smallest value of $\omega_k(t)$ is given.

$\frac{t}{T}$	$\frac{\varsigma(t)}{\ \varsigma(t)\ }$		
	L	M	H
L	1.1	0.9	1.1
M	0.6	0.5	0.7
H	0.1	0.2	0.3

(a)

$\frac{t}{T}$	$\frac{\varsigma(t)}{\ \varsigma(t)\ }$		
	L	M	H
L	0.5	0.4	0.6
M	0.4	0.3	0.5
H	0.1	0.2	0.2

(b)

Fig. 4. The fuzzy rule table for determining (a) $w_k(t)$, and (b) $\beta(t)$

2) *Cross-mutated operation*: In this section, a cross-mutated (CM) operation is introduced. This new CM operation is merged the idea of crossover and mutation operation of genetic algorithm. The aim of the CM operation is force the particle escape the local optima. Furthermore, a control parameter $\beta(t)$ is introduced into the CM operation, and the operation becomes more adaptive. This control parameter is governed by some fuzzy rules with human knowledge. By introducing the CM operation, the performance of the proposed PSO method is improved.

The details of the CM operation is as follows. Every velocity of particle element of the swarm will have a chance to undergo CM operation governed by a probability of cross-mutated operation, $p_{cm} \in [0, 1]$, which is defined by user. For each velocity of particle element, a random number R_{cm} between 0 and 1 will be generated such that if it is less than or equal to p_{cm} , the CM operation will take place on that element. The sensitivity of the p_{cm} with experimental results and the analysis will be discussed in later. The choice of the p_{cm} will affect the quality of solution.

The resulting velocity of particle element under the CM operation is given by:

$$\bar{v}_j^i(t) = (1 - \beta(t)) v_j^i(t) + \beta(t) \tilde{v}_j^i(t) \quad (15)$$

where

$$\tilde{v}_j^i(t) = \frac{1}{4} \{ r_3 \cdot (\rho_{\max_j} - \rho_{\min_j}) - \rho_{\max_j} \} \quad (16)$$

$v_j^i(t)$ is determined by (7), $\tilde{v}_j^i(t)$ is a random velocity of particle element and the value of this velocity is randomly generated and bounded with 0.25 of the dynamic range of the particle element. $r_3 \in [-1, 1]$ is a uniform random number. In (15), the resulting velocity of particle element $\bar{v}_j^i(t)$ is combined with the information of $v_j^i(t)$ and $\tilde{v}_j^i(t)$. This information exchanging process is like as crossover operation. However, in CM operation, $\bar{v}_j^i(t)$ is changed (mutated) one-by-one which is like as mutation operation. Therefore, we called it is cross-mutated (CM) operation.

In (15), the control parameter $\beta(t)$ provides a balance to control the resulting velocity $\bar{v}_j^i(t)$ converge toward $v_j^i(t)$ or $\tilde{v}_j^i(t)$. If $\beta(t)$ is approaching 0, the $\bar{v}_j^i(t)$ will tends to the $v_j^i(t)$. Conversely, when $\beta(t)$ is approaching 1, the $\bar{v}_j^i(t)$ will tends to the $\tilde{v}_j^i(t)$. The proposed random velocity $\tilde{v}_j^i(t)$ in (16) has ability to force the particle element to escape the local optima with a random movement. In this operation, the value of the $\beta(t)$ is governed by nine fuzzy-rules. The fuzzy inference system is same as before we mentioned. The inputs of the fuzzy system are same. They are $\varsigma(t)/\|\varsigma(t)\|$ and t/T . Now, there are 9 linguistic fuzzy rules and the fuzzy rule table for determine $\beta(t)$ is shown in Fig. 4 (b).

The rationale of the selected fuzzy rules is similar to the rules of adaptive inertia weight in last sub-section. The brief description of the choosing fuzzy rules for CM operation is given as follows: the value of $\beta(t)$ is determined by $\varsigma(t)/\|\varsigma(t)\|$ and t/T , and some fuzzy rules. As mention before, the value of the t/T represents the iteration stage. We can see as t/T is “L”, where the searching process is in early stage and as t/T is “H”, where the searching process is in later stage, the values of $\beta(t)$ in early stage are larger than the values of $\beta(t)$ in later stage. It is because a significant random velocity (higher value of $\beta(t)$ in (15)) provides a global exploration in early stage. Conversely, the effect of the random velocity should be reduced in later stage for fine-tuning (local exploitation) with smaller value of $\beta(t)$.

Till now, we have discussed about the effect of the $\varsigma(t)/\|\varsigma(t)\|$, the concept is same as the rules of adaptive inertia weight. In the early stage, when $\varsigma(t)/\|\varsigma(t)\|$ is “L”, that implies the location of each particle is closed. Thus, we need to set the value of $\beta(t)$ to be larger than that when $\varsigma(t)/\|\varsigma(t)\|$ is “M”. The reason lies in the fact that a higher chance that the solution being trapped into a local optimum. Conversely, in the later stage, the searching process undergoes fine-tuning process (local exploitation) to find the optimal solution. Because of this, when the value of $\varsigma(t)/\|\varsigma(t)\|$ is “L” that implies the position of most particles are closed (similar) and near the best solution. Thus, a smallest value of $\beta(t)$ is given.

In (7), the particle velocity is limited by a maximum value $v_{\max}$. The parameter $v_{\max}$ determines the resolution with which regions are to be searched between the present

position and the target position. This limit enhances the local exploitation of the problem space and it realistically simulates the incremental changes of human learning. If $v_{\max}$ is too high, particles might fly past good solutions. If $v_{\max}$ is too small, particles may not explore sufficiently beyond local solutions. Empirically, $v_{\max}$ is often set at 10% to 20% of the dynamic range of the element on each dimension.

A new swarm $X(t)$ is updated after the velocity of all particles are updated. To ensure all particle elements x_j^i in $X(t)$ fall within the range $[\rho_{\min}, \rho_{\max}]$, the following conditions are considered as follows: If $x_j^i(t) > \rho_{\max_j}$, then the updated $x_j^i(t)$ should equal to $\rho_{\max_j}$. Similarly, If $x_j^i(t) < \rho_{\min_j}$, then the updated $x_j^i(t)$ should equal to $\rho_{\min_j}$. Here, $\rho_{\min} = [\rho_{\min_1} \ \rho_{\min_2} \ \cdots \ \rho_{\min_n}]$ and $\rho_{\max} = [\rho_{\max_1} \ \rho_{\max_2} \ \cdots \ \rho_{\max_n}]$; $\rho_{\min_j}$ and $\rho_{\max_j}$ are minimum and maximum values of $x_j^i(t)$ respectively and $j = 1, 2, \dots, n$.

After an updated swarm is generated. Next, convert all the particles in the swarm to job permutation $\pi(t)$ by using the sequence-order system based on the roulette wheel mechanism. After going through the process of the sequence-order system, a set of job sequence will be formed and then undergo swap based local search operation for local exploration on a job permutation space.

3) *Swap based-Local search*: Local searches are very important for the exploitation. In this paper, a swap-based local search method is designed for fine-tuning on the flow shop scheduling problem. The operation of this local search is simply to swap the selected two job order number. The operation of swap based local searching method is described as follows.

- (1) Given a sequence of jobs π , where $\pi = J_1, J_2, \dots, J_{i-1}, J_i, J_{i+1}, \dots, J_n$.
- (2) Randomly picks 2 jobs, i.e., J_i and J_{i+1} .
- (3) Swap the selected 2 jobs.

After the process, the new sequence of jobs should be $J_1, J_2, \dots, J_{i-1}, J_{i+1}, J_i, \dots, J_n$. An example of swap based-local search method is given in Fig.5. In this figure, the job 7 and the job 1 are selected to undergo the swap process. After going through the swap operation, a new job sequence is generated. By using (1)-(5), a updated makespan C_{max} will be evaluated. The optimization process will repeat until a termination condition is met. The iterative process will be terminated when a defined number of iteration is met.

IV. COMPUTATIONAL RESULTS

To test the performance of the proposed FPSOCM to flow shop scheduling problem, computational simulation is carried out with a suite of flow shop benchmark problems [22].

A. Experimental setup

The performance of HPSOWM [8], HPSOM [5], HGAPSO [19], HGPSO [18], IPSO [20], and the proposed FPSOCM on solving the flow-shop benchmarking problems are evaluated.

The following simulation conditions are in use:

For All PSOs:

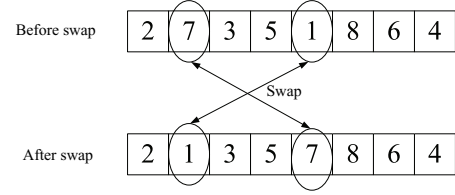


Fig. 5. The example of the swap-based local search method.

- Swarm size (γ): 100
- Number of runs: 20
- Acceleration constant φ_1 : 2.05
- Acceleration constant φ_2 : 2.05
- Maximum velocity v_{max} : 0.2
- Initial population: it is generated uniformly at random

For FPSOCM:

- Probability of cross-mutated operation (p_{cm}):
 $p_{cm} = 0.05$ for Car3, Car8;
 $p_{cm} = 0.01$ for Car1, Car2, Car4, Car5, Car6, Car7;

For HPSOWM, HPSOM, and HGAPSO:

- Probability of mutation operation (p_m): it is chosen by trial and error through experiments for good performance for all functions. ($p_m = 0.2$ for Car1 - Car8)

For HPSOWM:

- Shape parameter of the wavelet mutation: it is chosen by trial and error through experiments for good performance for all functions
- Parameter g of the wavelet mutation: 10000

For HGPSO:

- Learning rate: it is chosen by trial and error through experiments for good performance for all functions

For HGAPSO:

- Probability of crossover operation: 0.8

B. Results

In this section, the results for the 8 flow shop problem are given to validate the FPSOCM, which is proven to perform well on flow shop scheduling problem. The experimental results in terms of the average relative error (ARE), best relative error (BRE), and worst relative error (WRE) are summarized in Table I. The number of iterations 100 is adopted in all approaches. From the table, we can see that FPSOCM algorithm obtains a smaller ARE, BRE and WRE in all problems. In problem Car6 which is a 8 jobs 9 machines flow shop scheduling problem, the ARE of the FPSOCM is 0. Comparing with other algorithms, only FPSOCM can reach the optimal value. In Problem Car3, the ARE of FPSOCM is 0.06 which imply around 2.5 - 6 times improvement compared

(A)

Method	Problem			
	Car1 $n, m = 11, 5; C^* = 7038$	Car2 $n, m = 13, 4; C^* = 7166$	Car3 $n, m = 12, 5; C^* = 7312$	Car4 $n, m = 14, 4; C^* = 8003$
FPSOCM	ARE	0	0.06	0
	BRE	0	0	0
	WRE	0	0.602	0
HPSOWM	ARE	0	0.157	0
	BRE	0	0	0
	WRE	0	1.203	0
HPSOM	ARE	0	0.260	0
	BRE	0	0	0
	WRE	0	1.504	0
HGAPSO	ARE	0	0.356	0
	BRE	0	0	0
	WRE	0	1.504	0
HGPSO	ARE	0	0.345	0.012
	BRE	0	0	0
	WRE	2.930	1.504	0.062
IPSO	ARE	0	0.343	0
	BRE	0	0	0
	WRE	0	1.792	0

(B)

Method	Problem			
	Car5 $n, m = 10, 6; C^* = 7720$	Car6 $n, m = 8, 9; C^* = 8505$	Car7 $n, m = 7, 7; C^* = 6590$	Car8 $n, m = 8, 8; C^* = 8366$
FPSOCM	ARE	0.019	0	0
	BRE	0	0	0
	WRE	0.389	0	0
HPSOWM	ARE	0.161	0.153	0.024
	BRE	0	0	0
	WRE	0.622	0.764	1.960
HPSOM	ARE	0.282	0.115	0
	BRE	0	0	0
	WRE	1.308	0.764	0
HGAPSO	ARE	0.549	0.076	0.152
	BRE	0	0	0
	WRE	1.308	0.764	1.960
HGPSO	ARE	0.194	0.223	0.294
	BRE	0	0	0
	WRE	0.609	0.764	1.960
IPSO	ARE	0.343	0.483	0.144
	BRE	0	0	0
	WRE	1.308	2.787	1.960

n : Number of jobs

m : Number of machines

C^* : The optimal value known so far

TABLE I

COMPARISON RESULTS OF FPSOCM, HPSOWM, HPSOM, HGAPSO, HGPOS, AND IPSO. ALL RESULTS ARE AVERAGED ONES OVER 20 RUNS (A: CAR1-4, B: CAR5-8).

with other algorithm. Similarly, in Problem Car5, the ARE of FPSOCM is 0.019 which imply around 8 - 18 times improvement. So FPSOCM algorithm for flow shop scheduling problem to minimize makespan is more effective.

V. CONCLUSION

In this paper, we have proposed a hybrid particle swarm optimization which incorporates a fuzzy system and a new cross-mutated operation to tackle job shop scheduling problem. An adaptive inertia weight of PSO is presented. It is determined by a fuzzy system. Moreover, in order to solve the local optimum problem of PSO, a new operation namely cross-mutated operation is proposed. With these proposed operations, the solution quality is improved. The performance of the proposed hybrid PSO is compared with other hybrid PSOs with respect to the same set of flow scheduling problems. The results demonstrate the effectiveness of the proposed algorithm for the flow shop scheduling problem.

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