

Manufacturing Modeling Using An Evolutionary Fuzzy Regression

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Abstract— Fuzzy regression is a commonly used approach for modeling manufacturing processes in which the availability of experimental data is limited. Fuzzy regression can address fuzzy nature of experimental data in which fuzziness is not avoidable while carrying experiments. However, fuzzy regression can only address linearity in manufacturing process systems, but nonlinearity, which is unavoidable in the process, cannot be addressed. In this paper, an evolutionary fuzzy regression which integrates the mechanism of a fuzzy regression and genetic programming is proposed to generate manufacturing process models. It intends to overcome the deficiency of the fuzzy regression, which cannot address nonlinearities in manufacturing processes. The evolutionary fuzzy regression uses genetic programming to generate the structural form of the manufacturing process model based on tree representation which can address both linearity and nonlinearities in manufacturing processes. Then it uses a fuzzy regression to determine outliers in experimental data sets. By using experimental data excluding the outliers, the fuzzy regression can determine fuzzy coefficients which indicate the contribution and fuzziness of each term in the structural form of the manufacturing process model. To evaluate the effectiveness of the evolutionary fuzzy regression, a case study regarding modeling of epoxy dispensing process is carried out.

Keywords- fuzzy regression, manufacturing process modeling

I. INTRODUCTION

In manufacturing processes, process variables at each of the many processing steps in a manufacturing line need to be controlled, and all process variables related to the desired output need to be optimized to maintain tight control. Development of a manufacturing process model, which represents relationship between process variables and desired outputs, is essential.

While only a small amount of experimental data is available, fuzzy regression [1, 4, 10, 21-23] has the distinct advantage of being able to generate models in manufacturing process systems, which have a high degree of fuzziness. For examples, Schaible and Lee used the fuzzy linear regression

method to model the vertical CVD process [20]. Lai and Chang applied fuzzy regression to model the die casting process [1]. Ip et al. introduced the fuzzy linear regression to develop models for epoxy dispensing [8]. Three different approaches of fuzzy linear regression were summarized in Chang and Ayyub [2]. However, the existing fuzzy regression approaches cannot be used to develop manufacturing process models which are nonlinear in nature. In fact, nonlinear behaviors of manufacturing process commonly exist. If nonlinearities are integrated into the approach of fuzzy regression, more accurate models can be developed.

Furthermore, it is widely recognized that the quality of model development declines when outliers in experimental data exist, but very few studies have attempted to detect outliers when developing fuzzy regression models. Chen [6] proposed a method to detect outliers involving crisp inputs and fuzzy outputs. The method detects the difference in width between the spread of fuzzy data and the spread of fuzzy output. However, experimental data and the results of manufacturing processes involve crisp values of experimental settings and crisp values of experimental responses. Therefore, this method cannot be applied to manufacturing process modeling.

To overcome these deficiencies, we propose a genetic fuzzy regression, which can be used to generate manufacturing process models with nonlinear nature. The genetic fuzzy regression uses the general outcomes of genetic programming to construct models based on a tree structure representation in which both the interaction and higher order terms can be considered so as to address nonlinearity. Then it uses the mechanism of fuzzy regression to detect the outliers from experimental data sets based on an indicator of outliers. The fuzzy regression then estimates the contribution of each branch of the tree in order to determine the fuzzy coefficient of each term of the model by using the experimental data sets excluding the outliers. As nonlinear terms can be generated and represented in the branches of the tree based on the

genetic fuzzy regression, nonlinear terms can be generated in the manufacturing process models. Furthermore, as the fuzzy regression is used to determine fuzzy coefficients of the manufacturing process model, only a small amount of data is required, which is practical in the manufacturing process.

The effectiveness of the proposed genetic fuzzy regression approach is evaluated by modeling the epoxy dispensing process for electronic packages, which is used in various electronic packaging processes such as integrated circuit (IC) encapsulation and die-bonding [16]. Epoxy dispensing is a highly non-linear process that involves extremely complex inter-relationships among the epoxy properties, process conditions and overall encapsulation quality [7]. The genetic fuzzy regression is used to develop models for this manufacturing process. Modeling results based on genetic fuzzy regression is compared with those based on the fuzzy linear regression methods of Tanaka [9] and Peters [8], which have been employed to model the epoxy dispensing processes.

II. GENETIC FUZZY REGRESSION

The genetic fuzzy regression approach generates the manufacturing process model which relates output responses and process variables in fuzzy regression form. When defining, the genetic fuzzy regression needs to:

- specify the form of the manufacturing process model in fuzzy regression form
- determine the fuzzy coefficients which characterize the model
- define the algorithm of genetic fuzzy regression

A. Form of process model

The general form of the manufacturing process models, which involves nonlinear terms between process variables are represented as fuzzy regression form as follows:

$$\tilde{y} = \tilde{f}_0 + \sum_{i=1}^N \tilde{f}_i(x_i) + \sum_{i=1}^N \sum_{j=1}^N \tilde{f}_{ij}(x_i, x_j) + \dots + \tilde{f}_{1,2,\dots,N}(x_1, x_2, \dots, x_N) \quad (1)$$

where $\tilde{y}$ is the output response, x_i is the i -th process variable, $\tilde{f}_0$ is a fuzzy bias term and $\tilde{f}_i(x_i)$, $\tilde{f}_{ij}(x_i, x_j)$, ... represent univariate fuzzy components and bivariate fuzzy components respectively. A higher order high-dimensional Kolmogorov-Gabor polynomial is one of the forms of (1), which can be written as follows:

$$\begin{aligned} \tilde{y} = \tilde{f}_{NR}(x) = & \tilde{A}_0 + \sum_{i_1=1}^N \tilde{A}_{i_1} x_{i_1} + \sum_{i_1=1}^N \sum_{i_2=1}^N \tilde{A}_{i_1 i_2} x_{i_1} x_{i_2} + \\ & \dots + \sum_{i_1=1}^N \dots \sum_{i_d=1}^N \tilde{A}_{i_1 \dots i_d} \prod_{j=1}^d x_{i_j} \end{aligned} \quad (2)$$

where $\tilde{A}_0 = (a_0^c, a_0^s)$, $\tilde{A}_1 = (a_1^c, a_1^s)$, $\tilde{A}_2 = (a_2^c, a_2^s)$, ..., $\tilde{A}_N = (a_N^c, a_N^s)$, $\tilde{A}_{11} = (a_{11}^c, a_{11}^s)$, $\tilde{A}_{12} = (a_{12}^c, a_{12}^s)$, ..., $\tilde{A}_{NN} = (a_{NN}^c, a_{NN}^s)$, ..., $\tilde{A}_{N\dots N} = (a_{N\dots N}^c, a_{N\dots N}^s)$.

The fuzzy regression model (2) can be rewritten as follows:

$$\tilde{y} = \tilde{A}'_0 x'_0 + \tilde{A}'_1 x'_1 + \tilde{A}'_2 x'_2 \dots \tilde{A}'_{N_{NR}} x'_{N_{NR}} \quad (3)$$

$$\text{or } \tilde{y} = (a_0^c, a_0^s) x'_0 + (a_1^c, a_1^s) x'_1 + (a_2^c, a_2^s) x'_2 + \dots + (a_{N_{NR}}^c, a_{N_{NR}}^s) x'_{N_{NR}} \quad (4)$$

where $1+N_{NR}$ is the number of terms of (1), (2) and (3); $\tilde{A}'_0 = \tilde{A}_0$, $\tilde{A}'_1 = \tilde{A}_1$, $\tilde{A}'_2 = \tilde{A}_2$, ..., $\tilde{A}'_{N_{NR}} = \tilde{A}_{N\dots N}$; $x'_0 = 1$, $x'_1 = x_1$, $x'_2 = x_2$, ..., $x'_{N_{NR}} = x_1 \cdot x_2 \cdot \dots \cdot x_d$; and $\tilde{A}'_0 = (a_0^c, a_0^s)$, $\tilde{A}'_1 = (a_1^c, a_1^s)$, ..., $\tilde{A}'_{N_{NR}} = (a_{N_{NR}}^c, a_{N_{NR}}^s)$. $\tilde{A}'_i$ and x'_i are called the fuzzy coefficients and the transformed variables respectively, where $i=0,1,2,\dots,N_{NR}$.

The vectors of the fuzzy coefficients are defined as follows:

$$\tilde{A}' = (\tilde{A}'_0, \tilde{A}'_1, \dots, \tilde{A}'_{N_{NR}}) = ((a_0^c, a_0^s), (a_1^c, a_1^s), \dots, (a_{N_{NR}}^c, a_{N_{NR}}^s)), \quad (5)$$

$$a^c = (a_0^c, a_1^c, \dots, a_{N_{NR}}^c), \quad (6)$$

$$\text{and } a^s = (a_0^s, a_1^s, \dots, a_{N_{NR}}^s). \quad (7)$$

The vector of the transformed variables is defined as follows:

$$x' = (x'_0, x'_1, x'_2, \dots, x'_{N_{NR}}). \quad (8)$$

Using the vectors of the fuzzy coefficients and the vector of the transformed variables, (3) can be rewritten as follows:

$$\tilde{y} = \tilde{A}' \cdot x'^T \quad (9)$$

B. Determination of fuzzy coefficients

The fuzzy regression is proposed to determine the fuzzy coefficients of the manufacturing process models which are structured in the form of (4). Based on Peters' fuzzy regression [19], the fuzzy coefficients of the manufacturing process model are determined by solving the linear programming (LP) problem formulated as (10a) to (10f):

$$\max \bar{\lambda} = \frac{1}{M} \sum_{i=1}^M \lambda_i \quad (10a)$$

$$\text{s.t. } (1 - \bar{\lambda})p_0 - \sum_{i=1}^M \sum_{j=0}^N a_j^s |x_{ij}| \geq -d_0 \quad (10b)$$

$$(1 - \lambda_i)p_i - \sum_{j=0}^N a_j^c x_{ij} - \sum_{j=0}^N a_j^s |x_{ij}| \leq y_i, i=1,2,\dots,M \quad (10c)$$

$$(1 - \lambda_i)p_i + \sum_{j=0}^N a_j^c x_{ij} + \sum_{j=0}^N a_j^s |x_{ij}| \geq y_i, i=1,2,\dots,M \quad (10d)$$

$$\lambda_i \leq 1, i=1,2,\dots,M \quad (10e)$$

$$a_j^s \geq 0, j=0,1,\dots,N_{NS} \quad (10f)$$

where d_0 represents the desired value of the objective function, p_0 is the tolerance of the desired lower bound and p_i is the width of the tolerance interval of y_i . $\bar{\lambda}$ is the arithmetic mean of all λ_i . The parameters p_0 and p_i are determined in a context-dependent way according to the decision maker's experience and knowledge. A very low value of the special λ_i

in a Peters' fuzzy linear regression model indicates that the corresponding data set y_i is far outside the interval and can be treated as an outlier. Therefore λ_i ($i=1,2,\dots,M$) can be employed to determine whether or not the i th data set is an outlier [11]. With a threshold value of λ_0 , the y_i is defined as an outlier if $\lambda_i \leq \lambda_0$ for $i=1,2,\dots,M$, and the outliers are removed one by one from the training data sets during the process of developing manufacturing process models. The pseudocode of the fuzzy regression is shown below:

```

Determine the values of  $d_0, p_0$  and  $p_i$ ;
Select an appropriate threshold value  $\lambda_0$ ;
Solve the LP problem (10a)-(10f) using the 1-st to the
     $M$ -th data sets;
While (  $\min_{i=1,\dots,M} \lambda_i \leq \lambda_0$  ) do {
    Solve the LP problem (10a)-(10f) using the 1-st to the
         $M$ -th data sets excluding the  $k$ -th data set;
    // where  $\lambda_k$  is the small one among all the  $\lambda_i$  with
         $i=1, 2, \dots, M$ ;
    Remove the  $k$ -th data set from the  $M$  data sets
     $M=M-1$ ;
}
Return the final fuzzy coefficients

```

In the fuzzy regression, the first step is to determine the values of d_0, p_0 and p_i [19], which can be used to control the spread of the interval of the fuzzy regression model. A threshold value λ_0 is then defined as follows:

$$\lambda_0 = \frac{1}{2} \left(\max_{i=1,\dots,M} \lambda_i + \min_{i=1,\dots,M} \lambda_i \right) \quad (11)$$

where $\max_{i=1,\dots,M} \lambda_i$ and $\min_{i=1,\dots,M} \lambda_i$ are calculated using all the data sets. In the while-loop of the fuzzy regression, all λ_i with $i=1,2,\dots,M$, are found by solving the linear programming problem (10a)-(10f). If all λ_i are larger than λ_0 , the fuzzy coefficients are returned as the final solution, and the fuzzy regression is terminated. Otherwise, the LP problem (10a)-(10f) is solved by excluding the k -th data set, where λ_k are the smallest among all the λ_i with $i=1,\dots,M$. The k -th data set is then removed from the M data sets, and the number of data sets becomes $M-1$. The operations in the while-loop continues until all λ_i with $i=1,\dots,M$ are larger than λ_0 .

In the fuzzy regression, the dependent data belongs to a certain range based on the mechanism of Peters' fuzzy regression [19]. Outliers are compensated for by data that lies within the interval, and the estimated interval is determined by using all of the data rather than using only the "worst" data. Therefore a new variable is introduced to represent the degree to which the solution belongs to the set of "good solutions" (i.e. degree of membership).

Since some terms in (1) may be redundant, prudent selection of significant terms or orders is advisable if a more economical and adequate model is desired. In the intelligent fuzzy regression approach, genetic programming is used to generate the fuzzy regression model which consists of significant terms.

C. Pseudocode of Evolutionary Fuzzy Regression

Genetic programming is proposed to determine the structures of process models in the evolutionary fuzzy regression, and its pseudocode is shown as follows.

```

 $t=0$ 
Initialize  $\Omega(t)=[\theta_1(t), \theta_2(t), \dots, \theta_{POP}(t)]$ 
Assign fuzzy coefficients to all  $\theta_i(t)$ 
//  $\Omega(t)$  is the population of the  $t$ -th generation.
//  $\theta_i(t)$  is the  $i$ -th individual of  $\Omega(t)$ .
Evaluate all  $\theta_i(t)$  according to a fitness function
while (termination condition not fulfilled) do {
    Parent Selection  $\Omega(t+1)$ 
    Crossover  $\Omega(t+1)$ 
    Mutation  $\Omega(t+1)$ 
    Determine fuzzy coefficients in all  $\theta_i(t+1)$  by
        using
        Fuzzy regression discussed in Section 2B
    Evaluate all  $\theta_i(t+1)$ 
     $\Omega(t)=\Omega(t+1)$ 
     $t=t+1$ 
}

```

The evolutionary fuzzy regression begins by creating a random initial population $\Omega(t)$ with POP individuals $\theta_i(t)$, while $t=0$. Each individual $\theta_i(t)$ is in the form of a tree structure [12, 13], which can be used to represent the structure of the manufacturing process model as defined in (4). Hierarchical trees are composed of functions F and terminals T [12, 13]. The manufacturing process model (4) contains only the three arithmetic operations, $+$, $-$ and $*$, thus F is represented as $F = \{+, -, *\}$. The set of terminals is defined as $T = \{x_1, x_2, \dots, x_N\}$. In the tree, operations from the function set F are used as internal nodes, and arguments from the terminal set T are used as terminal nodes. For example, a hierarchical tree can be expressed as:

$$(x_1 * x_1) - (x_2 * x_2) + (x_1 * x_2 * x_4)$$

which is equivalent to:

$$x_1^2 - x_2^2 + x_1 x_2 x_4$$

The fuzzy regression as described in Section IIB is used to assign the fuzzy coefficients to each individual $\theta_i(t)$. All individuals are evaluated according to a defined fitness function, which is aimed at evaluating the goodness-of-fitness of the manufacturing process model. The mean absolute error (MAE) of the j -th individual can be calculated based on (12).

$$MAE_j = 100\% \times \sqrt{\frac{1}{M} \sum_{k=1}^M \left| \frac{y_k - F_j(\mathbf{x}_k)}{y_k} \right|}, \quad (12)$$

where F_j is the fuzzy regression model represented by the j -th individual, $(y_k, \mathbf{x}_k) = (y_k, (x_{k1}, x_{k2}, \dots, x_{kN}))$ is the k -th training data set which excludes the outliers detected by the fuzzy regression, and M is the number of training data sets excluding the outliers used to develop the manufacturing process model.

(12) is commonly known as an indicator of training error in a model. It reflects how well the model fits the training data sets. However, a model may contain many unnecessary and complex terms. A complex over-parameterized model with a large number of parametrical terms reduces the transparency

and interpretation of the model. To prevent the evolutionary fuzzy regression from generating models that are too complex, a fitness function is designed to balance the trade-off between the reduction of complexity and the model accuracy. In this paper, penalty terms are introduced into the fitness function of the evolutionary fuzzy regressions [17]. The fitness of the j -th individual is denoted as follows:

$$fitness_j = \frac{1 - MAE_j}{(1 + \exp(c_1(L_j - c_2)))} \quad (13)$$

where $fitness_j$ is the fitness value, L_j is the number of nodes of the j -th individual, and c_1 and c_2 are both penalty terms.

The parent selection process then uses the goodness-of-fitness of each individual to determine the selection of potential individuals to perform crossover or mutation. Finally, the new individuals with the determined fuzzy coefficients are evaluated using the fitness function to create a new population $\Omega(t+1)$. The process continues until the pre-defined termination condition has been fulfilled.

III. EVALUATION OF GENETIC FUZZY REGRESSION

In this section, the evaluation of the performance of the proposed evolutionary fuzzy regression is illustrated by modeling a fluid dispensing process [34] for the encapsulation of IC chips for electronic packages. The modeling results are compared with those based on Tanaka and Watada's fuzzy regression (FR-Tanaka) [43] and Peters' fuzzy regression (FR-Peters) [36]. All the algorithms, GP-FR, FR-Tanaka and FR-Peters were implemented using Matlab. The GP parameters in evolutionary fuzzy regression are shown as follows: population size = 50; maximum number of evaluated individuals = 5000; generation gap = 0.9; probability of crossover = 0.5; probability of mutation = 0.5.

Epoxy dispensing is a common process for performing the encapsulation of IC chips for electronic packages [15]. Modeling the epoxy dispensing process is critical to understanding the behavior of the process and to achieving optimization of the process. However, the epoxy dispensing process is difficult to characterize due to the complex behavior of the epoxy encapsulant and the existence of uncertainties inherent to epoxy dispensing systems [3, 5, 7]. In the following, the modeling of epoxy dispensing for IC chip encapsulation is described. In the epoxy dispensing machines for IC chip encapsulation, normally, silicon chips are covered using an X-Y numerically controlled dispensing system that delivers epoxy encapsulant through a needle by a pump. The material is commonly dispensed in a pattern, working from the center out. An epoxy dam around the die site and the second wire bond points can be made to contain the flow of material and produce a uniform-looking part as shown in Figure 1 in the appendix.

The epoxy dispenser consists of three essential parts, the syringe with epoxy, the time-pressure dispensing machine and the nozzle's head. The epoxy was injected from the syringe vertically by the time-pressure dispensing machine. The dropped epoxy paste was collected horizontally on the grinded plastic block. To minimize the discrepancies of the

experiments, the vertical distance between the syringe and glass slides was kept identically at 1.5cm in all the experiments. The quality characteristic, encapsulation weight (mg), was investigated. The weights of the epoxy pastes were measured by a precise electronic weighing instrument.

With the assistance of the company supporting this research, three process parameters, which are significant to the encapsulation weight y , were identified as follows:

- the dispensing time (5s to 9s), x_1 , which is the duration of the air pressure controlled by the time-pressure dispensing machine;
- the compressed air pressure (0.4Mpa to 0.6Mpa), x_2 , which is the amount of pressurized air compressed by the time-pressure dispensing machine; and
- the diameters of the nozzle head (0.61mm, 0.84 mm or 0.137 mm), x_3 .

Fifty-four experiments were carried out based on a 3^k full factorial design with 2. In the factorial design, 3 levels of dispensing time (x_1), 3 levels of compressed air pressure (x_2) and 3 levels of nozzle head diameters of (x_3) were adopted. Forty-eight out of the fifty-four experimental data sets and results were used to develop the process models of the epoxy dispensing process while the remaining seven were used to test the developed models. The outliers were included in all the training data sets. The 1st, 18th, 28th and 45th data sets were considered outliers in modeling the encapsulation weight. Both the training and testing data sets of the epoxy dispensing process are shown in Table 1 in the appendix.

Using the 47 training data sets for the encapsulation weight as shown in Table 4, the manufacturing process model for encapsulation weight as determined by FR-Tanaka is as follows:

$$y = (-0.70, -0.00) + (0.24, -0.00) \cdot x_1 + (0.03, 0.16) \cdot x_2 + (0.63, 0) \cdot x_3 \quad (14)$$

The manufacturing process model for encapsulation weight found by FR-Peters is as follows:

$$y = (-0.54, 0.15) + (0.20, 0.87) \cdot x_1 + (0.01, 0.14) \cdot x_2 + (0.55, 0.56) \cdot x_3 \quad (15)$$

Using the same experimental data sets shown in Table 4, the model for the encapsulation weight found by the GP-FR is shown as follows:

$$y = (-0.03, 1.42) + (0.06, 0.01) \cdot x_1 \cdot x_2 \cdot x_3^4 \quad (16)$$

It can be found that only linear terms can be generated by the FR-Tanaka and FR-Peters, and the higher order and interaction terms of the process models can be generated by the proposed evolutionary fuzzy regression. Therefore nonlinearity can be addressed by the manufacturing process models generated the proposed evolutionary fuzzy regression. To evaluate the effectiveness of the three fuzzy regression methods, 6 validation tests were conducted. Tables 2 show the results of the validation tests yielded by the developed models for the epoxy dispensing process. From the table, it can be found that the smallest errors can be obtained by the proposed evolutionary fuzzy regression rather than the other two methods in all the validation tests of both models of the

encapsulation weight. In addition, the mean errors and variances of errors obtained by the evolutionary fuzzy regression are the smallest.

IV. CONCLUSION

In this paper, an evolutionary fuzzy regression approach which integrates the mechanisms of genetic programming and fuzzy regression is proposed for modeling manufacturing processes. The proposed evolutionary fuzzy regression intends to compensate the limitation of the traditional fuzzy regression which can only address linearity in manufacturing systems but cannot address nonlinearity. Apart from this, the proposed evolutionary fuzzy regression approach can also be used to detect outliers from data sets such that models with better capability of prediction can be developed. To evaluate the effectiveness of the proposed evolutionary fuzzy regression approach, the evolutionary fuzzy regression, FR-Tanaka and FR-Peters were all applied to model the epoxy-dispensing process in which outliers exist in the data sets. Modeling results based on the three approaches were compared. The results indicate that the smallest prediction errors and errors in variance can be achieved by evolutionary fuzzy regression than by the commonly used fuzzy regression methods, FR-Peters, and FR-Tanaka. The achievement of better results can be explained by the introduction of nonlinear terms in the manufacturing process models developed based on evolutionary fuzzy regression, and the exclusion of outliers.

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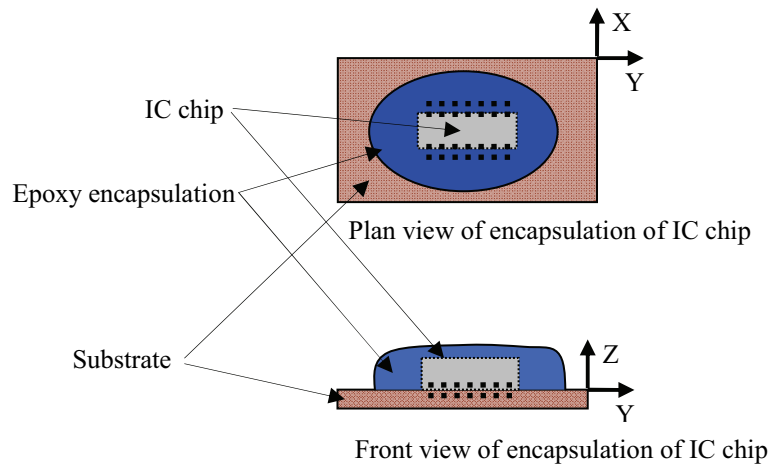


Figure 1 Encapsulation of IC chip

Table 1 Validation tests of the models of the encapsulation weight

Data set	Actual value	FR-Tanaka		FR-Peters		GP-FR	
		Predicted value	Abs error (%)	Predicted value	Abs error (%)	Predicted value	Abs error (%)
9	0.60	0.46	22.93	0.37	38.79	0.60	0.16
13	0.47	0.40	13.24	0.34	27.14	0.46	0.64
21	0.05	0.14	166.56	0.11	110.87	0.05	6.39
32	0.07	0.16	142.67	0.13	92.52	0.07	12.92
42	0.38	0.42	8.50	0.33	13.98	0.39	2.62
52	0.50	0.44	11.52	0.35	29.79	0.50	0.20
Mean relative error (%)		60.90		52.18		3.82	
Variance of relative errors (%)		5.35×10^3		1.57×10^3		25.50	

Table 2 Data sets for the epoxy dispensing system

Experimental data	Pressure (Mpa)	Dispensing time (sec)	Diameter of nozzle (mm)	Weight of epoxy (g)
1	0.5	5	1.19	0.278
2	0.4	9	1.19	0.383
3	0.6	7	0.61	0.025
4	0.5	5	0.61	0.015
5	0.4	9	0.61	0.023
6	0.5	9	0.84	0.075
7	0.6	5	0.84	0.045
8	0.4	7	0.84	0.05
9	0.5	9	0.61	0.022
10	0.6	5	0.61	0.019
11	0.4	5	0.84	0.033
12	0.5	7	0.84	0.066
13	0.6	9	0.84	0.096
14	0.4	5	1.19	0.215
15	0.5	7	1.19	0.405
16	0.6	9	1.19	0.612
17	0.6	7	0.84	0.075
18	0.5	5	0.84	0.046
19	0.4	9	0.84	0.06
20	0.6	7	1.19	0.4845
21	0.6	5	1.19	0.341
22	0.4	7	1.19	0.31
23	0.4	5	0.61	0.014
24	0.5	7	0.61	0.018
25	0.6	9	0.61	0.027
26	0.4	7	0.84	0.051
27	0.5	9	0.61	0.029
28	0.6	5	0.61	0.021
29	0.4	7	0.61	0.02
30	0.5	9	1.19	0.492
31	0.5	5	0.84	0.042
32	0.4	9	0.84	0.062
33	0.6	7	1.19	0.466
34	0.5	5	1.19	0.274
35	0.4	9	1.19	0.381
36	0.6	7	0.61	0.022
37	0.5	5	0.61	0.017
38	0.4	9	0.61	0.017
39	0.5	9	0.84	0.083
40	0.6	5	0.84	0.059
41	0.4	5	0.61	0.0075
42	0.5	7	0.61	0.022
43	0.6	9	0.61	0.027
44	0.4	5	0.84	0.038
45	0.5	7	0.84	0.064
46	0.6	9	0.84	0.1
47	0.4	5	1.19	0.214
48	0.5	7	1.19	0.381
49	0.6	9	1.19	0.602
50	0.6	7	0.84	0.078
51	0.6	5	1.19	0.337
52	0.4	7	1.19	0.298
53	0.4	7	0.61	0.018
54	0.5	9	1.19	0.497