

Mixed H_2/H_∞ Optimization with Discrete Smith Predictor for Fuzzy Decentralized Control of Nonlinear Interconnected Discrete Dynamic Systems with Large Delay

Name: Chih-Lyang Hwang, Senior Member, IEEE; e-mail: chih-lyang_hwang@hotmail.com
Department of Electrical Engineering of National Taiwan University of Science and Technology, Taiwan, R.O.C.

Abstract – Each subsystem of a nonlinear interconnected discrete dynamic delayed system is approximated by a weighted combination of L pulse transfer function delayed systems (PTFDS). To deal with large time-delay of the controlled system with acceptable performance, a two-degree-of-freedom control is designed. At beginning, the H_2 – norm of the difference between the transfer function of a reference model and the closed-loop transfer function of the j th PTFDS of subsystem i is minimized to obtain a suitable frequency response. In addition, the H_∞ – norm of the weighted sensitivity function between the output disturbance and its corresponding output of the j th PTFDS is simultaneously minimized to reduce its effect. In summary, the proposed control not only accomplishes the above-mentioned optimizations but also attenuates the effect of time-delay. Furthermore the comparisons between with and without discrete Smith predictor are presented. The main contributions of this paper are three folds. First, the large nominal time-delay is compensated by a discrete Smith predictor. Secondly, the proposed control can achieve the suitable response of the closed-loop system with the attenuation of output disturbance and time-varying delay. Thirdly, an observer is not needed and the discrete controller is more suitable for the implementation.

Keywords: Nonlinear interconnected discrete dynamic delayed system, Decentralized control, Fuzzy linear model, H_2 – optimization, H_∞ – optimization, Discrete Smith predictor.

I. INTRODUCTION

Time-delay or dead time is frequently encountered in many dynamic systems. The delay of information transmission between different subsystems, such as engine idle speed control [1], damping control in power system [2], playout or monitoring [3], an internet-based control [4], [6] and the image processing for the dynamic target tracking of visual servo system [7], teleoperation [8]-[10], results in a time-delay system. By implementing feedback control in (especial for large) time-delay system it always results in certain restrictions on achievable system bandwidth and the maximum available gain [1]-[10]. Based on the nominal delay of each subsystem, in this paper the reduced effect of time-varying delay is achieved by Smith predictor [9]-[12].

In the past three decades, the performance properties of interconnected systems have been widely studied [12]. However, the control performance of nonlinear interconnected systems has yet received sufficient attention. Due to complicated physical configuration and high dimensionality

of interconnected systems, the implementation of these systems with centralized control is neither economically plausible nor technically possible. In centralized control scheme, there are drawbacks such as the complexities in the design and debugging process. These in turn require scheduling of data collection and data storage etc. As a result, decentralized control scheme is considered a more favorable alternative for interconnected systems. However due to the inherent nonlinear interconnections among subsystems and it does not exist any efficient decentralized control scheme in dealing with interconnected nonlinear systems (e.g., the attenuation of uncertainty) [12], it needs to resort to new control scheme for such interconnected nonlinear system.

Recently, the use of fuzzy control for nonlinear systems has drawn many interests. Many successful applications have been reported in the literatures (e.g., [13], [14]). In addition, fuzzy T-S model with delayed control systems have been discussed (e.g., [15], [16]). In many of the literatures on delayed control systems, N linear (or bilinear) continuous-time (or discrete-time) state-space or transfer function systems for different fuzzy sets are considered. Under these circumstances, an alternative method is considered by using different pulse transfer functions with the same time delay to model the consequent parts of every subsystem. The controller design is therefore directed, simplified and more implementable. In addition, certain work does not attenuate the effect of time-delay. Facing these challenges, in this paper, we propose a controller design via mixed H_2/H_∞ optimization with discrete Smith predictor to obtain a robust and desired system performance.

Mixed H_2/H_∞ control problem has become a popular research topic in recent years [12], [17]. The response shaping based on H_2 – optimization to reduce the sluggish and oscillatory effects is often adopted [18]. On the other hand, H_∞ – optimization is usually used as the robust control methodology for diminishing the effect of disturbance [5], [6], [17]. As a result, the mixed H_2/H_∞ control problem is developed to robust system stability and to improve system performance [12]. Differences between this paper and previous papers [12] and [17] are described as follows: The paper in [17] did not discuss the time-delay property and its compensation; the paper in [12] discussed the controller design in the continuous time system and did not compare the

controller with and without Smith predictor.

For an easy implementation of the proposed controller, a discrete design ([17], [19]) is considered. At the outset, a specific pulse transfer function system (PTFDS) with respect to a specific operating point is obtained by implementing the least-square parameter estimation technique. After that a nominal and large time-delay is also estimated. To enhance system performance a two-degree-of-freedom controller with discrete Smith predictor is employed to each PTFDS. Subsequently, the H_2 - norm of the difference between an assigned reference model and the closed-loop transfer function of the j th PTFDS of subsystem i is minimized to attain a suitable frequency response without causing any oscillating and sluggish effects. However, the system performance may be deteriorated or even unstable due to the output disturbance caused by the interconnections among subsystems, the interactions from other PTFDSs, the error from fuzzy modeling, and/or the nonlinear time-varying uncertainties. To prevent this, the H_∞ - norm of the weighted sensitivity of the j th PTFDS between the output disturbance and its output is minimized to reduce its effect. Moreover, a specific weighted function can be selected to reject certain mode of output disturbance or to attenuate its effect in the system frequency response. In this operation it does not need to solve the Diophantine equation; it has the computational advantage especially for low-order system.

II. MATHEMATICAL PRELIMINARIES

A polynomial $\in \mathfrak{R}(z)$ is defined as $A(d) = a_{n_a} d^{n_a} + a_{n_a-1} d^{n_a-1} + \dots + a_0$, where a_i for $i=0,1,\dots,n_a$ denotes bounded coefficients, n_a stands for the system degree (i.e., if $a_{n_a} \neq 0$, $\deg\{A(d)\} = n_a$), and $d = z^{-1}$ denotes the backward shift operator (i.e., $z^{-1}y(k) \equiv y(k-1)$) or a complex variable in the z -transform. The z -transform of a discrete-time signal $y(k)$ is denoted by $Y(z^{-1}) = Z\{y(k)\}$. If $a_0 = 1$ it is called a monotonic polynomial. The notations of $A_+(d)$ and $A_-(d)$ denote respectively the stable and unstable part of $A(d)$. The polynomial $A_+(d)$ is assumed to be monotonic to obtain a unique factorization. Define $\|G(d)\|_2 = \|G(e^{-jw})\|_2 = \left[\int_{-\infty}^{\infty} |G(e^{-jw})|^2 dw / 2\pi \right]^{1/2}$, where $G \in H_2$, the set H_2 consists of all analytic functions G in $|z| \geq 1$. A rational function $G(d)$ in H_2 is called inner if $|G(e^{-jw})| = 1$ for almost all w , and it is outer if it has no zeros in $|z| \geq 1$. For any rational functions $G(d)$ and $H(d)$ in H_2 , if $G(d)$ is inner function and $\|G(d)H(d)\|_2$ is well defined, then $\|G(d)H(d)\|_2 = \|H(d)\|_2$. When a rational function multiplies by an inner function it preserves the value of the norm [12], [17]. The rational

function $A_-(d)/\tilde{A}_-(d)$ is a stable, causal, and all-pass operator, i.e., $|A_-(e^{-jw})/\tilde{A}_-(e^{-jw})| = 1$ for all w , and $\tilde{A}_-(d) = d^{n_a} A_-(d^{-1})$. The notation $\|A(e^{-jw})\|_\infty = \text{ess. sup}_w |A(e^{-jw})|$ is defined. The symbol $A_i^j(d)$ denotes a polynomial with bounded coefficients, its upper subscript j and lower subscript i respectively represent the polynomial of the j th PTFDS of subsystem i . The following lemma is given to explain the result of the minimax optimization for the j th PTFDS of subsystem i .

Lemma 1 [12], [17]: (i) The optimal $C_{z_i}^j = [W_i^j(d)V_i^j(d)]^*$, which minimizes $\|W_i^j(d)V_i^j(d)\|_\infty$, is of an all-pass form:

$$[W_i^j(d)V_i^j(d)]^* = \begin{cases} \rho_i^j \tilde{\Phi}_i^j(d)/\Phi_i^j(d), & \text{if } n_{b_{i-1}}^j = n_{\phi_i}^j + 1 \geq 1 \\ 0, & \text{if } n_{b_{i-1}}^j = 0 \end{cases}$$

where the polynomial $\Phi_i^j(d)$ is monic and stable.

(ii) The constants ρ_i^j and ϕ_i^j ($1 \leq m \leq n_{\phi_i}^j$) are real and are uniquely determined by the interpolation constraints (29) as described later. Furthermore, the minimized $\|W_i^j(d)V_i^j(d)\|_\infty$ is given by

$$\min \|W_i^j(d)V_i^j(d)\|_\infty = \left\| [W_i^j(d)V_i^j(d)]^* \right\|_\infty = |\rho_i^j|.$$

III. PROBLEM FORMULATION

Consider the following nonlinear interconnected discrete dynamic delayed system (NIDDDS) Σ that contains N subsystems Σ_i for $i=1,2,\dots,N$:

$$y_i(k) = f_{ii}[\varphi_i(k-\tau_i(k))] + \sum_{j=1, j \neq i}^N f_{ij}[\varphi_j(k-\tau_j(k))], \quad (1)$$

where $\varphi_i(k) = [y_i(k-1) \dots y_i(k-n) \ u_i(k-1) \dots u_i(k-m)]$; $y_i(k)$ and $u_i(k)$ denote the system output and the system input of the i th subsystem at time k , respectively; are the; $f_{ii}[\varphi_i(k-\tau_i(k))]$ denotes the unknown but smooth nonlinear delay function of the i th subsystem; $\tau_i(k) \geq 0$ is the time-varying delay of the i th subsystem; $f_{ij}[\varphi_j(t-\tau_j(k))]$, $j \neq i$, $i, j=1,2,\dots,N$, represent the smooth but unknown interconnections of the i th subsystem Σ_i , brought about by other subsystems Σ_m , where $m=1,\dots,N$, and $m \neq i$.

Based on the approximation of the fuzzy linear model [12], a fuzzy dynamic model to represent the local linear input/output relations of the i th subsystem (see Fig. 1) is described by the fuzzy IF-THEN rules. The j th rule of the fuzzy dynamic model for the i th subsystem Σ_i with uncertainties is expressed in the following form:

System Rule j: IF $z_{i_1}(k)$ is $M_{i_1}^j$... and $z_{i_{m_i}}(k)$ is $M_{i_{m_i}}^j$ THEN

$$y_i(k) = d^{\bar{\tau}_i} B_i^j(d) u_i(k) / A_i^j(d) + \sum_{m=1, m \neq i}^N d^{\tau_{im}(k)} B_{im}^j(d) u_m(k) / A_{im}^j(d) + \sum_{m=1, m \neq i}^N \Delta_{im}^j(u_m(k), k) + w_i(k), \quad (2)$$

where $j = 1, 2, \dots, L$, $i = 1, 2, \dots, N$; $\bar{m}_i \leq n_{a_i}^j + n_{b_i}^j + 1$, $A_i^j(1) = 1$; L is the number of IF-THEN rules; $z_{l_i}(k), \dots, z_{\bar{m}_i}(k)$ are premise variables, which are functions of $y_i(k-1), \dots, y_i(k-n)$, $u_i(k-1), \dots, u_i(k-m)$; $\bar{\tau}_i$ is a positive integer denoting the nominal delay of subsystem i (in this paper, it is generally several hundred); $y_i(k)$ also denotes the output from the j th IF-THEN rules; $d^{\bar{\tau}_i} B_i^j(d) u_i(k) / A_i^j(d)$ is the j th nominal and known PTFDS of subsystem i ; $\sum_{m=1, m \neq i}^N d^{\tau_{im}(k)} B_{im}^j(d) u_m(d) / A_{im}^j(d)$ denotes the linear coupling terms; $\sum_{m=1, m \neq i}^N \Delta_{im}^j(u_m(k), k)$ denotes the nonlinear time-varying uncertainties, that are relatively bounded by $c_0 + \sum_{i=1}^N c_i \|u_i(k)\|_2$, where $c_i \geq 0$, $i = 0, 1, \dots, N$, and caused by the interconnections of other subsystems and the interactions from other PTFDSs; and $w_i(k)$ denotes the approximation error of the fuzzy model of the i th subsystem, e.g., time-varying delay, parameter variations. Assume that $A_i^j(d)$ and $B_i^j(d)$ for $i=1, 2, \dots, N$, $j=1, 2, \dots, L$ are coprime. One of main advantages of this form is that no state estimator is required; then the control design becomes simple and effective.

The output of the overall fuzzy system of the i th subsystem is inferred as follows:

$$y_i(k) = \sum_{j=1}^L \mu_i^j(k) \left\{ d^{\bar{\tau}_i} B_i^j(d) u_i(k) / A_i^j(d) + \tilde{d}_{o_i}^j(k) \right\} \quad (3)$$

where

$$\tilde{d}_{o_i}^j(k) = \sum_{m=1, m \neq i}^N \left\{ d^{\tau_{im}(k)} B_{im}^j(d) u_m(k) / A_{im}^j(d) + \Delta_{im}^j(u_m(k), k) \right\} + w_i(k) \quad (4)$$

$$\mu_i^j(k) = h_i^j(k) / \sum_{j=1}^L h_i^j(k) \text{ and } h_i^j(k) = \prod_{m=1}^{\bar{m}_i} M_{i_m}^k(z_{i_m}(k)). \quad (5)$$

It is assumed that

$$h_i^j(k) \geq 0, \sum_{j=1}^L \mu_i^j(k) = 1 \quad (6)$$

for $i = 1, 2, \dots, N$, $j = 1, 2, \dots, L$, and $\forall k$.

It is also assumed that the fuzzy controller shares the same fuzzy sets with the fuzzy system (2).

Control Rule j : IF $z_{l_i}(k)$ is $M_{l_i}^j$... and $z_{\bar{m}_i}(k)$ is $M_{\bar{m}_i}^j$ THEN

$$u_i(k) = -S_i^j(d) y(k) / R_i^j(d) + T_i^j(d) r_i(k) / R_i^j(d) - S_i^j(d) B_i^j(d) (1 - d^{\bar{\tau}_i}) u_i(k) / (R_i^j(d) A_i^j(d)) \quad (7)$$

where the polynomials $R_i^j(d)$, $S_i^j(d)$ and $T_i^j(d)$ are determined to achieve the control of the j th PTFDS of subsystem i , and the last term in (7) denotes the Smith predictor to deal with the nominal system delay. The overall fuzzy control of the i th subsystem is then inferred as follows:

$$u_i(k) = \sum_{j=1}^L \mu_i^j(k) \left\{ -S_i^j(d) y(k) / R_i^j(d) + T_i^j(d) r_i(k) / R_i^j(d) - S_i^j(d) B_i^j(d) (1 - d^{\bar{\tau}_i}) u_i(k) / (R_i^j(d) A_i^j(d)) \right\}. \quad (8)$$

The nominal closed-loop characteristic polynomial of the j th PTFDS of subsystem i is also described as follows:

$$A_{c_i}^j(d) = A_i^j(d) R_i^j(d) + B_i^j(d) S_i^j(d). \quad (9)$$

Hence, the closed-loop output response of the i th subsystem, i.e., $y_i(k)$, is derived from (3) and (7); i.e.,

$$\begin{aligned} y_i(k) &= \sum_{j=1}^L \mu_i^j(k) \left\{ d^{\bar{\tau}_i} B_i^j(d) [u_i^j(k) + u_i(k) - u_i^j(k)] / A_i^j(d) + \hat{d}_{o_i}^j(k) \right\} \\ &= \sum_{j=1}^L \mu_i^j(k) \left\{ d^{\bar{\tau}_i} B_i^j(d) [-S_i^j(d) y_i(k) + T_i^j(d) r_i(k)] / [A_i^j(d) R_i^j(d)] \right. \\ &\quad \left. + \psi_i^j(k) + \hat{d}_{o_i}^j(k) \right\} \\ &= \sum_{j=1}^L \mu_i^j(k) \left\{ d^{\bar{\tau}_i} B_i^j(d) [-S_i^j(d) y_i(k) + T_i^j(d) r_i(k)] / [A_i^j(d) R_i^j(d)] \right. \\ &\quad \left. + d_{o_i}^j(k) \right\} \end{aligned} \quad (10a)$$

where $u_i^j(k)$ denotes the control input of the j th PTFDS of subsystem i (i.e., (7)),

$$\psi_i^j(k) = d^{\bar{\tau}_i} B_i^j(d) [u_i(k) - u_i^j(k)] / A_i^j(d),$$

$$d_{o_i}^j(k) = \hat{d}_{o_i}^j(k) + \psi_i^j(k). \quad (10b)$$

The signal $\psi_i^j(k)$ denotes the interaction on the j th PTFDS of subsystem i resulting from other PTFDSs, including the error between nominal delay and time-varying delay. In summary, $d_{o_i}^j(k)$ denotes the output disturbance of the j th PTFDS of subsystem i caused by the interconnections stemming from other subsystems, the interactions resulting from other PTFDSs, the approximation error of fuzzy-model, and the nonlinear time-varying uncertainties. In general, the corresponding output disturbance is not small and even including various frequency components. In this situation, an effective controller is required to achieve satisfactory performance.

After some mathematical manipulations of (10a) and using (7), the following is achieved.

$$\sum_{j=1}^L \frac{\mu_i^j(k) A_{c_i}^j(d)}{A_i^j(d) R_i^j(d)} \left\{ y_i(k) - \frac{d^{\bar{\tau}_i} B_i^j(d) T_i^j(d)}{A_{c_i}^j(d)} r_i(k) - \frac{A_i^j(d) R_i^j(d)}{A_{c_i}^j(d)} d_{o_i}^j(k) \right\} = 0. \quad (11)$$

Hence, a sufficient condition for the validity of (11) is the term in the braces $\{\cdot\}$ of (11) to be equal to zero; i.e.,

$$y_i(k) = d^{\bar{\tau}_i} B_i^j(d) L_i^j(d) r_i(k) - V_i^j(d) d_{o_i}^j(k) \quad (12a)$$

where

$$L_i^j(d) = B_i^j(d) T_i^j(d) / A_{c_i}^j(d),$$

$$V_i^j(d) = A_i^j(d) R_i^j(d) / A_{c_i}^j(d). \quad (12b)$$

The aforementioned output disturbance is supposed to be bounded as follows (see (4), (10b)):

$$\|d_{o_i}^j(k)\|_2 \leq \alpha_{o_i}^j + \sum_{m=1}^N \alpha_{m_i}^j \|u_i(k)\|_2 \quad (13)$$

where $\alpha_{o_i}^j$ and $\alpha_{m_i}^j$, $i, j, m = 1, 2, \dots, N$ are bounded [12].

Assume that the input/output relationship between $r_i(t)$ and $\bar{y}_i^j(t)$, for a stable reference model of the j th PTFDS of subsystem i , is as follows:

$$\bar{y}_i^j(k) = d^{\bar{\tau}_i} G_i^j(d) r_i(k) / F_i^j(d) \quad (14)$$

where $G_i^j(d)$ and $F_i^j(d)$ for $i = 1, 2, \dots, N, j = 1, 2, \dots, L$ are coprime, and $F_i^j(d)$ is a stable monic polynomial with degree $n_{f_i}^j$. The purpose of using a reference model is to shape the response of the closed-loop of the j th PTFDS of subsystem i . For tracking a set-point, $G_i^j(1)/F_i^j(1) = 1$ is assigned.

For synthesizing the problem, two cost functions in the H_2 - norm and H_∞ - norm spaces are defined as follows [12], [17]:

$$C_{l_i}^j = \|d^{\bar{\tau}_i} [G_i^j(d)/F_i^j(d) - B_{-i}^j(d)L_i^j(d)]\|_2 \quad (15)$$

$$\text{and } C_{2_i}^j = \|W_i^j(d)V_i^j(d)\|_\infty$$

For $d_{o_i}^j(k) = 0$, the optimal model matching to the cost function $C_{l_i}^j$ is addressed as follows: Based on the norm-preserving property of the inner function (i.e., $\|e^{-jw\bar{\tau}_i}\| = 1 \forall w$), from eqn. (15) $C_{l_i}^j$ is reduced to the following equation:

$$C_{l_i}^j = \|G_i^j(d)/F_i^j(d) - B_{-i}^j(d)L_i^j(d)\|_2. \quad (16)$$

At beginning, the polynomial $B_{-i}^j(d)$ is factorized as

$$B_{-i}^j(d) = B_{-i}^j(d^{-1}) \cdot \tilde{B}_{-i}^j(d) / B_{-i}^j(d^{-1}) \quad (17)$$

where $B_{-i}^j(d)/\tilde{B}_{-i}^j(d)$ is an inner function and the remaining $\tilde{B}_{-i}^j(d)$ is an outer function. Then

$$\begin{aligned} C_{l_i}^j &= \|G_i^j(d)/F_i^j(d) - L_i^j(d)\tilde{B}_{-i}^j(d)B_{-i}^j(d)/\tilde{B}_{-i}^j(d)\|_2 \\ &= \|B_{-i}^j(d)/\tilde{B}_{-i}^j(d) [\tilde{B}_{-i}^j(d)G_i^j(d)/(F_i^j(d)B_{-i}^j(d)) - L_i^j(d)\tilde{B}_{-i}^j(d)]\|_2. \end{aligned} \quad (18)$$

Similarly, based on the norm-preserving property of the inner function, (18) is simplified as follows:

$$(C_{l_i}^j)^2 = \|\tilde{B}_{-i}^j(d)G_i^j(d)/(F_i^j(d)B_{-i}^j(d)) - L_i^j(d)\tilde{B}_{-i}^j(d)\|_2^2. \quad (19)$$

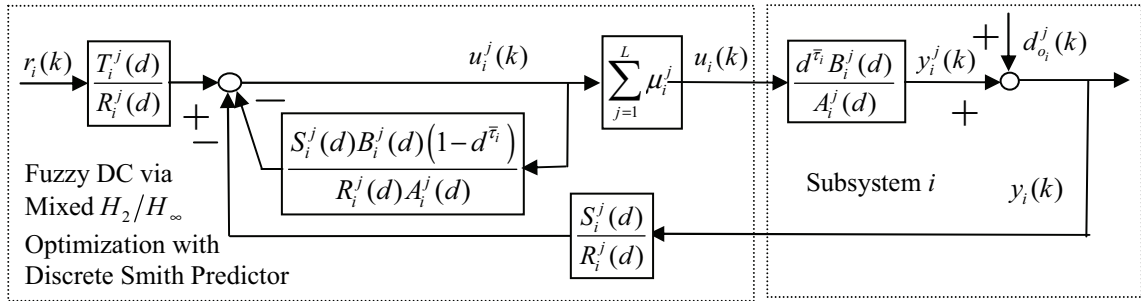


Fig.1. The overall control block diagram.

where the weighted function $W_i^j(d)$ denotes a suitable rational function for the j th PTFDS of subsystem i .

The objectives of this paper are expressed as follows (cf. Fig. 1): Search for a control function (i.e., the polynomials $R_i^j(d)$, $S_i^j(d)$, and $T_i^j(d)$) so as to simultaneously satisfy the following two requirements: (i) For $d_{o_i}^j(k) = 0$, the cost function $C_{l_i}^j$ is minimized, (ii) The cost function $C_{2_i}^j$ is simultaneously minimized.

IV. CONTROLLER DESIGN USING MIXED H_2/H_∞ OPTIMIZATION

There are two processes in the design of the decentralized fuzzy controller via mixed H_2/H_∞ optimization for the NIDDDs (1).

A. Minimization of $C_{l_i}^j$

Decomposing the first term yields

$$\begin{aligned} &B_{-i}^j(d^{-1})G_i^j(d)/(F_i^j(d)B_{-i}^j(d)) \\ &= \bar{F}_i^j(d)/F_i^j(d) + \bar{B}_i^j(d)/B_{-i}^j(d) \end{aligned} \quad (20)$$

Rewrite (20) as

$$\tilde{B}_{-i}^j(d)G_i^j(d) = B_{-i}^j(d)\bar{F}_i^j(d) + F_i^j(d)\bar{B}_i^j(d)$$

where $\bar{F}_i^j(d)$ and $\bar{B}_i^j(d)$ are some polynomials to be determined, which are interrelated respectively with the control polynomial $T_i^j(d)$ and the optimum value of $C_{l_i}^j$.

$$\text{or } \tilde{B}_{-i}^j(d)G_i^j(d) - B_{-i}^j(d)\bar{F}_i^j(d) \Big|_{d=z_1^j, \dots, z_{n_{f_i}}^j} = 0 \quad (21)$$

where $z_1^j, \dots, z_{n_{f_i}}^j$ are the zeros of $F_i^j(d) = 0$. Then the polynomial $\bar{B}_i^j(d)$ can be obtained from (21) by long division. Substituting (20) into (19) gives

$$(C_{l_i}^j)^2 = \|\bar{F}_i^j(d)/F_i^j(d) + \bar{B}_i^j(d)/B_{-i}^j(d) - L_i^j(d)\tilde{B}_{-i}^j(d)\|_2^2 \quad (22)$$

where the second term in the right hand side of (22) is

unstable, and the remaining terms are stable. By orthogonality, it follows that

$$\begin{aligned} (C_{l_i}^j)^2 &= \left\| \bar{F}_i^j(d)/F_i^j(d) - L_i^j(d)\tilde{B}_{-i}^j(d) \right\|^2 \\ &+ \left\| \bar{B}_{-i}^j(d)/B_{-i}^j(d) \right\|_2^2 \geq \left\| \bar{B}_{-i}^j(d)/B_{-i}^j(d) \right\|_2^2. \end{aligned} \quad (23)$$

Hence, the optimal $L_i^j(d)^*$ and $C_{l_i}^{j*}$ are achieved as follows:

$$L_i^j(d)^* = \bar{F}_i^j(d) / (F_i^j(d)\tilde{B}_{-i}^j(d)) \quad (24)$$

$$C_{l_i}^{j*} = \left\| \bar{B}_{-i}^j(d)/B_{-i}^j(d) \right\|_2. \quad (25)$$

Comparing (24) and (12b) and using (9) it gives

$$A_i^j(d)R_i^j(d) + B_i^j(d)S_i^j(d) = B_{+i}^j(d)\tilde{B}_{-i}^j(d)F_i^j(d)X_i^j(d) \quad (26)$$

$$T_i^j(d) = \bar{F}_i^j(d)X_i^j(d) \quad (27)$$

where $X_i^j(d)$ is a stable polynomial. If the reference model contains the unstable zeros of the nominal system, i.e., $G_i^j(s) = B_{-i}^j(d)\bar{G}_i^j(d)$, for some polynomials $\bar{G}_i^j(d)$, then the perfect model matching (i.e., $C_{l_i}^{j*} = 0$) can be obtained. Then from (20) it has,

$$\bar{F}_i^j(d) = B_{-i}^j(d^{-1})\bar{G}_i^k(s) \text{ and } \bar{B}_{-i}^j(d) = 0. \quad (28)$$

B. Minimization of $C_{2_i}^j$

In this subsection, the task is to find the polynomials $R_i^j(d)$ and $S_i^j(d)$ such that the optimal $C_{2_i}^j$ is obtained. From previous studies [12], [17], the optimal $\left\| W_i^j(d)V_i^j(d) \right\|_\infty$ must satisfy the following interpolation constraints:

$$V_i^j(p_i^j) = 0, j=1,2,\dots,n_{a_i}^j, \quad 1-V_i^j(z_i^j) = 0, j=1,2,\dots,n_{b_{-i}}^j \quad (29)$$

where $p_i^j (1 \leq j \leq n_{a_i}^j)$ and $z_i^j (1 \leq j \leq n_{b_{-i}}^j)$ denote the zeros of $A_{-i}^j(d)$ and $B_{-i}^j(d)$, respectively.

Remark 1: If the inner loop of Fig. 1 disappears (e.g., [17]), then the characteristic polynomial of the closed-loop system in (9) is replaced by the following equation:

$$A_{c_i}^j(d) = A_i^j(d)R_i^j(d) + d^{\bar{\tau}_i}B_i^j(d)S_i^j(d). \quad (30)$$

Then the second interpolation constraint of (19) is modified as follows:

$$1-V_i^j(z_i^j) = 0, j=1,2,\dots,n_{b_{-i}}^j + \bar{\tau}_i \quad (31)$$

where $z_i^j (1 \leq j \leq n_{b_{-i}}^j + \bar{\tau}_i)$ denote the zeros of $d^{\bar{\tau}_i}B_{-i}^j(d)$. The degree of $\Phi_i^j(d)$ becomes $n_{\phi_i}^j = n_{b_{-i}}^j - 1 + \bar{\tau}_i$, which is large especially for large time-delay system (e.g., $\bar{\tau}_i$ equals several hundred for the internet-based piezoelectric actuator driven table system). In summary, the degrees of control polynomials increase and the effect of time-delay is dominant for the control without using Smith predictor.

Based on the result of Lemma 1 and the constraint (29), the following equation is achieved

$$V_i^j(d) = \rho_i^j W_{d_i}^j(d) A_{-i}^j(d) \tilde{\Phi}_i^j(d) / [W_{n_i}^j(d) \tilde{A}_{-i}^j(d) \Phi_i^j(d)] \quad (32)$$

where $W_i^j(d) = W_{n_i}^j(d)/W_{d_i}^j(d)$, $W_{n_i}^j(d)$ is a stable polynomial, and $W_{d_i}^j(d)$ contains the zeros in the region $|z| \geq 1$ for rejecting the corresponding output disturbance. In addition, the constraint (29) gives the following result:

$$W_{n_i}^j(d) \tilde{A}_{-i}^j(d) \Phi_i^j(d) - \rho_i^j W_{d_i}^j(d) A_{-i}^j(d) \tilde{\Phi}_i^j(d) = B_{-i}^j(d) Q_i^j(d) \quad (33)$$

where $n_{\phi_i}^j = n_{b_{-i}}^j - 1$ and $n_{q_i}^j = n_{w_{n_i}}^j + n_{a_{-i}}^j - 1$. Or, we can rewrite (33) as (34)

$$W_{n_i}^j(z_i^j) \tilde{A}_{-i}^j(z_i^j) \Phi_i^j(z_i^j) - \rho_i^j W_{d_i}^j(z_i^j) A_{-i}^j(z_i^j) \tilde{\Phi}_i^j(z_i^j) = 0. \quad (34)$$

By the solutions of ρ_i^j and $\Phi_i^j(d)$ for (32), the following equations are obtained from (12), (33), and (34)

$$A_{c_i}^j(d) = A_{+i}^j(d) W_{n_i}^j(d) \tilde{A}_{-i}^j(d) \Phi_i^j(d) B_{+i}^j(d) \Gamma_i^j(d) \quad (35)$$

$$R_i^j(d) = \rho_i^j W_{d_i}^j(d) \tilde{\Phi}_i^j(d) B_{+i}^j(d) \Gamma_i^j(d). \quad (36)$$

By comparing (35) and (26) and with the fact (9), one has

$$\Gamma_i^j(d) = \tilde{B}_{-i}^j(d) F_i^j(d) \Omega_i^j(d) \quad (37)$$

$$X_i^j(d) = A_{+i}^j(d) W_{n_i}^j(d) \tilde{A}_{-i}^j(d) \Phi_i^j(d) \Omega_i^j(d) \quad (38)$$

where $\Omega_i^j(d)$ is a stable polynomial. Then from (35)-(38)

$$A_{c_i}^j(d) = B_{+i}^j(d) \tilde{B}_{-i}^j(d) F_i^j(d) A_{+i}^j(d) W_{n_i}^j(d) \tilde{A}_{-i}^j(d) \Phi_i^j(d) \Omega_i^j(d) \quad (39)$$

$$R_i^j(d) = \rho_i^j W_{d_i}^j(d) \tilde{\Phi}_i^j(d) B_{+i}^j(d) \tilde{B}_{-i}^j(d) F_i^j(d) \Omega_i^j(d). \quad (40)$$

Substituting (36), (37), and (32) into (9) yields

$$S_i^j(d) = A_{+i}^j(d) Q_i^j(d) \tilde{B}_{-i}^j(d) F_i^j(d) \Omega_i^j(d). \quad (41)$$

From (27) and (38), the following polynomial $T_i^j(d)$ is obtained:

$$T_i^j(d) = \bar{F}_i^j(d) A_{+i}^j(d) W_{n_i}^j(d) A_{-i}^j(d) \Phi_i^j(d) \Omega_i^j(d). \quad (42)$$

That is, the characteristic polynomial of the closed-loop and the control polynomials of the j th PTFDS of subsystem i for the fuzzy mixed H_2/H_∞ optimization in (15) are attained through eqns. (39)-(42).

The following theorem discusses the stability of the closed-loop interconnected system (1).

Theorem 1: Suppose the proposed controller (8) is applied to the NIDDDS (1) or an equivalent fuzzy model system (3). Suppose also the inequality (13) with the condition (43) is satisfied. Then the control system in Fig. 1 is L_2^N -stable.

$$\left(\Gamma_1^j \right)_{ii} = 1 - \alpha_i^j \left\| S_i^j(e^{-j\omega}) A_{-i}^j(e^{-j\omega}) / A_{c_i}^j(e^{-j\omega}) \right\|_2 > 0, \quad (43a)$$

for $i=1,2,\dots,N, j=1,2,\dots,L$

$$\Gamma_1^j(e^{-j\omega}) > 0 \quad (43b)$$

where $\left(\Gamma_1^j(e^{j\omega}) \right)_{mi} = -\alpha_i^j \left\| S_i^j(e^{-j\omega}) A_{-i}^j(e^{-j\omega}) / A_{c_i}^j(e^{-j\omega}) \right\|_2$
 $0 < (\Gamma_1^k)_{ii} \leq 1$ and $0 \leq -(\Gamma_1^j)_{ij} < 1, i \neq j$, for $i, j=1,2,\dots,N$.

Proof: It can be refer to the reference [12].

C. The Proposed Control Algorithm

The design procedure of the proposed fuzzy decentralized fuzzy controller via mixed H_2/H_∞ optimization with discrete Smith predictor is as follows:

Step 1: Factorize the nominal subsystem i as $A_i^j(d) = A_{+i}^j(d)A_{-i}^j(d)$ and $B_i^j(d) = B_{+i}^j(d)B_{-i}^j(d)$.

Step 2: Assign a suitable reference model $F_i^j(d)$ and $G_i^j(d)$ for the j th PTFDS of subsystem i .

Step 3: Choose an appropriate weighted function for the second cost function (15), i.e., $W_{d_i}^j(d)$ and $W_{n_i}^j(d)$.

Step 4: Based on (21), the polynomials $\bar{F}_i^j(d)$ and $\bar{B}_i^j(d)$ are obtained. Then the optimal $C_{l_i}^j = \|\bar{B}_i^j(d)/B_{-i}^j(d)\|_2$ is attained. If the matching condition is satisfied, then $\bar{F}_i^j(d) = \bar{B}_{-i}^j(d)\bar{G}_i^j(d)$ and $\bar{B}_i^j(d) = 0$.

Step 5: From (34), the scalar ρ_i^j and the polynomial $\Phi_i^j(d)$ are obtained. Then the optimal $C_{2_i}^j = \rho_i^j$ is attained. Based on the long division for (33), the polynomial $Q_i^j(d)$ is obtained.

Step 6: The characteristic polynomial of the closed-loop and the control polynomials of the j th PTFDS of subsystem i , i.e., $A_{c_i}^j(d)$, $\{R_i^j(d), S_i^j(d), T_i^j(d)\}$, for the fuzzy mixed H_2/H_∞ optimization in (15) are from (39)-(42).

VI. CONCLUSIONS

A decentralized control of nonlinear interconnected discrete dynamic systems with large delay using fuzzy mixed H_2/H_∞ optimization with discrete Smith predictor was developed. The past effort on the use of fuzzy T-S delayed control did not consider the robust tracking performance (e.g., the attenuation of uncertainty and delay, and the compensation of large delay). In this paper, a two degree-of-freedom optimization technique with discrete Smith predictor was employed to control each PTFDS. A suitable frequency response without the oscillating and sluggish phenomena was obtained. The effect of output disturbance caused by the interconnections coming from other subsystems, the approximation error of the i th subsystem, and the interactions resulting from other PTFDSs, was also minimized. Furthermore, the control implementation did not need to calculate the Diophantine equation. There were also computational advantages, especially for low-order system. No fuzzy observer was required; the proposed discrete controller was more suitable for the implementation. Further application to an internet-based piezoelectric actuator driven table system [17] will be progressed.

REFERENCES

- [1] C. Bengea, X. Li and R. A. DeCarlo, "Combined controller-observer design for uncertain time delay systems with application to engine idle speed control," *ASME J. Dyna. Syst. Meas. & Contr.*, vol. 126, pp. 772-780, Dec. 2004.
- [2] R. Majumder, B. Chaudhuri, B. C. Pal and Q.C. Zhong, "A unified Smith predictor approach for power system damping control design using remote signals," *IEEE Trans. Contr. Syst. Technol.*, vol. 13, no. 13, pp. 1063-1068, Nov. 2005.
- [3] M. K. Ranganathan and L. Kilmartin, "Neural and fuzzy computation techniques for playout delay adaptation in VoIP networks," *IEEE Trans. Neural Networks*, vol. 16, no. 5, pp. 1174-1194, Sep. 2005.
- [4] C. L. Hwang, L. J. Chang and Y. S. Yu, "Network-based fuzzy decentralized sliding-mode control for car-like mobile robots," *IEEE Trans. Ind. Electron.*, vol. 54, no. 1, pp. 574-585, Feb. 2007.
- [5] B. Jiang, Z. Mao and P. Shi, " H_∞ - filter design for a class of networked control system via T-S fuzzy model approach," *IEEE Trans. Fuzzy Syst.*, vol. 18, no. 1, pp. 201-208, Feb. 2010.
- [6] C. Huang, Y. Bai and X. Liu, "H-infinity state feedback control for networked cascade control systems with uncertain delay," *IEEE Trans. Ind. Inform.*, vol. 6, no. 1, pp. 62-72, Feb. 2010.
- [7] C. L. Hwang and C. Y. Shih, "A distributed active-vision network-space approach for trajectory tracking and obstacle avoidance of a car-like wheeled robot," *IEEE Trans. Ind. Electron.*, vol. 56, no. 3, pp. 846-855, Mar. 2009.
- [8] E. J. Rodríguez-Seda, D. Lee and M. W. Spong, "Experimental comparison study of control architectures for bilateral teleoperators," *IEEE Trans. Robotics*, vol. 25, no. 6, pp. 1304-1318, Dec. 2009.
- [9] C. L. Lai and P. L. Hsu, "Design the remote control system with the time-delay estimator and the adaptive Smith predictor," *IEEE Trans. Ind. Inform.*, vol. 6, no. 1, pp. 73-80, Feb. 2010.
- [10] K. Natori, T. Tsuji, K. Ohnishi, A. Hase and K. Jezernik, "Time-delay compensation by communication disturbance observer for bilateral teleoperation under time-varying delay," *IEEE Trans. Ind. Electron.*, vol. 57, no. 3, pp. 1050-1059, Mar. 2010.
- [11] M. Ono, H. Shibasaki, K. Matsumoto, K. Sasaki and Y. Ishida, "Discrete modified Smith predictor based on optimal control method for a plant with an integrator," *IEEE SMC2010*, pp. 630-635, 2010.
- [12] C. L. Hwang and C. C. Wong, "Fuzzy mixed H_2/H_∞ optimized design of decentralized control for nonlinear interconnected dynamic delay systems," *IEEE FUZZ2008*, Hong Kong, pp. 168-173, June 1st-6th, 2008.
- [13] R. J. Wai, M. A. Kuo and J. D. Lee, "Design of fuzzy adaptive sliding-mode control for nonlinear two-axis inverted pendulum servomechanism," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 5, pp. 1232-1244, Oct. 2008.
- [14] H. Zhang, C. Dang and C. Li, "Decentralized H_∞ filter design for discrete-time interconnected fuzzy systems," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 6, pp. 1428-1440, Dec. 2009.
- [15] B. Chen, X. P. Lin, S. C. Tong and C. Lin, "Observer-based stabilization of T-S fuzzy systems with input delay," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 3, pp. 652-663, Jun. 2008.
- [16] C. Peng, D. Yue, T.C. Yang and E. G. Tian, "On delay-dependent approach for robust stability and stabilization of T-S fuzzy systems with constant delay and uncertainties," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 5, pp. 1143-1156, Oct. 2009.
- [17] C. L. Hwang and S. Y. Han, "Fuzzy mixed H^2/H^∞ optimization-based decentralized model reference control and application to piezo-driven XY table systems," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 2, pp. 145-160, Apr. 2007.
- [18] J. Chen, S. Hara and G. Chen, "Best tracking and regulation performance under control energy constraint," *IEEE Trans. Automat. Contr.*, vol. 48, no. 8, pp. 1320-1336, Aug. 2003.
- [19] J. Dong, Y. Wang and G. H. Yang, "Output feedback fuzzy controller design with local nonlinear feedback laws for discrete-time nonlinear systems," *IEEE Trans. Syst. Man & Cybern., Pt. B*, vol. 40, no. 6, pp. 1447-1459, Dec. 2010.