

A SVD Approach to H_∞ Decentralized Static Output Feedback Fuzzy Control Design for Nonlinear Interconnected Systems

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Abstract—This study introduces H_∞ decentralized static output feedback fuzzy control design for nonlinear interconnected systems via T-S fuzzy models. In general, due to the interactions among subsystems, it is difficult to design an H_∞ decentralized output feedback controller for nonlinear interconnected systems. A singular value decomposition (SVD) method is proposed in this study to solve the H_∞ decentralized static output feedback fuzzy control problem. By the proposed SVD method, the problem of H_∞ decentralized static output feedback fuzzy control design for nonlinear interconnected systems is characterized in terms of solving an eigenvalue problem (EVP). This EVP can be easily solved by using a LMI-based optimization method. Finally, simulation example is given to illustrate the design procedure and robust performance of the proposed methods.

Keywords— H_∞ ; decentralized; static output feedback; SVD; LMI; EVP.

I. INTRODUCTION

Due to the physical configuration and high dimensionality of interconnected systems, a centralized control is neither economically feasible nor even necessary. Therefore, a decentralized control scheme is preferred in control design of the large-scale interconnected systems. In other words, the decentralized control scheme attempts to avoid difficulties in complexity of system design. In the past three decades, there are many applications of linear decentralized methodologies to spacecraft dynamics, power systems, industrial processes, transportation networks and others [1], [15]. On the other hand, there are few studies concerning with the stabilization control design for the interconnected nonlinear systems because of the complexity among them [24], [13], [14]. Due to the effects of nonlinear interconnection among subsystems, there is still no efficient way to deal with the decentralized control problem of nonlinear interconnected systems, especially for the output feedback control case.

In the past few years, there has been rapidly growing interest in decentralized fuzzy control for the nonlinear interconnected systems using T-S fuzzy model. However, most of them are state feedback cases [10], [9], [21], [22]. On the other hand, there are few studies concerned with the systematic synthesis of output feedback control design for nonlinear interconnected

systems using the T-S fuzzy models [18], [19]. The output feedback control design for nonlinear systems using the T-S fuzzy models is more general and complex than the state feedback one. When dealing with output feedback control design, either a dynamic output feedback (fuzzy observer-based) case or a static output feedback case is considered. For the dynamic output feedback case, a fuzzy observer is also involved as well as the fuzzy controller [20], [8]. In other words, for the fuzzy observer-based output feedback control design, a fuzzy observer should also be involved to estimate the system states using the output information and a fuzzy controller is constructed using the estimated states. For the static output feedback case [12], [3], [4], [5], [11], [6], [23], the fuzzy controller depends on the system outputs only without the complex structures of the fuzzy observer. In general, the control design of static output feedback is more straightforward but less flexible than that of dynamic output feedback and the structure of static output feedback controller is much simpler than that of dynamic output feedback controller. This study introduces H_∞ decentralized static output feedback fuzzy control design for nonlinear interconnected systems via T-S fuzzy models. A SVD method is proposed in this study to solve the H_∞ decentralized static output feedback fuzzy control design for nonlinear interconnected systems, which can be characterized in terms of solving an EVP for each subsystem. The EVP can be easily solved by using a genetic algorithm and a LMI-based optimization method. The SVD approach provides different aspect from the linear transformation approach [12], [11] and moreover the proposed SVD method is very simple and easy to follow.

The main contributions of this paper are stated as follows: (1) The H_∞ output feedback control problem for nonlinear interconnected systems is studied via a decentralized static output feedback fuzzy controller. (2) A SVD method is proposed to solve H_∞ decentralized static output feedback fuzzy control problem which can be characterized in terms of solving an EVP for each subsystem. By the proposed SVD method, the decentralized static fuzzy control gains for each subsystem can be obtained efficiently.

The paper is organized as follows. The system description is presented in Section II. In section III, the H_∞ decentralized static feedback fuzzy control design for the nonlinear inter-

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connected systems is introduced. In Section IV, simulation example is provided to demonstrate the design procedures. Finally, concluding remarks are made in Section V. In what follows, $M_{[m \times n]}$ denotes a $m \times n$ matrix and $I_{[n \times n]}$ and $0_{[m \times n]}$ denote a $n \times n$ identity matrix and a $m \times n$ zero matrix, respectively. If dimensions of matrices are not particularly defined, they are of appropriate and clear dimensions.

II. SYSTEM DESCRIPTION

A T-S fuzzy dynamic model [16] has been proposed to represent locally linear input/output relations for nonlinear systems. This fuzzy dynamic model is described by fuzzy If-Then rules and will be employed here to deal with the control design problem of a nonlinear interconnected system S which is composed of N subsystems S_i ($i = 1, \dots, N$). The k^{th} rule of the fuzzy model of subsystem S_i is proposed as the following form [10], [9], [21], [22], [18], [19]:

Plant Rule k :

$$\begin{aligned} \text{If} \quad & z_{i1}(t) \text{ is } F_{i1}^k \dots \text{and } z_{iq_i}(t) \text{ is } F_{iq_i}^k \\ & \dot{x}_i(t) = A_{ik}x_i(t) + B_{1ik}u_i(t) \\ \text{Then} \quad & + \sum_{j=1, j \neq i}^N [A_{ijk}x_j(t)] + B_{2ik}w_i(t) \\ & y_i(t) = C_i x_i(t) \text{ for } k = 1, \dots, L_i \end{aligned} \quad (1)$$

where $x_i(t) = [x_{i1}(t), \dots, x_{in_i}(t)]^T \in R^{n_i}$ denotes the vector of state, $u_i(t) = [u_{i1}(t), \dots, u_{im_i}(t)]^T \in R^{m_i}$ denotes the vector of control input, $w_i(t)$ denotes the vector of the external disturbance, $y_i(t) = [y_{i1}(t), \dots, y_{ip_i}(t)]^T \in R^{p_i}$ denotes the vector of output, $F_{i1}^k, \dots, F_{iq_i}^k$ are the fuzzy sets, L_i is the number of If-Then rules of the subsystem S_i , the matrices $A_{ik}, B_{1ik}, A_{ijk}, B_{2ik}$, and C_i are of appropriate dimensions, and $z_{i1}(t), \dots, z_{iq_i}(t)$ are the premise variables.

Remark 1: For different subsystem, the number of rules may be different. In other words, L_i can be different for different subsystem. ■

Remark 2: Since in this study the static output feedback control is considered, the premise variables $z_{i1}(t), \dots, z_{iq_i}(t) \in y_i(t)$ with $q_i \leq p_i$ for subsystem S_i are assumed. ■

Remark 3: In this study, $C_i \in R^{p_i \times n_i}$ ($i = 1, \dots, N$) with full row rank are assumed, i.e., $\text{rank}(C_i) = p_i$. Hence, a singular value decomposition (SVD) of the output matrix C_i for the subsystem S_i can be defined as

$$C_i = U_i S_i V_i^T = U_i S_i V_i^{-1} \quad (2)$$

where

$$\begin{aligned} U_i &\in R^{p_i \times p_i} \\ S_i &= \begin{bmatrix} \Sigma_{i[p_i \times p_i]} & 0_{[p_i \times (n_i - p_i)]} \end{bmatrix} \in R^{p_i \times n_i} \\ V_i &\in R^{n_i \times n_i} \end{aligned}$$

and

$$\Sigma_i = \text{diag} \{ \bar{\sigma}_{i1}, \bar{\sigma}_{i2}, \dots, \bar{\sigma}_{ip_i} \} \in R^{p_i \times p_i}$$

with singular values $\bar{\sigma}_{i1} \geq \bar{\sigma}_{i2} \geq \dots \geq \bar{\sigma}_{ip_i} > 0$. Note that, since U_i and V_i are orthogonal matrices, $U_i^T = U_i^{-1}$, $V_i^T =$

V_i^{-1} , and then $V_i^T V_i = V_i V_i^T = V_i^{-1} V_i = V_i V_i^{-1} = I_{[n_i \times n_i]}$. ■

The overall fuzzy model of subsystem S_i can be rearranged as the following form:

$$\begin{aligned} \dot{x}_i(t) &= \sum_{k=1}^{L_i} h_k(z_i(t)) \{ A_{ik} x_i(t) + B_{1ik} u_i(t) \\ &\quad + \sum_{j=1, j \neq i}^N [A_{ijk} x_j(t)] + B_{2ik} w_i(t) \} \\ y_i(t) &= \sum_{k=1}^{L_i} h_k(z_i(t)) [C_i x_i(t)] \end{aligned} \quad (3)$$

where $\mu_k(z_i(t)) = \prod_{q=1}^{q_i} F_{iq}^k(z_{iq}(t))$, $h_k(z_i(t)) = \frac{\mu_k(z_i(t))}{\sum_{k=1}^{L_i} \mu_k(z_i(t))}$,

$z_i(t) = [z_{i1}(t), \dots, z_{iq_i}(t)]$, and $F_{iq}^k(z_{iq}(t))$ is the grade of membership of $z_{iq}(t)$ in F_{iq}^k ($q = 1, \dots, q_i$). It is assumed that $\mu_k(z_i(t)) \geq 0$ and $\sum_{k=1}^{L_i} \mu_k(z_i(t)) > 0$ for $k = 1, 2, \dots, L_i$ for all t . Therefore, we get $h_k(z_i(t)) \geq 0$ for $k = 1, 2, \dots, L_i$ and $\sum_{k=1}^{L_i} h_k(z_i(t)) = 1$.

The aim of this study is to propose a decentralized static output feedback fuzzy controller for nonlinear interconnected systems. Suppose the following decentralized static output feedback fuzzy controller is proposed to deal with the nonlinear interconnected subsystem S_i :

Control Rule s :

$$\begin{aligned} \text{If} \quad & z_{i1}(t) \text{ is } F_{i1}^s \dots \text{and } z_{iq_i}(t) \text{ is } F_{iq_i}^s \\ \text{Then} \quad & u_i(t) = K_{is} y_i(t) \end{aligned} \quad (4)$$

for $s = 1, 2, \dots, L_i$. The overall decentralized static output feedback fuzzy controller for the i^{th} subsystem is represented as follows:

$$\begin{aligned} u_i(t) &= \sum_{s=1}^{L_i} h_s(z_i(t)) K_{is} y_i(t) \\ &= \sum_{s=1}^{L_i} h_s(z_i(t)) [K_{is} C_i x_i(t)] \end{aligned} \quad (5)$$

where K_{is} is the static fuzzy control gain for the s^{th} control rule and the i^{th} interconnected subsystem S_i .

After some manipulation, the i^{th} subsystem can be expressed as the following compact form:

$$\begin{aligned} \dot{x}_i(t) &= \sum_{j=1, j \neq i}^N \sum_{k=1}^{L_i} h_k(z_i(t)) \sum_{s=1}^{L_i} h_s(z_i(t)) [\bar{A}_{iks} x_i(t) \\ &\quad + A_{ijk} x_j(t) + B_{2ik} w_i(t)] \end{aligned} \quad (6)$$

where

$$\begin{aligned} \bar{A}_{iks} &= A_{ik} + B_{1ik} K_{is} C_i \\ &= A_{ik} + B_{1ik} K_{is} U_i S_i V_i^{-1}. \end{aligned}$$

III. H_∞ DECENTRALIZED STATIC OUTPUT FEEDBACK FUZZY CONTROL SYNTHESIS FOR NONLINEAR INTERCONNECTED SYSTEMS

The purpose of this study is to determine a decentralized static output feedback fuzzy controller in (5) for the system in (6) with the guaranteed H_∞ performance for all $w_i(t) \in L_2$. Let us consider the H_∞ performance for subsystem S_i as follows [18], [19], [12], [5], [11], [23]

$$\frac{\int_0^{t_f} x_i^T(t) \Psi_i x_i(t) dt}{\int_0^{t_f} w_i^T(t) w_i(t) dt} \leq \rho_i^2 \quad (7)$$

or

$$\int_0^{t_f} x_i^T(t) \Psi_i x_i(t) dt \leq \rho_i^2 \int_0^{t_f} w_i^T(t) w_i(t) dt \quad (8)$$

where Ψ_i is symmetric positive definite weighting matrix, t_f is terminal time of control, ρ_i is a prescribed attenuation level for the subsystem S_i . The physical meaning of (7) or (8) is that the effect of any $w_i(t) \in L_2$ on state $x_i(t)$ must be attenuated below a desired level ρ_i^2 from the viewpoint of energy, no matter what $w_i(t)$ is, i.e., the L_2 gain from $w_i(t)$ to $x_i(t)$ must be less than or equal to a prescribed value ρ_i^2 . The H_∞ performance with a prescribed attenuation level is useful for a robust stabilization design without knowledge of $w_i(t)$. Thereafter, the attenuation level ρ_i^2 for each subsystem can also be minimized so that the H_∞ performance in (8) is reduced as small as possible.

Hence, if initial conditions are also considered, the H_∞ performance for subsystem S_i in (8) can be modified as follows

$$\begin{aligned} & \int_0^{t_f} x_i^T(t) \Psi_i x_i(t) dt \\ & \leq x_i^T(0) \tilde{P}_i x_i(0) + \rho_i^2 \int_0^{t_f} w_i^T(t) w_i(t) dt \end{aligned} \quad (9)$$

where $\tilde{P}_i = (V_i^{-1})^T \bar{P}_i (V_i^{-1})$ is a symmetric positive definite weighting matrix.

For simplicity, the notation $\begin{bmatrix} M_{11} & * \\ M_{21} & M_{22} \end{bmatrix} \triangleq \begin{bmatrix} M_{11} & M_{21}^T \\ M_{21} & M_{22} \end{bmatrix}$ is adopted throughout this paper. Then, we get the following main result for the H_∞ decentralized static output feedback fuzzy control.

Theorem 1: In the nonlinear interconnected systems (6), if there exist symmetric positive matrices Q_i with

$$Q_i = \begin{bmatrix} Q_{i1[p_i \times p_i]} & 0 \\ 0 & Q_{i2[(n_i - p_i) \times (n_i - p_i)]} \end{bmatrix} \in R^{n_i \times n_i}$$

and matrices $\bar{Y}_{is}$ with

$$\bar{Y}_{is} = \begin{bmatrix} Y_{is[m_i \times p_i]} & 0_{[m_i \times (n_i - p_i)]} \end{bmatrix} \in R^{m_i \times n_i}$$

such that the following LMIs

$$\begin{bmatrix} \{\bar{M}_1 + \bar{M}_1^T\} & * & * & * \\ \bar{A}_{ijk}^T & 0 & * & * \\ \bar{B}_{2ik}^T & 0 & -\rho_i^2 I & * \\ Q_i & 0 & 0 & -\Psi_{vi}^{-1} \end{bmatrix} \leq 0 \quad (10)$$

hold for $i, j = 1, 2, \dots, N$ ($j \neq i$) and $k \leq s = 1, 2, \dots, L_i$ s.t. $h_k(\cdot) \cap h_s(\cdot) \neq \emptyset$, where

$$\begin{aligned} \bar{M}_1 &= \frac{(\bar{A}_{ik} Q_i + \bar{B}_{1ik} \bar{Y}_{is}) + (\bar{A}_{is} Q_i + \bar{B}_{1is} \bar{Y}_{ik})}{2} \\ \bar{A}_{ik} &= V_i^{-1} A_{ik} V_i \\ \bar{B}_{1ik} &= V_i^{-1} B_{1ik} \\ \bar{A}_{ijk} &= V_i^{-1} A_{ijk} \\ \bar{B}_{2ik} &= V_i^{-1} B_{2ik} \\ \Psi_{vi} &= V_i^T \Psi_i V_i = V_i^{-1} \Psi_i V_i, \end{aligned}$$

then (i) the H_∞ control performance in (9) is guaranteed for a prescribed ρ_i^2 in the case of $w_i(t) \neq 0$ and (ii) the interconnected systems S is stable in the sense of Lyapunov in the case of $w_i(t) = 0$. Moreover, the static fuzzy control gains can be obtained as

$$K_{is} = Y_{is} Q_{i1}^{-1} \Sigma_i^{-1} U_i^{-1}.$$

Proof: From (9), we obtain

$$\begin{aligned} & \int_0^{t_f} x_i^T(t) \Psi_i x_i(t) dt \\ &= \int_0^{t_f} \{x_i^T(t) \Psi_i x_i(t) + \frac{d}{dt} [x_i^T \tilde{P}_i x_i]\} dt \\ & \quad + x_i^T(0) \tilde{P}_i x_i(0) - x_i^T(t_f) \tilde{P}_i x_i(t_f) \\ &\leq x_i^T(0) \tilde{P}_i x_i(0) \\ & \quad + \int_0^{t_f} \{x_i^T(t) \Psi_i x_i(t) + \frac{d}{dt} [x_i^T \tilde{P}_i x_i]\} dt \\ &= x_i^T(0) \tilde{P}_i x_i(0) \\ & \quad + \sum_{j=1, j \neq i}^N \sum_{k=1}^{L_i} h_k^2(z_i(t)) \int_0^{t_f} \{x_i^T(t) \Psi_i x_i(t) \\ & \quad + \eta^T \begin{bmatrix} \{\Psi_i \\ + \tilde{P}_i (\frac{\bar{A}_{ikk} + \bar{A}_{ikk}}{2}) & * & * \\ + (\frac{\bar{A}_{ikk} + \bar{A}_{ikk}}{2})^T \tilde{P}_i \} & 0 & * \\ \bar{A}_{ijk}^T \tilde{P}_i & 0 & -\rho_i^2 I \\ \bar{B}_{2ik}^T \tilde{P}_i & 0 & -\rho_i^2 I \end{bmatrix} \eta \\ & \quad \} dt + 2 \sum_{j=1, j \neq i}^N \sum_{k=1}^{L_i} h_k(z_i(t)) \sum_{k \leq s}^{L_i} h_s(z_i(t)) \\ & \quad \times \int_0^{t_f} \{x_i^T(t) \Psi_i x_i(t) \\ & \quad + \eta^T \begin{bmatrix} \{\Psi_i \\ + \tilde{P}_i (\frac{\bar{A}_{iks} + \bar{A}_{iks}}{2}) & * & * \\ + (\frac{\bar{A}_{iks} + \bar{A}_{iks}}{2})^T \tilde{P}_i \} & -I & * \\ \bar{A}_{ijk}^T \tilde{P}_i & 0 & -\rho_i^2 I \\ \bar{B}_{2ik}^T \tilde{P}_i & 0 & -\rho_i^2 I \end{bmatrix} \eta \\ & \quad \} dt + \rho_i^2 \int_0^{t_f} w_i^T(t) w_i(t) dt \end{aligned} \quad (11)$$

where $\eta^T = [x_i^T(t), x_j^T(t), w_i^T(t)]$. From (11), if

$$\begin{bmatrix} \{\Psi_i + (V_i^{-1})^T \bar{P}_i (V_i^{-1}) (\frac{\bar{A}_{iks} + \bar{A}_{isk}}{2}) \\ + (\frac{\bar{A}_{iks} + \bar{A}_{isk}}{2})^T (V_i^{-1})^T \bar{P}_i (V_i^{-1})\} & * & * \\ \bar{A}_{ijk}^T (V_i^{-1})^T \bar{P}_i (V_i^{-1}) & 0 & * \\ \bar{B}_{2ik}^T (V_i^{-1})^T \bar{P}_i (V_i^{-1}) & 0 & -\rho_i^2 I \end{bmatrix} \leq 0 \quad (12)$$

for $i, j = 1, 2, \dots, N$ ($j \neq i$) and $k \leq s = 1, 2, \dots, L_i$, then we obtain

$$\begin{aligned} & \int_0^{t_f} x_i^T(t) \Psi_i x_i(t) dt \\ & \leq x_i^T(0) \bar{P}_i x_i(0) + \rho_i^2 \int_0^{t_f} w_i^T(t) w_i(t) dt. \end{aligned} \quad (13)$$

By pre-multiplying $\begin{bmatrix} V_i & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}^T$ and post-multiplying $\begin{bmatrix} V_i & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$ to (12), we obtain

$$\begin{bmatrix} \{V_i^T \Psi_i V_i \\ + \bar{P}_i (V_i^{-1}) (\frac{\bar{A}_{iks} + \bar{A}_{isk}}{2}) V_i \\ + V_i^T (\frac{\bar{A}_{iks} + \bar{A}_{isk}}{2})^T (V_i^{-1})^T \bar{P}_i\} & * & * \\ \bar{A}_{ijk}^T (V_i^{-1})^T \bar{P}_i & 0 & * \\ \bar{B}_{2ik}^T (V_i^{-1})^T \bar{P}_i & 0 & -\rho_i^2 I \end{bmatrix} \leq 0. \quad (14)$$

By the following definitions,

$$\begin{aligned} \bar{A}_{ik} &= V_i^{-1} A_{ik} V_i \\ \bar{B}_{1ik} &= V_i^{-1} B_{1ik} \\ \bar{A}_{ijk} &= V_i^{-1} A_{ijk} \\ \bar{B}_{2ik} &= V_i^{-1} B_{2ik} \\ \Psi_{vi} &= V_i^T (\Psi_i) V_i = V_i^{-1} (\Psi_i) V_i, \end{aligned}$$

(14) can be expressed as the following compact form

$$\begin{bmatrix} \Psi_{vi} + \bar{P}_i \bar{M}_2 + \bar{M}_2^T \bar{P}_i & * & * \\ \bar{A}_{ijk}^T \bar{P}_i & 0 & * \\ \bar{B}_{2ik}^T \bar{P}_i & 0 & -\rho_i^2 I \end{bmatrix} \leq 0. \quad (15)$$

$$\text{where } \bar{M}_2 = (\frac{\bar{A}_{ik} Q_i + \bar{B}_{1ik} K_{is} U_i S_i}{2} + (\bar{A}_{is} Q_i + \bar{B}_{1is} K_{ik} U_i S_i)).$$

By pre-multiplying $\begin{bmatrix} Q_i & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$ and post-multiplying

$$\begin{bmatrix} Q_i & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \text{ to (15), we obtain}$$

$$\begin{bmatrix} \{Q_i \Psi_{vi} Q_i + \bar{M}_3 + \bar{M}_3^T\} & * & * \\ \bar{A}_{ijk}^T & 0 & * \\ \bar{B}_{2ik}^T & 0 & -\rho_i^2 I \end{bmatrix} \leq 0 \quad (16)$$

where

$$\bar{M}_3 = (\frac{\bar{A}_{ik} Q_i + \bar{B}_{1ik} K_{is} U_i S_i Q_i + (\bar{A}_{is} Q_i + \bar{B}_{1is} K_{ik} U_i S_i Q_i)}{2}) \text{ and}$$

$$Q_i = \bar{P}_i^{-1} = Q_i^T > 0.$$

Note that, by the Schur complements [2], (16) is equivalent to

$$\begin{bmatrix} \{\bar{M}_3 + \bar{M}_3^T\} & * & * & * \\ \bar{A}_{ijk}^T & 0 & * & * \\ \bar{B}_{2ik}^T & 0 & -\rho_i^2 I & * \\ Q_i & 0 & 0 & -\Psi_{vi}^{-1} \end{bmatrix} \leq 0. \quad (17)$$

For the simplicity of design, let

$$Q_i = \begin{bmatrix} Q_{i1} & 0 \\ 0 & Q_{i2} \end{bmatrix} \quad (18)$$

and then

$$\begin{aligned} & \bar{B}_{1ik} K_{is} U_i S_i Q_i \\ &= \bar{B}_{1ik} K_{is} U_i [\Sigma_i \quad 0] \begin{bmatrix} Q_{i1} & 0 \\ 0 & Q_{i2} \end{bmatrix} \\ &= \bar{B}_{1ik} [K_{is} U_i \Sigma_i Q_{i1} \quad 0] \\ &= \bar{B}_{1ik} [Y_{is} \quad 0] = \bar{B}_{1ik} \bar{Y}_{is} \end{aligned} \quad (19)$$

where

$$\begin{aligned} Y_{is} &= K_{is} U_i \Sigma_i Q_{i1} \\ \bar{Y}_{is} &= [Y_{is} \quad 0]. \end{aligned}$$

By substituting (18) and (19) into (17), we obtain the following LMIs

$$\begin{bmatrix} \{\bar{M}_1 + \bar{M}_1^T\} & * & * & * \\ \bar{A}_{ijk}^T & 0 & * & * \\ \bar{B}_{2ik}^T & 0 & -\rho_i^2 I & * \\ Q_i & 0 & 0 & -\Psi_{vi}^{-1} \end{bmatrix} \leq 0. \quad (20)$$

$$\text{where } \bar{M}_1 = \frac{(\bar{A}_{ik} Q_i + \bar{B}_{1ik} \bar{Y}_{is}) + (\bar{A}_{is} Q_i + \bar{B}_{1is} \bar{Y}_{ik})}{2}.$$

If we can solve $Q_i = \text{diag}\{Q_{i1}, Q_{i2}\}$ and $\bar{Y}_{is} = [Y_{is} \quad 0]$ from the above LMIs, the decentralized static fuzzy control gains can be obtained as

$$K_{is} = Y_{is} Q_{i1}^{-1} \Sigma_i^{-1} U_i^{-1}. \quad (21)$$

Therefore, the H_∞ control performance is achieved with a prescribed ρ_i^2 in the case of $w_i(t) \neq 0$. In the case of $w_i(t) = 0$, let us define a Lyapunov function for the subsystem S_i in (6) as

$$LF_i(t) = x_i^T(t) \tilde{P}_i x_i(t)$$

where $\tilde{P}_i = (V_i^{-1})^T \bar{P}_i (V_i^{-1})$ is a symmetric positive definite weighting matrix. Thus, the Lyapunov function for the overall interconnected system S can be defined as

$$LF(t) = \sum_{i=1}^N LF_i(t) = \sum_{i=1}^N \{x_i^T(t) \tilde{P}_i x_i(t)\}.$$

The time derivative of the Lyapunov function $LF(t)$ is

$$\begin{aligned}
& \frac{d}{dt}(LF(t)) \\
&= \sum_{i=1}^N \sum_{j=1, j \neq i}^N \sum_{k=1}^{L_i} h_k(z_i(t)) \sum_{s=1}^{L_i} h_s(z_i(t)) \\
&\quad \times \{ [\bar{A}_{iks} x_i(t) + A_{ijk} x_j(t)]^T \tilde{P}_i x_i(t) \\
&\quad + x_i^T(t) \tilde{P}_i [\bar{A}_{iks} x_i(t) + A_{ijk} x_j(t)] \} \\
&= \sum_{i=1}^N \sum_{j=1, j \neq i}^N \sum_{k=1}^L h_k(z_i(t)) \sum_{s=1}^L h_s(z_i(t)) \\
&\quad \times \{ \bar{\eta}^T \begin{bmatrix} \tilde{P}_i \bar{A}_{iks} + \bar{A}_{ijs}^T \tilde{P}_i & * \\ A_{ijk}^T \tilde{P}_i & 0 \end{bmatrix} \bar{\eta} \} \\
&= \sum_{i=1}^N \sum_{j=1, j \neq i}^N \sum_{k=1}^{L_i} h_k^2(z_i(t)) \\
&\quad \times \{ \bar{\eta}^T \begin{bmatrix} \tilde{P}_i (\frac{\bar{A}_{ikk} + \bar{A}_{ikk}}{2}) \\ + (\frac{\bar{A}_{ikk} + \bar{A}_{ikk}}{2})^T \tilde{P}_i & * \\ A_{ijk}^T \tilde{P}_i & 0 \end{bmatrix} \bar{\eta} \} \\
&\quad + 2 \sum_{i=1}^N \sum_{j=1, j \neq i}^N \sum_{k=1}^{L_i} h_k(z_i(t)) \sum_{k < s}^{L_i} h_s(z_i(t)) \\
&\quad \times \{ \bar{\eta}^T \begin{bmatrix} \tilde{P}_i (\frac{\bar{A}_{iks} + \bar{A}_{isk}}{2}) \\ + (\frac{\bar{A}_{iks} + \bar{A}_{isk}}{2})^T \tilde{P}_i & * \\ A_{ijk}^T \tilde{P}_i & 0 \end{bmatrix} \bar{\eta} \} \quad (22)
\end{aligned}$$

where $\bar{\eta}^T = [x_i^T(t), x_j^T(t)]$. From (22), if

$$\begin{bmatrix} \tilde{P}_i (\frac{\bar{A}_{iks} + \bar{A}_{isk}}{2}) + (\frac{\bar{A}_{iks} + \bar{A}_{isk}}{2})^T \tilde{P}_i & * \\ A_{ijk}^T \tilde{P}_i & 0 \end{bmatrix} \leq 0 \quad (23)$$

for $i, j = 1, 2, \dots, N$ ($j \neq i$) and $k \leq s = 1, \dots, L_i$, then $\frac{d}{dt}(LF(t)) \leq 0$. It is easy to show that (12), which is equivalent to (10) if Q_i in (18) is specified, implies (23) and then the interconnected system S is stable in the sense of Lyapunov, if the LMIs in (10) hold. This completes the proof. ■

As mentioned, to obtain better robust performance, the attenuation level ρ_i^2 for each subsystem can be minimized so that the H_∞ performance in (9) is reduced as small as possible. Therefore, the H_∞ static output feedback fuzzy control problem for the nonlinear interconnected systems can be formulated as the following minimization problem which is also called eigenvalue problem (EVP).

$$\begin{aligned}
& \min \quad \sum_{i=1}^N \rho_i^2 \\
& \text{subject to} \quad Q_i = Q_i^T > 0, \bar{Y}_{is}, \text{ and (10)}. \quad (24)
\end{aligned}$$

Remark 4: By the above formulation in (24), the attenuation level ρ_i^2 for each subsystem may not be the minimum but the sum of the attenuation level ρ_i^2 , i.e., $\sum_{i=1}^N \rho_i^2$, is minimal, which means the formulation in (24) finds the attenuation level ρ_i^2 as small as possible for each subsystem by minimizing $\sum_{i=1}^N \rho_i^2$ with equal weighting for each subsystem. For the simplicity of design, one may assume $\rho_1 = \rho_2 = \dots = \rho_N =$

ρ . In such a case, the EVP in (24) can be simplified as follows:

$$\begin{aligned}
& \min \quad \rho^2 \\
& \text{subject to} \quad Q_i = Q_i^T > 0, \bar{Y}_{is}, \text{ and (10)}. \quad (25)
\end{aligned}$$

■

According to the analysis above, the H_∞ decentralized static output feedback fuzzy control design is summarized as follows:

Design Procedure:

- Step 1: Construct the fuzzy plant rules in (1).
- Step 2: Perform the SVD for the output matrices $C_i = U_i S_i V_i^T = U_i [\Sigma_i \ 0] V_i^T$ ($i = 1, \dots, N$) in (2).
- Step 3: Solve the EVP in (24) and then obtain the corresponding minimal attenuation level ρ_i^2 and the corresponding $Q_i = \text{diag}\{Q_{i1}, Q_{i2}\}$ and $\bar{Y}_{is} = [Y_{is} \ 0]$ (thus decentralized static fuzzy control gains $K_{is} = Y_{is} Q_{i1}^{-1} \Sigma_i^{-1} U_i^{-1}$ can also be obtained), simultaneously.
- Step 4: Construct the H_∞ decentralized static output feedback fuzzy controller in (5).

IV. SIMULATION EXAMPLE

In this section, the stabilization problem of two interconnected systems S_1 and S_2 using the proposed decentralized static output feedback fuzzy control scheme is considered. For the subsystem S_1 , a modified Lorenz's equation with input, outputs, and interconnected terms from subsystem S_2 is given by

$$S_1 : \begin{cases} \dot{x}_{11} = -10x_{11} + 10x_{12} + 0.1x_{22} \\ \quad + u_1(t) + 0.1w_{11} \\ \dot{x}_{12} = 28x_{11} - x_{12} - x_{11}x_{13} \\ \quad - 0.3x_{21} + 0.1w_{12} \\ \dot{x}_{13} = x_{11}x_{12} - 8/3x_{13} + 0.05x_{21} \\ \quad - 0.01x_{22} + 0.1w_{13} \\ y_{11} = x_{11} \\ y_{12} = 5x_{12} + 2x_{13} \end{cases} \quad (26)$$

where external disturbance

$$\begin{aligned}
w_1(t) &= [w_{11}, w_{12}, w_{13}]^T \\
&= [50 \sin(10t)e^{-0.5t}, 50 \cos(10t)e^{-t}, \\
&\quad 50[\sin(10t) + \cos(10t)]e^{-0.2t}]^T \in L_2.
\end{aligned}$$

For this subsystem S_1 , the modified Lorenz's equation can be exactly represented by the following two-rule fuzzy model under $x_{11} (= y_{11}) \in [-\kappa_1, \kappa_1]$ [17].

Rule 1 : IF y_{11} is F_{11}^1

THEN $\dot{x}_1(t) = A_{111}x_1(t) + A_{121}x_2(t) + B_{111}u_1(t) + B_{211}w_1(t)$, and $y_1(t) = C_1x_1(t)$,

Rule 2 : IF y_{11} is F_{11}^2

THEN $\dot{x}_1(t) = A_{122}x_1(t) + A_{122}x_2(t) + B_{112}u_1(t) + B_{212}w_1(t)$, and $y_1(t) = C_1x_1(t)$,

$$\text{where } x_1(t) = [x_{11}, x_{12}, x_{13}]^T, y_1(t) = [y_{11}, y_{12}]^T,$$

$$A_{11} = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -\kappa_1 \\ 0 & \kappa_1 & -8/3 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & \kappa_1 \\ 0 & -\kappa_1 & -8/3 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 2 \end{bmatrix},$$

$$A_{12k} = \begin{bmatrix} 0 & 0.1 \\ -0.3 & 0 \\ 0.05 & -0.01 \end{bmatrix},$$

$$B_{11k} = [1 \ 0 \ 0]^T,$$

$$\text{and } B_{21k} = 0.1I_{[3 \times 3]} \ (k = 1, 2)$$

where $F_{11}^1 = \frac{1}{2}(1 + \frac{y_{11}}{\kappa_1}) = h_1(y_{11})$, $F_{11}^2 = \frac{1}{2}(1 - \frac{y_{11}}{\kappa_1}) = h_2(y_{11})$, and $\kappa_1 = 30$.

For the subsystem S_2 , a modified Duffing forced-oscillation model with input, output, and interconnected terms from subsystem S_1 is given by

$$S_2 : \begin{cases} \dot{x}_{21} = x_{22} + 0.05x_{11} - 0.01x_{12} \\ \quad + 6\cos(t) + 0.5u_2^*(t) + 0.5w_{21} \\ \dot{x}_{22} = -x_{21}^3 - 0.1x_{22} - 0.01x_{12} + 0.04x_{13} \\ \quad + 12\cos(t) + u_2^*(t) + 0.03w_{22} \\ y_{21} = 10x_{21} \end{cases} \quad (27)$$

where external disturbance $w_2(t) = [w_{21}, w_{22}]^T = [N_1 e^{-0.5t}, N_2 e^{-t}]^T \in L_2$, where $N_1 = 200\text{sign}[\sin(10t)]$ and $N_2 = 200\text{sign}[\cos(10t)]$ are assumed to be periodic square wave with period $\frac{2\pi}{10}$ and magnitude ± 200 . For this subsystem S_2 , the modified Duffing forced-oscillation model can be exactly represented by the following two-rule fuzzy model under $x_{21} (= 0.1y_{21}) \in [-\kappa_2, \kappa_2]$ [17].

Rule 1 : IF $0.1y_{21}$ is F_{21}^1

THEN $\dot{x}_2(t) = A_{21}x_2(t) + A_{211}x_1(t) + B_{121}u_2(t) + B_{221}w_2(t)$, and $y_2(t) = C_2x_2(t)$,

Rule 2 : IF $0.1y_{21}$ is F_{21}^2

THEN $\dot{x}_2(t) = A_{22}x_2(t) + A_{212}x_1(t) + B_{122}u_2(t) + B_{222}w_2(t)$, and $y_2(t) = C_2x_2(t)$,

where $x_2(t) = [x_{21}, x_{22}]^T$, $y_2(t) = y_{21}$, $u_2(t) = 12\cos(t) + u_2^*(t)$,

$$A_{21} = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} 0 & 1 \\ -\kappa_2^2 & -0.1 \end{bmatrix},$$

$$C_2 = [10 \ 0],$$

$$A_{21k} = \begin{bmatrix} 0.05 & -0.01 & 0 \\ 0 & -0.01 & 0.04 \end{bmatrix},$$

$$B_{12k} = [0.5 \ 1]^T,$$

$$\text{and } B_{22k} = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.03 \end{bmatrix} \ (k = 1, 2)$$

where $F_{21}^1 = (1 - \frac{(0.1y_{21})^2}{\kappa_2^2}) = h_1(y_{21})$, $F_{21}^2 = \frac{(0.1y_{21})^2}{\kappa_2^2} = h_2(y_{21})$, and $\kappa_2 = 50$.

Remark 5: For the simplicity of design, we can assume $\rho_1 = \rho_2 = \rho$ for these two subsystems in this example. ■

There is no efficient algorithm in Matlab [7] to solve the nonstrict LMIs in (10), therefore one may replace the diagonal zero entry in (10) by a negative value to solve the nonstrict LMIs problem in (24). Consequently, the solutions of the nonstrict LMIs in (10) can be obtained correspondingly by solving the following strict LMIs if ε^2 is small enough, for example $\varepsilon^2 \leq 10^{-8}$.

$$\begin{bmatrix} \bar{M}_1 & * & * & * \\ \bar{A}_{ijk}^T & -\varepsilon^2 I & * & * \\ \bar{B}_{2ik}^T & 0 & -\rho_i^2 I & * \\ Q_i & 0 & 0 & -\Psi_{vi}^{-1} \end{bmatrix} < 0 \quad (28)$$

To obtain feasible solutions for the strict LMIs in (28), the weighting matrices Ψ_1 and Ψ_2 should be small enough. In this example, $\varepsilon^2 = 10^{-8}$ and the weighting matrices are chosen as follows

$$\Psi_1 = 5 \times 10^{-8} I_{[3 \times 3]}, \quad \Psi_2 = 5 \times 10^{-8} I_{[2 \times 2]}.$$

Then, following the **Design Procedure** in the above section, the SVD of C_1 and C_2 is given as follows:

$$C_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 5.3852 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T \\ \times \begin{bmatrix} 0 & 1 & 0 \\ 0.9285 & 0 & -0.3714 \\ 0.3714 & 0 & 0.9285 \end{bmatrix},$$

$$C_2 = [1] [10 \ 0] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T.$$

Solving the EVP in (25), one can obtain the corresponding minimal attenuation level $\rho_{\min}^2 = 3 \times 10^{-8}$ and

$$Q_1 = \text{diag}\{Q_{11}, Q_{12}\} \\ = 10^7 \times \begin{bmatrix} 3.6452 & -0.6262 & 0 \\ -0.6262 & 0.4863 & 0 \\ 0 & 0 & 3.519 \end{bmatrix} \\ Q_2 = \text{diag}\{Q_{21}, Q_{22}\} \\ = 10^6 \times \begin{bmatrix} 0.0003 & 0 \\ 0 & 1.9996 \end{bmatrix} \\ \bar{Y}_{11} = 10^8 \times \begin{bmatrix} -2.3573 & -7.0707 & 0 \end{bmatrix} \\ \bar{Y}_{12} = 10^8 \times \begin{bmatrix} -2.7502 & -3.8084 & 0 \end{bmatrix} \\ \bar{Y}_{21} = \begin{bmatrix} -2.0096 \times 10^6 & 0 \end{bmatrix} \\ \bar{Y}_{22} = \begin{bmatrix} -1.3050 \times 10^6 & 0 \end{bmatrix}.$$

The decentralized static output feedback fuzzy control gains can be found as follows

$$K_{11} = [-197.3995 \ -7.4974], \\ K_{12} = [-113.0383 \ -5.0066], \\ K_{21} = -713.0568, \ K_{22} = -463.0563.$$

Therefore, the proposed decentralized static output feedback fuzzy controller is constructed as

$$u_i(t) = \sum_{s=1}^2 h_s(y_{s1}) K_{is} y_i(t), \quad (i = 1, 2).$$

Ignoring the effect of initial conditions, the H_∞ performance in this example can be written as follows

$$5 \times 10^{-8} \int_0^{t_f} x_i^T(t) x_i(t) dt \leq 3 \times 10^{-8} \int_0^{t_f} w_i^T(t) w_i(t) dt$$

which is equivalent to

$$\frac{\int_0^{t_f} x_i^T(t) x_i(t) dt}{\int_0^{t_f} w_i^T(t) w_i(t) dt} \leq 0.6$$

The following table shows the results for the different choice of ε^2 and Ψ_i , respectively.

ε^2	Ψ_i	$\rho_{\min}^2$
10^{-8}	5×10^{-8}	3×10^{-8}
10^{-9}	5×10^{-9}	3×10^{-9}
$10^{-9.5}$	$5 \times 10^{-9.5}$	9.571×10^{-10}

Remark 6: In the above example, if $\varepsilon^2 < 10^{-9.5}$, no feasible solution can be found. ■

Figs. 1-4 present the simulation results with initial conditions $(x_{11}(0), x_{12}(0), x_{13}(0)) = (5, 10, -5)$ and $(x_{21}(0), x_{22}(0)) = (4.5, -5)$. The trajectories of state variables $x_{11}(t)$, $x_{12}(t)$, and $x_{13}(t)$ are shown in Fig. 1 while the trajectories of state variables $x_{21}(t)$ and $x_{22}(t)$ are shown in Fig. 2. The control inputs $u_1(t)$ and $u_2^*(t) = u_2(t) - 12 \cos(t)$ are applied at $t = 2$ (sec) and shown in Fig. 3 and Fig. 4, respectively. From Fig. 1 and Fig. 2, one observes that the states are very oscillating before $t = 2$ (sec) but all states are stabilized rapidly after the control inputs are applied. From the simulation results, the proposed H_∞ decentralized static output feedback fuzzy control scheme can solve the output feedback control problem for nonlinear interconnected systems effectively and systematically with the aid of LMI-based optimization method.

V. CONCLUSIONS

In this study, H_∞ decentralized static output feedback fuzzy control design for nonlinear interconnected systems is considered via T-S fuzzy models. Furthermore, the stability of the nonlinear interconnected systems is also guaranteed by the proposed decentralized static output feedback control scheme. A SVD method is proposed to solve the H_∞ static output feedback stabilization problem for the nonlinear interconnected systems, which can be characterized in terms of an EVP for each subsystem. By the proposed SVD method, the H_∞ decentralized static fuzzy control gains can be obtained efficiently. The proposed decentralized static output feedback fuzzy control scheme is very simple without complex control algorithms. Therefore, it is suitable for practical applications. Simulation example is given to illustrate the design procedure and robust performance of the proposed method.

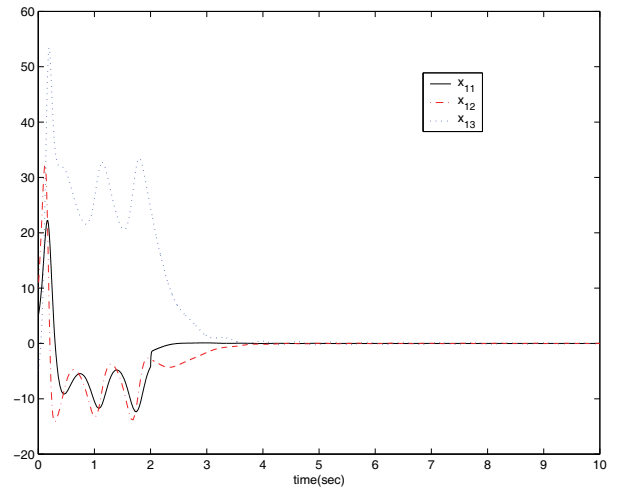


Fig. 1. The trajectories of state variables $x_{11}(t)$ (black), $x_{12}(t)$ (red), and $x_{13}(t)$ (blue).

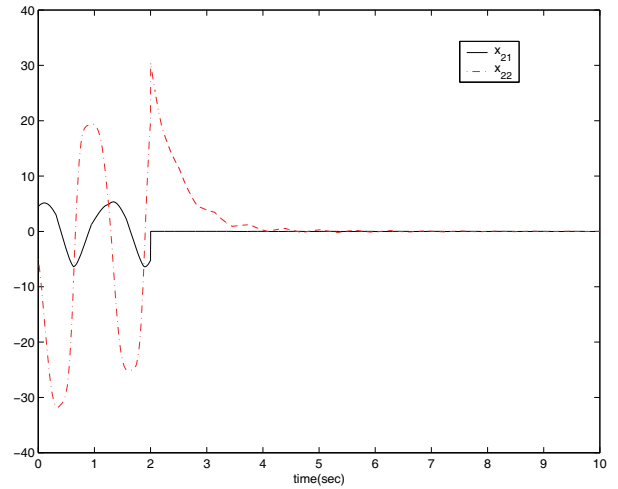


Fig. 2. The trajectories of state variables $x_{21}(t)$ (black) and $x_{22}(t)$ (red).

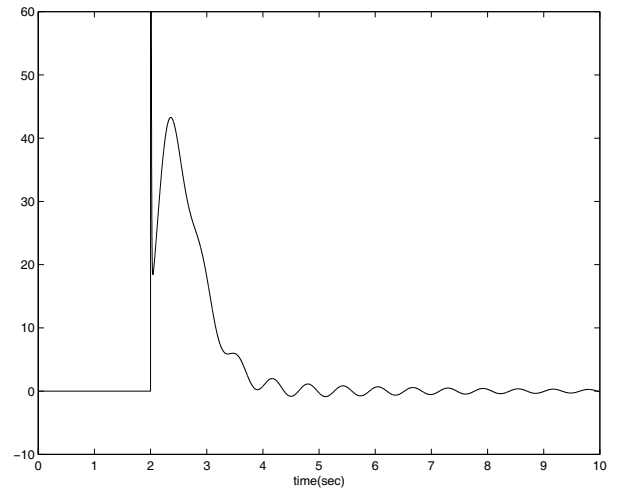


Fig. 3. The control input $u_1(t)$.

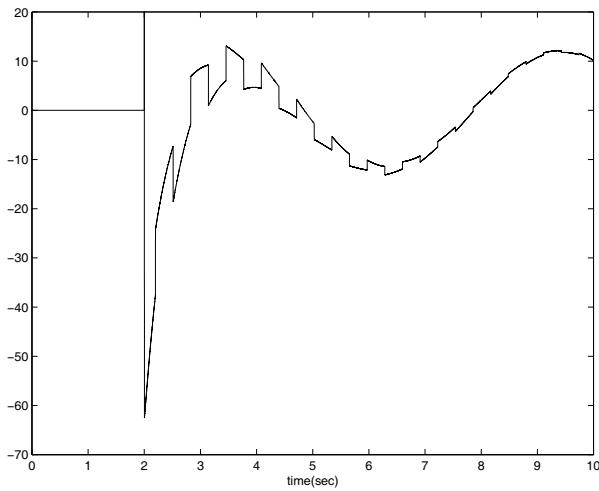


Fig. 4. The control input $u_2^*(t)$.

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