

Robust H_∞ Fuzzy Observer for A Class of Time-Delay Discrete Fuzzy Bilinear Systems with Parameter Uncertainties

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Abstract—In this paper, a robust H_∞ fuzzy observer is proposed for a class of time-delay discrete fuzzy bilinear systems with parameter uncertainties. Utilizing the Lipschitz conditions, a linear matrix inequality approach is developed and a sufficient condition is obtained to design the robust H_∞ fuzzy observer. Besides, the robust H_∞ fuzzy observer for time-delay discrete fuzzy bilinear systems with parameter uncertainties can be obtained through the LMIs without any pre-assigned gain matrices. Finally, a numerical example is illustrated the feasibility and effectively of proposed observer design method.

Keywords- Time-delay, fuzzy observer, linear matrix inequalities; fuzzy bilinear system.

I. INTRODUCTION

During the last decade, fuzzy technique has attracted great attention from both the academic and industrial applications. Among various fuzzy controls, a so-called Takagi–Sugeno (T–S) fuzzy model [1-3] becomes a popular and convenient tool in analyzing the nonlinear system. Through fuzzy modeling procedure, the T-S fuzzy model can be obtained from a nonlinear system. In addition, the obtained T-S fuzzy model is composed of many linear models; therefore, there are many linear system theories which can be utilized to analyze the T-S fuzzy model. By examining controller design, the parallel distributed compensation (PDC) scheme is adopted for each rule to stabilize the overall T-S fuzzy model. For the problems of stability and stabilization of T-S fuzzy models, the Lyapunov direct method [4-7] is mainly investigated to yield the stability/stabilization conditions. Furthermore, the solutions can be found via linear matrix inequalities (LMIs) techniques [8-11] for the T-S fuzzy control system under consideration. Due to the systematic design procedure, different methodologies have been also proposed for the stabilization problem of T-S fuzzy model [4], [12-13].

Even though T-S fuzzy model is composed of linear models blended together with nonlinear scalar positive functions sharing the convex sum property, and can be utilized the linear system theory to analyze the T-S fuzzy model. But unfortunately, there are many real-world nonlinear systems

cannot be adequately approximated by linear models [14], [15]. Due to this reason, [16] first proposed a novel fuzzy modeling method to modelize the nonlinear system into a T-S fuzzy bilinear system (FBS). Besides, [17] studied the H_∞ control problem for a class of discrete fuzzy bilinear systems (DFBSs). Furthermore, [18-20] explored the stabilization problem of T-S FBS with time-delay. In addition to the stabilization problem, [21] investigated the observer design for T-S FBS. [22] explored the output feedback control and proposed an observer for T-S time-delay FBS, but in this paper, some matrices still needs to be pre-assigned for obtaining the observer gains. Moreover, there is no any study proposed a LMI method to obtain the observer gains without any pre-assigned matrices for time-delay FBS/DFBS. Furthermore, there is no any literature to investigate the robust H_∞ fuzzy observer for time-delay DFBS with parameter uncertainties and additive disturbance. Hence, we will explore the robust fuzzy H_∞ observer problem in this paper. The main contributions of this paper include i) Proposing a robust fuzzy H_∞ observer for time-delay DFBSs with parameter uncertainties; ii) The stability conditions for time-delay DFBSs with parameter uncertainties can be formulated as LMIs, and iii) The robust fuzzy H_∞ observer gain for time-delay DFBSs with parameter uncertainties can be obtained from LMIs without any pre-assigned parameter/matrix.

Following the introduction, the paper is organized as follows. In Section II the observer design problem for the time-delay DFBS with parameter uncertainties is introduced. Applying the Lipschitz conditions and some variable transformations, the sufficient conditions for time-delay DFBSs with parameter uncertainties can be transformed into the forms of LMI in Section III. In Section IV, a simulation example is provided to illustrate the effectiveness of the proposed design approach. Finally, in Section V, conclusions are drawn.

II. PROBLEM DESCRIPTION

The T-S fuzzy dynamic model is described by fuzzy IF-THEN rules, which locally represent input-output relations of

The authors would like to thank the National Science Council for financially supporting this research under contract NSC-99-2221-E-027-101 and NSC-98-2622-E-168-013-CC3.

nonlinear systems. Similar to [2], [4], the i th rule of the uncertain DFBS with disturbance can be represented by the following form:

Plant Rule i : IF $s_1(k)$ is F_{1i} and $\dots$ and $s_g(k)$ is F_{ig}

THEN

$$x(k+1) = (A_i + \Delta A_i)x(k) + (A_{di} + \Delta A_{di})x(k-\tau) + N_i x(k)u(k) + N_{di} x(k-\tau)u(k) + D_{1i} w(k) \quad (1a)$$

$$y(k) = (C_i + \Delta C_i)x(k) + (C_{di} + \Delta C_{di})x(k-\tau) + D_{2i} w(k) \quad (1b)$$

$$z(k) = R_i e(k) + R_{di} e(k-\tau) \quad (1c)$$

$$x(k) = \phi(k) \quad \forall k \in [-\tau, 0], \quad i = 1, \dots, r$$

where F_{ij} is the fuzzy set, and r is the number of IF-THEN rules; $s_1(t), s_2(t), \dots, s_g(t)$, are the premise variables. $x(k) \in R^{n \times 1}$ is the system state; $u(k) \in R^{1 \times 1}$ is the input and satisfies $|u(k)| \leq 1$ for all $k \in \mathbf{Z}$, $w(k) \in R^{q \times 1}$ is the exogenous disturbance which belongs to $l_2[0, \infty)$; $\tau > 0$ is an integer representing the time delay of the fuzzy system; $y(k) \in R^{p \times 1}$ is the measured output; $z(k) \in R^{m \times 1}$ is the controlled output of the error state, $A_i, A_{di}, N_i, N_{di}, C_i, C_{di}, D_{1i}, D_{2i}, R_i$, and R_{di} are known real matrices with appropriate dimensions. $\Delta A_i, \Delta A_{di}, \Delta C_i$, and ΔC_{di} are unknown matrices representing time-varying parameter uncertainties, and are assumed to be of the following matched form:

Assumption 1: [8,] [10]

$$[\Delta A_i \quad \Delta A_{di}] = G_{1i} F_{1i} [E_{1i} \quad E_{2i}] \quad (2a)$$

$$[\Delta C_i \quad \Delta C_{di}] = G_{2i} F_{2i} [E_{3i} \quad E_{4i}] \quad (2b)$$

where $G_{1i}, G_{2i}, E_{1i}, E_{2i}, E_{3i}$, and E_{4i} are known real constant matrices of appropriate dimensions, and $F_{1i}(k)$, and $F_{2i}(k)$ are unknown matrix function with Lebesgue-

measurable elements and satisfy $F_{1i}^T(k)F_{1i}(k) \leq I$ and $F_{2i}^T(k)F_{2i}(k) \leq I$. The parameter uncertainties $\Delta A_i, \Delta A_{di}, \Delta C_i$, and ΔC_{di} are said to be admissible if Assumption 1 hold. Similar to [4] and [10], using a center average defuzzifier, product inference, and singleton fuzzifier, the discrete fuzzy bilinear system with time-delay can be expressed as (3a)-(3c).

$$\text{Where } \mu_i(s(k)) = \prod_{j=1}^g F_{ij}(s(k)) \quad , \quad h_i(s(k)) = \frac{\mu_i(s(k))}{\sum_{j=1}^r \mu_j(s(k))} \quad ,$$

$F_{ij}(s_j(k))$ is the grade of membership of $s_j(k)$ in F_{ij} , and $s(k) = [s_1(k) \quad s_2(k) \quad \dots \quad s_n(k)]^T$. Some basic properties of

$\mu_i(s(k))$ are $\mu_i(s(k)) \geq 0$ and $\sum_{i=1}^r \mu_i(s(k)) > 0$. It is clear that

$h_i(s(k)) \geq 0$ and $\sum_{j=1}^r h_j(s(k)) = 1$. According to the conception

of observer for linear system, we consider the H_∞ fuzzy observer as follows:

$$\begin{aligned} \hat{x}(k+1) = & \sum_{i=1}^r h_i(s(k)) (A_i \hat{x}(k) + A_{di} \hat{x}(k-\tau) + N_i \hat{x}(k)u(k) \\ & + N_{di} \hat{x}(k-\tau)u(k) + L_i [y(k) - C_i \hat{x}(k) - C_{di} \hat{x}(k-\tau)]) \end{aligned} \quad (4)$$

where $\hat{x}(k)$ is the estimated state vector of $x(k)$, and $L_i \in R^{n \times 1}$ are observer parameters to be determined. Besides, $N_i^*(u(k), x(k), x(k-\tau), k) = (N_i x(k)u(k) + N_{di} x(k-\tau)u(k))$ is assumed to satisfy the following global Lipschitz conditions: $N_i^*(u(k), 0, 0, k) = 0$, and

$$\|N_i^*(u(k), x, \underline{x}, k) - N_i^*(u(k), \hat{x}, \hat{\underline{x}}, k)\| \quad (5)$$

For all $k \in \mathbf{Z}$, $u(k) \in R^{1 \times 1}$ and $x, \underline{x}, \hat{x}, \hat{\underline{x}} \in R^{n \times 1}$, where $\bar{N}_i, \bar{N}_{di}$ are known matrices with appropriate dimensions.

$$\begin{aligned} x(k+1) = & \sum_{i=1}^r \mu_i(s(k)) ((A_i + \Delta A_i)x(k) + (A_{di} + \Delta A_{di})x(k-\tau) + N_i x(k)u(k) + N_{di} x(k-\tau)u(k) + D_{1i} w(k)) / \sum_{i=1}^r \mu_i(s(k)) \\ = & \sum_{i=1}^r h_i(s(k)) ((A_i + \Delta A_i)x(k) + (A_{di} + \Delta A_{di})x(k-\tau) + N_i x(k)u(k) + N_{di} x(k-\tau)u(k) + D_{1i} w(k)) \end{aligned} \quad (3a)$$

$$\begin{aligned} y(k) = & \sum_{i=1}^r \mu_i(s(k)) ((C_i + \Delta C_i)x(k) + (C_{di} + \Delta C_{di})x(k-\tau) + D_{2i} w(k)) / \sum_{i=1}^r \mu_i(s(k)) \\ = & \sum_{i=1}^r h_i(s(k)) ((C_i + \Delta C_i)x(k) + (C_{di} + \Delta C_{di})x(k-\tau) + D_{2i} w(k)) \end{aligned} \quad (3b)$$

$$\begin{aligned} z(k) = & \sum_{i=1}^r \mu_i(s(k)) (R_i e(k) + R_{di} e(k-\tau)) / \sum_{i=1}^r \mu_i(s(k)) \\ = & \sum_{i=1}^r h_i(s(k)) (R_i e(k) + R_{di} e(k-\tau)) \end{aligned} \quad (3c)$$

Let $e(k) = x(k) - \hat{x}(k)$; from (1) and (4) so that we have the fuzzy observer error dynamics as follows:

$$\begin{aligned} e(k+1) = & \sum_{i=1}^r \sum_{j=1}^r h_i(s(k))h_j(s(k)) \left((A_i - L_i C_j) e(k) \right. \\ & + (A_{di} - L_i C_{dj}) e(k-\tau) + \delta_{fi}(k) + G_{li} \delta_{ai}(k) \\ & \left. + L_i G_{2j} \delta_{ej}(k) + (D_{li} + L_i D_{2j}) w(k) \right) \end{aligned} \quad (6)$$

where

$$\delta_{ai} = F_{li} (E_{li} x(k) + E_{2i} x(k-\tau)) \quad (7a)$$

$$\delta_{ci} = F_{2i} (E_{3i} x(k) + E_{4i} x(k-\tau)) \quad (7b)$$

$$\begin{aligned} \delta_{fi} = & (N_i x(k) u(k) + N_{di} x(k-\tau) u(k)) \\ & - (N_i \hat{x}(k) u(k) + N_{di} \hat{x}(k-\tau) u(k)) \\ = & (N_i e(k) + N_{di} e(k-\tau)) u(k) \end{aligned} \quad (7c)$$

According to (2), (5) and (7a)-(7c), we have the following results

$$\delta_{ai}^T(k) \delta_{ai}(k) \leq \begin{bmatrix} x(k) \\ x(k-\tau) \end{bmatrix}^T \Lambda_{ai} \begin{bmatrix} x(k) \\ x(k-\tau) \end{bmatrix} \quad (8a)$$

$$\delta_{ci}^T(k) \delta_{ci}(k) \leq \begin{bmatrix} x(k) \\ x(k-\tau) \end{bmatrix}^T \Lambda_{ci} \begin{bmatrix} x(k) \\ x(k-\tau) \end{bmatrix} \quad (8b)$$

$$\delta_{fi}^T(k) \delta_{fi}(k) \leq \begin{bmatrix} e(k) \\ e(k-\tau) \end{bmatrix}^T \Lambda_{fi} \begin{bmatrix} e(k) \\ e(k-\tau) \end{bmatrix} \quad (8c)$$

where

$$\Lambda_{ai} = \begin{bmatrix} E_{li}^T E_{li} & E_{li}^T E_{2i} \\ E_{2i}^T E_{li} & E_{2i}^T E_{2i} \end{bmatrix} \quad (9a)$$

$$\Lambda_{ci} = \begin{bmatrix} E_{3i}^T E_{3i} & E_{3i}^T E_{4i} \\ E_{4i}^T E_{3i} & E_{4i}^T E_{4i} \end{bmatrix} \quad (9b)$$

$$\Lambda_{fi} = \begin{bmatrix} \bar{N}_i^T \bar{N}_i & \bar{N}_i^T \bar{N}_{di} \\ \bar{N}_{di}^T \bar{N}_i & \bar{N}_{di}^T \bar{N}_{di} \end{bmatrix}. \quad (9c)$$

The purpose of this paper is to design the robust fuzzy observer parameters L_i such that the fuzzy observer error dynamics (6) are globally stable independent of the time delay τ and the following conditions H_∞ -norm upper-bound constraint is simultaneously guaranteed:

$$\|z(k)\|_{[0, \infty)} \leq \gamma \|w(k)\|_{[0, \infty)} \quad (10)$$

for all nonzero $w(k) \in l_2[0, +\infty)$ and all admissible uncertainties.

III. MAIN RESULTS

In this section, an LMI approach will be developed to solve the problem of robust H_∞ fuzzy observer for T-S time-delay DFBS with parameter uncertainties formulated in the previous section. Before discussing the theorem, we first give the following results which will be used in our main results.

Lemma 1: [6], [7], [23]: Given any matrices X , Y and ε with appropriate dimensions such that $\varepsilon > 0$ we have

$$X^T Y + X Y^T \leq \varepsilon X^T X + \varepsilon^{-1} Y^T Y.$$

Theorem 1: Consider the uncertain DFBS with disturbance (1). Given a prescribed H_∞ performance index $\gamma > 0$, if there exist positive scalars ε_{1ij} , ε_{2ij} , λ , and ρ_{ij} and matrices $P_1 > 0$; $P_2 > 0$; $Q_1 > 0$; $Q_2 > 0$ and X_i , satisfying the LMIs (11) and (12), shown at the next page, then the fuzzy observer error dynamics (6) with the fuzzy observer (4) with $L_i = P_1^{-1} X_i$ is globally asymptotically stable and with an H_∞ performance bound $\gamma > 0$.

Proof: Based on the Lyapunov direct method, Lipschitz conditions, and Lemma 1, one can obtain the LMIs (11) and (12). (Due to the limitation of space, we omit the proof.)

IV. NUMERICAL EXAMPLE

Consider the time-delay DFBS with parameter uncertainty as follows:

Rule 1:

IF x_1 is about -1

THEN

$$\begin{aligned} x(k+1) = & (A_1 + \Delta A_1) x(k) + (A_{d1} + \Delta A_{d1}) x(k-\tau) \\ & + (N_1 + \Delta N_1) x(k) u(k) \\ & + (N_{d1} + \Delta N_{d1}) x(k-\tau) u(k) + D_{11} w(k) \end{aligned} \quad (13a)$$

$$y(k) = (C_1 + \Delta C_1) x(k) + (C_{d1} + \Delta C_{d1}) x(k-\tau) + D_{21} w(k)$$

$$z(k) = R_1 e(k) + R_{d1} e(k-\tau)$$

Rule 2:

IF x_1 is about 1

THEN

$$\begin{aligned} x(k+1) = & (A_2 + \Delta A_2) x(k) + (N_2 + \Delta N_2) x(k) u(k) \\ & + (A_{d2} + \Delta A_{d2}) x(k-\tau) \\ & + (N_{d2} + \Delta N_{d2}) x(k-\tau) u(k) + D_{12} w(k) \end{aligned} \quad (13b)$$

$$y(k) = (C_2 + \Delta C_2) x(k) + (C_{d2} + \Delta C_{d2}) x(k-\tau) + D_{22} w(k)$$

$$z(k) = R_2 e(k) + R_{d2} e(k-\tau).$$

$$\left[\begin{array}{ccc} \Pi_{1ii} & 0 & 0 \\ 0 & \Pi_{2ii} & 0 \\ 0 & 0 & \Pi_{3ii} \end{array} \right] + \zeta_1 I \quad \Xi_{ii}^T(k) \quad \mathbf{0} \\ \Xi_{ii}(k) \quad -P \quad PS_{ii} \\ \mathbf{0} \quad S_{ii}^T P \quad -\left[\begin{array}{cc} \varepsilon_{1ii} I & 0 \\ 0 & \varepsilon_{2ii} I \end{array} \right] \end{array} \right] < 0 \quad (1 \leq i \leq r) \quad (11)$$

$$\left[\begin{array}{ccc} \Psi_{1ij} & 0 & 0 \\ 0 & \Psi_{2ij} & 0 \\ 0 & 0 & \Psi_{3ij} \end{array} \right] + \zeta_2 I \quad \Xi_{ij}^T(k) \quad \mathbf{0} \\ \Xi_{ij}(k) \quad -\left[\begin{array}{cc} P & 0 \\ 0 & P \end{array} \right] \quad \left[\begin{array}{cc} P & 0 \\ 0 & P \end{array} \right] S_{ij} \\ \mathbf{0} \quad S_{ij}^T \left[\begin{array}{cc} P & 0 \\ 0 & P \end{array} \right] \quad -\left[\begin{array}{cccc} \varepsilon_{1ij} I & 0 & 0 & 0 \\ 0 & \varepsilon_{2ij} I & 0 & 0 \\ 0 & 0 & \varepsilon_{1ji} I & 0 \\ 0 & 0 & 0 & \varepsilon_{2ji} I \end{array} \right] \end{array} \right] < 0 \quad (1 \leq i < j \leq r) \quad (12)$$

where $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$, $Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}$, $\Pi_{1ii} = \begin{bmatrix} P_1 - Q_1 & 0 \\ 0 & Q_1 \end{bmatrix} - [\varepsilon_{1ii}(\Lambda_{ai} + \Lambda_{fi}) + \varepsilon_{2ii}\Lambda_{ai} + \lambda\Lambda_{ci}]$, $S_{ii} = \begin{bmatrix} S_i & 0 \\ 0 & S_i \end{bmatrix}$,

$$S_{ij} = \begin{bmatrix} S_i & 0 & 0 & 0 \\ 0 & S_j & 0 & 0 \\ 0 & 0 & S_i & 0 \\ 0 & 0 & 0 & S_j \end{bmatrix}, S_i = [G_{1i} \quad I], \Lambda_{ci} = \begin{bmatrix} E_{3i}^T E_{3i} & E_{3i}^T E_{4i} \\ E_{4i}^T E_{3i} & E_{4i}^T E_{4i} \end{bmatrix}, \Pi_{2ii} = \begin{bmatrix} P_2 - Q_2 & 0 \\ 0 & Q_2 \end{bmatrix} - (\varepsilon_{2ii}\Lambda_{fi} + \Lambda_{ri}), \Pi_{3ii} = \begin{bmatrix} \lambda I & 0 \\ 0 & \gamma^2 I \end{bmatrix},$$

$$\Psi_{1ij} = 2 \begin{bmatrix} P_1 - Q_1 & 0 \\ 0 & Q_1 \end{bmatrix} - [\varepsilon_{1ij}(\Lambda_{ai} + \Lambda_{fi}) + \varepsilon_{1ji}(\Lambda_{aj} + \Lambda_{fj}) + \varepsilon_{2ij}\Lambda_{ai} + \varepsilon_{2ji}\Lambda_{aj} + \lambda(\Lambda_{ci} + \Lambda_{cj})],$$

$$\Psi_{2ij} = 2 \begin{bmatrix} P_2 - Q_2 & 0 \\ 0 & Q_2 \end{bmatrix} - (\varepsilon_{2ij}\Lambda_{fi} + \varepsilon_{2ji}\Lambda_{fj} + \Lambda_{rij}), \Psi_{3ij} = \begin{bmatrix} \lambda I & 0 & 0 \\ 0 & \lambda I & 0 \\ 0 & 0 & 2\gamma^2 I \end{bmatrix},$$

$$\Xi_{ii}(k) = \Xi_{ii} = \begin{bmatrix} P_1 A_i & P_1 A_{di} & 0 & 0 & 0 & P_1 D_{1i} \\ 0 & 0 & P_2 A_i - X_i C_i & P_2 A_{di} - X_i C_{di} & X_i G_i & P_2 D_{1i} + X_i D_{2i} \end{bmatrix},$$

$$\Xi_{ij}(k) = \Xi_{ij} = \begin{bmatrix} P_1 A_i & P_1 A_{di} & 0 & 0 & 0 & 0 & P_1 D_{1i} \\ 0 & 0 & P_2 A_i - X_i C_j & P_2 A_{di} - X_i C_{dj} & 0 & X_i G_{2j} & P_2 D_{1i} + X_i D_{2j} \\ P_1 A_j & P_1 A_{dj} & 0 & 0 & 0 & 0 & P_1 D_{1j} \\ 0 & 0 & P_2 A_j - X_j C_i & P_2 A_{dj} - X_j C_{di} & X_j G_{2i} & 0 & P_2 D_{1j} + X_j D_{2i} \end{bmatrix},$$

$$\Lambda_{ri} = \begin{bmatrix} R_i^T R_i & R_i^T R_{di} \\ R_{di}^T R_i & R_{di}^T R_{di} \end{bmatrix}, \Lambda_{rij} = \frac{1}{2} \begin{bmatrix} (R_i + R_j)^T (R_i + R_j) & (R_i + R_j)^T (R_{di} + R_{dj}) \\ (R_{di} + R_{dj})^T (R_i + R_j) & (R_{di} + R_{dj})^T (R_{di} + R_{dj}) \end{bmatrix}.$$

where $A_1 = \begin{bmatrix} 0.4 & 0.01 \\ 0.02 & -0.55 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0.35 & 0.02 \\ -0.2 & -0.5 \end{bmatrix}$,
 $N_1 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.1 \end{bmatrix}$, $N_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $A_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$,
 $A_{d2} = \begin{bmatrix} -0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}$, $N_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.05 \end{bmatrix}$, $N_{d2} = \begin{bmatrix} 1 & 0 \\ 0 & 0.03 \end{bmatrix}$,
 $D_{11} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix}$, $D_{12} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $C_1 = [0.1 \ 0]$,
 $C_2 = [0.1 \ 0]$, $C_{d1} = [-0.2 \ -0.3]$, $C_{d2} = [0.1 \ -0.1]$,
 $D_{21} = [0.2 \ 0.7]$, $D_{22} = [0.3 \ 0.5]$, $R_1 = [0 \ 0.2]$,
 $R_2 = [0.3 \ 0]$, $R_{d1} = [-0.2 \ -0.1]$, $R_{d2} = [0.1 \ -0.2]$ and
 $\tau = 1$. The membership functions of the state x_1 are defined as
 $\mu_1(k) = (1 - x_1) / 2$ and $\mu_2(k) = (1 + x_1) / 2$.

Based on Assumption 1, we can define the matrices G_{ij} , $F_i(t)$, E_{1i} , E_{2i} , E_{3i} , and E_{4i} , where $i, j = 1, 2$, as follows.

$$G_{11} = \begin{bmatrix} 0.01 & -0.1 \\ 0.1 & 0.1 \end{bmatrix}, \quad G_{12} = \begin{bmatrix} 0.02 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, \quad G_{21} = [0.02 \ 0.1],$$

$$G_{22} = [0.01 \ 0.1]$$

$$F_1(t) = F_2(t) = 0.1 \times \begin{bmatrix} \text{rand} & 0 \\ 0 & \text{rand} \end{bmatrix}$$

where *rand* is a random number taken from a normal distribution over $[-1, 1]$.

$$E_{11} = E_{12} = E_{31} = E_{32} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$E_{21} = E_{22} = E_{41} = E_{42} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}.$$

By solving the solution to the stability conditions in Theorem 1 using MATLAB LMI toolbox, with $\gamma = 1$, $\lambda = 1$ and $\zeta_1 = \zeta_2 = 0.0001$, where $i, j = 1, 2$, we have

$$P_1 = \begin{bmatrix} 0.4332 & 0.0061 \\ 0.0061 & 0.4188 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.5421 & -0.0191 \\ -0.0191 & 0.4226 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.1333 & -0.0008 \\ -0.0008 & 0.0894 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.1458 & -0.0245 \\ -0.0245 & 0.1054 \end{bmatrix},$$

$$X_1 = \begin{bmatrix} -0.1373 \\ -0.1201 \end{bmatrix}, \quad X_2 = \begin{bmatrix} -0.0312 \\ -0.1830 \end{bmatrix} \text{ and the robust } H_\infty \text{ fuzzy}$$

observers is (4) with

$$L_1 = P_1^{-1} X_1 = \begin{bmatrix} -0.2991 \\ -0.2824 \end{bmatrix}$$

$$L_2 = P_1^{-1} X_2 = \begin{bmatrix} -0.0629 \\ -0.4361 \end{bmatrix}.$$

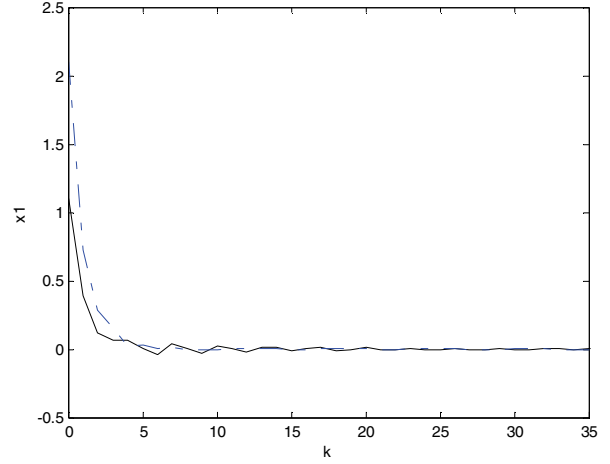


Fig. 1. Responses of $x_1(k)$ and $\hat{x}_1(k)$ for time-delay DFBS with parameter uncertainties.

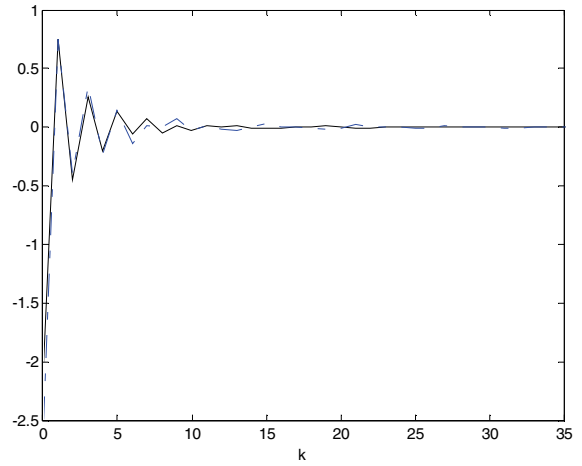


Fig. 2. Responses of $x_2(k)$ and $\hat{x}_2(k)$ for time-delay DFBS with parameter uncertainties.

Applying the controller proposed in [20] with $\rho = 0.3$ and control gain $K_1 = [-0.275 \ -0.35]$ and $K_2 = [-0.35 \ -0.25]$ to time-delay DFBS with parameter uncertainties (13). The time-delay DFBS with parameter uncertainties was simulated by choosing a system's initial conditions $x(0) = [1.1 \ -2.0]^T$ and observer's initial condition $\hat{x}(0) = [2.1 \ -2.5]^T$ and the exogenous disturbance input $w(k) = [0.5e^{-0.1k} \sin(2k) \ 0.3e^{-0.08k} \cos(k)]^T$ are illustrated in Fig. 1 and Fig. 2. From the simulation results, one can find that the state variables of time-delay DFBSs with parameter uncertainties (13) and their estimated by time-delay DFBS observer are almost the same after a short transient instant. Fig. 3 shows the control input. Fig. 4 shows that the designed fuzzy observer ensures the robust asymptotic stability of fuzzy observer error dynamics (6) and guarantees a prescribed H_∞ performance level.

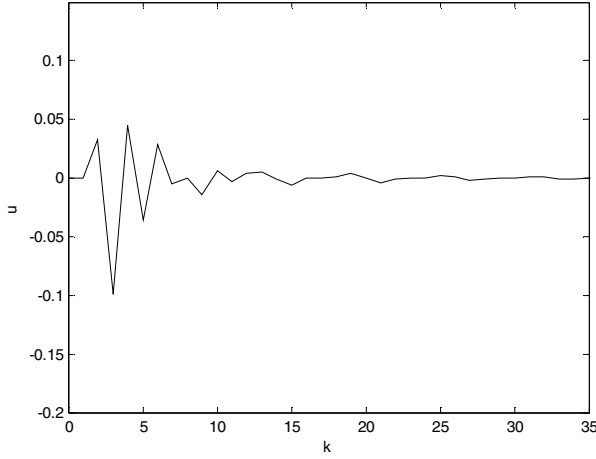


Fig. 3. Control input $u(k)$ for time-delay DFBS with parameter uncertainties.

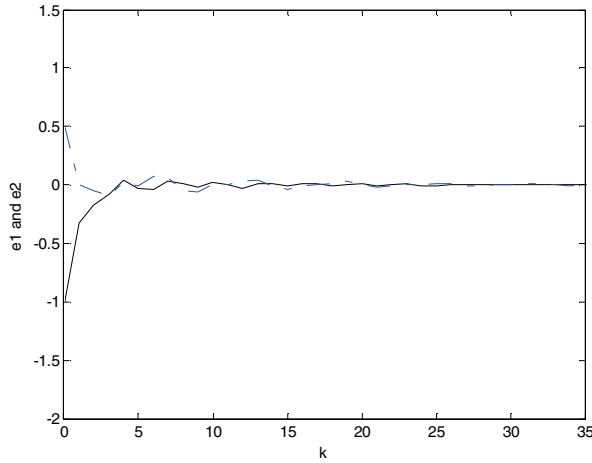


Fig. 4. Responses of e_1 and e_2 for initial conditions $x(0)=[1.1 \ -2.0]^T$ and $\hat{x}(0)=[2.1 \ -2.5]^T$.

V. CONSLUTION

In this paper, a robust H_∞ fuzzy observer has been presented for a class of time-delay DFBSs with parameter uncertainties. Based on the Lipschitz condition, the design problem of the H_∞ fuzzy observer can be reformulated as an LMI problem. Besides, the robust H_∞ fuzzy observer for time-delay DFBSs with parameter uncertainties can be obtained from the proposed LMIs without any pre-assigned matrices. The simulation has shown the effectiveness and feasibility of the proposed method to the reconstruction of the state.

ACKNOWLEDGMENT

The work is supported by the National Science Council, Taiwan, under grants NSC-99-2221-E-027-101 and NSC-98-2622-E-168-013-CC3.

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