

Multi-level Multi-Objective Decision Problem through Fuzzy Random Regression based Objective Function

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Abstract—A multi-level decision making problem confronts several issues especially in coordinating decision in hierarchic processes and in compromising conflicting objectives for each decision level. Therefore, its mathematical model plays a pivotal role in solving such problem, and is influencing to the final result. However, it is sometimes difficult to estimate the coefficients of objective functions of the model in real situations specifically when the statistical data contain random and fuzzy information. Thus, decision making scheme should provide an appropriate method to handle the presence of such uncertainties. Hence, this paper proposes a fuzzy random regression method to estimate the coefficients value for the objective functions of multi-level multi-objective model. The algorithm is constructed to obtain a satisfaction solution, which fulfills at least weak Pareto optimality. A numerical example illustrates the proposed solution procedure.

Keywords- *fuzzy random regression model, multi-level problem; multi-objective; additive fuzzy goal programming.*

I. INTRODUCTION

Organizational decision making is usually based on the multi-level structure that consists of the top-level decision maker and subsequent levels of decision making power. At least, one decision maker (DM) is located at each level and controls a decision for optimizing their level or unit's objectives. Decision making process in multi-level organization should pay an attention to the central administrations issues and the coordination among lower-level activities. Although the execution of decision moves from an upper level to a lower level, the reaction of a lower-level decision maker affects the optimization of the decision at an upper level decision maker ([1]; [2]; [3]). In such cases, a proper distribution of decision powers with respect to problem belongs to the DM in the respective level must be addressed in most multi-level decision situations.

The structure of multi-objective problem in each level of organization increases the complexities of decision making and planning. DM has to resolve several conflicting objectives at different levels and preference which cause dissatisfaction among decision makers. Although the standard mathematical

programming problems are able to solve the multi-objective problem, it is usually considering only one DM in a single-level structure. Thus, multi-objective optimizations in the multi-level structure are required to address this problem. Mathematical programming that takes a multi-level structure, with each level controlling several conflicting objectives is known as multi-level multi-objective programming ([4]; [5]).

In a multi-level decision-making context, each DM which represents decision-making unit should have a motivation to cooperate with others. The satisfactions of a DM at other levels are important and must be considered to accomplish overall organization's goal. Therefore, a satisfaction approach is needed to support the decision-making in multi-level structure that confronts a multiple objective problem at each level. Even though the standard mathematical programming was used to solve the multi-objective problem, these kinds of model are basically unable to accommodate multi-objective problem in multi-level structure due to the presence of decision makers in each level of decision structure.

Moreover, one often needs to make a decision under uncertainty in the real-situation decision making problems. In this study, uncertainties are characterized by fuzzy and random information. The uncertainties were observed during the model settings where the statistical data used to estimate the coefficients may include either random or fuzzy information. We explain the situation as follows. Assume that a profit of crop is reliant on weathers, which this scenarios occur in random. The profit itself may be estimated as a fuzzy number as well. That is, the profit is expressed by a fuzzy random variable. Furthermore, if multiple decision makers are involved in evaluating the same subject, for instance in the multi-attribute evaluation problem, the difference of decision maker's evaluation also should be considered ([6]; [7]). Hence, such situation may take place in real situation and should be handled sufficiently.

Additionally, estimating the model coefficients is essential under the coexistence of fuzzy and random information in the decision evaluation. Though, the coefficients usually are not exactly known, as relevant data are sometimes not given or are

sometimes difficult to obtain. However, it makes these decisions crucial and influential to the model's result and the occurrence of errors in the determination of the coefficients might ruin the model formulation [8]. Consequently, the model coefficients are then important in the model settings and should be determined.

Motivated by the above situations, in this paper the satisfaction-based optimization method of additive FGP is introduced to solve multi-level multi-objective (MLMO) problem where the objective functions are developed from Fuzzy Random Regression (FRR) model in the initial phase of model setting. The methodology presented in this paper embraces two objectives. First, the FRR model is used to estimate the coefficients and develop the fuzzy random based objective functions for multi-objective model settings. Second, the additive FGP method is then used to solve the multi-objective linear problem in the multi-level decision making structures.

The remainder of this paper is divided into six sections. Section II describes the construction of fuzzy random objective function. Section III explains important components of FGP based on additive model. Section IV describes the algorithm for solving MLMO decision making with fuzzy random objective function built in prior. An illustrative example of production planning is presented in Section V. Discussions and conclusions are given in Sections VI and VII.

II. BUILDING FUZZY RANDOM OBJECTIVE FUNCTION

Building appropriate objective function in the multi-objective model problem is important and influence the overall model result. Thus this section is spent to explain the solution to estimate the model coefficient and further develop a fuzzy random objective function by using FRR model [6].

The FRR model is constructed based on fuzzy random variables and its confidence intervals. To describe the fuzzy random variables, let Pos be a possibility measure defined on the power set $P(\Gamma)$ in a universe Γ . Let $\Re$ be the set of real numbers. A function $Y : \Gamma \rightarrow \Re$ is a fuzzy variable defined on Γ [9]. The possibility distribution μ_Y of Y is defined by $\mu_Y(t) = Pos\{Y = t\}$, $t \in \Re$ which is the possibility of the event $\{Y = t\}$. The possibility and necessity of the event $\{Y \leq r\}$ for fuzzy variable Y with possibility distribution μ_Y , are given respectively in the following forms:

$$\begin{aligned} Pos\{Y \leq r\} &= \sup_{t \leq r} \mu_Y(t), \\ Nec\{Y \leq r\} &= 1 - \sup_{t > r} \mu_Y(t). \end{aligned} \quad (1)$$

The expected value of a fuzzy variable is defined as follows:

Definition 1: Let Y be a fuzzy variable. The expected value of Y where the two integrals are finite is defined as

$$\begin{aligned} E[Y] &= \int_0^\infty \left(\frac{1}{2} \left[1 + \sup_{t \geq r} \mu_Y(t) - \sup_{t < r} \mu_Y(t) \right] \right) dr \\ &\quad - \int_{-\infty}^0 \left(\frac{1}{2} \left[1 + \sup_{t \leq r} \mu_Y(t) - \sup_{t > r} \mu_Y(t) \right] \right) dr, \end{aligned} \quad (2)$$

From (1), the expected value of Y is

$$E[Y] = \frac{1}{4} (a^l + 2c + a^r) \quad \text{when } Y \text{ is a triangular fuzzy}$$

number (c, a^l, a^r) . Fuzzy random variables and their expected value operators are defined as follows.

Definition 2: Let (Ω, Σ, Pr) be a probability space and F_ν , a collection of fuzzy variables defined on possibility space $(\Gamma, P(\Gamma), Pos)$. A fuzzy random variable is a map $X : \Omega \rightarrow F_\nu$ such that for any Borel subset B of $\Re$, $Pos\{X(\omega) \in B\}$ is a measureable function of ω .

Let X be a fuzzy random variable on Ω . For any fuzzy random variable X on Ω , the expected value of the fuzzy variable $X(\omega)$ is denoted by $E[X(\omega)]$ [10].

The expected value of the fuzzy random variable X is then defined as the mathematical expectation of the random variable $E[X(\omega)]$.

Definition 3: Let X be a fuzzy random variable defined on a probability space (Ω, Σ, Pr) . The expected value of X is defined as

$$E[X] = \int_\Omega \left[\int_0^\infty \left(\frac{1}{2} [a] \right) dr - \int_{-\infty}^0 \left(\frac{1}{2} [b] \right) dr \right] Pr(d\omega) \quad (3)$$

where

$$a = 1 + \sup_{t \geq r} \mu_{X(\omega)^{(r)}} - \sup_{t < r} \mu_{X(\omega)^{(r)}} \quad \text{and}$$

$$b = 1 + \sup_{t \leq r} \mu_{X(\omega)^{(r)}} - \sup_{t > r} \mu_{X(\omega)^{(r)}}.$$

Definition 4: Let X be a fuzzy random variable defined on a probability space (Ω, Σ, Pr) with expected value e . The variance of X is defined based on $e = E[X]$ from Definition 3, as follows:

$$var[X] = E[(X - e)^2]. \quad (4)$$

The fuzzy random data Y_i and X_{ik} for all $i = 1, \dots, N$ and

$k = 1, \dots, K$ are defined as $Y_i = \bigcup_{t=1}^{M_{Y_i}} \{(Y_i^t, Y_i^{t,l}, Y_i^{t,r})_A, p_i^t\}$, and

$X_{ik} = \bigcup_{t=1}^{M_{X_{ik}}} \{(X_{ik}^t, X_{ik}^{t,l}, X_{ik}^{t,r})_A, q_{ik}^t\}$, with probability p_i^t and

q_{ik}^t for $i = 1, \dots, N$, $k = 1, \dots, K$ and $t = 1, \dots, M$ or $t = 1, \dots, M_{X_{ik}}$, respectively.

Let us denote a fuzzy linear model with fuzzy coefficients $\tilde{C}_i, \dots, \tilde{C}_K$ as $\bar{Y}_i = \tilde{C}_i X_{i1} + \dots + \tilde{C}_K X_{iK}$. Each $\bar{Y}_i$ denotes an estimate of the output and

$\tilde{C}_k = \left(\frac{1}{2} [\tilde{c}_k^l + \tilde{c}_k^r], \tilde{c}_k^l, \tilde{c}_k^r \right)_\Delta$ are symmetric triangular fuzzy coefficients for given triangular fuzzy random data X_{ik} for $i = 1, \dots, N$ and $k = 1, \dots, K$.

The input data $X_{ik} = (x_{ik}, x_{ik}^l, x_{ik}^r)_\Delta$ and output data $\bar{Y}_i = (y_i, y_i^l, y_i^r)_\Delta$ for all $i = 1, \dots, N$ and $k = 1, \dots, K$, are fuzzy random variables. Therefore, the following relation should hold:

$$\bar{Y}_i = \tilde{C}_i X_{i1} + \dots + \tilde{C}_K X_{iK} \supset_{FR} Y_i, \quad i = 1, \dots, N, \quad (5)$$

where $\supset_{FR}$ is a fuzzy random inclusion relation. The inclusion relation is based on the expectation-based inclusion and the fuzzy random regression was converted to the expected value regression model with fuzzy inclusion relation at level h [11].

By using the one-sigma ($1 \times \sigma$) confidence interval, the interval is obtained by the expectation and variance of a fuzzy random variable as follows:

$$I[e_X, \sigma_X] \underline{\Delta} [E(X) - \sqrt{\text{var}(X)}, E(X) + \sqrt{\text{var}(X)}] \quad (6)$$

Hence, the FRR model with $1 \times \sigma$ - confidence intervals is expressed as follows:

$$\begin{aligned} \min_A J(C) &= \sum_{k=1}^K (\tilde{C}_k^l - \tilde{C}_k^r) \\ \tilde{C}_k^r &\geq \tilde{C}_k^l, \\ Y_i &= \tilde{C}_i I[e_{X_{i1}}, \sigma_{X_{i1}}] + \dots + \tilde{C}_K I[e_{X_{iK}}, \sigma_{X_{iK}}] \supset_{FR} Y_i \\ &\text{for } i=1, \dots, N \quad k=1, \dots, K. \end{aligned} \quad (7)$$

The FRR model with the confidence interval is given in the following form:

$$\bar{Y}_i^* = \tilde{C}_K I[e_{X_{iK}} + \sigma_{X_{iK}}], \quad i = 1, \dots, N \quad (8)$$

Using the fuzzy random variables-based regression model, we estimate the coefficient ξ_{ij} for decision parameters. The coefficients of FRR model results in a close interval $[c_j^l, c_j^r]$.

Considering the center value of $\hat{\xi}_{ij} = \frac{(c_j^l + c_j^r)}{2}$, the fuzzy random based objective function is built as

$$F_j(x) = \{\bar{Y}_i = \xi_{ij}(X_i)\} \quad (9)$$

Hence, the coefficients c_{ij} of objective function are identified through regression model that is based on a hybrid uncertainty including the fuzziness and the randomness that co-exist in formal decision problem, and these objective functions are further used in the setting of multi-objective model.

III. MLMO DECISION MAKING BASED ON FUZZY GOAL PROGRAMMING: AN ADDITIVE MODEL.

Let us consider an organization that has a multi-level programming problem with multi-objective function $f_i(x)$ for $i = 1, \dots, p$. Assume that the decisions are made

sequentially beginning with top level DM_1 , which controls a vector $x_1 \in X_1$, down through DM_p , which controls a vector $x_p \in X_p$, where x_i is nonempty subset of $\Re^{n_i}$, $i = 1, \dots, p$, $n_i = n_1 + \dots + n_p$ and $x_i = x_1, \dots, x_p$.

The decision maker DM_i at the i^{th} level has authority over the decision variable x_i . Mathematically, the multi-level multi-objective linear programming problem is a nested optimization problem ([1]; [12]) and has the following structures:

$$\left. \begin{aligned} &\text{Find } x \text{ so as to satisfy} \\ &\min_{x_1 \in X_1} F_1(x) = \min_{x_1} \{f_{11}(x), \dots, f_{1m_1}(x)\} \\ &\text{where } x_1 \text{ solves} \\ &\min_{x_2 \in X_2} F_2(x) = \min_{x_2} \{f_{21}(x), \dots, f_{2m_2}(x)\} \\ &\vdots \\ &\text{where } x_2, \dots, x_p \text{ solves} \\ &\min_{x_p \in X_p} F_p(x) = \min_{x_p} \{f_{p1}(x), \dots, f_{pm_p}(x)\} \\ &\text{subject to :} \\ &x \in X \\ &X = \{x \in \Re^n : Ax(\leq, \geq, =)b, x \geq 0\} \end{aligned} \right\} \begin{array}{l} \text{level 1} \\ \text{level 2} \\ \vdots \\ \text{level } p \end{array} \quad (10)$$

where $f_{ij}(x) = c_1^{ij}x_1 + \dots + c_p^{ij}x_p$, $i = 1, \dots, p$, $j = 1, \dots, m_i$, is the objective function. c_k^{ij} are the constants, and A_i are coefficients matrices of size $m \times n_i$. n_i is the number of decision makers involved.

The FGP [13] based optimization solution is utilized to solve problem model (10). The goal value g_{ij} and tolerance value μ_{ij} are determined by computing the individual best and least solution to obtain the satisfaction degree [14].

The linear membership functions for the fuzzy objective $\mu_{f_{ij}}(x)$ for maximization case of $F_i(x) \gtrless g_i$ can be formulated as follows:

$$\mu_{F_i}(x) = \begin{cases} 1 & \text{if } f_{ij}(x) \geq \mu_{ij} \\ \frac{f_{ij}(x) - \mu_{ij}}{g_{ij} - \mu_{ij}} & \text{if } \mu_{ij} \leq f_{ij}(x) \leq g_{ij} \\ 0 & \text{if } f_{ij}(x) \leq \mu_{ij} \end{cases} \quad (11)$$

In a multi-level decision-making situation in which the decisions made at each level are influenced by the upper level's decision, there exist decision variables that are controlled by upper-level decision makers. Let t_k^{iL} and t_k^{iR} be the lower bound and upper bound of tolerance values for the decision vectors considered by the i^{th} level decision maker, respectively.

The triangular fuzzy numbers of decision vectors x_{ik} are $(x_{ik}^* - t_k^{i_L}, x_{ik}^*, x_{ik}^* + t_k^{i_R})$. Thus, the linear membership functions for each of the K_i components of the decision vector $x^{i*} = (x_1^{i*}, \dots, x_p^{i*})$ controlled by the decision makers of the upper $p-1$ levels can be formulated as follows:

$$\mu_{x_{ik}}(x) = \begin{cases} \frac{x_{ik} - (x_{ik}^* - t_k^{i_L})}{t_k^{i_L}} & \text{if } x_{ik}^* - t_k^{i_L} \leq x_{ik} \leq x_{ik}^* \\ \frac{(x_{ik}^* + t_k^{i_R}) - x_{ik}}{t_k^{i_R}} & \text{if } x_{ik}^* \leq x_{ik} \leq x_{ik}^* + t_k^{i_R} \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

Let V represent the fuzzy achievement function consisting of membership functions for fuzzy objectives μ_i . Recall the fuzzy random objective function of Equation (9), the additive FGP model for multi-level decision-making with fuzzy random objective function is written as follows:

$$\begin{aligned} \max \quad & V(\mu_p) = \sum_{j=1}^{m_1} \mu_{1j} + \dots + \sum_{j=1}^{m_{p-1}} \mu_{pj} + \sum_{k=1}^{n_{p-1}} [\mu(x)_{p-1k}^L + \mu(x)_{p-1k}^R] \\ \text{subject to} \quad & \mu_{F_i}(x) = \frac{\mu_{ij} - (g_{ij}^L x_1 + \dots + g_{ij}^L x_p)}{g_{ij} - \mu_{ij}}, \quad i = 1, \dots, p, j = 1, \dots, m_i, \\ & \mu_{x_{ik}}^L(x) = \frac{x_{ik} - (x_{ik}^* - t_k^{i_L})}{t_k^{i_L}}, \quad i = 1, \dots, p-1, k = 1, \dots, m_{K_i}, \\ & \mu_{x_{ik}}^R(x) = \frac{(x_{ik}^* + t_k^{i_R}) - x_{ik}}{t_k^{i_R}}, \quad i = 1, \dots, p-1, k = 1, \dots, m_{K_i}, \\ & A_1 x_1 + \dots + A_p x_p (\leq, \geq, =) b, \quad x \geq 0, \\ & \mu_i \in [0, 1], \quad i = 1, \dots, m. \end{aligned} \quad (13)$$

Because the objectives are fuzzy unlike the conventional goal programming that minimizes deviation, the fuzzy decision function consisting of μ_i must be maximized.

IV. THE ADDITIVE FUZZY GOAL PROGRAMMING FOR MLMO PROBLEM BASED ON FUZZY RANDOM OBJECTIVE FUNCTION

The process flow of decision-making under multi-level additive FGP model is illustrated in the following algorithm. The additive FGP is separately solved for the i^{th} level multi-objective program where $i = 1, \dots, p-1$.

Phase I: Building fuzzy random objective functions

- 1) Determine all the objectives for MLMO problem.
- 2) Solve each objective described in the model to find the estimate's coefficients. For each of the goal constraints $f_{ij}(x)$,
 - prepare the fuzzy random input-output data.
 - calculate the one sigma confidence interval in terms of Equation (6).
 - perform FRR model by means of Equation (7).

- 3) Build fuzzy random based objective function as Equation (9) including the confidence interval value.
- 4) Formulate the MLMO linear program model (10) to illustrate the model for the respective problem. Substitute the fuzzy random-based objective function as Equation (9) that is derived from Step 3) in the MLMO model.

Phase II: Solving MLMO problems

For each objective established at each level in the MLMO Model (10), compute the individual solutions to determine the target goal g_{ij} and tolerance μ_{ij} , respectively.

A: First-Level Additive-FGP

- 1) Assume $p = l$. Set $l = 1$.
- 2) Create the fuzzy achievement function V_{μ_1} consisting of μ_{ij} for $i = 1, \dots, p$ $j = 1, \dots, m_i$ in level 1.
- 3) Create the fuzzy objective membership function $\mu_{F_i}(x)$ with the aspiration value g_{ij} and μ_{ij} for the fuzzy goals of the first level controlled by DM_1 .
- 4) Formulates the additive FGP model based on Model (10) for the first-level problem to obtain a satisfactory solution.
- 5) Send the obtained solutions of the objective function $x^{1*} = (x_1^{1*}, \dots, x_p^{1*})$ decisions to the next lower level.

B: Second-Level towards p^{th} -Level Additive-FGP.

- 1) Set $l = l + 1$. Determine the obtained solutions $x^{l-1*} = (x_1^{l-1*}, \dots, x_p^{l-1*})$ decisions from the higher level $l-1$.
- 2) Let the decision maker decide the lower bound and upper bound of the tolerance values $t_k^{i_L}$ and $t_k^{i_R}$ on the decision vector $x^{l-1*} = (x_1^{1*}, \dots, x_p^{1*})$.
- 3) Develop the fuzzy achievement function V_{μ_p} consisting of $\mu_{f_{ij}}$ for $i = 1, \dots, l$, $j = 1, \dots, m_l$ and $\mu_{x_{ik}}$ for $i = 1, \dots, l-1$; $k = 1, \dots, n_{l-1}$.
- 4) Create the fuzzy objectives membership function $\mu_{f_i}(x)$ with the aspiration values g_{ij} and μ_{ij} for the fuzzy goals of the first level controlled by the DM_1 .
- 5) Develop the decision vector membership function $\mu_{x_{ik}}(x)$, $i = 1, \dots, p-1$, $k = 1, \dots, K_i$ based on Equation (12).
- 6) Formulate the additive-FGP Model (10) of the p -level problem to obtain a satisfactory solution to the first-level FGP problem.
- 7) Solve the model to obtain $x^{l*} = (x_1^{l*}, \dots, x_p^{l*})$.
- 8) Repeat step 1 until $l < p$, to generate a satisfactory solution $x^{l*} = (x_1^{l*}, \dots, x_p^{l*})$ for the MLMO linear programming problem.

TABLE I. THE OBJECTIVES

Decision Maker	The Objectives
Government level, DM_1 (upper-level)	f_{11} Maximize the export revenue
	f_{12} Maximize the national level profit
	f_{13} Minimize the capital,
State level, DM_2 (middle-level)	f_{21} Maximize the production volume
	f_{22} Maximize the profit for state level
	f_{23} Minimize the cost of production
City level, DM_3 (lower-level)	f_{31} Maximize the production volume
	f_{32} Minimize the cost of production

V. NUMERICAL EVALUATIONS AND DISCUSSIONS

The following multi-objective linear programming problem is illustrated to demonstrate the proposed procedure. The problem situation is described as follows. One export-oriented country is concentrating on producing three important products x_1 , x_2 and x_3 which are manufactured by company C_m , $m = 1, \dots, M$. This company has distributed branch B_m , $m = 1, \dots, M$, in city level for producing the products.

There are 3 levels of decision making related to this case and each level are responsible to accomplish the objectives that are decided in prior. Table 1 summarizes the objectives decided in each level.

Let us assume f_{ij} represent the DM_i objective in each level. The multi-level multi-objective problem can be summarized as follows:

Find x so as to satisfy:

[Level1]

$$\min_{x_1} F_1(x) = \{f_{11}(x), f_{12}(x), f_{13}(x)\},$$

where x_2 and x_3 solve

[Level2]

$$\min_{x_2} F_2(x) = \{f_{21}(x), f_{22}(x), f_{23}(x)\},$$

where x_3 solves

[Level3]

$$\min_{x_3} F_3(x) = \{f_{31}(x), f_{32}(x)\},$$

Subject to constraints:

$$\begin{aligned} \text{raw} & 3.815x_1 + 0.910x_2 + 0.220x_3 \leq 87.75; \\ \text{labor} & 0.650x_1 + 0.900x_2 + 0.125x_3 \leq 4.425; \\ \text{mills} & 17.50x_1 + 2.160x_2 + 4.775x_3 \leq 95.20; \\ \text{capital} & 1.350x_1 + 0.980x_2 + 0.890x_3 \leq 20.15; \end{aligned} \quad (15)$$

Phase I: Building fuzzy random objective functions

The initial step in this phase is the data preparation to estimate the decision coefficients through a fuzzy random-based regression and further develop the objective function.

The previously collected data set is then pre-processed ([7]; [15]). In this example, three types of product are produced from three different production plants. The probabilities are assigned as the proportion of product being produced in i^{th} plant to the total production numbers.

An FRR model (7) is used to estimate the weights of the decision parameters x_1 , x_2 and x_3 and to develop the objective function for each objective that was defined in Table 1. Using the fuzzy random data format of input-output data, we solve the linear program (7) and obtain the estimated coefficients c_i^l and c_i^r for all objectives. For the other objective functions we run the same procedure. The interval $[c_i^l, c_i^r]$ shows the estimated coefficients for the decision parameters.

The objective functions are developed based on the central value of the interval coefficients as follows:

$$F_1 = \begin{cases} f_{11} = (2.078)_T I[e_{x_1}, \sigma_{x_1}] + (0.260)_T I[e_{x_2}, \sigma_{x_2}] + (0.170)_T I[e_{x_3}, \sigma_{x_3}] \\ f_{12} = (1.010)_T I[e_{x_1}, \sigma_{x_1}] + (1.700)_T I[e_{x_2}, \sigma_{x_2}] + (0.476)_T I[e_{x_3}, \sigma_{x_3}] \\ f_{13} = (0.438)_T I[e_{x_1}, \sigma_{x_1}] + (0.796)_T I[e_{x_2}, \sigma_{x_2}] + (0.512)_T I[e_{x_3}, \sigma_{x_3}] \end{cases} \quad (16)$$

$$F_2 = \begin{cases} f_{21} = (1.126)_T I[e_{x_1}, \sigma_{x_1}] + (0.100)_T I[e_{x_2}, \sigma_{x_2}] + (0.097)_T I[e_{x_3}, \sigma_{x_3}] \\ f_{22} = (0.856)_T I[e_{x_1}, \sigma_{x_1}] + (1.473)_T I[e_{x_2}, \sigma_{x_2}] + (0.443)_T I[e_{x_3}, \sigma_{x_3}] \\ f_{23} = (0.380)_T I[e_{x_1}, \sigma_{x_1}] + (0.737)_T I[e_{x_2}, \sigma_{x_2}] + (0.277)_T I[e_{x_3}, \sigma_{x_3}] \end{cases} \quad (17)$$

$$F_3 = \begin{cases} f_{31} = (0.921)_T I[e_{x_1}, \sigma_{x_1}] + (0.050)_T I[e_{x_2}, \sigma_{x_2}] + (0.526)_T I[e_{x_3}, \sigma_{x_3}] \\ f_{32} = (0.380)_T I[e_{x_1}, \sigma_{x_1}] + (0.737)_T I[e_{x_2}, \sigma_{x_2}] + (0.216)_T I[e_{x_3}, \sigma_{x_3}] \end{cases} \quad (18)$$

The three sets of objective functions (16), (17) and (18) represent the objective functions on each decision making level. At this step, we end the Phase I on the development of objective functions. Next, we move to the second phase to solve the MLMO problem through an additive-FGP approach.

Phase II: Solving MLMO problem

The MLMO problem (10) can be rewritten by considering the result from Phase I as follows:

$$\min_{x_1} = \begin{cases} f_{11} = 2.078x_1 + 0.260x_2 + 0.170x_3, \\ f_{12} = 1.010x_1 + 1.700x_2 + 0.476x_3, \\ f_{13} = 0.438x_1 + 0.796x_2 + 0.512x_3. \end{cases}$$

where x_2 and x_3 solves,

$$\min_{x_2} = \begin{cases} f_{21} = 1.126x_1 + 0.100x_2 + 0.097x_3, \\ f_{22} = 0.856x_1 + 1.473x_2 + 0.443x_3, \\ f_{23} = 0.380x_1 + 0.737x_2 + 0.277x_3. \end{cases} \quad (19)$$

where x_3 solves,

$$\min_{x_3} = \begin{cases} f_{31} = 0.921x_1 + 0.050x_2 + 0.526x_3, \\ f_{32} = 0.380x_1 + 0.737x_2 + 0.216x_3. \end{cases}$$

subject to : system constraint s (15);
 $x_i \geq 0$;

The first step for solving MLMO problem starts with computation of the individual solutions to determine the goal g_{ij} and its tolerance u_{ij} of each objective function. Table 2

tabulates the individual solutions of all objectives functions for the three levels of the MLMO problem.

Based on the procedure stated in Phase II, the equivalent linear program (19) for the first-level decision making as follows:

$$\begin{aligned} \max &= u_1 + u_2 + u_3; \\ u_1 &= 0.183x_1 + 0.023x_2 + 0.015x_3; \\ u_2 &= 0.078x_1 + 0.131x_2 + 0.036x_3; \\ u_3 &= 1 - (0.038x_1 + 0.069x_2 + 0.044x_3); \\ \text{system constraint s (15);} \\ x_i &\geq 0. \end{aligned} \quad (20)$$

LINGO© computer software is used to run the equivalent ordinary linear programming model. The solution of the first level is $x^{1*} = (x_1^{1*}, x_2^{1*}, x_3^{1*}) = (5.30, 1.08, 0.00)$. Let DM_1 decide $x_1^1 = 5.30$ from the first level solution and the negative and positive tolerance are decided to be $t_1^{1L} = 0.5$ and $t_1^{1R} = 0.5$ respectively. The second level solution proceeds as follows:

$$\begin{aligned} \max &= u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + ux_1^L + ux_1^R; \\ u_1 &= 0.183x_1 + 0.023x_2 + 0.015x_3; \\ u_2 &= 0.078x_1 + 0.131x_2 + 0.036x_3; \\ u_3 &= 1 - (0.038x_1 + 0.069x_2 + 0.044x_3); \\ u_4 &= 0.183x_1 + 0.016x_2 + 0.015x_3; \\ u_5 &= 0.072x_1 + 0.125x_2 + 0.037x_3; \\ u_6 &= 1 - (0.054x_1 + 0.106x_2 + 0.040x_3); \\ ux_1^L &= 2x_1 - 9.6; \\ ux_1^R &= 9.6 - 2x_1; \\ \text{system constraint s (15);} \\ x_i &\geq 0. \end{aligned} \quad (21)$$

The solution of the second level is observed as $x^{2*} = (x_1^{2*}, x_2^{2*}, x_3^{2*}) = (4.80, 1.19, 1.80)$. Let DM_2 decide $x_2^2 = 1.19$, $t_1^{2L} = 0.75$ and $t_1^{2R} = 0.25$. The solution for the third level is shown in the following equivalent linear programming problem.

$$\begin{aligned} \max &= u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7 \\ &+ u_8 + ux_1^L + ux_1^R + ux_2^L + ux_2^R; \\ u_1 &= 0.183x_1 + 0.023x_2 + 0.015x_3; \\ u_2 &= 0.078x_1 + 0.131x_2 + 0.036x_3; \\ u_3 &= 1 - (0.038x_1 + 0.069x_2 + 0.044x_3); \\ u_4 &= 0.183x_1 + 0.016x_2 + 0.015x_3; \\ u_5 &= (0.072x_1 + 0.125x_2 + 0.037x_3); \\ u_6 &= 1 - (0.054x_1 + 0.106x_2 + 0.040x_3); \\ u_7 &= 0.087x_1 + 0.004x_2 + 0.009x_3; \\ u_8 &= 1 - (0.065x_1 + 0.127x_2 + 0.037x_3); \\ ux_2^R &= 2x_1 - 9.6; \\ ux_2^L &= 9.6 - 2x_1; \\ ux_2^R &= 1.333x_2 - 0.586; \\ ux_2^L &= 3.76 - 4x_2; \\ \text{system constraint s (15);} \\ x_i &\geq 0. \end{aligned} \quad (22)$$

The satisfactory solution of the MLMO problem is $x^{3*} = (x_1^{3*}, x_2^{3*}, x_3^{3*}) = (4.80, 0.69, 0.00)$. The procedure brings to an end as $l = p = 3$, and the satisfactory solution is obtained.

TABLE II. INDIVIDUAL SOLUTIONS

f_{ij}	f_i^-	f_i^+
f_{11}	0.00	11.30
f_{12}	0.00	12.89
f_{13}	0.00	11.50
f_{21}	0.00	6.12
f_{22}	0.00	11.74
f_{23}	0.00	6.92
f_{31}	0.00	10.48
f_{32}	0.00	5.77

TABLE III. FUZZY RANDOM BASED COEFFICIENTS

Decision Making Level	Goal	Fuzzy Random-Based Coefficient		
		Product 1	Product 2	Product 3
1	Export revenue	[2.031, 2.215]	[0.200, 0.320]	[0.120, 0.220]
	Profit	[0.935, 1.086]	[1.206, 2.194]	[0.420, 0.532]
	Capital	[0.365, 0.512]	[0.735, 0.857]	[0.412, 0.613]
2	Production	[1.098, 1.154]	[0.000, 0.200]	[0.092, 0.103]
	Profit	[0.856, 0.856]	[1.006, 1.940]	[0.354, 0.352]
	Cost	[0.305, 0.455]	[0.680, 0.794]	[0.203, 0.532]
3	Production	[0.872, 0.971]	[0.000, 0.100]	[0.082, 0.970]
	Cost	[0.305, 0.455]	[0.680, 0.794]	[0.200, 0.232]

TABLE IV. MLMO ADDITIVE-FGP RESULTS

FGP solution		The Solution
Objective	Achievement	
$f_{11} = 10.154$	$\mu_{11} = 0.89$	11.30
$f_{12} = 6.021$	$\mu_{12} = 0.46$	12.89
$f_{13} = 2.652$	$\mu_{13} = 0.77$	11.50
$f_{21} = 5.474$	$\mu_{21} = 0.89$	6.12
$f_{22} = 5.125$	$\mu_{22} = 0.43$	11.74
$f_{23} = 2.333$	$\mu_{23} = 0.67$	6.92
$f_{31} = 4.458$	$\mu_{31} = 0.42$	10.48
$f_{32} = 2.84$	$\mu_{32} = 0.60$	5.77

VI. RESULTS AND DISCUSSIONS

In this section, the results of the industrial problem case are explained. The FRR Model (7) was applied to the datasets of the manufacturing production. The coefficients are obtained as shown in Table III, where each coefficient is represented in interval-valued form. This result illustrates the coefficients for each attribute and shows the range of their evaluations. The coefficients ξ_i are the fuzzy random coefficients for the decision variables x_i .

The coefficients values derived by the FRR approach are represented by the interval valued form which shows the fuzzy judgments and supports the range of values around expected value that is likely to contain the estimation target. Table IV summarizes the objective function value and the achievement of each objective function. Objective f_{11} and f_{21} in the first and second level achieve about 90% which maximized the export revenues. However, the objective of maximizing the profit only achieved about 46% and 43% in the government and state level. 77% goal achievement is gained for the objective to maximize the capital in the government level. From the results, decision makers can perform decision analysis to consider the solution derived by the mathematical approach. The decision maker can reevaluate the solution and change the decision and repeat the process. Since the proposed method is based on satisfaction approach, the decision makers may involve in evaluating the results to achieve better satisfying solution.

VII. CONCLUSIONS

In this study, an additive FGP approach is used to solve multi-level multi-objective decision problem. The initial model setting specifically the objective functions were developed by FRR model to treat the fuzziness and randomness in the evaluation. In the solution, each multi-objective problem at each level was solved separately, starting with the highest level. The main advantage of this algorithm is to allow a decision maker to select the desired satisfactory value from the higher level that is then used in the following level. The procedure also enables the decision maker of the respective decision making unit to decide the decision value by means of the mathematical solution based on their satisfaction. Additionally, the proposed method uses one-sigma confidence intervals when developing the objective functions which include the mean interval values of all samples in the model. Using the confidence intervals, we could provide a more complete description of the information in the data about the estimation target. Our model also supports the range of values around expected value that is likely to contain the estimation target.

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