

Fractional Order Chaotic System Tracking Design Based on Adaptive Hybrid Intelligent Control

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Abstract—In this paper, an adaptive hybrid fuzzy neural network (FNN) controller is proposed to achieve prescribed tracking performance of fractional order chaotic systems. Based on the trade-off between plant knowledge and control knowledge, a weighting factor can be adjusted by combining the indirect adaptive FNN control effort and the direct FNN adaptive control effort. Nonlinear fractional order chaotic response system is fully illustrated to track the trajectory generated from fractional order chaotic drive system. The numerical results show that tracking error and control effort can be made smaller and the proposed hybrid intelligent control scheme is more flexible during the design process.

Keywords- Fractional order chaotic systems; fuzzy neural network (FNN), adaptive hybrid control.

I. INTRODUCTION

In recent years, fractional calculus deals with derivatives and integrations of arbitrary order [30]-[32] and numerous applications of fractional order systems in many areas of science and engineering have been proposed such as control theory, electromagnetic waves, viscoelastic systems, diffusion signal processing and bio-engineering [33]-[36]. It is observed that the description of some systems is more accurate when the fractional derivative is used. Nowadays, many fractional-order differential systems behave chaotically, such as the fractional-order Chua's system [37], [38], the fractional-order Duffing system [39], [40], the fractional-order Lu system [41], the fractional-order Chen's system [42], [43], the fractional-order cellular neural network [15]-[16], the fractional-order neural network [17].

Owing to its potential applications in secure communication and control processing [18], study of chaos tracking, synchronization, in fractional order dynamical systems and related phenomena is receiving growing attention [19], [20]. The tracking problem of fractional order chaotic systems is first investigated by Deng and Li [21] who carried out tracking

in case of the two fractional Lü systems. Afterwards, they studied chaos tracking of the Chen system with a fractional order in a different manner [22]-[24].

Furthermore, the advent of fuzzy set techniques provides us with a powerful tool to solve demanding real word problem with uncertain and unpredictable environments. Based on the universal approximation theorem, [1]-[5] (fuzzy logic controllers are general enough to perform any nonlinear control actions) there is rapidly growing interest in systematic design methodologies for a class of nonlinear systems using fuzzy adaptive control schemes. By equipping with a training algorithm an adaptive fuzzy controller is synthesized from a collection of fuzzy IF-THEN rules and the parameters of the membership functions characterizing the linguistic terms in the IF-THEN rules changed according to some adaptive law for the purpose of controlling a plant to track a reference trajectory.

Like the conventional adaptive control, the adaptive FNN control is classified into direct and indirect FNN adaptive control categories [27]-[29]. A direct adaptive FNN controller uses fuzzy logic systems as controller in which linguistic fuzzy control rules can be directly incorporated into the controller. On the other hand, an indirect adaptive FNN controller uses fuzzy descriptions to model the plant in which fuzzy IF-THEN rules describing the plant can be directly incorporated into the indirect FNN controller. Moreover, a hybrid adaptive FNN controller can be constructed using a weighting factor to sum together the control efforts from indirect adaptive FNN controller and direct adaptive FNN controller [44],[45].

In this paper, by combining the approximate mathematical model, linguistic model description and linguistic control rules into a single adaptive FNN controller, an adaptive hybrid fuzzy neural network (FNN) controller is proposed to achieve prescribed tracking performance of fractional order chaotic systems. A new adaptive H^∞ fuzzy control algorithm

incorporated Lyapunov stability criterion is proposed so that not only the stability of adaptive fuzzy control system is guaranteed but also the influence of the approximation error and external disturbance on the tracking error can be attenuated to an arbitrarily prescribed level.

This paper is organized as follows: In section II, an introduction to fractional derivative and its relation to the approximation solution will be addressed. A brief description of the fuzzy neural networks is presented in section III. Section IV generally proposes adaptive hybrid FNN control of uncertain fractional order systems in presence of uncertainty and its stability analysis. In Section V, application of the proposed method on fractional order expression chaotic system is investigated. Finally, the simulation results and conclusion will be presented in Section VI.

II. BASIC DEFINITION AND PRELIMINARIES FOR FRACTIONAL ORDER SYSTEMS

The fractional calculus has been known more than 300 years, since the development of regular calculus. It is a generalization of integration and differentiation to non-integer order fundamental operator, denoted by ${}_a D_t^q$, where a and t are the limits of the operator. This operator is a notation for taking both the fractional integral and functional derivative in a single expression defined as

$${}_a D_t^q = \begin{cases} \frac{d^q}{dt^q}, & q > 0 \\ 1 & q = 0 \\ \int_a^t (d\tau)^{-q}, & q < 0 \end{cases} \quad (1)$$

There are some basic definitions for the general fractional and the commonly used definitions are Grunwald-Letnikov and Riemann-Liouville. The Grunwald-Letnikov definition is expressed as

$${}_a D_t^q f(t) = \lim_{h \rightarrow 0} \sum_{j=0}^{\left[\frac{t-a}{h} \right]} (-1)^j \binom{q}{j} f(t-jh) \quad (2)$$

where $[\cdot]$ is the integer part. The simplest and easiest definition is Riemann-Liouville definition given as

$${}_a D_t^q f(t) = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{q-n+1}} d\tau \quad (3)$$

where n is the first integer which is not less q , i.e., $n-1 < q < n$, and Γ is the Gamma function.

The numerical simulation of a fractional differential equation is not simple as that of an ordinary differential equation. In this paper, the algorithm which is an improved version of Adams-Bashforth-Moulton algorithm [12]-[13] to find an approximation for fractional order systems based on predictor-correctors [12], [14] is given. Consider the following differential equation

$${}_0 D_t^q y(t) = r(y(t), t), \quad 0 \leq t \leq T \quad \text{and} \quad y^{(k)}(0) = y_0^{(k)}, \quad k = 0, 1, 2, \dots, m-1 \quad (4)$$

where

$${}_0 D_t^q y(t) = \begin{cases} \frac{1}{\Gamma(m-q)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{q-m+1}} d\tau, & m-1 < q < m \\ \frac{d^m}{dt^m} y(t) & q = m \end{cases} \quad (5)$$

and is the first integer larger the q . The solution of the

equation (4) is equivalent to Volterra integral equation [34] described as

$$y(t) = \sum_{k=0}^{[q]-1} y_0^{(k)} \frac{t^k}{k!} + \frac{1}{\Gamma(q)} \int_0^t (t-\lambda)^{q-1} r(y(\lambda), \lambda) d\lambda \quad (6)$$

Let $h = T/N$, $t_n = nh$, $n = 0, 1, 2, \dots, N$. Then (6) can be discretized as follows.

$$y_h(t_{n+1}) = \sum_{k=0}^{[q]-1} y_0^{(k)} \frac{t_{n+1}^k}{k!} + \frac{h^q}{\Gamma(q+2)} r(y_h^p(t_{n+1}), t_{n+1}) + \frac{h^q}{\Gamma(q+2)} \sum_{j=0}^n a_{j,n+1} r(y_h(t_j), t_j) \quad (7)$$

where predict value $y_h^p(t_{n+1})$ is determined by

$$y_h^p(t_{n+1}) = \sum_{k=0}^{[q]-1} y_0^{(k)} \frac{t_{n+1}^k}{k!} + \frac{h^q}{\Gamma(q)} \sum_{j=0}^n b_{j,n+1} r(y_h(t_j), t_j) \quad (8)$$

and

$$a_{j,n+1} = \begin{cases} n^{q+1} - (n-q)(n+1)^q, & j = 0 \\ (n-j+2)^{q+1} + (n-j)^{q+1} - 2(n-j+1)^{q+1} & 1 \leq j \leq n \\ 1 & j = n+1 \end{cases} \quad (9)$$

$$b_{j,n+1} = \frac{h^q}{q} ((n+1-j)^q - (n-j)^q) \quad (10)$$

The approximation error is given as

$$\max_{j=0,1,2,\dots,N} |y(t_j) - y_h(t_j)| = O(h^p) \quad (11)$$

where $p = \min(2, 1+q)$. Therefore, the numerical solution of a fraction order system can be obtained by applying the above mentioned algorithm.

III. A BRIEF DESCRIPTION OF THE FUZZY NEURAL NETWORKS [6]-[9], [25]

Fuzzy logic systems are knowledge-based or rule-based systems which consist of so called fuzzy IF-THEN rules. The basic configuration the T-S FNN system includes a fuzzy rule base, which comprises of a collection of fuzzy IF-THEN rules in the following form:

$R^{(l)} : \text{IF } x_1 \text{ is } F_1^l, \text{ and } \dots, \text{ and } x_n \text{ is } F_n^l, \text{ THEN } y_l = q_0^l +$

$$q_1^l x_1 + \dots + q_n^l x_n = \underline{\theta}_l^T [1 \ x^T]^T \quad (12)$$

where F_i^l are fuzzy sets and $\underline{\theta}_l^T = [q_0^l, q_1^l, \dots, q_n^l]$ is a vector of the adjustable weight factors of the consequence part of the fuzzy rule. Also y_l is a fuzzy inference engine to combine the fuzzy IF-THEN rules in the fuzzy rule base into a mapping from an input linguistic vector $\underline{x}^T = [x_1, x_2, \dots, x_n] \in R^n$ to an output variable $y \in R$. Let M be the number of the fuzzy IF-THEN rules. The output of the fuzzy logic systems with central average defuzzifier, product inference and singleton fuzzifier can be expressed as

$$y(\underline{x}) = \frac{\sum_{l=1}^M \phi^l \cdot y_l}{\sum_{l=1}^M \phi^l} = \frac{\sum_{l=1}^M \phi^l \cdot \underline{\theta}_l^T [1 \ x^T]^T}{\sum_{l=1}^M \phi^l} \quad (13)$$

where $\mu_{F_i^l}(x_i)$ is the membership function value of the fuzzy variable x_i and $\phi^l = \prod_{i=1}^n \mu_{F_i^l}(x_i)$ is the true value of the l^{th} implication. Equation (13) can be rewritten as

$$y(\underline{x}) = \underline{\theta}^T \underline{\psi}(\underline{x}) \quad (14)$$

where $\underline{\theta}^T = [\theta_1^T \ \theta_2^T \ \dots \ \theta_M^T]$ is an adjustable parameter vector and $\underline{\psi}^T(\underline{x}) = [\psi^1(\underline{x}), \psi^2(\underline{x}), \dots, \psi^M(\underline{x})]$ is a fuzzy basis function vector defined as

$$\psi^i(\underline{x}) = \frac{\phi^i[1 \ \underline{x}^T]}{\sum_{l=1}^M \phi^l} \quad (15)$$

When the inputs are fed into the T-S FNN, the true value v^j of the l th implication is computed. Applying the common defuzzification strategy, the output of the neural networks expressed as (13) is pumped out. The overall configuration of the T-S FNN is shown in Fig.1 [27]-[29].

Based on the universal approximation theorem [26], the above fuzzy logic system is capable of uniformly approximating any well-defined nonlinear function over a compact set U_c to any degree of accuracy. Also it is straightforward to show that a multi-output system can always be approximated by a group of single-output approximation systems.

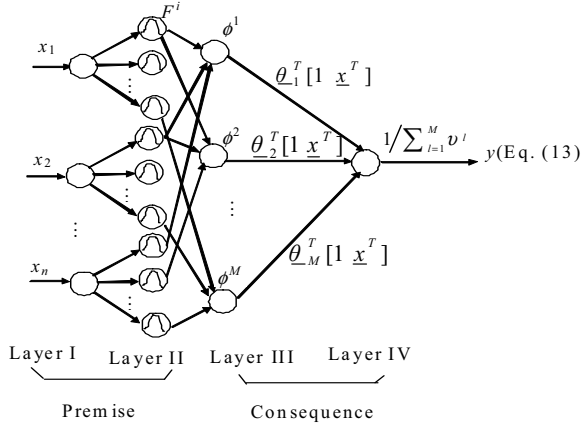


Figure1 The configuration of the T-S fuzzy-neural networks.

IV. ADAPTIVE HYBRID FUZZY H^∞ TRACKING CONTROL OF UNCERTAIN FRACTIONAL ORDER CHAOTIC SYSTEMS

In this section, we study adaptive H^∞ tracking control of uncertain fractional order chaotic systems, i.e., to force output trajectory of the response system to track output trajectory of the drive system.

Consider a fractional order chaotic dynamic system

$$\begin{aligned} x^{(nq)} &= f(\underline{x}, t) + g(\underline{x}, t)u + d(t) \\ y &= x_1 \end{aligned} \quad (16)$$

where $\underline{x} = [x_1, x_2, \dots, x_n]^T = [x, x^{(q)}, x^{(2q)}, \dots, x^{((n-1)q)}]^T$ is the state vector, $f(\underline{x}, t)$ and $g(\underline{x}, t)$ are unknown but bounded nonlinear functions which express system dynamics, $d(t)$ is the external bounded disturbance and $u(t)$ is the control input. The control objective is to force the system output y to follow a bounded reference signal y_d which is the output trajectory of a drive system, under the constraint that all signals involved must be bounded. To begin with, the reference signal vector $\underline{y}_d$ and the tracking error vector $\underline{e}$ will be defined as

$$\underline{y}_d = [y_d, y_d^{(q)}, \dots, y_d^{((n-1)q)}]^T \in R^n,$$

$$\underline{e} = \underline{y}_d - \underline{x} = [e, e^{(q)}, e^{(2q)}, \dots, e^{((n-1)q)}]^T \in R^n, e^{(iq)} = y_d^{(iq)} - x^{(iq)} = y_d^{(iq)} - y^{(iq)}$$

Let $\underline{k} = [k_1, k_2, \dots, k_n]^T \in R^n$ to be chosen such that $p(s) = s^{(nq)} + k_n s^{(n-1)q} + \dots + k_1$ is a Hurwitz polynomial. If the functions $f(\underline{x}, t)$ and $g(\underline{x}, t)$ are known and the system is free of external disturbance d , then control law of the certainty equivalent controller is obtained as [38]

$$\bar{u} = \frac{1}{g(\underline{x}, t)} (-f(\underline{x}, t) + y_d^{(nq)} + \underline{k}^T \underline{e}) \quad (17)$$

Substituting (17) into (16), we have [30]

$$e^{(nq)} + k_n e^{(n-1)q} + \dots + k_1 e = 0$$

which is the main objective of control, $\lim_{t \rightarrow \infty} e(t) = 0$.

However, $f(\underline{x}, t)$ and $g(\underline{x}, t)$ are unknown and external disturbance, $d(t) \neq 0$, the ideal control effort (17) cannot be implemented. We replace $f(\underline{x}, t)$ and $g(\underline{x}, t)$ by the fuzzy logic system $f(\underline{x}|\underline{\theta}_f)$ and $g(\underline{x}|\underline{\theta}_g)$ in specified form as (14), i.e.,

$$f(\underline{x}|\underline{\theta}_f) = \xi^T(\underline{x})\underline{\theta}_f, \quad g(\underline{x}|\underline{\theta}_g) = \xi^T(\underline{x})\underline{\theta}_g \quad (18)$$

Here the fuzzy basis function $\xi(\underline{x})$ depends on the fuzzy membership functions and is supposed to be fixed, while $\underline{\theta}_f$ and $\underline{\theta}_g$ are adjusted by adaptive laws based on Lyapunov stability criterion.

However, depending on plant knowledge and control knowledge, a hybrid adaptive FNN controller can be constructed by incorporating both fuzzy description and fuzzy control rules using a weighting factor α to combine the indirect adaptive FNN controller and the direct adaptive FNN controller. Based on the trade-off between plant knowledge and control knowledge, the weighting factor $\alpha \in [1, 1]$ can be adjusted. Therefore, the total control effort can be expressed as

$$u = \alpha u_i + (1 - \alpha) u_d \quad (19)$$

where the direct adaptive FNN controller u_d and the indirect adaptive FNN controller u_i are given as follows:

$$u_d = u_d(\underline{x}|\underline{\theta}) - \frac{u_a}{g(\underline{x}|\underline{\theta})} \quad (20)$$

$$u_i = \frac{1}{g(\underline{x}|\underline{\theta}_g)} [-f(\underline{x}|\underline{\theta}_f) + y_d^{(nq)} + \underline{k}^T \underline{e} + u_a] \quad (21)$$

where the robust compensator u_a is employed to attenuate the external disturbance and the fuzzy approximation errors and $u_d(\underline{x}|\underline{\theta})$ is obtained by fuzzy logic system specified as

$$u_d(\underline{x}|\underline{\theta}_D) = \xi^T(\underline{x})\underline{\theta}_D \quad (22)$$

By substituting (19) into (16), the error dynamic equation can be obtained in state space representation as

$$\begin{aligned} \dot{\underline{e}}^{(q)} &= A \underline{e} + B \{ \alpha [f(\underline{x}|\underline{\theta}_f) - f(\underline{x}, t)] + [g(\underline{x}|\underline{\theta}_g) - g(\underline{x}, t)] u_i \\ &\quad - (1 - \alpha) (g(\underline{x}, t) [u_d(\underline{x}|\underline{\theta}) - \bar{u}] + u_a - d(t)) \} \end{aligned} \quad (23)$$

where
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ -k_1 & -k_2 & -k_3 & -k_4 & \dots & -k_{(n-1)} & -k_n \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

The optimal parameter estimations $\underline{\theta}_f^*$, $\underline{\theta}_g^*$ and $\underline{\theta}_D^*$ are defined as

$$\underline{\theta}_f^* = \arg \min_{\underline{\theta}_f \in \Omega_f} [\sup_{x \in \Omega_x} |f(x|\underline{\theta}_f) - f(x,t)|] \quad (24)$$

$$\underline{\theta}_g^* = \arg \min_{\underline{\theta}_g \in \Omega_g} [\sup_{x \in \Omega_x} |g(x|\underline{\theta}_g) - g(x,t)|] \quad (25)$$

and

$$\underline{\theta}_D^* = \arg \min_{\underline{\theta}_D \in \Omega_D} [\sup_{x \in \Omega_x} |u_D(x|\underline{\theta}_D) - \bar{u}|] \quad (26)$$

where Ω_f , Ω_g , Ω_D and Ω_x are constraint sets of suitable bounds on $\underline{\theta}_f$, $\underline{\theta}_g$, $\underline{\theta}_D$ and x respectively and they are defined as $\Omega_f = \{\underline{\theta}_f | |\underline{\theta}_f| \leq M_f\}$, $\Omega_g = \{\underline{\theta}_g | |\underline{\theta}_g| \leq M_g\}$, $\Omega_D = \{\underline{\theta}_D | |\underline{\theta}_D| \leq M_D\}$ and $\Omega_x = \{x | |x| \leq M_x\}$, where M_f , M_g , M_D and M_x are positive constants.

By using (24)-(26), error dynamic equation (23) can be rewritten as

$$\begin{aligned} \underline{e}^{(q)} = A\underline{e} + B\{ & \alpha[f(x|\underline{\theta}_f) - f(x|\underline{\theta}_f^*)] + [g(x|\underline{\theta}_g) - g(x|\underline{\theta}_g^*)]u_i \\ & - (1-\alpha)g(x,t)[u(x|\underline{\theta}_D) - u(x|\underline{\theta}_D^*)] + u_a + \omega \} \end{aligned} \quad (27)$$

where the minimum approximation errors is defined as

$$\begin{aligned} \omega = & \alpha[f(x|\underline{\theta}_f^*) - f(x,t)] + [g(x|\underline{\theta}_g^*) - g(x,t)]u_i \\ & - (1-\alpha)g(x,t)[u_D(x|\underline{\theta}_D^*) - \bar{u}] - d(t) \end{aligned} \quad (28)$$

If $\tilde{\theta}_f = \underline{\theta}_f - \underline{\theta}_f^*$, $\tilde{\theta}_g = \underline{\theta}_g - \underline{\theta}_g^*$ and, $\tilde{\theta}_D = \underline{\theta}_D - \underline{\theta}_D^*$, we have

$$\underline{e}^{(q)} = A\underline{e} + B[\xi(x)^T \tilde{\theta}_f + \xi(x)^T \tilde{\theta}_g u_i + \xi(x)^T \tilde{\theta}_D + u_a + \omega] \quad (29)$$

Following the proceeding consideration, the following theorem can be obtained.

Theorem: Consider the fractional order SISO nonlinear chaotic system (16) with control input (19), if the robust compensator u_a and the fuzzy-based adaptive laws are chosen as

$$u_a = -\frac{1}{r} B^T P \underline{e} \quad (30)$$

$$\underline{\theta}_f^{(q)} = -r_1 \xi(x) B^T P \underline{e} \quad (31)$$

$$\underline{\theta}_g^{(q)} = -r_2 \xi(x) B^T P \underline{e} u_i \quad (32)$$

$$\underline{\theta}_D^{(q)} = r_3 \xi(x) g(x,t) B^T P \underline{e} \quad (33)$$

where $r > 0, r_i > 0, i = 1 \sim 3$, and $P = P^T > 0$ is the solution of the following Ricatti-like equation

$$PA + A^T P + Q - PB \left(\frac{2}{r} - \frac{1}{\rho^2} \right) B^T P = 0 \quad (34)$$

where $Q = Q^T > 0$ is a prescribed weighting matrix. Therefore, the H^∞ tracking performance can be achieved for a prescribed attenuation level ρ and all the variables of the closed-loop system are bounded.

Proof: In order to analyze the closed-loop stability, the Lyapunov function candidate is chosen as

$$V = \frac{1}{2} \underline{e}^T P \underline{e} + \frac{\alpha}{2r_1} \tilde{\theta}_f^T \tilde{\theta}_f + \frac{\alpha}{2r_2} \tilde{\theta}_g^T \tilde{\theta}_g + \frac{(1-\alpha)}{2r_3} \tilde{\theta}_D^T \tilde{\theta}_D \quad (35)$$

Taking the derivative of the (35) with respect to time, we get

$$\begin{aligned} V^{(q)}(t) = & \frac{1}{2} (\underline{e}^{(q)}(t))^T P \underline{e}(t) + \frac{1}{2} \underline{e}^T(t) P \underline{e}^{(q)}(t) \\ & + \frac{\alpha}{r_1} (\tilde{\theta}_f^T)(\tilde{\theta}_f^{(q)}) + \frac{\alpha}{r_2} (\tilde{\theta}_g^T)(\tilde{\theta}_g^{(q)}) + \frac{1-\alpha}{r_3} (\tilde{\theta}_D^T)(\tilde{\theta}_D^{(q)}) \end{aligned} \quad (36)$$

$$\begin{aligned} V^{(q)} = & \frac{1}{2} \underline{e}^T A^T P \underline{e} + \left[\alpha (\xi^T \tilde{\theta}_f + \xi^T \tilde{\theta}_g u_i) - (1-\alpha) g(x,t) \xi^T \tilde{\theta}_D + \omega \right]^T B^T P \underline{e} \\ & + \frac{1}{2} \underline{e}^T P A \underline{e} + \underline{e}^T P B \left[\alpha (\xi^T \tilde{\theta}_f + \xi^T \tilde{\theta}_g u_i) - (1-\alpha) g(x,t) \xi^T \tilde{\theta}_D + u_a + \omega \right] \\ & + \frac{\alpha}{r_1} \tilde{\theta}_f^T \tilde{\theta}_f^{(q)} + \frac{\alpha}{r_2} \tilde{\theta}_g^T \tilde{\theta}_g^{(q)} + \frac{(1-\alpha)}{r_3} \tilde{\theta}_D^T \tilde{\theta}_D^{(q)} \\ = & \frac{1}{2} \underline{e}^T (PA + A^T P - \frac{2}{r} P B B^T P) \underline{e} + (\alpha \underline{e}^T P B \xi^T(x) + \frac{\alpha}{\gamma_1} \tilde{\theta}_f^{(q)T}) \tilde{\theta}_f \\ & + (\alpha \underline{e}^T P B \xi^T(x) u_i + \frac{\alpha}{\gamma_2} \tilde{\theta}_g^{(q)T}) \tilde{\theta}_g - \left((1-\alpha) \underline{e}^T P B g(x,t) \xi^T(x) - \frac{(1-\alpha)}{\gamma_3} \tilde{\theta}_D^{(q)T} \right) \tilde{\theta}_D \\ & + \frac{1}{2} (\omega^T B^T P \underline{e} + \underline{e}^T P B \omega) \end{aligned} \quad (37)$$

From the robust compensator u_a and the fuzzy-based adaptive laws are given (30)-(33), after simple manipulation, we have

$$\begin{aligned} V^{(q)}(t) = & -\frac{1}{2} \underline{e}^T Q \underline{e} - \frac{1}{2\rho^2} \underline{e}^T P B B^T \underline{e} + \underline{e}^T P B \omega \\ = & -\frac{1}{2} \underline{e}^T Q \underline{e} - \frac{1}{2} \left(\frac{1}{\rho} B^T P \underline{e} - \rho \omega \right)^T \left(\frac{1}{\rho} B^T P \underline{e} - \rho \omega \right) + \frac{1}{2} \rho^2 \omega^T \omega \\ \leq & -\frac{1}{2} \underline{e}^T Q \underline{e} + \frac{1}{2} \rho^2 \omega^T \omega \end{aligned} \quad (38)$$

Integrating (38) from $t=0$ to $t=T$, we have

$$V(T) - V(0) \leq -\frac{1}{2} \int_0^T (\underline{e}^T Q \underline{e} + \frac{1}{2} \rho^2 \omega^T \omega) dt \quad (39)$$

Since $V(T) \geq 0$, (39) can be rewritten as follows:

$$\int_0^T \underline{e}^T Q \underline{e} dt \leq \underline{e}^T(0) P \underline{e}(0) + \underline{\theta}^T(0) \underline{\theta}(0) + \rho^2 \int_0^T \omega^T \omega dt \quad (40)$$

Therefore, the H^∞ tracking performance can be achieved. The proof is completed.

V. SIMULATION EXAMPLE

In this section, we will apply our adaptive hybrid H^∞ FNN controller to force the fractional order chaotic response system to track the trajectory of the fractional order chaotic drive system.

Example: The fractional order chaotic drive and response systems are given as follows:

Drive System:

$$\begin{cases} D^{(q)} y_1 = y_2 \\ D^{(q)} y_2 = 1.2 y_1 - y_2 - y_1^2 + 0.5 \cos(t) \end{cases}$$

Response System:

$$\begin{cases} D^{(q)}x_1 = x_2 \\ D^{(q)}x_2 = x_1 - 1.8x_2 - x_1^2 + 0.9\cos(t) + u(t) + d(t) \end{cases}$$

where external disturbance $d(t) = 0.1\sin(t)$. The main objective is to control the trajectories of the response system to track the reference trajectories obtained from the drive system. The initial conditions of drive and response systems are chosen as $\begin{bmatrix} y_1(0) \\ y_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, respectively. values of $q = 0.95$ are considered. All design constants are specified as $k_1 = k_2 = 1$, $r_1 = 200$, $r_2 = 40$, $r_3 = 100$, $r_4 = 10$, $r = 0.005$, $\rho = 0.05$ and step size $h = 0.01$.

Case 1. $q=0.95$, $\alpha=0.7$: For $q = 0.95$ and free of control input, the phase portrait of the drive and response systems is given in Fig. 2.

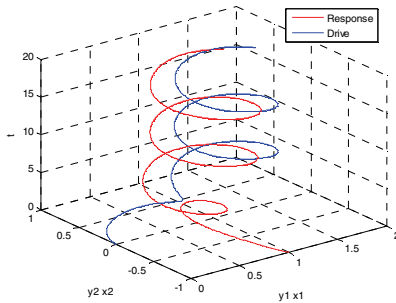


Figure 2. Phase portrait of chaotic drive and response systems.

The membership functions for x_i , $i = 1, 2$ are shown in Fig. 3 and are selected as follows:

$$\begin{aligned} \mu_{F_1^i}(x_i) &= \exp\left[-0.5\left(\frac{x_i - 2}{0.8}\right)^2\right], \quad \mu_{F_2^i}(x_i) = \exp\left[-0.5\left(\frac{x_i - 1.5}{0.8}\right)^2\right], \\ \mu_{F_3^i}(x_i) &= \exp\left[-0.5\left(\frac{x_i - 1}{0.8}\right)^2\right], \quad \mu_{F_4^i}(x_i) = \exp\left[-0.5\left(\frac{x_i - 0.5}{0.8}\right)^2\right], \\ \mu_{F_5^i}(x_i) &= \exp\left[-0.5\left(\frac{x_i}{0.8}\right)^2\right], \quad \mu_{F_6^i}(x_i) = \exp\left[-0.5\left(\frac{x_i + 0.5}{0.8}\right)^2\right], \\ \mu_{F_7^i}(x_i) &= \exp\left[-0.5\left(\frac{x_i + 1}{0.8}\right)^2\right] \end{aligned}$$

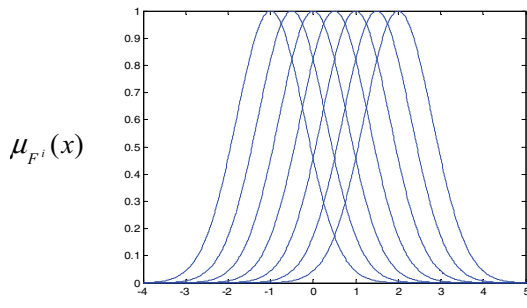


Figure 3. The membership function for x_i , $i = 1, 2$.

From the adaptive laws (33)-(36) and the robust compensator (32), the control effort of the response system can be obtained as

$$u = \alpha u_i + (1 - \alpha) u_d \quad (41)$$

Fig. 4 and 5 show the trajectories of the states x_1 , y_1 and x_2 , y_2 , respectively. Control effort trajectory is given in Fig 6 and phase portrait, tracking performance, of the drive and response systems is shown in Fig. 7. Fig. 8 shows the graph of $V^{(q)}(t)$ which is always negative defined and consequently is stable.

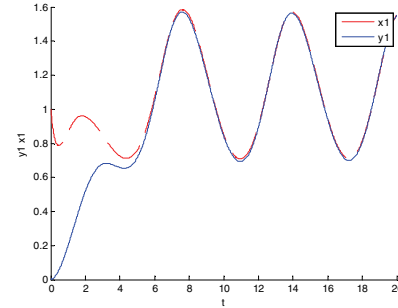


Figure 4. The trajectories of the states x_1 and y_1 .

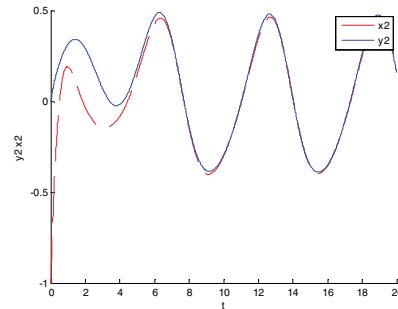


Figure 5. The trajectories of the states x_2 and y_2 .

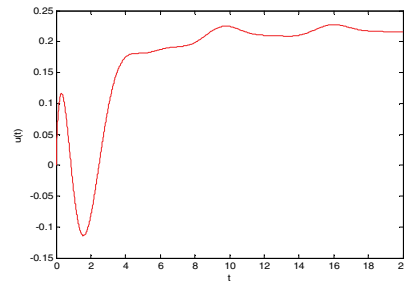


Figure 6. Trajectory of the control effort.

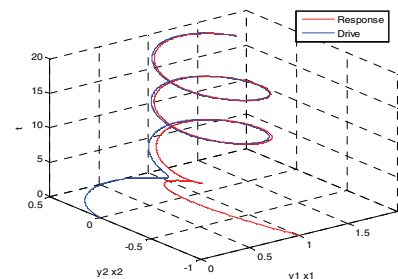


Figure 7. Phase portrait, tracking performance, of the drive and response systems.

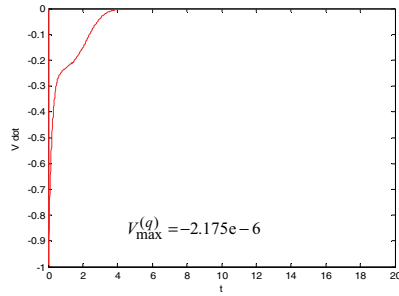


Figure 8. The graph of $V^{(q)}(t)$.

In order to show the robustness of the proposed adaptive fuzzy H^∞ control, the control effort is activated at 5 second. The phase portrait, tracking performance, of the drive and response systems is given in Fig. 9. Fig. 10 and 11 show the trajectories of the states x_1 , y_1 and x_2 , y_2 , respectively. We can see that a fast synchronization of drive and response is achieved as the control effort is activated. Control effort trajectory is given in Fig 12.

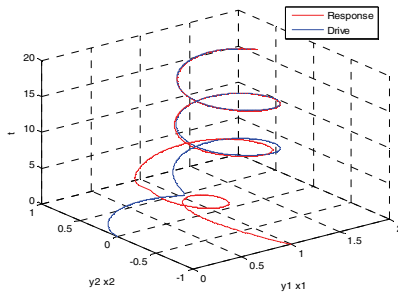


Figure 9. Phase portrait, tracking performance, of the drive and response systems.

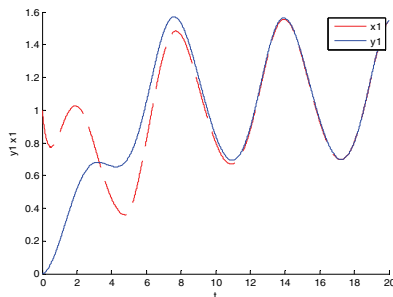


Figure 10. The trajectories of the states x_1 and y_1 .

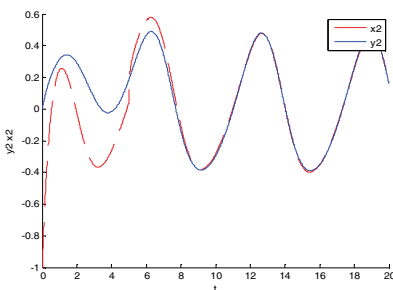


Figure 11. The trajectories of the states x_2 and y_2 .

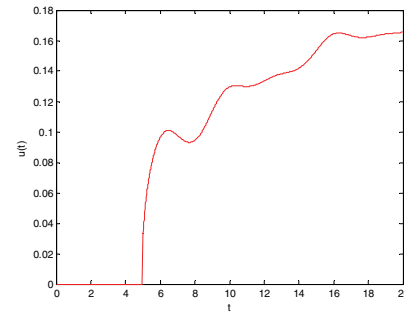


Figure 12. Trajectory of the control effort.

VI. CONCLUSIONS

A novel adaptive hybrid FNN controller is proposed to achieve prescribed tracking performance of fractional order chaotic systems in this paper. It is a flexible design methodology by the trade-off between plant knowledge and control knowledge using a weighting factor α adopted to sum together the control effort from indirect adaptive FNN controller and direct adaptive FNN controller. Based on the Lyapunov synthesis approach, free parameters of the adaptive fuzzy controller can be tuned on line by output feedback control law and adaptive laws. The simulation example, the output trajectory of the fractional order chaotic response system to tracking the trajectory of the fractional order chaotic drive system, is given to demonstrate the effectiveness of the proposed methodology. The significance of the adaptive fuzzy H^∞ control in the simulations for different values of q is manifest. Simulation results show that a fast synchronization of the drive and response systems can be achieved and q is reduced the chaos is seen reduced, i.e., the synchronization error is reduced, accordingly.

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