

Handling Fuzzy Decision Making Problem based on Linguistic Information and Intersection Concept

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Abstract—Multi-criteria decision-making (MCDM) is one of the most widely used decision methodologies. Because every kind of MCDM approach has its strong point and weakness, it is hard to make sure that what kind of MCDM approach is suitable to a specific problem. Therefore, a new decision making method is proposed in this paper based on linguistic information and intersection concept which is called linguistic intersection method (LIM). The linguistic variables are used to express the opinion of each decision-maker. There are four MCDM methods such as TOPSIS, ELECTRE, PROMETHEE, and VIKOR are included in the linguistic intersection method. First, each MCDM approach is used to determine the ranking order of all alternatives in accordance with the linguistic evaluations by decision-makers. And then, the intersection set is determined for the better alternatives of all methods. Third, the final ranking order of alternatives in the intersection set can be determined by the proposed method. This study presented an example to implement and compare the proposed method with individual linguistic MCDM method. Finally, some conclusions and future research will be discussed at the end of this paper.

Keywords- MCDM; Linguistic variables; linguistic intersection method.

I. INTRODUCTION

Decision-Making is the procedure to find the best action among a set of feasible actions [1]. Multi-criteria decision-making (MCDM) is one of the most widely used decision methodologies in the sciences, business, government and engineering worlds. MCDM methods can help to improve the quality of decisions by making the decision-making process more explicit, rational, and efficient [2].

Technique for order performance by similarity to ideal solution (TOPSIS) is first developed by Hwang and Yoon (1981) for solving a MCDM problem [3]. The concept of TOPSIS is choosing the best alternative according to the relative position [the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS)] in all of the alternatives. Using TOPSIS method can make an efficacious and total ranking order of alternatives. But the drawback of TOPSIS is that it can not make a pairwise comparison and judge the degree of difference among all alternatives.

ELECTRE method is a highly developed multi-criteria analysis model which takes into account the uncertainty and vagueness in the decision process [4]. It is based on the axiom of partial comparability which is suitable for alternative

selection. But the drawback of ELECTRE is that it is not easy to obtain the total ranking order of alternatives.

PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) is a multi-criteria decision making method [5]. It is well adapted to problems where a finite number of alternative actions are to be ranked by considering several, sometimes conflicting, criteria [6]. There are six basic types of preference function in PROMETHEE method, so decision makers can establish flexible standard according to the requirement of particular decision making problem with respect to each criterion by PROMETHEE method.

VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje in Serbian, means Multicriteria Optimization and Compromise Solution) is a multi criteria decision making method developed by Opricovic [6]. VIKOR determines a compromise solution that provides the maximum group utility for the majority and a minimum of individual regret for the opponent [7]. So, VIKOR can find a compromise priority ranking order of alternatives according to the selected criteria [8, 9]. According to the VIKOR method, the compromise ranking order could be performed by comparing the measure of closeness to the ideal alternative through the process of ranking and selecting a set of alternatives in the presence of conflicting criteria [7].

The information for choosing best alternative is including quantitative information and qualitative information. Quantitative information is easy to describe by crisp value. A more realistic approach may be to use linguistic assessments instead of crisp values [10]. The 2-tuple linguistic representation model is based on the concept of symbolic translation [11, 12]. It is an effective method to reduce the mistakes of information translation and avoid information loss by computing with words [13].

Because every kind of MCDM approach has its strong point and weakness, it is hard to make sure that what kind of MCDM approach is suitable to a specific problem. However, choosing the wrong MCDM approach to make decision will reduce the effectiveness and quality of decision-making. In order to avoid this problem, this study presented a linguistic intersection method (LIM) to handle the multi- criteria decision making problem in a fuzzy environment.

II. EVALUATION INFORMATION

The quantitative and qualitative information will be considered in the process of decision making problem.

Qualitative information can be expressed by 2-tuple linguistic variable. The membership function of 2-tuple linguistic variable can be expressed as triangle fuzzy number [14]. There are two types of 2-tuple linguistic variable are applied in this study (shown as Table 1 and Figures 1-2).

TABLE 1. DIFFERENT TYPES OF LINGUISTIC VARIABLES

Linguistic variable	
Type I:	Extremely Poor (s_0^5), Poor (s_1^5), Fair (s_2^5), Good (s_3^5), Extremely Good (s_4^5)
Type II:	Extremely Poor (s_0^7), Poor (s_1^7), Medium Poor (s_2^7), Fair (s_3^7), Medium Good (s_4^7), Good (s_5^7), Extremely Good (s_6^7)

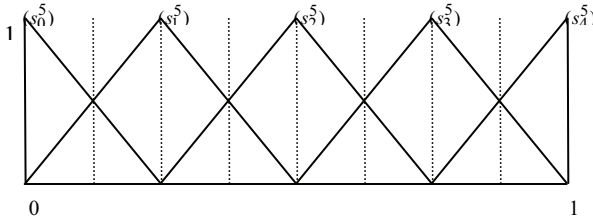


Figure 1. Membership functions of linguistic variables at type 1 ($t=1$)

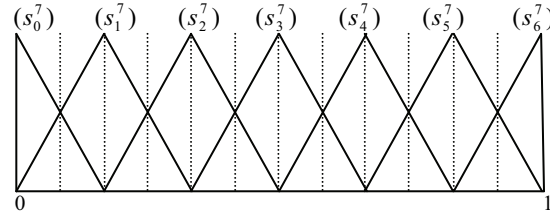


Figure 2. Membership functions of linguistic variables at type 2 ($t=2$)

Let $S = \{s_0, s_1, s_2, \dots, s_g\}$ be a finite and totally ordered linguistic term set. A 2-tuple linguistic variable can be expressed as (s_i, α_i) , where s_i is the central value of i -th linguistic term in S and α_i is a numerical value representing the difference between calculated linguistic term and the closest index label in the initial linguistic term set. The symbolic translation function Δ is presented to translate β into a 2-tuple linguistic variable [14]. Then, the symbolic translation process is applied to translate β ($\beta \in [0, 1]$) into a 2-tuple linguistic variable as [15]

$$\Delta(\beta) = (s_i, \alpha_i) \quad (1)$$

where $i = \text{round}(\beta \times g)$, $\alpha_i = \beta - \frac{i}{g}$ and $\alpha_i \in [-\frac{1}{2g}, \frac{1}{2g}]$.

A reverse function Δ^{-1} is defined to return an equivalent numerical value β from 2-tuple linguistic information (s_i, α_i) . It can be represented as follows [15].

$$\Delta^{-1}(s_i, \alpha_i) = (i/g) + \alpha_i = \beta \quad (2)$$

Let $x = \{(r_1, \alpha_1), (r_2, \alpha_2), \dots, (r_n, \alpha_n)\}$ be a 2-tuple linguistic variable set and $W = \{(w_1, \alpha_{w1}), (w_2, \alpha_{w2}), \dots, (w_n, \alpha_{wn})\}$ be

the set of linguistic weight of each linguistic variable. The linguistic arithmetic mean $\bar{X}$ is computed as [16]

$$\bar{X} = \Delta \left(\frac{1}{n} \sum_{i=1}^n \Delta^{-1}(r_i, \alpha_i) \right) = (s_m, \alpha_m) \quad (3)$$

The linguistic weighted arithmetic mean is computed as [16]

$$\bar{X}^w = \Delta \left(\frac{\sum_{i=1}^n (\Delta^{-1}(r_i, \alpha_i) * \Delta^{-1}(w_i, \alpha_{wi}))}{\sum_{i=1}^n \Delta^{-1}(w_i, \alpha_{wi})} \right) = (s_m^w, \alpha_m^w) \quad (4)$$

In general, decision makers would use different kind of 2-tuple linguistic variables based on their knowledge or experiences to express their opinions [17]. A transformation function is needed to transfer these 2-tuple linguistic variables from different kind of linguistic sets to a standard linguistic set at unique domain. In the method of Herrera and Martinez [18], the domain of the linguistic variables will increase as the number of linguistic variable is increased. To overcome this drawback, a translation function is applied here to transfer a crisp number or 2-tuple linguistic variable to a standard linguistic term at the unique domain [15]. Suppose that the interval $[0, 1]$ is the unique domain. The linguistic variable sets with different semantics (or types) will be defined by partitioning the interval $[0, 1]$. Transforming a crisp number β ($\beta \in [0, 1]$) into i -th linguistic term $(s_i^{n(t)}, \alpha_i^{n(t)})$ of type t as

$$\Delta_t(\beta) = (s_i^{n(t)}, \alpha_i^{n(t)}) \quad (5)$$

where $i = \text{round}(\beta \times g_t)$, $\alpha_i^{n(t)} = \beta - \frac{i}{g_t}$, $g_t = n(t) - 1$ and n

(t) is the number of linguistic variable of type t . Transforming i -th linguistic term of type t into a crisp number β as

$$\Delta_t^{-1}(s_i^{n(t)}, \alpha_i^{n(t)}) = \frac{i}{g_t} + \alpha_i^{n(t)} = \beta \quad (6)$$

where $g_t = n(t) - 1$ and $\alpha_i^{n(t)} \in [-\frac{1}{2g_t}, \frac{1}{2g_t}]$.

Therefore, the transformation from i -th linguistic term $(s_i^{n(t)}, \alpha_i^{n(t)})$ of type t to k -th linguistic term $(s_k^{n(t+1)}, \alpha_k^{n(t+1)})$ of type $t+1$ can be expressed as

$$\Delta_{t+1}(\Delta_t^{-1}(s_i^{n(t)}, \alpha_i^{n(t)})) = (s_k^{n(t+1)}, \alpha_k^{n(t+1)}) \quad (7)$$

where $g_{t+1} = n(t+1) - 1$ and $\alpha_k^{n(t+1)} \in [-\frac{1}{2g_{t+1}}, \frac{1}{2g_{t+1}}]$.

III. PROPOSED METHOD

In general, the content of decision making can include:

- (1) A set of alternatives is called $A = \{A_1, A_2, \dots, A_m\}$
- (2) A set of criteria is called $C = \{C_1, C_2, \dots, C_n\}$, there are n criteria which we will use. The quantitative criteria are from C_1 to C_Z . The qualitative criteria are from C_{Z+1} to C_n .
- (3) A set of decision makers is called $D = \{E_1, E_2, \dots, E_k\}$.
- (4) $\tilde{W} = [\tilde{w}_j]_n, j=1, 2, \dots, n$, $\tilde{w}_j$ can be represented as the linguistic weight of j -th criterion.

(5) $D = [x_{ij}]_{m \times n}, i=1,2,\dots,m, j=1,2,\dots,n$ is multi criteria decision making matrix. It can be represented as

$$D = [x_{ij}]_{mn} = \begin{matrix} & C_1 & \dots & C_z & C_{z+1} & \dots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ \dots \\ A_m \end{matrix} & \begin{bmatrix} x_{11} & \dots & x_{1z} & \tilde{x}_{1z+1} & \dots & \tilde{x}_{1n} \\ x_{21} & \dots & x_{2z} & \tilde{x}_{2z+1} & \dots & \tilde{x}_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x_{m1} & \dots & x_{mz} & \tilde{x}_{mz+1} & \dots & \tilde{x}_{mn} \end{bmatrix} \end{matrix} \quad (9)$$

In quantitative criteria, x_{ij} represents the performance of i -th alternative with respect to j -th criterion. We use crisp value CV_{ij} to represent x_{ij} . In qualitative criteria, $\tilde{x}_{ij}$ represents the linguistic performance of i -th alternative with respect to j -th criterion. Decision makers can use 2-tuple linguistic variable to express their opinions about linguistic performance. Let $F_j^k(A_i) = (S_{ij}^k, \alpha_{ij}^k)$ represent the linguistic performance of i -th alternative respect to j -th criterion which is expressed by k -th decision maker.

The formula of transfer crisp value CV_{ij} to a 2 tuple linguistic variable is as follows:

$$F_j(A_i) = \Delta^{-1} \left(\left(CV_{ij} - \min_i(CV_{ij}) \right) / \left(\max_i(CV_{ij}) - \min_i(CV_{ij}) \right) \right) \quad (10)$$

The formula of aggregating all of the opinions about the linguistic performance of i -th alternative with respect to j -th criterion is as follows:

$$F_j(A_i) = \Delta \left(\frac{1}{K} \sum_{k=1}^K \Delta^{-1}(S_{ij}^k, \alpha_{ij}^k) \right) = (S_{ij}, \alpha_{ij}) \quad (11)$$

The formula of aggregating all of the opinions about the linguistic weight of j -th criterion is as follows:

$$\tilde{W}_j = \Delta \left(\frac{1}{K} \sum_{k=1}^K \Delta^{-1}(S_{jk}^w, \alpha_{jk}^w) \right) = (S_j^w, \alpha_j^w) \quad (12)$$

A. Linguistic TOPSIS

The formula of linguistic weighted matrix is as follows:

$$\tilde{V} = [\tilde{v}_{ij}]_{m \times n}, i=1,2,\dots,m, j=1,2,\dots,n \quad (13)$$

where $\tilde{v}_{ij} = \Delta(\Delta^{-1}(\tilde{x}_{ij}) * \Delta^{-1}(\tilde{w}_j))$.

The formula of positive ideal solution and negative ideal solution is as $\tilde{A}^+ = (\tilde{v}_1^*, \tilde{v}_2^*, \dots, \tilde{v}_n^*)$ and $\tilde{A}^- = (\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-)$,

where $\tilde{v}_j^* = \Delta \left(\max_i \left(\Delta^{-1}(\tilde{v}_{ij}) \right) \right)$ and $\tilde{v}_j^- = \Delta \left(\min_i \left(\Delta^{-1}(\tilde{v}_{ij}) \right) \right)$.

The formula of calculating the distance between the positive ideal solution and negative ideal solution is as follows:

$$d_i^+ = d(\tilde{A}_i, \tilde{A}^+) = \sqrt{\sum_{j=1}^n \left(\max_i \left(\Delta^{-1}(\tilde{v}_{ij}) \right) - \Delta^{-1}(\tilde{v}_{ij}^*) \right)^2} \quad (14)$$

$$d_i^- = d(\tilde{A}_i, \tilde{A}^-) = \sqrt{\sum_{j=1}^n \left(\min_i \left(\Delta^{-1}(\tilde{v}_{ij}) \right) - \Delta^{-1}(\tilde{v}_{ij}^-) \right)^2} \quad (15)$$

The closeness coefficient can be computed as

$$CC_i = d_i^- / (d_i^+ + d_i^-), \quad i=1,2,\dots,m \quad (16)$$

The ranking order of alternatives can be determined in accordance with the closeness coefficient. If $CC_i > CC_j$, then alternative A_i is better than A_j .

B. Linguistic ELECTRE

According to the ELECTRE, the threshold of criteria which is including preference threshold p_j , indifference threshold q_j and veto threshold v_j . The concordance index $C_j(A_i, A_l)$ represents the degree of alternative A_i is better than A_l with respect to j -th criterion. It can be computed as

$$C_j(A_i, A_l) = \begin{cases} 1, & \Delta^{-1}(\tilde{x}_{ij}) \geq \Delta^{-1}(\tilde{x}_{lj}) - q_j \\ \frac{\Delta^{-1}(\tilde{x}_{ij}) - \Delta^{-1}(\tilde{x}_{lj}) + p_j}{p_j - q_j}, & \Delta^{-1}(\tilde{x}_{ij}) - q_j \geq \Delta^{-1}(\tilde{x}_{lj}) \geq \Delta^{-1}(\tilde{x}_{ij}) - p_j \\ 0, & \Delta^{-1}(\tilde{x}_{ij}) \leq \Delta^{-1}(\tilde{x}_{lj}) - p_j \end{cases} \quad (17)$$

The overall concordance index $C(A_i, A_l)$ can be computed as

$$C(A_i, A_l) = \sum_{j=1}^n \left(\Delta^{-1}(\tilde{w}_j) \right) \left(\sum_{k=1}^n \Delta^{-1}(\tilde{w}_k) \right) C_j(A_i, A_l) \quad (18)$$

The discordance index $D_j(A_i, A_l)$ represents the degree of alternative A_i is not better than A_l with respect to j -th criterion. It can be computed as

$$D_j(A_i, A_l) = \begin{cases} 1, & \Delta^{-1}(\tilde{x}_{ij}) \leq \Delta^{-1}(\tilde{x}_{lj}) - v_j \\ \frac{\Delta^{-1}(\tilde{x}_{ij}) - p_j - \Delta^{-1}(\tilde{x}_{lj})}{v_j - p_j}, & \Delta^{-1}(\tilde{x}_{ij}) - p_j \geq \Delta^{-1}(\tilde{x}_{lj}) \geq \Delta^{-1}(\tilde{x}_{ij}) - v_j \\ 0, & \Delta^{-1}(\tilde{x}_{ij}) \geq \Delta^{-1}(\tilde{x}_{lj}) - p_j \end{cases} \quad (19)$$

The credibility matrix $S(A_i, A_l)$ can be computed as

$$S(A_i, A_l) = \begin{cases} C(A_i, A_l), & \text{if } D_j(A_i, A_l) \leq C(A_i, A_l) \quad \forall j \\ C(A_i, A_l) \prod_{j \in J(A_i, A_l)} \frac{1 - D_j(A_i, A_l)}{1 - C(A_i, A_l)}, & \text{otherwise} \end{cases} \quad (20)$$

where $J(A_i, A_l)$ represents the set of criteria which satisfy the discordance index of the criteria is larger than the overall concordance index

The degree value of alternative A_i is better than all of the other alternatives can be computed as

$$\phi_e^+(A_i) = \sum_{A_l \in A} S(A_i, A_l) \quad (21)$$

The degree value of all of the other alternatives is better than alternative A_i can be computed as

$$\phi_e^-(A_i) = \sum_{A_l \in A} S(A_l, A_i) \quad (22)$$

Calculate the net flow of alternative A_i as

$$\phi_e(A_i) = \phi_e^+(A_i) - \phi_e^-(A_i) \quad (23)$$

Normalized the net flow of alternatives as

$$OTI_e(A_i) = (\phi_e(A_i)/m + 1)/2 \quad (24)$$

According to the OTI_e , we can determine the ranking order of all alternatives.

C. Linguistic PROMETHEE

Determine the preference functions and thresholds of all criteria, and calculate the individual preference value of alternative A_r is better than alternative A_s with respect to each criterion. In this paper, we use two kinds of preference function.

Level criterion with linear preference is

$$H(\tilde{x}_{rj}, \tilde{x}_{sj}) = \begin{cases} 1, \Delta^{-1}(\tilde{x}_{rj}) - \Delta^{-1}(\tilde{x}_{sj}) > p \\ \frac{1}{2}, q \leq \Delta^{-1}(\tilde{x}_{rj}) - \Delta^{-1}(\tilde{x}_{sj}) \leq p \\ 0, q \leq \Delta^{-1}(\tilde{x}_{rj}) - \Delta^{-1}(\tilde{x}_{sj}) \end{cases} \quad (25)$$

Criterion with linear preference and indifference area is

$$H(\tilde{x}_{rj}, \tilde{x}_{sj}) = \begin{cases} 1, p < \Delta^{-1}(\tilde{x}_{rj}) - \Delta^{-1}(\tilde{x}_{sj}) \\ \frac{\Delta^{-1}(\tilde{x}_{rj}) - q}{p - q}, q \leq \Delta^{-1}(\tilde{x}_{rj}) - \Delta^{-1}(\tilde{x}_{sj}) \leq p \\ 0, \Delta^{-1}(\tilde{x}_{rj}) - \Delta^{-1}(\tilde{x}_{sj}) < q \end{cases} \quad (26)$$

Calculate the overall preference value to represent alternative A_r is better than A_s as

$$\pi(A_r, A_s) = \frac{n}{j=1} \left(\Delta^{-1}(\tilde{w}_j) / \sum_{k=1}^n \Delta^{-1}(\tilde{w}_k) \right) * H_j(\tilde{x}_{rj}, \tilde{x}_{sj}) \quad (27)$$

The degree about alternative A_r is better than all of the other alternatives as

$$\phi_p^+(A_r) = \sum_{b \in A} \pi(A_r, b) \quad (28)$$

The degree about all of the other alternatives is better than alternative A_r as

$$\phi_p^-(A_r) = \sum_{b \in A} \pi(b, A_r) \quad (29)$$

Calculate the net flow of alternative A_r as

$$\phi_p(A_r) = \phi_p^+(A_r) - \phi_p^-(A_r) \quad (30)$$

And then, normalized the net flow of alternative A_r as

$$OTI_p(A_r) = (\phi_p(A_r)/m) + 1/2 \quad (31)$$

According to the OTI_p , we can determine the ranking order of all alternatives can rank all of the alternatives.

D. Linguistic VIKOR

The linguistic positive-ideal solution (F_j^*) of each general criterion can be calculated as

$$F_j^* = \Delta \left(\max_i \Delta^{-1}(F_j(A_i)) \right), \forall A_i \in A \quad (32)$$

The linguistic negative-ideal solution (F_j^-) of each general criterion can be calculated as

$$F_j^- = \Delta \left(\min_i \Delta^{-1}(F_j(A_i)) \right), \forall A_i \in A \quad (33)$$

The group utility for the majority (S_i) of each alternative can be calculated as

$$S_i = \sum_{j=1}^n \Delta^{-1}(\tilde{w}_j) * \frac{\Delta^{-1}(F_j^*) - \Delta^{-1}(F_j(A_i))}{\Delta^{-1}(F_j^*) - \Delta^{-1}(F_j^-)} \quad \forall i \quad (34)$$

The individual regret rating for the opponent R_i of each alternative can be calculated as

$$R_i = \max_j \left(\Delta^{-1}(\tilde{w}_j) * \frac{\Delta^{-1}(F_j^*) - \Delta^{-1}(F_j(A_i))}{\Delta^{-1}(F_j^*) - \Delta^{-1}(F_j^-)} \right) \quad \forall i \quad (35)$$

R_i represents as the maximum regret by choosing i-th alternative as solution according to choose the worst performance in general criteria.

The value (Q_i) of each alternative can be calculated as

$$Q_i = v * \frac{S_i - S^*}{S^- - S^*} + (1 - v) * \frac{R_i - R^*}{R^- - R^*} \quad \forall i \quad (36)$$

$$S^* = \min_i S_i, S^- = \max_i S_i, R^* = \max_i R_i, R^- = \min_i R_i$$

v represents the decision making coefficient, v is between 0 and 1. According to the Q_i , we can determine the ranking order of all alternatives.

E. Linguistic Intersection Method (LIM)

We can consider linguistic TOPSIS, linguistic ELECTRE, linguistic PROMETHEE and linguistic VIKOR methods as four experts who provide information of each alternative, respectively. Smart manager will make a decision by considering the suggestions of experts simultaneously. Let $\Omega_t = \{A_i | \text{Rank}(CC_i) \leq SN\}$, $\Omega_e = \{A_i | \text{Rank}(OTI_e(A_i)) \leq SN\}$, $\Omega_p = \{A_i | \text{Rank}(OTI_p(A_i)) \leq SN\}$ and $\Omega_v = \{A_i | \text{Rank}(Q_i) \leq SN\}$. Ω_t , Ω_e , Ω_p and Ω_v represents the best alternative set of linguistic TOPSIS, ELECTRE, PROMETHEE and VIKOR method. The $\text{Rank}(CC_i)$, $\text{Rank}(OTI_e(A_i))$, $\text{Rank}(OTI_p(A_i))$ and $\text{Rank}(Q_i)$ represent the alternative ranking order of each MCDM approach. SN represents the number of alternatives which manager considers the maximum volume of alternatives where alternative is better exists in each MCDM approach. We execute the intersection from four kinds of set as $\Omega_{can} = \Omega_t \cap \Omega_e \cap \Omega_p \cap \Omega_v$. The Ω_{can} represents the common set which is agreeable by four experts (MCDM methods) and can be defined as consensus alternative set. According to the consensus alternative set, normalized the closeness coefficient value as

$$P_t(A_i) = \begin{cases} \frac{CC_i}{\sum_{A_i \in \Omega_{can}} CC_i}, A_i \in \Omega_{can} \\ 0, A_i \notin \Omega_{can} \end{cases} \quad (37)$$

Normalized the OTI_e as

$$P_e(A_i) = \begin{cases} \frac{OTI_e(A_i)}{\sum_{A_i \in \Omega_{can}} OTI_e(A_i)}, A_i \in \Omega_{can} \\ 0, A_i \notin \Omega_{can} \end{cases} \quad (38)$$

Normalized the OTI_p as

$$P_p(A_i) = \begin{cases} \frac{OTI_p(A_i)}{\sum_{A_i \in \Omega_{can}} OTI_p(A_i)}, A_i \in \Omega_{can} \\ 0, A_i \notin \Omega_{can} \end{cases} \quad (39)$$

Normalized the Q_i , as

$$P_v(A_i) = \begin{cases} \frac{(1-Q_i)}{\sum_{A_i \in \Omega_{can}} (1-Q_i)}, & A_i \in \Omega_{can} \\ 0, & A_i \notin \Omega_{can} \end{cases} \quad (40)$$

Let $P_{imp}(A_i)$ be the comprehensive performance of each alternative in the consensus alternative set. It can be integrated from four kinds of MCDM methods as

$$P_{imp}(A_i) = v_t * P_t(A_i) + v_e * P_e(A_i) + v_p * P_p(A_i) + v_v * P_v(A_i) \quad (41)$$

where $v_t + v_e + v_p + v_v = 1$. The v_t , v_e , v_p and v_v represent the importance of four methods from the viewpoint of manager.

IV. NUMERICAL EXAMPLE

Suppose a notable gift manufacturer wants to sell famous gift. The manufacturer wants to entry the market of China, so they have to choose a sales channel company for outsourcing retail business. There are seven criteria are considered such as the market share rate of the sales channel company (C_1), gross profit margin (C_2), inventory turnover ratio (C_3), current ratio (C_4), brand image (C_5), retail place (C_6) and enterprise culture (C_7). Brand image, retail place and enterprise culture are qualitative criteria. Gift manufacturer employs four enterprise consultants to evaluate twelve sales channel companies. The process of linguistic intersection method is shown as follows.

Step1. Collect quantitative information of twelve sales channel companies as Table 2.

TABLE 2. THE QUANTITATIVE INFORMATION

	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆
C ₁	4%	0.7%	3%	2.7%	1.6%	2.1%
C ₂	10.2%	9.8%	7%	6.6%	12%	8%
C ₃	7	6.8	8	4.2	7.5	6
C ₄	1.5	1.2	1.4	1.8	2	1.5
	A ₇	A ₈	A ₉	A ₁₀	A ₁₁	A ₁₂
C ₁	4.2%	1.2%	4%	1.6%	3%	0.5%
C ₂	7.2%	6.4%	11.2%	5.6%	8%	7.5%
C ₃	4.3	6.5	7	4.5	8	5.6
C ₄	1.4	1.3	2	2.1	2.2	1.8

Step2. Transform quantitative information of twelve sales channel companies into 5 scale linguistic variable.

Step3. Four enterprise consultants choose linguistic variable flexibly to express their opinions about performance of each sales channel company with respect to each qualitative criterion as Table 3 and the weight of each criterion as Table 4. Enterprise consultants E₁ and E₂ choose 5 scale linguistic variable to express their opinions. Enterprise consultants E₃ and E₄ use 7 scale linguistic variable to express their opinions.

TABLE 3. THE WEIGHT LINGUISTIC VARIABLES

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇
E ₁	(s ₃ ⁷ ,0)	(s ₅ ⁷ ,0)	(s ₄ ⁷ ,0)	(s ₆ ⁷ ,0)	(s ₆ ⁷ ,0)	(s ₅ ⁷ ,0)	(s ₆ ⁷ ,0)

E ₂	(s ₆ ⁷ ,0)	(s ₇ ⁷ ,0)	(s ₆ ⁷ ,0)	(s ₆ ⁷ ,0)	(s ₃ ⁷ ,0)	(s ₆ ⁷ ,0)	(s ₄ ⁷ ,0)
E ₃	(s ₂ ⁵ ,0)	(s ₂ ⁵ ,0)	(s ₃ ⁵ ,0)	(s ₄ ⁵ ,0)	(s ₁ ⁵ ,0)	(s ₄ ⁵ ,0)	(s ₂ ⁵ ,0)
E ₄	(s ₁ ⁵ ,0)	(s ₃ ⁵ ,0)	(s ₂ ⁵ ,0)	(s ₂ ⁵ ,0)	(s ₂ ⁵ ,0)	(s ₁ ⁵ ,0)	(s ₃ ⁵ ,0)

TABLE 4. THE RATING LINGUISTIC VARIABLES

Criteria		E ₁	E ₂	E ₃	E ₄		E ₁	E ₂	E ₃	E ₄
C ₅	A ₁	(s ₄ ⁵ ,0)	(s ₁ ⁵ ,0)	(s ₂ ⁷ ,0)	(s ₆ ⁷ ,0)	A ₇	(s ₁ ⁵ ,0)	(s ₃ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
	A ₂	(s ₁ ⁵ ,0)	(s ₄ ⁵ ,0)	(s ₃ ⁷ ,0)	(s ₃ ⁷ ,0)	A ₈	(s ₄ ⁵ ,0)	(s ₄ ⁵ ,0)	(s ₃ ⁷ ,0)	(s ₃ ⁷ ,0)
	A ₃	(s ₃ ⁵ ,0)	(s ₁ ⁵ ,0)	(s ₂ ⁷ ,0)	(s ₂ ⁷ ,0)	A ₉	(s ₃ ⁵ ,0)	(s ₀ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
	A ₄	(s ₄ ⁵ ,0)	(s ₃ ⁵ ,0)	(s ₃ ⁷ ,0)	(s ₆ ⁷ ,0)	A ₁₀	(s ₃ ⁵ ,0)	(s ₃ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₃ ⁷ ,0)
	A ₅	(s ₀ ⁵ ,0)	(s ₃ ⁵ ,0)	(s ₃ ⁷ ,0)	(s ₇ ⁷ ,0)	A ₁₁	(s ₂ ⁵ ,0)	(s ₂ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
	A ₆	(s ₂ ⁵ ,0)	(s ₀ ⁵ ,0)	(s ₆ ⁷ ,0)	(s ₇ ⁷ ,0)	A ₁₂	(s ₄ ⁵ ,0)	(s ₀ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
C ₆	A ₁	(s ₂ ⁵ ,0)	(s ₁ ⁵ ,0)	(s ₃ ⁷ ,0)	(s ₄ ⁷ ,0)	A ₇	(s ₃ ⁵ ,0)	(s ₂ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
	A ₂	(s ₂ ⁵ ,0)	(s ₃ ⁵ ,0)	(s ₅ ⁷ ,0)	(s ₃ ⁷ ,0)	A ₈	(s ₄ ⁵ ,0)	(s ₄ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
	A ₃	(s ₀ ⁵ ,0)	(s ₀ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₆ ⁷ ,0)	A ₉	(s ₁ ⁵ ,0)	(s ₀ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
	A ₄	(s ₄ ⁵ ,0)	(s ₄ ⁵ ,0)	(s ₅ ⁷ ,0)	(s ₇ ⁷ ,0)	A ₁₀	(s ₀ ⁵ ,0)	(s ₁ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
	A ₅	(s ₃ ⁵ ,0)	(s ₀ ⁵ ,0)	(s ₆ ⁷ ,0)	(s ₇ ⁷ ,0)	A ₁₁	(s ₄ ⁵ ,0)	(s ₀ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
	A ₆	(s ₁ ⁵ ,0)	(s ₃ ⁵ ,0)	(s ₄ ⁷ ,0)	(s ₇ ⁷ ,0)	A ₁₂	(s ₀ ⁵ ,0)	(s ₀ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
C ₇	A ₁	(s ₄ ⁵ ,0)	(s ₀ ⁵ ,0)	(s ₃ ⁷ ,0)	(s ₄ ⁷ ,0)	A ₇	(s ₂ ⁵ ,0)	(s ₀ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
	A ₂	(s ₃ ⁵ ,0)	(s ₂ ⁵ ,0)	(s ₃ ⁷ ,0)	(s ₆ ⁷ ,0)	A ₈	(s ₀ ⁵ ,0)	(s ₄ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
	A ₃	(s ₃ ⁵ ,0)	(s ₂ ⁵ ,0)	(s ₆ ⁷ ,0)	(s ₇ ⁷ ,0)	A ₉	(s ₂ ⁵ ,0)	(s ₀ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₃ ⁷ ,0)
	A ₄	(s ₀ ⁵ ,0)	(s ₄ ⁵ ,0)	(s ₃ ⁷ ,0)	(s ₇ ⁷ ,0)	A ₁₀	(s ₁ ⁵ ,0)	(s ₂ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
	A ₅	(s ₀ ⁵ ,0)	(s ₄ ⁵ ,0)	(s ₆ ⁷ ,0)	(s ₇ ⁷ ,0)	A ₁₁	(s ₀ ⁵ ,0)	(s ₃ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₇ ⁷ ,0)
	A ₆	(s ₃ ⁵ ,0)	(s ₂ ⁵ ,0)	(s ₄ ⁷ ,0)	(s ₇ ⁷ ,0)	A ₁₂	(s ₁ ⁵ ,0)	(s ₀ ⁵ ,0)	(s ₇ ⁷ ,0)	(s ₄ ⁷ ,0)

Step4. Manager determines the number of better alternatives (SN) in each MCDM approach. By the purpose of fairness, we set the number of better alternatives (SN) in each MCDM is the same and equal to six (fifty percent of number of alternatives). And then, the selected alternative set by TOPSIS is $\Omega_t = \{A_1, A_2, A_9, A_{11}, A_5, A_3\}$, the selected alternative set by ELECTRE is $\Omega_e = \{A_5, A_{11}, A_4, A_1, A_9, A_8\}$, the selected alternative set by PROMETHEE method is $\Omega_p = \{A_5, A_{11}, A_1, A_4, A_9, A_8\}$ and the selected alternative set by VIKOR is $\Omega_v = \{A_{11}, A_5, A_9, A_1, A_4, A_6\}$. Final, we calculate the intersection set as $\Omega_{can} = \Omega_t \cap \Omega_e \cap \Omega_p \cap \Omega_v = \{A_1, A_5, A_9, A_{11}\}$.

Step5. Manager determines the relative importance of TOPSIS $v_t = 1/4$, ELECTRE $v_e = 1/4$, PROMETHEE $v_p = 1/4$ and VIKOR $v_v = 1/4$. And calculate the relative evaluation value of TOPSIS(P_t), ELECTRE(P_e), PROMETHEE(P_p) and VIKOR(P_v). And then, calculate the overall performance of each alternative P_{imp} (see Table 11). Finally, the ranking order of alternatives is $A_{11} > A_5 > A_1 > A_9$.

V. MCDM METHOD COMPARISON

In order to justify the effectiveness of proposed method, this paper compares the ranking result of the LIM with original linguistic TOPSIS, linguistic ELECTRE, linguistic PROMETHEE and linguistic VIKOR. It finds that alternative A_1 , A_5 , A_9 and A_{11} which are in intersection set are relatively stable and stand on the top 5 positions, no matter what kind of MCDM approach we use. On the other hand, the alternatives which are not in intersection set don't have a relatively consistent rank in every kind of linguistic MCDM method. If we consider that the sum ranking order of each alternative in each linguistic MCDM method, the final ranking order of alternatives is the same as LIM method (see Table 5).

TABLE 5 THE RANKING ORDER INFORMATION

Alternative	Rank of TOPSIS	Rank of ELECTRE	Rank of PROMETHEE	Rank of VIKOR	Sum of Rank	Overall Rank
A_{11}	4	2	2	1	9	1
A_5	5	1	1	2	9	1
A_1	1	4	3	4	12	3
A_9	3	5	5	3	16	4
A_4	10	3	4	5	22	5
A_2	2	7	7	11	27	6
A_3	6	8	8	8	30	7
A_8	12	6	6	9	33	8
A_6	11	10	10	6	37	9
A_7	9	9	9	10	37	9
A_{12}	8	12	11	7	38	11
A_{10}	7	11	10	12	40	12

VI. CONCLUSION AND FUTURE RESEARCH

In this paper, a linguistic intersection method (LIM) is presented to handle the MCDM problems in the fuzzy environment. The advantage of linguistic intersection method is illustrated as follows.

1. The picked alternative which is executed by LIM in accordance with the agreement of each MCDM approach. It can promote effectiveness of decision making.
2. Although LIM is a little complexity in executing decision making process, but all of the process is easy to calculate and can be done by computer program.
3. Linguistic TOPSIS, linguistic ELECTRE, linguistic PROMETHEE and linguistic VIKOR are used to determine the performance of each alternative in this study. In reality, LIM is an extendable method which can extend by adding new developed MCDM approach by empowering new developed MCDM approach.
4. The LIM can effective deal with MCDM problems based

on quantitative and qualitative information simultaneously.

In the future, an interactive program will be developed to handle the fuzzy MCDM problems based on linguistic intersection method (LIM).

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