

Generalized Fuzzy Imaginary Ideals of Rings

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Abstract—Our aim in this paper is to introduce and study the new type of fuzzy ideals of a ring called generalized fuzzy imaginary right (resp . left) ideals and the direct products of them . The equivalence relation of them is given , besides , the properties of their intersection, union and level sets are discussed . Finally, we also give the properties of their homomorphic preimage.

Keywords—fuzzy imaginary ideals ;generalized fuzzy imaginary ideals ; $(\in, \in \vee q_{(\lambda, \mu)})$ - fuzzy imaginary ideals ; level sets ;homomorphic preimage

I. INTRODUCTION

The theory of fuzzy sets which was introduced by Zadeh in 1965[1] is applied to many mathematical branches. Rosenfeld inspired the fuzzification of algebraic structures and introduced the notion of fuzzy subgroups in 1971 [2] . Since then, many scholars studied the theory of fuzzy algebra and have published a series of papers [3-6]. In 1982, W. J. Liu defined and studied fuzzy subrings as well as fuzzy ideals in rings [7]. In 1990, H.C.Ma gave the definition of imaginary ideals of rings and obtained some results[8] , then in 1992, he discussed the relation between the imaginary ideals, pseudo ideals and ideals of rings [9], besides, some relevant examples were also given by him.

In 1992, Indian scholar S. K. Bhakat and P. Das introduced the concept of (α, β) -fuzzy subgroup[10] by using "belong to $(\in)$ " relation and "quais-coincident with(q)"relation between the fuzzy point and the fuzzy sets. In fact ,it's an important and useful generalization of Rosenfeld's fuzzy subgroup. In 1996, S. K. Bhakat and P .Das introduced the concept of (α, β) -fuzzy subring in [11], B . J .Zhao and H. G .Chi has introduced the concept of (λ, μ) -generalized fuzzy subring in [12]. Z . H. Liao et al extended Rosenfeld's fuzzy algebra, Bhakat and Das's fuzzy algebra and $(\bar{\in}, \bar{\in} \vee \bar{q})$ -fuzzy algebra to $(\in, \in \vee q_{(\lambda, \mu)})$ - fuzzy algebra, and do series of researches [13-16] .

This paper is the continuation of the above work, we give the definition of $(\in, \in \vee q_{(\lambda, \mu)})$ - fuzzy imaginary right (resp. left) ideals of rings, generalized fuzzy imaginary right (resp. left) ideals of rings.The properties of their intersection, union and level sets et al are also discussed. Besides , we find the equivalence relation between $(\in, \in \vee q_{(\lambda, \mu)})$ - fuzzy imaginary right (resp. left) ideals and generalized fuzzy imaginary right (resp. left) ideals . Finally ,the property of homomorphic preimage is given .

II. PRELIMINARIES

From now on , we write R and H for rings.

Definition 2.1[8] Let A be a subring of R , then A is said to be a imaginary right (resp. left) ideal of R if for all $x \in A$ and $y \in R$, $xy \in A$ (resp. $yx \in A$) implies $y \in A$.

We say A a imaginary ideal of R if A is both a imaginary right ideal and a imaginary left ideal of R .

Definition2.2[14] Let $\alpha, \lambda, \mu \in [0, 1]$ and $\lambda < \mu$, if $A(x) \geq \alpha$ then a fuzzy point x_α belongs to a fuzzy subset A , denoted by $x_\alpha \in A$; if $\lambda < \alpha$ and $A(x) + \alpha > 2\mu$ then a fuzzy point x_α generalized quasicoincident with a fuzzy subset A , denoted by $x_\alpha q_{(\lambda, \mu)} A$.If $x_\alpha \in A$ or $x_\alpha q_{(\lambda, \mu)} A$, then $x_\alpha \in \vee q_{(\lambda, \mu)} A$.

Definition2.3 [3] Let $R_i (1 \leq i \leq n)$ be rings and the direct product:

$$\prod_{1 \leq i \leq n} R_i = \{(a_1, a_2, \dots, a_n) | a_i \in R_i\} .$$

Then $\prod_{1 \leq i \leq n} R_i$ is a ring under the operations as follows:

$$(a_1, a_2, \dots, a_n) + (b_1, b_2, \dots, b_n) = (a_1 + b_1, a_2 + b_2, \dots, a_n + b_n) ;$$

$$(a_1, a_2, \dots, a_n)(b_1, b_2, \dots, b_n) = (a_1 b_1, a_2 b_2, \dots, a_n b_n) .$$

Definition2.4 [3] Let $A_i (1 \leq i \leq n)$ be fuzzy subsets of R_i , then a fuzzy subset $\prod_{1 \leq i \leq n} A_i$ is defined as $(\prod_{1 \leq i \leq n} A_i)(x_1, x_2, \dots, x_n) = \inf_{1 \leq i \leq n} A_i(x_i)$ called fuzzy direct product.

Definition 2.5[3] A fuzzy subset A of R is called a generalized fuzzy subring of R if for all $x, y \in R$, the following conditions are satisfied ,

$$(1) A(x+y) \vee \lambda \geq A(x) \wedge A(y) \wedge \mu ;$$

$$(2) A(-x) \vee \lambda \geq A(x) \wedge \mu ;$$

$$(3) A(xy) \vee \lambda \geq A(x) \wedge A(y) \wedge \mu .$$

Theorem 2.1[3] Let A be a fuzzy subset of R , then A is a generalized fuzzy subring of R if and only if $A_\alpha \neq \Phi$ is a sub - ring of R for all $\alpha \in (\lambda, \mu]$.

Theorem 2.2[6] Let A and B be generalized fuzzy subrings of R , then $A \cap B$ is a generalized fuzzy subring of R .

Theorem 2.3 [6] A is a subring of R if and only if χ_A is a generalized fuzzy subring of R .

Theorem 2.4[3] Let $f: R \rightarrow H$ be a homomorphism, if B is a generalized fuzzy subring of H , then $f^{-1}(B)$ is a generalized fuzzy subring of R .

III. $(\in, \in \vee q_{(\lambda, \mu)})$ - FUZZY IMAGINARY IDEALS OF RINGS

Definition 3.1 Let A be a generalized fuzzy subring of R , then A is called a generalized fuzzy imaginary right (resp. left) ideal of R if for all $x, y \in R$, $A(y) \vee \lambda \geq A(x) \wedge A(xy) \wedge \mu$ ($A(y) \vee \lambda \geq A(x) \wedge A(yx) \wedge \mu$).

We say A a generalized fuzzy imaginary ideal of R if A is both a generalized fuzzy imaginary right ideal and a generalized fuzzy imaginary left ideal of R .

Definition 3.2 Let A be an $(\in, \in \vee q_{(\lambda, \mu)})$ - fuzzy subring of R , $\alpha \in (\lambda, 1]$, then A is called an $(\in, \in \vee q_{(\lambda, \mu)})$ - fuzzy imaginary right (resp. left) ideal of R , if $x_\alpha \in A$, $y \in R$ and $(xy)_\alpha \in A$ (resp. $(yx)_\alpha \in A$) implies $y_\alpha \in \vee q_{(\lambda, \mu)} A$.

We say A an $(\in, \in \vee q_{(\lambda, \mu)})$ - fuzzy imaginary ideal of R if A is both an $(\in, \in \vee q_{(\lambda, \mu)})$ - fuzzy imaginary right ideal and an $(\in, \in \vee q_{(\lambda, \mu)})$ - fuzzy imaginary left ideal of R .

Theorem 3.1 A fuzzy subset A is an $(\in, \in \vee q_{(\lambda, \mu)})$ - fuzzy subring of R if and only if A is a generalized fuzzy subring of R .

The proof is omitted .

Theorem 3.2 Let A be a fuzzy subset of R , then the following conditions are equivalent:

- (1) A is an $(\in, \in \vee q_{(\lambda, \mu)})$ - fuzzy imaginary right (resp. left) ideal of R ;
- (2) A is a generalized fuzzy imaginary right (resp. left) ideal of R ;
- (3) Nonempty set A_α is a imaginary right (resp. left) ideal of R for all $\alpha \in (\lambda, \mu]$.

Proof. We prove the theorem for imaginary right ideal. Analogous arguments apply to imaginary left ideal.

(1) $\Rightarrow$ (2)

We obtain that A is a generalized fuzzy subring of R based on theorem 3.1. Assume that there exists $x_0, y_0 \in R$ such that

$A(y_0) \vee \lambda < A(x_0) \wedge A(x_0 y_0) \wedge \mu$, choose α such that $A(y_0)$

$\vee \lambda < \alpha < A(x_0) \wedge A(x_0 y_0) \wedge \mu$, then $A(y_0) < \alpha$, $A(x_0) > \alpha$, $A(x_0 y_0) > \alpha$ and $\lambda < \alpha < \mu$, so $(x_0)_\alpha \in A$, $(x_0 y_0)_\alpha \in A$, thus $(y_0)_\alpha \in \vee q_{(\lambda, \mu)} A$ based on definition 3.2 .But $A(y_0) < \alpha$, the-

refore $A(y_0) + \alpha < \alpha + \alpha \leq 2\mu$, a contradiction . So A is a generalized fuzzy imaginary right ideal of R .

(2) $\Rightarrow$ (1)

From theorem 3.1, we know that A is an $(\in, \in \vee q_{(\lambda, \mu)})$ -

fuzzy subring of R . For all $x, y \in R$ and $\alpha \in (\lambda, 1]$, if $x_\alpha \in A$ and $(xy)_\alpha \in A$, then $A(x) \geq \alpha$ and $A(xy) \geq \alpha$. Since A is a generalized fuzzy imaginary right ideal of R , then $A(y) \vee \lambda \geq A(x) \wedge A(xy) \wedge \mu \geq \alpha \wedge \mu$. If $\alpha > \mu$, then $A(y) \vee \lambda \geq \mu$, by $\lambda < \mu$, so $A(y) \geq \mu$, then $A(y) + \alpha \geq \mu + \alpha > 2\mu$, i.e.,

$y_\alpha \in \vee q_{(\lambda, \mu)} A$. On the other hand, if $\alpha \leq \mu$, then we can obtain that $A(y) \geq \alpha$, i.e., $y_\alpha \in A$, so $y_\alpha \in \vee q_{(\lambda, \mu)} A$. Thus A is an $(\in, \in \vee q_{(\lambda, \mu)})$ - fuzzy imaginary right ideal of R .

(2) $\Rightarrow$ (3)

we know that A_α is a subring of R based on theorem 2.1 .

For all $x \in A_\alpha$ and $y \in R$, if $xy \in A_\alpha$, then $A(x) \geq \alpha$ and $A(xy) \geq \alpha$. Since A is a generalized fuzzy imaginary right ideal of R , then for all $\alpha \in (\lambda, \mu]$, we have $A(y) \vee \lambda \geq A(x) \wedge A(xy) \wedge \mu \geq \alpha \wedge \mu = \alpha$, by $\lambda < \alpha$, so $A(y) \geq \alpha$, i.e., $y \in A_\alpha$. Therefore A_α is a imaginary right ideal of R for all $\alpha \in (\lambda, \mu]$.

(3) $\Rightarrow$ (2)

We obtain that A is a generalized fuzzy subring of R based on theorem 2.1 . Assume that there exists $x_0, y_0 \in R$ such that

$A(y_0) \vee \lambda < A(x_0) \wedge A(x_0 y_0) \wedge \mu$, choose α such that $A(y_0) \vee \lambda < \alpha < A(x_0) \wedge A(x_0 y_0) \wedge \mu$, then $A(y_0) < \alpha$, $A(x_0) > \alpha$, $A(x_0 y_0) > \alpha$ and $\lambda < \alpha < \mu$ i.e., $x_0 \in A_\alpha$ and $x_0 y_0 \in A_\alpha$, since A_α is a imaginary right ideal of R , then $y_0 \in A_\alpha$, i.e., $A(y_0) \geq \alpha$, a contradiction to $A(y_0) < \alpha$. Therefore A is a generalized fuzzy imaginary right ideal of R .

Theorem 3.3 Let A and B be generalized fuzzy imaginary right

(resp. left) ideals of R , then $A \cap B$ is a generalized fuzzy imaginary right (resp. left) ideals of R .

Proof. We prove the theorem for imaginary right ideal. Analogous arguments apply to imaginary left ideal.

We obtain that $A \cap B$ is a generalized fuzzy subring of R based on theorem 2.2, for all $x, y \in R$, we have $(A \cap B)(y) \vee \lambda = (A(y) \wedge B(y)) \vee \lambda = (A(y) \vee \lambda) \wedge (B(y) \vee \lambda) \geq (A(x) \wedge A(xy) \wedge \mu) \wedge (B(x) \wedge B(xy) \wedge \mu) = (A \cap B)(x) \wedge (A \cap B)(xy) \wedge \mu$. Therefore $A \cap B$ is a generalized fuzzy imaginary right ideal of R .

Corollary 3.1 Let $A_i (i \in I)$ be generalized fuzzy imaginary right (resp. left) ideals of R , then $\bigcap_{i \in I} A_i$ is a generalized fuzzy imaginary right (resp. left) ideal of R .

Theorem 3.4 Let $A_i (i \in I)$ be generalized fuzzy imaginary right (resp . left) ideals of R and for all $i, j \in I$, $A_i \subseteq A_j$ or $A_j \subseteq A_i$, then $\bigcup_{i \in I} A_i$ is a generalized fuzzy imaginary right (resp. left) ideal of R .

Proof. We prove the theorem for imaginary right ideal. Analogous arguments apply to imaginary left ideal.

Firstly , we prove that $\bigvee_{i \in I} (A_i(x) \wedge A_i(y) \wedge \mu) = (\bigcup_{i \in I} A_i)(x) \wedge (\bigcup_{i \in I} A_i)(y) \wedge \mu$. Obviously , $\bigvee_{i \in I} (A_i(x) \wedge A_i(y) \wedge \mu) \leq (\bigcup_{i \in I} A_i)(x) \wedge (\bigcup_{i \in I} A_i)(y) \wedge \mu$. Assume that $\bigvee_{i \in I} (A_i(x) \wedge A_i(y) \wedge \mu) \neq (\bigcup_{i \in I} A_i)(x) \wedge (\bigcup_{i \in I} A_i)(y) \wedge \mu$, then there exists r such that $\bigvee_{i \in I} (A_i(x) \wedge A_i(y) \wedge \mu) < r < (\bigcup_{i \in I} A_i)(x) \wedge (\bigcup_{i \in I} A_i)(y) \wedge \mu$. Since $A_i \subseteq A_j$ or $A_j \subseteq A_i$ for all $i, j \in I$, then exists $k \in I$ such that $r < A_k(x) \wedge A_k(y) \wedge \mu$. On the other hand , $A_i(x) \wedge A_i(y) \wedge \mu < r$ for all $i \in I$, a contradiction . Therefore $\bigvee_{i \in I} (A_i(x) \wedge A_i(y) \wedge \mu) = (\bigcup_{i \in I} A_i)(x) \wedge (\bigcup_{i \in I} A_i)(y) \wedge \mu$.

Next , for all $x, y \in R$,

$$\begin{aligned} (1) & (\bigcup_{i \in I} A_i)(x+y) \vee \lambda = \bigvee_{i \in I} A_i(x+y) \vee \lambda = \bigvee_{i \in I} (A_i(x+y) \vee \lambda) \geq \bigvee_{i \in I} (A_i(x) \wedge A_i(y) \wedge \mu) = (\bigcup_{i \in I} A_i)(x) \wedge (\bigcup_{i \in I} A_i)(y) \wedge \mu; \\ (2) & (\bigcup_{i \in I} A_i)(-x) \vee \lambda = \bigvee_{i \in I} A_i(-x) \vee \lambda = \bigvee_{i \in I} (A_i(-x) \vee \lambda) \geq \bigvee_{i \in I} (A_i(x) \wedge \mu) = (\bigcup_{i \in I} A_i)(x) \wedge \mu; \\ (3) & (\bigcup_{i \in I} A_i)(xy) \vee \lambda = \bigvee_{i \in I} A_i(xy) \vee \lambda = \bigvee_{i \in I} (A_i(xy) \vee \lambda) \geq \bigvee_{i \in I} (A_i(x) \wedge A_i(y) \wedge \mu) = (\bigcup_{i \in I} A_i)(x) \wedge (\bigcup_{i \in I} A_i)(y) \wedge \mu; \end{aligned}$$

$$(4) \quad (\bigcup_{i \in I} A_i)(y) \vee \lambda = \bigvee_{i \in I} A_i(y) \vee \lambda = \bigvee_{i \in I} (A_i(y) \vee \lambda) \geq \bigvee_{i \in I} (A_i(x) \wedge A_i(xy) \wedge \mu) = (\bigcup_{i \in I} A_i)(x) \wedge (\bigcup_{i \in I} A_i)(xy) \wedge \mu. \text{ Therefore } \bigcup_{i \in I} A_i$$

is a generalized fuzzy imaginary right ideal of R .

Theorem 3.5 Let A_1 and A_2 be generalized fuzzy imaginary right (resp . left) ideals of R_1 and R_2 respectively , then $A_1 \times A_2$ is a generalized fuzzy imaginary right (resp . left) ideal of $R_1 \times R_2$.

Proof. We prove the theorem for imaginary right ideal. Analogous arguments apply to imaginary left ideal.

Firstly, we prove that $A_1 \times A_2$ is a generalized fuzzy subring of $R_1 \times R_2$. For all $x, y \in R_1 \times R_2$, where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Since A_1 and A_2 are generalized fuzzy subrings of R_1 and R_2 respectively, so we have $(A_1 \times A_2)(x+y) \vee \lambda = (A_1 \times A_2)((x_1, x_2) + (y_1, y_2)) \vee \lambda = (A_1 \times A_2)((x_1 + y_1, x_2 + y_2)) \vee \lambda = (A_1(x_1 + y_1) \wedge A_2(x_2 + y_2)) \vee \lambda = (A_1(x_1 + y_1) \vee \lambda) \wedge (A_2(x_2 + y_2) \vee \lambda) \geq (A_1(x_1) \wedge A_1(y_1) \wedge \mu) \wedge (A_2(x_2) \wedge A_2(y_2) \wedge \mu) = (A_1 \times A_2)((x_1, x_2)) \wedge (A_1 \times A_2)((y_1, y_2)) \wedge \mu = (A_1 \times A_2)(x) \wedge (A_1 \times A_2)(y) \wedge \mu$. Similarly, we can prove that $(A_1 \times A_2)(-x) \vee \lambda \geq (A_1 \times A_2)(x) \wedge \mu$ and $(A_1 \times A_2)(xy) \vee \lambda \geq (A_1 \times A_2)(x) \wedge (A_1 \times A_2)(y) \wedge \mu$. So $A_1 \times A_2$ is a generalized fuzzy subring of $R_1 \times R_2$.

Next, for all $x, y \in R_1 \times R_2$, where $x = (x_1, x_2)$ and $y = (y_1, y_2)$, $(A_1 \times A_2)(y) \vee \lambda = (A_1 \times A_2)((y_1, y_2)) \vee \lambda = (A_1(y_1) \wedge A_2(y_2)) \vee \lambda = (A_1(y_1) \vee \lambda) \wedge (A_2(y_2) \vee \lambda) \geq (A_1(x_1) \wedge A_1(x_1 y_1) \wedge \mu) \wedge (A_2(x_2) \wedge A_2(x_2 y_2) \wedge \mu) = (A_1 \times A_2)((x_1, x_2)) \wedge (A_1 \times A_2)((x_1 y_1, x_2 y_2)) \wedge \mu = (A_1 \times A_2)((x_1, x_2)) \wedge (A_1 \times A_2)((x_1, x_2)(y_1, y_2)) \wedge \mu = (A_1 \times A_2)(x) \wedge (A_1 \times A_2)(xy) \wedge \mu$.

Therefore $A_1 \times A_2$ is a generalized fuzzy imaginary right ideal of $R_1 \times R_2$.

Theorem 3.6 Let A_i be generalized fuzzy imaginary right (resp . left) ideals of R_i , then $\prod_{1 \leq i \leq n} A_i$ is a generalized fuzzy imaginary right (resp . left) ideal of $\prod_{1 \leq i \leq n} R_i$.

Proof. We prove the theorem for imaginary right ideal. Analogous arguments apply to imaginary left ideal.

Firstly , we prove that $\prod_{1 \leq i \leq n} A_i$ is a generalized fuzzy subring of $\prod_{1 \leq i \leq n} R_i$. For all $x, y \in \prod_{1 \leq i \leq n} R_i$, where $x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$, then $(\prod_{1 \leq i \leq n} A_i)(x+y) \vee \lambda = \inf_{1 \leq i \leq n} A_i(x+y) \vee \lambda = \inf_{1 \leq i \leq n} (A_i(x+y) \vee \lambda) \geq \inf_{1 \leq i \leq n} (A_i(x_i) \wedge A_i(y_i) \wedge \mu) = \inf_{1 \leq i \leq n} A_i(x_i) \wedge \inf_{1 \leq i \leq n} A_i(y_i) \wedge \mu$

$= (\prod_{1 \leq i \leq n} A_i)(x) \wedge (\prod_{1 \leq i \leq n} A_i)(y) \wedge \mu$. Similarly, $(\prod_{1 \leq i \leq n} A_i)(-x) \vee \lambda = \inf A_i(-x_i) \vee \lambda = \inf(A_i(-x_i) \vee \lambda) \geq \inf(A_i(x_i) \wedge \mu) = \inf A_i(x_i) \wedge \mu = (\prod_{1 \leq i \leq n} A_i)(x) \wedge \mu$. We can prove $\prod_{1 \leq i \leq n} A_i(xy) \vee \lambda \geq (\prod_{1 \leq i \leq n} A_i)(x) \wedge (\prod_{1 \leq i \leq n} A_i)(y) \wedge \mu$ using the same method. Therefore $\prod_{1 \leq i \leq n} A_i$ is a generalized fuzzy subring of $\prod_{1 \leq i \leq n} R_i$.

Next, For all $x, y \in \prod_{1 \leq i \leq n} R_i$, where $x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$, then $(\prod_{1 \leq i \leq n} A_i)(y) \vee \lambda = \inf A_i(y_i) \vee \lambda = \inf(A_i(y_i) \vee \lambda) \geq \inf(A_i(x_i) \wedge A_i(x_i y_i) \wedge \mu) = \inf A_i(x_i) \wedge \inf A_i(x_i y_i) \wedge \mu = (\prod_{1 \leq i \leq n} A_i)(x) \wedge (\prod_{1 \leq i \leq n} A_i)(xy) \wedge \mu$.

Thus $\prod_{1 \leq i \leq n} A_i$ is a generalized fuzzy imaginary right ideal of $\prod_{1 \leq i \leq n} R_i$.

Theorem 3.7 Let A be a subset of R , then χ_A is a generalized fuzzy imaginary right (resp. left) ideal of R if and only if A is a imaginary right (resp. left) ideal of R .

Proof. We prove the theorem for imaginary right ideal. Analogous arguments apply to imaginary left ideal.

We know that A is a subring of R based on theorem 2.3.

For all $x \in A$ and $y \in R$, if $xy \in A$, then $\chi_A(xy) = 1$ and $\chi_A(x) = 1$. Since χ_A is a generalized fuzzy imaginary right ideal of R , so $\chi_A(y) \vee \lambda \geq \chi_A(x) \wedge \chi_A(xy) \wedge \mu = \mu$, by $\lambda < \mu$, so $\chi_A(y) \geq \mu = 1$, then $y \in A$. Therefore A is a imaginary right ideal of R .

Conversely, we can easily obtain that χ_A is a generalized fuzzy subring of R based on theorem 2.3. Assume that there exists $x_0, y_0 \in R$ such that $\chi_A(y_0) \vee \lambda < \chi_A(x_0) \wedge \chi_A(x_0 y_0) \wedge \mu$, choose α such that $\chi_A(y_0) \vee \lambda < \alpha < \chi_A(x_0) \wedge \chi_A(x_0 y_0) \wedge \mu$, then $\chi_A(y_0) < \alpha$, $\chi_A(x_0) > \alpha$, $\chi_A(x_0 y_0) > \alpha$ and $\lambda < \alpha < \mu$ i.e., $x_0 \in A$ and $x_0 y_0 \in A$, since A is a imaginary right ideal of R , then $y_0 \in A$, so $\chi_A(y_0) = 1 > \alpha$, a contradiction. Thus χ_A is a generalized fuzzy imaginary right ideal of R .

Theorem 3.8 Let A be a generalized fuzzy imaginary right (resp. left) ideal of R , then $A_{\lambda} = \{x \mid A(x) > \lambda\}$ is a imaginary right (resp. left) ideal of R .

Proof. We prove the theorem for imaginary right ideal. Analogous arguments apply to imaginary left ideal.

Since A is a generalized fuzzy imaginary right ideal of R , then for every $x, y \in A_{\lambda}$, we have $A(x+y) \vee \lambda \geq A(x) \wedge$

$A(y) \wedge \mu > \lambda$, $A(-x) \vee \lambda \geq A(x) \wedge \mu > \lambda$, and $A(xy) \vee \lambda \geq A(x) \wedge A(y) \wedge \mu > \lambda$, i.e., $A(x+y) > \lambda$, $A(-x) > \lambda$ and $A(xy) > \lambda$, so $x+y, xy, -x \in A_{\lambda}$. Therefore A_{λ} is a subring of R .

For all $x \in A_{\lambda}$ and $y \in R$, if $xy \in A_{\lambda}$, then $A(xy) > \lambda$ and $A(x) > \lambda$. Since A is a generalized fuzzy imaginary right ideal of R , then $A(y) \vee \lambda \geq A(x) \wedge A(xy) \wedge \mu > \lambda$, so $A(y) > \lambda$, i.e., $y \in A_{\lambda}$. Therefore $A_{\lambda} = \{x \mid A(x) > \lambda\}$ is a imaginary right ideal of R .

Theorem 3.9 Let $f: R \rightarrow H$ be a homomorphism, if B is a generalized fuzzy imaginary right (resp. left) ideal of H , then $f^{-1}(B)$ is a generalized fuzzy imaginary right (resp. left) ideal of R .

Proof. We prove the theorem for imaginary right ideal. Analogous arguments apply to imaginary left ideal.

Based on theorem 2.4, we obtain that $f^{-1}(B)$ is a generalized fuzzy subring of R . For all $x, y \in R$, $f(x), f(y) \in H$, since B is a generalized fuzzy imaginary right ideal of H , then

$$\begin{aligned} & f^{-1}(B)(y) \vee \lambda \\ &= B(f(y)) \vee \lambda \\ &\geq B(f(x)) \wedge B(f(x)f(y)) \wedge \mu \\ &= B(f(x)) \wedge B(f(xy)) \wedge \mu \\ &= f^{-1}(B)(x) \wedge f^{-1}(B)(xy) \wedge \mu \end{aligned}$$

Therefore $f^{-1}(B)$ is a generalized fuzzy imaginary right ideal of R .

IV. CONCLUSIONS

In this paper, we gave the definition of $(\in, \in \vee q_{(\lambda, \mu)})$ -fuzzy imaginary right (resp. left) ideals of rings and generalized fuzzy imaginary right (resp. left) ideals of rings. Moreover, we found the equivalence relation between $(\in, \in \vee q_{(\lambda, \mu)})$ -fuzzy imaginary right (resp. left) ideals and generalized fuzzy imaginary right (resp. left) ideals. The properties of their intersection, union and level sets were also discussed. Finally, we gave the properties of their homomorphic preimage.

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