

Group Decision Focusing on Outliers

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Abstract—This paper investigates a group decision problem focusing on outliers in a group. The group decision in this paper is considered as approximation of all individual opinions, which are given normalized crisp or interval values. The difference of an individual opinion from the group opinion represents his/her compromise in order to reach a consensus so that it should be minimized. The first step is to detect outliers in a group and assign outlier degree to individual. The outlier degree is distance of an individual from the group of others and defined as the reduction of compromise by excluding him/her from the group. The outlier degree of a unique individual is more than those of other general individuals. Similarly to outlier degree, inlier degree is closeness of others to an individual and defined as the compromise of the group without him/her. The second step is to approximate individual opinions so as to be a group opinion. The weighted sum of compromises of all individuals is minimized introducing the outlier degrees as the weights of individuals. In this model, outliers are considered positively into account for a group opinion. Therefore, based on compromises of individuals, the models to define outlier degree and to obtain group opinion by approximating individual opinions are proposed in this paper.

Index Terms—Group decision making, Outlier, Compromise

I. INTRODUCTION

A group decision is often believed to be better than individual decision simply because of cumulating all members' knowledge, being able to check errors carefully and so on. Although it is mentioned that a group performs better than "average" individual but worse than "best" individual [1], the developed techniques of integrating and combining individual judgments are effective in part. Aside from the issues on managements of decision makers and their characters, the technique in group decision making can be applied not only when decision makers give their judgments but when various evaluations from different viewpoints, such as time series, are given. One of the advantages of group decision making over individual one is that it facilitates creativity and develops new ideas by inspiring each other [1]. It often happens that when the discussion is stable and static, a unique opinion stimulate it and it becomes active again. Individuals may realize that relatively extreme individual holds opinion closer to the group's most admired position and this comparison may act to move moderate individuals toward a more extreme position [2]. In this sense, a unique opinion might be worth being paid attention. In case of the group consisting of various stakeholders, powerful or major opinion sometimes breaks down the other small or minor opinions, even if they are important. Such a situation as depending on majority too much

might be controlled by including unique opinion positively in the decision. In this way, outlier helps to induce a fresh and fair decision. This paper emphasizes group opinions in which a unique opinion is positively reflected. While, for instance, the decision problem to determine next year's plan from past years' results, it is reasonable to consider an average and general year more than unique years. In the case where it is not necessary to pay attention to a unique individual and/or generality is important, inlier degree is considered, instead of outlier degree.

The situation that a group reaches a consensus is explained in various ways; such that all individuals are unanimous for the decision, they agree to support or practice the decision, or the decision meets their interests [3]. It is also assumed that the compromises of all individuals are limited to be as less as possible [4], [5]. So, the group opinion in this study is obtained, in other words, by approximating individual opinions so as to minimize the errors of estimated value from the observed value. The concept is similar to regression analysis, except that it assumes a linear or other regression model beforehand.

This paper consists as follows. In Section II, when individual opinions are given as crisp or interval, individual compromise in order to reach a consensus is measured as difference between individual and group evaluations. In Section III, from the viewpoint of compromises, outlier degree representing distance of an individual from the group of others is defined, and inlier degree as well. Then, in Section IV, the model to obtain a group opinion as the approximation of individual opinions is shown. The compromises of all individuals are weighted by their outlier degrees and minimized so as to reach a consensus. Finally, in Section V, the proposed compromise-weighted model is tested with two numerical examples on budget.

II. COMPROMISES OF INDIVIDUALS

At first, both the individual and group opinions are assumed as crisp evaluations of alternatives as in Table I. Individual i gives his/her opinion as evaluation of alternative j , w_{ij} , and for all alternatives they are normalized as $\sum_j w_{ij} = 1$. A group opinion in this problem setting is an approximation of individual opinions so that they are normalized as:

$$\sum_j w_j = 1.$$

The group opinion w_j can be smaller or greater than the

TABLE I
GIVEN INDIVIDUAL EVALUATIONS AND OBTAINED GROUP EVALUATIONS

Individual	Alternative					Sum
	1	...	j	...	n	
1	w_{11}	...	w_{1j}	...	w_{1n}	1
...	...	...	...	...	...	...
i	w_{i1}	...	w_{ij}	...	w_{in}	1
...	...	...	...	...	...	...
m	w_{m1}	...	w_{mj}	...	w_{mn}	1
Group	w_1	...	w_j	...	w_n	1

corresponding individual opinion w_{ij} ;

$$\begin{aligned} w_j &\leq w_{ij} & \text{or} & & w_j > w_{ij} \\ \Leftrightarrow w_j &= w_{ij} - \underline{d}_{ij} & \text{or} & & w_j = w_{ij} + \bar{d}_{ij}. \end{aligned}$$

The positive and negative deviations from a group opinion, $\bar{d}_{ij}$ and $\underline{d}_{ij}$, represent compromise of individual i in order to reach a consensus on alternative j . In other words, his/her compromised opinion is denoted as interval $[w_{ij} - \underline{d}_{ij}, w_{ij} + \bar{d}_{ij}]$ and should include the obtained group opinion as;

$$w_{ij} - \underline{d}_{ij} \leq w_j \leq w_{ij} + \bar{d}_{ij}.$$

In interval regression analysis, errors between observed values and estimated intervals by the regression model are minimized [6] and their relations are also considered in [7]. Here, no regression models are assumed so that the simple sum of errors between all the given individual opinions and the obtained group opinions should be minimized as:

$$\min \sum_i (\bar{d}_{ij} + \underline{d}_{ij}),$$

which is, in other words, the compromise of all individuals to reach the group decision.

Following these conditions, total compromise of the group is obtained as follows:

$$\begin{aligned} c = & \min \sum_{i,j} (\bar{d}_{ij} + \underline{d}_{ij}), \\ \text{s.t.} & \quad w_{ij} - \underline{d}_{ij} \leq w_j \quad \forall i, j, \\ & \quad w_j \leq w_{ij} + \bar{d}_{ij} \quad \forall i, j, \\ & \quad \sum_j w_j = 1 \quad \forall j, \\ & \quad w_j, \underline{d}_{ij}, \bar{d}_{ij} \geq 0 \quad \forall i, j, \end{aligned} \quad (1)$$

where the variables are group evaluations, w_j , and positive and negative compromises, $\bar{d}_{ij}$ and $\underline{d}_{ij}$. It is apparent that positive and/or negative compromise is 0, since both group and individual evaluations, w_{ij} and w_j , in the first two constraints are crisp. By approximating observed values; individual evaluations, so as to minimize errors of estimated values, group evaluations; total compromise of the group is obtained as sum of errors.

Next, both the individual and group opinions are extended into interval evaluations of alternatives as $W_{ij} = [\underline{w}_{ij}, \bar{w}_{ij}]$ and $W_j = [\underline{w}_j, \bar{w}_j]$, respectively. In order to normalize intervals, definition of interval probability, originally proposed in [8] and also used in [9], [10], is used.

Definition of Interval Probability The set of intervals denoted as $\{W_1, \dots, W_n\}$ where $W_i = [\underline{w}_i, \bar{w}_i]$ are called interval probabilities if and only if

$$\begin{aligned} 1) & \quad 0 \leq \underline{w}_i \leq \bar{w}_i, \quad \forall i, \\ 2) & \quad \sum_{i \neq j} \bar{w}_i + \underline{w}_j \geq 1, \quad \forall j, \\ 3) & \quad \sum_{i \neq j} \underline{w}_i + \bar{w}_j \leq 1, \quad \forall j. \end{aligned} \quad (2)$$

From (2), the following two inequalities hold:

$$\sum_i \underline{w}_i \leq 1 \text{ and } \sum_i \bar{w}_i \geq 1.$$

Then, (2) is regarded as the normality condition of intervals corresponding to the conventional one; $\sum_i w_i = 1$. It is noted that in interval probabilities there are many combinations of crisp values whose sum is one. The conventional probabilities, such as $\underline{w}_i = \bar{w}_i = w_i \geq 0 \quad \forall i$, also satisfy the requirements in (2) as follows: 1) is apparent and 2) and 3) are verified as

$$\sum_{i \neq j} w_i + w_j = \sum_i w_i = 1.$$

Therefore, the interval probability is an extension of the conventional probability.

As an extension of crisp evaluation, the compromised opinion with interval evaluation is denoted as interval $[\underline{w}_{ij} - \underline{d}_{ij}, \bar{w}_{ij} + \bar{d}_{ij}]$ which should include group opinion as:

$$\underline{w}_{ij} - \underline{d}_{ij} \leq \underline{w}_j \text{ and } \bar{w}_j \leq \bar{w}_{ij} + \bar{d}_{ij}.$$

Similarly to (1) with crisp evaluations, the total compromise is obtained as the optimal objective function value of the following problem:

$$\begin{aligned} c = & \min \sum_{i,j} (\bar{d}_{ij} + \underline{d}_{ij}), \\ \text{s.t.} & \quad \underline{w}_{ij} - \underline{d}_{ij} \leq \underline{w}_j \quad \forall i, j, \\ & \quad \bar{w}_j \leq \bar{w}_{ij} + \bar{d}_{ij} \quad \forall i, j, \\ & \quad \sum_{j \neq k} \bar{w}_j + \underline{w}_k \geq 1 \quad \forall k, \\ & \quad \sum_{j \neq k} \underline{w}_j + \bar{w}_k \leq 1 \quad \forall k, \\ & \quad \underline{d}_{ij}, \bar{d}_{ij} \geq 0 \quad \forall i, j, \\ & \quad 0 \leq \underline{w}_j \leq \bar{w}_j \quad \forall j. \end{aligned} \quad (3)$$

Focusing on the first two constraints, when the following relation is satisfied, i.e.:

$$\max_i \underline{w}_{ij} < \min_i \bar{w}_{ij}, \quad (4)$$

the solutions of (3), $\underline{w}_j$ and $\bar{w}_j$, may be interval included in interval $[\max_i \underline{w}_{ij}, \min_i \bar{w}_{ij}]$ and the positive and negative compromises are 0; $\underline{d}_{ij} = \bar{d}_{ij} = 0 \quad \forall i$. However, since interval evaluation reflects fluctuations of judgments on alternatives, relation (4), where the upper bound of interval evaluation given by any individual should be more than the lower bound given by anyone, is strict. It often happens that relation (4) is not satisfied, i.e.:

$$\max_i \underline{w}_{ij} \geq \min_i \bar{w}_{ij}.$$

In such a case, solutions of (3) are $\underline{w}_j = \bar{w}_j$ so as to satisfy the first two constraints and minimize the sum of positive and negative compromises. Then, when even individual opinions are given as interval evaluations, the group evaluations are

often obtained as crisp, much less when they are given as crisp. In case of crisp individual evaluations, the interval group evaluations are never obtained because two constraints contradict to be interval as follows.

$$\begin{cases} \underline{w}_j = \max_i w_{ij} \\ \overline{w}_j = \min_i w_{ij} \end{cases} \not\Rightarrow \underline{w}_j \leq \overline{w}_j.$$

Only in case that the given individual evaluations are similar enough to satisfy relation (4), the group evaluation is obtained as interval. In other cases, by assuming group evaluation as crisp, it is also rewritten as (1).

III. DETECTION OF OUTLIERS BASED ON COMPROMISES

The first step of this paper is to detect outliers in a group and determine their outlier degrees based on individual compromises. The outlier degree represents distance of an individual from the group of others and is measured as the difference between the compromises of groups with and without the individual. Therefore, the outlier degree of a unique individual is more than those of other general individuals. In this section, we assumed that crisp individual evaluations are given so that group opinions are also obtained as crisp. The total compromise of the group without individual k , is obtained as the optimal objective function value by the following problem based on (1).

$$\begin{aligned} c_k = & \min \sum_{i \neq k, j} (\bar{d}_{ij} + \underline{d}_{ij}), \\ \text{s.t. } & w_{ij} - \underline{d}_{ij} \leq w_j \quad \forall i \neq k, j, \\ & w_j \leq w_{ij} + \bar{d}_{ij} \quad \forall i \neq k, j, \\ & \sum_j w_j = 1, \\ & w_j, \underline{d}_{ij}, \bar{d}_{ij} \geq 0 \quad \forall i \neq k, j, \end{aligned} \quad (5)$$

where the optimal solutions satisfying the optimal objective function value are not unique because of LP problem, but they are not used in the following. For interval group evaluations, constraints are transformed as in (3).

The optimal objective function value c_k , which is smaller than that of (1), denotes the compromise remained in the group by excluding individual k . It is explained from two viewpoints and illustrated conceptually in Fig. 1, where large and small circles denote total compromises without and with individual k , respectively. From the viewpoints of individuals except k , compromise increases more from c_k to c as k is farther from them. While, from the viewpoint of individual k , compromises decreases more from c to c_k as other individuals are closer to him/her. The former viewpoint leads to outlier degree and the latter one leads to inlier degree, respectively.

As for outlier degree, the distance of individual k from the group of others is measured as reduction of total compromise by excluding him/her; $c - c_k$. Based on such a difference of total compromises with and without an individual in the group, outlier degree is defined as follows:

$$p_k = \frac{c - c_k}{\sum_i (c - c_i)} \quad \forall k, \quad (6)$$

where distances for all individuals are normalized so are individual and group evaluations. The outlier degree of a unique individual is more than those of general ones.

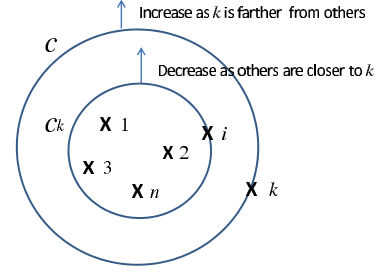


Fig. 1. Total compromises with and without an individual

Similarly to outlier degree, inlier degree is measured as the closeness of other individuals to individual k , which is the total compromise without k . Based on such a remained compromise by normalizing for all individuals, inlier degree is defined as follows.

$$q_k = \frac{c_k}{\sum_i c_i} \quad \forall k, \quad (7)$$

where a general individual has more inner degree than unique ones do.

IV. GROUP OPINIONS CONSIDERING OUTLIER DEGREES

The second step of this paper is to approximate individual opinions so as to be a group opinion. Group decision making is discussed in many literatures. For instance, in the well-known method, AHP (Analytic Hierarchy Process), by adding a decision makers' hierarchy to the top of the problem structure, the priority weights of alternatives are summed up reflecting the decision makers' weights obtained at the hierarchy [11], [12], [13]. Such a weighted summation technique is useful to take all decision makers' judgments into account. In order to determine the decision makers' weights, the corresponding pairwise comparison matrix should be given in AHP or they might be given directly. Here, the proposed outlier degree by (6) are introduced as the weight of an individual in the group so that no one needs to give the corresponding judgments. Then, the group evaluation is obtained as:

$$w_j = \sum_i p_i w_{ij} \quad \forall j, \quad (8)$$

where the weights of individuals are directly multiplied to their evaluations.

It is also practical to measure fairness of consensus by individual compromises. From individual viewpoint, his/her difference from a group decision is one of concerns since it is recognized just by comparing two values. Considering such a difference, in other words, an individual compromise, the other model to obtain group opinion is formulated as follows:

$$\begin{aligned} \min & \sum_{i,j} p_i (\bar{d}_{ij} + \underline{d}_{ij}), \\ \text{s.t. } & \text{constraints in (1),} \end{aligned} \quad (9)$$

where the objective function in (1) is transformed into outlier degree based weighted compromises so that (9) is called compromise-weighted model. In the sense of focusing on

TABLE II
RATIOS OF SIX EXPENSES OF FIVE TOWNS IN 2004, THEIR GROUP DECISION, AND THE MERGED CITY IN 2005

Year	Town	Budget size billion	Ratio					
			Expense 1	Expense 2	Expense 3	Expense 4	Expense 5	Expense 6
2004	A	2.4	0.232	0.267	0.046	0.253	0.092	0.109
2004	B	2.6	0.297	0.245	0.049	0.132	0.135	0.142
2004	C	4.8	0.237	0.203	0.083	0.222	0.129	0.126
2004	D	1.9	0.294	0.242	0.052	0.097	0.169	0.145
2004	E	1.7	0.236	0.438	0.042	0.100	0.097	0.087
	Group decision by (9)		0.237	0.267	0.049	0.192	0.129	0.126
	Group decision by (8)		0.254	0.298	0.053	0.160	0.120	0.116
2005	City	15.2	0.244	0.219	0.064	0.254	0.128	0.090

the difference between given individual and obtained group evaluations, individuals in a group can easily understand the procedure and accept the results. This model is based on the idea that outliers are taken positively into account for a group opinion.

Since outlier degree represents the distance of an individual from the group, the unique opinion is reflected in group opinion positively in (8) and (9). While, such a decision problem that unique opinion is not welcomed to the group one, inlier degrees are introduced as weights of individuals. The models are formulated by replacing outlier degrees p_i in (8) and (9) into inlier degrees q_i .

V. NUMERICAL EXAMPLES

From the viewpoint of a group decision making, two examples on planning budgets of local governments are tested. One is the case of new city consisting of several towns and its budget is estimated from the member towns' results [14]. In order not to neglect smaller or unique town's status, outlier degrees are used. The other is the case of prefecture running stable for recent years and its budget is estimated from its past years' results [15]. In order not to be affected by transient situation, inlier degrees are used. Although all the given ratios are crisp in both examples, in case of interval ratios, the similar procedure can be applied.

A. Group decision with outlier degrees

Table II shows ratios on six expenses, such as 1-personnel 2-public loan, 3-aid, 4-investment, 5-nonpersonnel and 6-subsidy, of five small towns in 2004 and their budget sizes. They were merged into a city in 2005 so that at that time the new city is forced to plan new budget referring to the results of five towns in 2004. This decision problem is used in this section. As show in Table II, the size of Town E is almost one third of the largest one, Town C, and Expense 2 of Town E is apparently different from other towns. Though, since small towns often have some difficulties to adapt a new situation, the status of Town E is also or more carefully considered. First, by (5) total compromise without each town is obtained and by (6) the outlier degrees are obtained as shown in Table III. The smallest budget Town E has the greatest outlier degree. Then, introducing outlier degrees as weights of towns, the approximations of five towns' ratios as a group decision are obtained by compromise-weighted model (9) and shown at the

TABLE III
OUTLIER DEGREES AND COMPROMISES OF TOWNS

Town	Outlier degree	Compromise		
		By (8)		By (9)
A	0.217	0.187	>	0.123
B	0.124	0.169	>	0.165
C	0.187	0.223	>	0.128
D	0.180	0.238	<	0.239
E	0.292	0.281	<	0.342
Sum of compromises		1.098	>	0.996

third bottom line of Table II. The obtained ratios equal to the medians of the corresponding five given ratios, except expense 4, whose obtained ratio is around the median so as to normalize for all expenses. To comparison, those by (8), where given ratio is weighted directly, and the results of the city in 2005 are shown at the bottom two lines of Table II. The budget size was enlarged by merging but the ratios by (9) are more similar to the real ratios than those by (8). The compromise, which is the difference of each town's ratio from the group ratio by both models are shown in Table III. The total compromise of Town E, which might be an outlier so that its weight is the largest among five towns, by (8) is restrained to be 0.281, less than 0.342 by (9). In details, Town E compromises on six expenses by (8) as $0.281=0.017+0.140+0.011+0.060+0.023+0.030$ and by (9) as $0.342=0.000+0.171+0.008+0.092+0.032+0.039$. As for Expense 2, on which Town E is different from other towns, its compromise by (8) is less than that by (9). These facts might refer that when individual towns' ratios are weighted directly as in (8), group ratio might be overfitted to Town E. By weighting individual towns' compromises as in (9), the compromise of Town E is fairly and properly reflected in the obtained group ratio.

B. Group decision with inlier degrees

Table IV shows ratios of four cost divisions, such as 1-personnel related cost 2-nonpersonnel related cost, 3-investment related cost, 4-other cost of the prefecture from 2004 to 2008 and their budget sizes. The decision problem for the prefecture is to plan its next fiscal year's budget referring to its past five years' results. As shown in Table IV, the differences of ratios between years are smaller comparing to Table II. It may not be necessary to pay attention to a unique year because of similar results; moreover, it may not be reasonable to be affected by transient year. Therefore, inlier

TABLE IV
RATIOS OF FOUR COST DIVISIONS FOR FIVE YEARS AND THEIR GROUP APPROXIMATION

Year	Budget size billion	Ratio				Sum of deviations
		Division 1	Division 2	Division 3	Division 4	
2008	372	0.382	0.289	0.272	0.057	
2007	354	0.379	0.278	0.287	0.056	
2006	388	0.361	0.304	0.271	0.064	
2005	376	0.370	0.264	0.275	0.091	
2004	413	0.345	0.298	0.288	0.070	
Group approximation by (9)		0.372	0.289	0.275	0.064	0.029
Group approximation by (8)		0.368	0.287	0.278	0.067	0.031
2009	352	0.374	0.281	0.287	0.057	

TABLE V
INLIER DEGREES AND DIFFERENCES FROM PAST RESULTS

Year	Inlier degree	Compromise		
		By (8)		By (9)
2008	0.222	0.031	>	0.020
2007	0.197	0.041	>	0.039
2006	0.213	0.034	>	0.030
2005	0.183	0.052	<	0.054
2004	0.184	0.045	<	0.053
Sum of compromises		0.204	>	0.196

degree of each year is obtained by (7) from total compromise without a year by (5) and shown in Table V. Although the inlier degrees of all years are around 0.2, it could be said that those of 2004 and 2005 are less than those of 2006, 2007 and 2008, the more recent three years. This might be because the budget has been changed gradually for years and reflecting the recent years' results in a plan is practical. Then, introducing inlier degrees as weights of years, the approximations of five years results as a group decision are obtained by compromise-weighted model (9) and to comparison by (8). They are shown at the second and third bottom lines of Table IV and differences from past years' result is shown in Table V. Those of 2008, 2007 and 2006 by (9) are smaller than those by (8). The obtained ratios by compromise-weighted model (9) are more similar to the more recent three years. Comparing the deviations between the ratios by (8) and (9) to real ratios of 2009 shown at the right row of Table IV, those by (9) are slightly smaller than those by (8).

VI. CONCLUSION

Focusing on outliers, group decision is obtained as the approximation of the given individual opinions. The models to define outlier degree and to obtain group opinion are proposed from the viewpoint of compromise. Since one of the major concerns of an individual in the group decision making is the difference of the obtained group opinion from his/her opinion, the compromise represented as such a difference is focused on in this paper. The outlier degree is used as an individual weight based on the idea that a unique opinion is sometimes worth being paid attention. It is measured as the distance of an individual from the group of others and defined as the reduction of total compromise by excluding the individual from the group. Similarly, inlier degree is used, when general opinions seem to be paid more attention. Then, the group

opinion is obtained by minimizing weighted compromises so as to reach a consensus. The easily recognized compromise for group members is a useful and reasonable measurement and the proposed models based on it are practical in real problems.

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