

Optimization of Gaussian Fuzzy Membership Functions and Evaluation of the Monotonicity Property of Fuzzy Inference Systems

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Abstract—In this paper, two issues relating to modeling of a monotonicity-preserving Fuzzy Inference System (FIS) are examined. The first is on designing or tuning of Gaussian Membership Functions (MFs) for a monotonic FIS. Designing Gaussian MFs for an FIS is difficult because of its spreading and curvature characteristics. In this study, the *sufficient conditions* are exploited, and the procedure of designing Gaussian MFs is formulated as a constrained optimization problem. The second issue is on the testing procedure for a monotonic FIS. As such, a testing procedure for a monotonic FIS model is proposed. Applicability of the proposed approach is demonstrated with a real world industrial application, i.e., Failure Mode and Effect Analysis. The results obtained are analysis and discussed. The outcomes show that the proposed approach is useful in designing a monotonicity-preserving FIS model.

Keywords—Fuzzy Inference System, monotonicity property, Gaussian membership functions, sufficient conditions, monotonicity testing

I. INTRODUCTION

Many real-world systems abide the monotonicity property between the input(s) and output(s) of a system. Consider an Fuzzy Inference System (FIS), $y = f(x_1, x_2, \dots, x_i, \dots, x_n)$, that fulfils the condition of monotonicity between its output, y , with respect to its input, x_i . The output monotonically increases or decreases as the input increases, i.e., $f(x_1, x_2, \dots, x_i^1, \dots, x_n) \leq f(x_1, x_2, \dots, x_i^2, \dots, x_n)$ or $f(x_1, x_2, \dots, x_i^1, \dots, x_n) \geq f(x_1, x_2, \dots, x_i^2, \dots, x_n)$, respectively, for $x_i^1 < x_i^2$. The monotonicity property is an additional qualitative information/knowledge that can be exploited to obtain an interpretable and optimized FIS model [1]. The importance of this line of study has been highlighted in a number of recent publications [1-8]. But, there are only a few articles that address the problem of designing a monotonicity-preserving FIS model [2]. A useful foundation of the study is the *sufficient conditions*, which comprise a set of mathematical conditions derived with the assumption that $dy/dx_i \geq 0$ or $dy/dx_i \leq 0$, for the monotonic increasing or decreasing condition, respectively, via *partial differentiation* and the *quotient rule* [3].

This paper attempts to address two issues related to modeling of a monotonicity-preserving FIS model. A common issue in FIS modeling is how fuzzy Membership Functions

(MFs) are designed and tuned. One of the difficulties of designing Gaussian MFs is because of their spreading and curvature characteristics [4]. Although the *sufficient conditions* have been adopted as the condition for designing MFs [2, 5-7], it is not clear how a good MF design should be. In [9], the use of computing with words in risk assessment has been reviewed, and it has been highlighted that designing MFs is a crucial problem. Thus, in this paper, the *sufficient conditions* are further extended, and applied to designing Gaussian MFs. As the design of Gaussian MFs is formulated as a constrained optimization problem, the extended *sufficient conditions* are used as the hard constraint for optimization. An objective function is also used to compare a set of candidates (Gaussian MFs) with other available information. In addition, a Genetic Algorithm (GA) is deployed to provide a solution for the optimization problem.

The second issue is how the monotonicity property of an FIS model can be measured or evaluated. From the literature, a monotonicity measure of data in neural network modeling was introduced by Daniels and Velikova [10]. However, the monotonicity measure in FIS modeling receives little attention. Thus, a method to measure the degree of fulfillment of the monotonicity property of an FIS model is proposed. The monotonicity measure is important because the exact condition(s) for an FIS to satisfy the monotonicity property is unknown, and the mathematical proof of monotonicity is difficult [8]. There is a possibility for an FIS to be monotonic even when the *sufficient conditions* are not fulfilled. Hence, the proposed monotonicity measure is useful to give an indication whether the designed FIS model is monotonic in practice. The effectiveness of our proposed techniques is demonstrated using a real industrial application, i.e., Failure Mode and Effect Analysis (FMEA) with an FIS-based Risk Priority Number (RPN) [11-12] model. This is a typical example for FIS-based risk assessment models [9] that require the monotonicity property [11].

This paper is organized as follows. In section II, a review on Gaussian MFs, FIS models, and the *sufficient conditions* are presented. In section III, the proposed technique for optimizing Gaussian fuzzy MFs is explained. In section IV, the proposed technique to measure the monotonicity property is described.

In section V, experimental results are presented and discussed. Finally, concluding remarks are given in section VI.

II. BACKGROUND OF GAUSSIAN FUZZY SETS, FUZZY INFERENCE SYSTEMS, AND THE SUFFICIENT CONDITIONS

A. Gaussian Fuzzy Sets

A Gaussian MF (see Fig. 1) can be represented as

$$\mu_G(x; c, \sigma) = e^{-[x-c]^2/2\sigma^2} \quad (1)$$

where c is the center of the fuzzy set, and σ parameterizes the width of the fuzzy set. The α -cut of a fuzzy set is a crisp set that contains all the elements of the universe set X that have a membership grade equals to or greater than α , where $1 \geq \alpha > 0$ (Note that a Gaussian MF is a non-zero MF). Based on the α -cut of a fuzzy set, the width of the fuzzy set, W_α , is determined. From Fig. 1, W_α is ac or cb , which can be determined by (2).

$$W_\alpha = ac = cb = \sqrt{(-\ln \alpha)(2\sigma^2)} \quad (2)$$

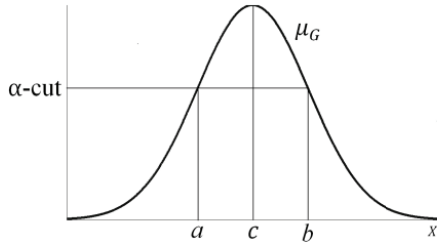


Figure 1. A Gaussian membership function

B. Fuzzy Inference Systems

The fuzzy production rules for an n -input FIS model, where $n > 0$, can be represented as follows.

$R^{j_1, j_2, \dots, j_n}$:

If $(x_1 \text{ is } A_1^{j_1}) \text{ AND } (x_2 \text{ is } A_2^{j_2}) \dots \text{ AND } (x_n \text{ is } A_n^{j_n})$,
THEN $(y \text{ is } B^{j_1, j_2, \dots, j_n}) \quad 1 \leq j_i \leq M_i$.

The AND operator in the rule antecedent part is the product function. For the x_i domain, its MFs are $\mu_i^1(x_i)$, $\mu_i^2(x_i)$, ..., and $\mu_i^{M_i}(x_i)$. The upper and lower limits for the universe of discourse of x_i are $\bar{x}_i$ and $\underline{x}_i$, respectively. The output is obtained by using the weighted average of a representative value, $b^{j_1, j_2, \dots, j_n}$ with respect to its compatibility grade, as in (3).

$$y = \frac{\sum_{j_n=1}^{M_n} \dots \sum_{j_2=1}^{M_2} \sum_{j_1=1}^{M_1} (\mu_1^{j_1}(x_1) \times \mu_2^{j_2}(x_2) \times \dots \times \mu_n^{j_n}(x_n) \times b^{j_1, j_2, \dots, j_n})}{\sum_{j_n=1}^{M_n} \dots \sum_{j_2=1}^{M_2} \sum_{j_1=1}^{M_1} (\mu_1^{j_1}(x_1) \times \mu_2^{j_2}(x_2) \times \dots \times \mu_n^{j_n}(x_n))} \quad (3)$$

where $b^{j_1, j_2, \dots, j_n}$ is the representative value of $B^{j_1, j_2, \dots, j_n}$, i.e., $b^{j_1, j_2, \dots, j_n} = \text{Rep}(B^{j_1, j_2, \dots, j_n})$. This value represents the overall location of the MF. It can be obtained via defuzzification, or be represented by the point whereby the membership value is 1.

C. Sufficient Conditions

The first derivative of an FIS model, as in (3), returns a weighted addition series. The *sufficient conditions* assume that all the components in the weighted addition series are always *greater than or equal to zero*, or *less than or equal to zero*. Two conditions can be derived [3], as follows.

Condition 1. At the rule antecedent part, $(d\mu_i^p(x)/dx)/\mu_i^p(x) \geq (d\mu_i^q(x)/dx)/\mu_i^q(x)$, where $p > q$. Note that $(d\mu_i(x)/dx)/\mu_i(x)$ is the ratio between the rate of change in the membership degree and the membership degree itself. The derivative of a Gaussian MF with respect to x is $G'(x) = -((x-c)/\sigma^2)G(x)$. Note that $(d\mu_i(x)/dx)/\mu_i(x)$ for a Gaussian MF, i.e., $(G'(x)/G(x))$, returns a linear function, i.e. $E(x) = G'(x)/G(x) = -(1/\sigma^2)x + (c/\sigma^2)$.

Condition 2. At the rule consequent part, $b^{j_1, j_2, \dots, j_i=p, \dots, j_n} - b^{j_1, j_2, \dots, j_i=q, \dots, j_n} \geq 0$ or $b^{j_1, j_2, \dots, j_i=p, \dots, j_n} - b^{j_1, j_2, \dots, j_i=q, \dots, j_n} \leq 0$ for $dy/dx_i \geq 0$ or $dy/dx_i \leq 0$, respectively. This condition suggests that a monotonic rule base is required.

III. OPTIMIZATION OF FUZZY MEMBERSHIP FUNCTIONS

A. Extension for Condition 1

Conditions 1 and **2** can be included as part of the FIS modeling process [2, 5-7]. **Condition 1** is an inequality that can be used to check the validity of Gaussian MF design i.e., a fuzzy partition. **Condition 2** can be used to check the validity of the rules in a monotonic FIS model. In this paper, the focus is on **Condition 1**.

In this work, $(d\mu_i^p(x)/dx)/\mu_i^p(x)$ is viewed as a projection of the Gaussian MFs, which allows the MFs to be visualized. As an example, two Gaussian MFs, $G_1 = e^{-[x-c_1]^2/2\sigma_1^2}$ and $G_2 = e^{-[x-c_2]^2/2\sigma_2^2}$, where $c_1 < c_2$, are considered. Their $G'(x)/G(x)$ ratios are $E_1(x) = -(1/\sigma_1^2)x + (c_1/\sigma_1^2)$ and $E_2(x) = -(1/\sigma_2^2)x + (c_2/\sigma_2^2)$, respectively. To satisfy **Condition 1**, $E_1(x) \leq E_2(x)$ is required for the defined universe of discourse. Hence, (4) and (5) are required.

$$E_1(\underline{x}_i) \leq E_2(\underline{x}_i) \quad (4)$$

$$E_1(\bar{x}_i) \leq E_2(\bar{x}_i) \quad (5)$$

B. Optimization of Gaussian Membership Functions

In this paper, the proposed extension for **Condition 1** (from section III(A)) is further exploited to be part of a Gaussian MF design process. It acts as the constraint for an optimization problem. For an input domain, x_i with n_i Gaussian MFs, the Gaussian MFs are parameterized by $c_1, c_2, \dots, c_{n_i}$ and $\sigma_1, \sigma_2, \dots, \sigma_{n_i}$. The design of the Gaussian MFs is formulated as a constrained optimization problem, as follows.

An objective function, $f(c_1, c_2, \dots, c_{n_i}, \sigma_1, \sigma_2, \dots, \sigma_{n_i})$, is optimized subjected to constraints as follow.

- #1. $E_q(\underline{x}_i: c_q, \sigma_q) \leq E_{q+1}(\underline{x}_i: c_{q+1}, \sigma_{q+1})$ for $q = 1, 2, \dots, n_{i-1}$
 #2. $E_q(\bar{x}_i: c_q, \sigma_q) \leq E_{q+1}(\bar{x}_i: c_{q+1}, \sigma_{q+1})$ for $q = 1, 2, \dots, n_{i-1}$

Constraints #1 and #2 are the conditions of feasible Gaussian MFs design for a monotonic FIS model. The search process can be achieved by various optimization techniques, e.g., a non-linear programming technique, GA, particle swarm optimization, or harmonic search.

C. A Case Study on Failure Mode and Effect Analysis

Failure Mode and Effect Analysis (FMEA) is a popular problem prevention methodology that can be interfaced with many engineering and reliability models [11]. FMEA uses a Risk Priority Number (RPN) model to evaluate the risk associated with each failure mode [11]. The RPN model considers three risk factors, i.e., severity (S), occurrence (O), and detect (D), and produces an RPN score ($RPN = g_{RPN}(S, O, D)$) [11]. The three input factors are estimated by the domain experts in accordance with the scale from “1” to “10” based on a set of commonly agreed evaluation criteria, which are presented with the scale tables [11].

In this paper, the focus is on designing an FIS-based RPN model [11-12] that satisfies the monotonicity property. The FIS-based RPN model considered S, O, and D as the inputs and the RPN as the output. The Gaussian MFs of S, O, and D are generated from the respective scale tables. As an example, Table 1 shows the scale table of S, which is used in a semiconductor manufacturing plant. There are 5 linguistic terms; each is represented by a Gaussian MF. The relationship between S, O, and D and the RPN are expressed with a set of fuzzy If-Then rules.

Table 1. The scale table for Severity

Rank	Linguistic Terms	Criteria	Lower Limit	Mid-point	Upper Limit
10	Very High (Liability)	Failure will affect safety or compliance to law.	9.5	10	-
9~8	High (Reliability / reputation)	Customer impact. Major reliability excursions.	7.5	8.5	9.5
7~6	Moderate (Quality / Convenience)	Impacts customer yield. Wrong package/par/markings.	5.5	6.5	7.5
5~2	Low (Special Handling)	Yield hit, Cosmetic.	1.5	3.5	5.5
1	None (Unnoticed)	Unnoticed.	-	1	1.5

In our previous work [5-6, 11], it has been demonstrated that an effective FIS-based RPN model should satisfy the monotonicity property. This is because the input attributes (i.e., the S, O, and D ratings) are defined in such a way that the higher the rating, the more critical the situation is. The output (i.e., the RPN score) is a measure of the failure risk. The monotonicity property is important to allow a valid comparison among all failure modes to be made.

D. Gaussian MFs Design

A qualitative scale table usually consists of several partitions, each with some criteria. Each partition can have different width, and it can be represented using three points, i.e., the lower limit, the mid-point, and the upper limit. The width of the n partition, $W_{partition}^n$, is defined as follows:

$$W_{partition}^n = upperlimit^n - midpoint^n = midpoint^n - lowerlimit^n \quad (6)$$

As an example, the scale table of S (Table 1) consists of five partitions, i.e., 1, 2-5, 6-7, 8-9, and 10. The lower limit, the mid-point, and the upper limit for the second partition ($n = 2$), i.e., 2-5, are 1.5, 3.5, and 5.5, respectively, while $W_{partition}^2$ is 2.

In this paper, we attempt to search for a set of Gaussian MFs with their widths nearest to the widths of the partitions in the scale table. The width of a Gaussian MFs can be defined using (2) with $\alpha = 0.5$ [13-14]. The issue is on how to formulate a useful objective function. In this paper, the investigation is focused on formulating an objective function, as in (7). It is a measure of total similarity of the generated MFs corresponding to the partitions of scale table. We further assume that $c_1, c_2, \dots, c_{n_i}$ are mid-points for these partitions which need not to be optimized.

$$f(c_1, c_2, \dots, c_{n_i}, \sigma_1, \sigma_2, \dots, \sigma_{n_i}) = f(\sigma_1, \sigma_2, \dots, \sigma_{n_i}) = \frac{1}{\sqrt{\sum_{n=1}^{n_i} (W_{partition}^n - W_{\alpha}^n)^2}} \quad (7)$$

E. Genetic Algorithm

The problem formulated can be solved using a GA, a population-based stochastic optimization technique. Each potential solution set, in the form of $(\sigma_1, \sigma_2, \dots, \sigma_{n_i})$, is represented as an individual. The GA is used to search for the solution set with the best fitness value.

Figure 2 shows the general procedure of the GA. The hard constraints, i.e., constraints #1 and #2, are considered as penalties of the GA objective function to be minimized, as in (8). We attempt to minimize (7), which is part of the GA objective function.

GA Objective function

$$= W_{penalty} \times \begin{cases} 0 & \text{if constraints \#1 and \#2 fulfilled} \\ 1 & \text{if constraints \#1 and \#2 not fulfilled} \end{cases} \quad (8)$$

$$+ f(\sigma_1, \sigma_2, \dots, \sigma_{n_i})$$

```
%Note: Initialize variable
Initial population (each individual is  $\sigma_1, \sigma_2, \dots, \sigma_{n_i}$ )
While stopping criteria is not triggered
{
  Evaluate each individual with (8)
  Record the best individual sets
  Crossover
  Mutation
  Evaluate the stopping criteria
}
```

Figure 2. Pseudo-code of the proposed GA-based MFs optimization model

IV. A MEASURE FOR THE MONOTONICITY PROPERTY

From the literature, the measure of the degree of monotonicity for a data set, D is defined by (9) [10]. The aim is to represent the monotonic relationship using a numerical value from 0 to 1. A value close to 1 indicates an increasing monotonic relationship among data. A value near to 0 indicates a decreasing monotonic relationship among data. A value near to 0.5 indicates no monotonic relationship among data.

$$\text{Degree of monotonicity} = \frac{\# \text{ Monotone pairs}(D)}{\# \text{ Comparable pairs}(D)} \quad (9)$$

In this paper, fulfillment of the monotonicity property of an FIS model is measured by comparing the output pairs of an FIS model. Instead of indicate the monotonic increasing or decreasing relationship of a data set as in [10], an index that indicates whether an FIS model observes the monotonicity property is proposed. The procedure is as follows.

Let $y = f(\bar{x})$ be represented by an n -dimension matrix y_{x_k} . y_{x_k} is the output of $f(x_k)$, correspond to a input(s) vector, $x_k = [x_1, x_2, \dots, x_n]$, $k = 1, 2, \dots, n$. The test procedure test is as follows.

- Determine the upper and lower limits of the universe of discourse for input(s), i.e., x_k , and denote them as $\bar{x}_k$ and $\underline{x}_k$, respectively.
- Divide x_k into n_k^t divisions. The grid size of x_k , $gs_k = (\bar{x}_k - \underline{x}_k) / n_k^t$. $y_{x_1, x_2, \dots, x_n}$ is denoted as a $n_1^t \times n_2^t \times \dots \times n_n^t$ matrix.
- Compare $y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + gs_i \times m_i}}$ and $y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + (gs_i + 1) \times m_i}}$, $m_i = 0, 1, 2, \dots, n_i^t - 1$, with a function denoted as $\text{monotone}(y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + gs_i \times m_i}})$. Equation (10) or (11) is adopted for a monotonic increasing or decreasing relationship, respectively.

$$\text{monotone}(y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + gs_i \times m_i}}) = \begin{cases} 1 & \text{if } y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + gs_i \times m_i}} \leq y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + (gs_i + 1) \times m_i}} \\ 0 & \text{if } y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + gs_i \times m_i}} > y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + (gs_i + 1) \times m_i}} \end{cases} \quad (10)$$

$$\text{monotone}(y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + gs_i \times m_i}}) = \begin{cases} 1 & \text{if } y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + gs_i \times m_i}} \geq y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + (gs_i + 1) \times m_i}} \\ 0 & \text{if } y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + gs_i \times m_i}} < y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + (gs_i + 1) \times m_i}} \end{cases} \quad (11)$$

- Obtain the *Monotone index* between y and x_i for an FIS model using (12).

$$\text{Monotone index} = \frac{\sum_{m_n=1}^{n_n^t} \dots \sum_{m_1=1}^{n_1^t} \sum_{m_i=1}^{n_i^t - 1} \left(\text{monotone}(y_{x_{k=1,2,3 \dots n, k \neq i, \underline{x}_i + gs_i \times m_i}}) \right)}{\sum_{m_n=1}^{n_n^t} \dots \sum_{m_1=1}^{n_1^t} \sum_{m_i=1}^{n_i^t - 1} (1)} \quad (12)$$

In short, first, all possible comparable sets of y and x_i are generated. The number of the monotonic pairs is counted. The degree of monotonicity is obtained with (12). As an example, the monotonicity property between S and the RPN for an FIS-based RPN can be measured as follows.

Comparable points in the form of $[S, O, D]$ are generated. Examples of comparable points are $[1, 1, 1]$ and $[2, 1, 1]$, $[2, 1, 1]$

and $[3, 1, 1], \dots, [9, 1, 1]$ and $[10, 1, 1], \dots, [1, 1, 2]$ and $[2, 1, 2], \dots$ and finally $[9, 10, 10]$ and $[10, 10, 10]$. The RPN scores for $[1, 1, 1]$ ($g_{RPN}(1, 1, 1)$) and $[2, 1, 1]$ ($g_{RPN}(2, 1, 1)$) are compared, and their fulfillment of monotonicity ($g_{RPN}(1, 1, 1) \leq g_{RPN}(2, 1, 1)$) are checked. The same goes to other comparable points. In this study, a total of 900 comparable points are generated between S and the RPN. The same procedure can be applied to O and the RPN, as well as D and the RPN. In total, there are 2700 (3×900) comparable points that have been generated for evaluation of the monotonicity property in the study.

V. EXPERIMENTAL RESULT AND DISCUSSIONS

Figure 3 depicts the objective function versus the number of generations of the GA. Figure 4 depicts the optimized Gaussian MFs of S using the best individual after 300 iterations. Figure 5 depicts the projection of the Gaussian MFs using **Condition 1**. From the results, it is obvious that the proposed techniques can be used to automatically generate a set of optimized Gaussian MFs, which are able to fulfill **Condition 1**. From Figure 4, it can be observed that the width of each Gaussian MF is close to the scale table of S .

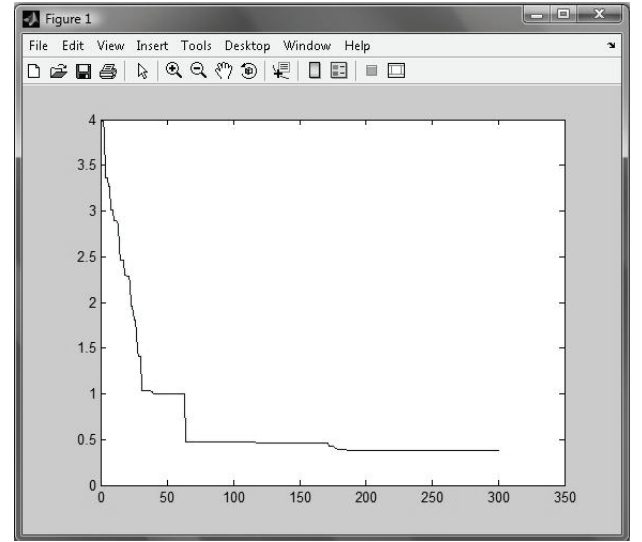


Figure 3 Objective function versus the number of generations of the GA-based MFs design procedure

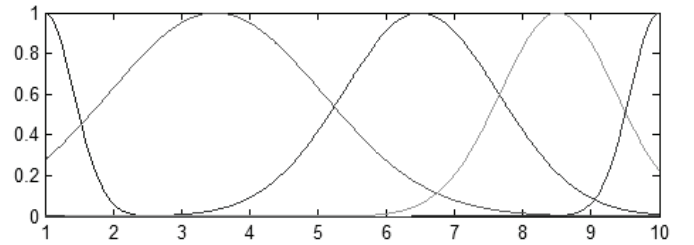


Figure 4 Projection of the MFs for Severity

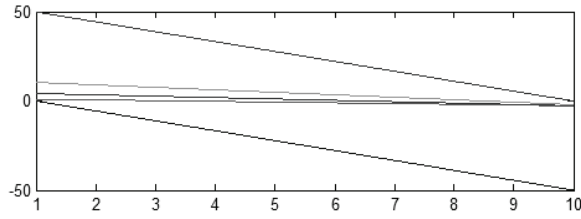


Figure 5 Optimized MFs for Severity.

Table 2 shows the scale table of D of the FIS-based RPN model. There are 6 linguistic terms, each is represented by a Gaussian MF.

Table 2 The scale table for Detect

Rank	Linguistic Terms	Criteria	Lower Limit	Mid-point	Upper Limit
10	Extremely Low	No Control available.	9.5	10	-
9	Very Low	Controls probably will not Detect	8.5	9	9.5
8~7	Low	Controls may not Detect excursion until reach next functional area.	6.5	7.5	8.5
6~5	Moderate	Controls are able to Detect within the same functional area	4.5	5.5	6.5
4~3	High	Controls are able to Detect within the same machine/module.	2.5	3.5	4.5
2~1	Very High	Controls will Detect excursions before next lot is produced. Prevent excursion from occurring	-	1	2.5

Figure 6 depicts the optimized Gaussian MFs for scale table D using the best individual after 300 iterations. From Figure 6, it can be seen that the width of each of Gaussian MF is close to the scale table of D. Figure 7 depicts the projection of the MFs using **Condition 1**.

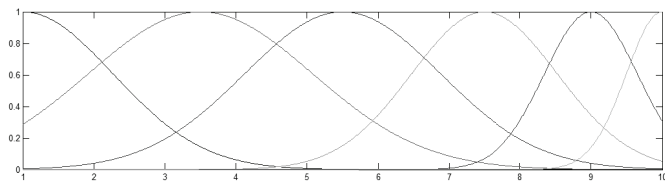


Figure 6 Projection of the MFs for Detect.

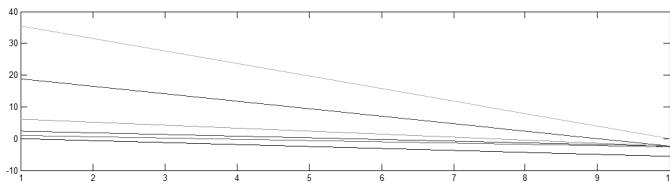


Figure 7 Optimized MFs for Detect.

In order to validate the Gaussian MFs designed using the proposed techniques, a data set collected from a real semiconductor manufacturing process, i.e., the test handler

process, is used for experimentation. Figures 8 and 9 depict the surface plots of the RPN versus S and O, while D is kept at 10 and 1, respectively. Note that monotonic surface plots are obtained.

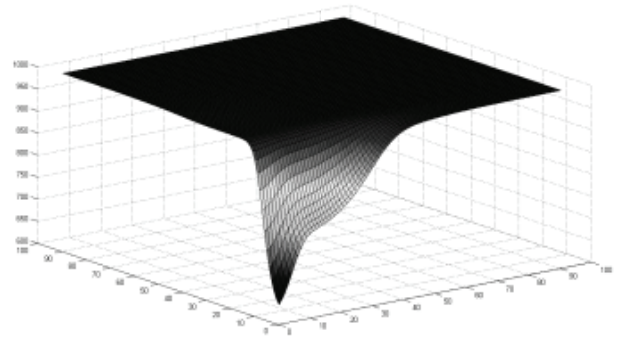


Figure 8 RPN versus S and O for the test handler process, while D=10

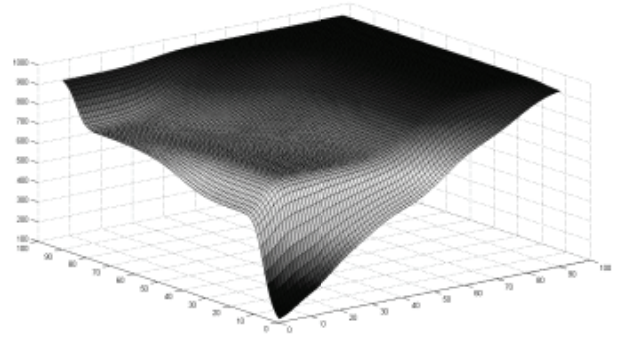


Figure 9 RPN versus S and O for the test handler process, while D=1

However, it is not straightforward to evaluate the monotonicity relationship between the RPN and S, O, and D. The surface plot may not be an effective method in this regard. As such, we propose to test the monotonic relationship using procedure in section IV. Here, comparable points are generated, and the *degree of monotonicity* is obtained. The results are summarized in Table 3. As can be seen, the *degree of monotonicity* is 1. This vindicates the monotonic increasing relationships between the RPN and S, O, and D. In another words, the monotonicity property is guaranteed.

Table 3 The scale table for Severity

Data set	#Comparable pairs	#Monotone pairs	Degree of monotonicity
Test handler	2700	2700	1

VI. SUMMARY

In this paper, the monotonicity property of an FIS model has been investigated, and a novel technique for designing

Gaussian MFs for a monotonic FIS model has been proposed. The technique shows how a set of Gaussian MFs can be generated from a qualitative scale table. The proposed technique is useful, as it allows the Gaussian MFs design procedure for a monotonic FIS model to be automated. The proposed method can be applied to various FIS modeling problems that involve a qualitative scale table, e.g., assessment and decision problems, as well as education assessment problem [7].

On the other hand, the use of a monotonicity measure in FIS modeling has been proposed in this paper. The usefulness of the proposed techniques has been demonstrated with a real industrial case study, i.e., an FIS-based RPN model for FMEA. Promising results have been obtained.

For further work, the use of other α values can be examined. Besides, other techniques such as nonlinear programming, particle swarm optimization, and harmony search can be adopted for optimization purposes. Applicability of the monotonic FIS models to other domains, e.g. education assessment, can be investigated too.

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