

A Hybrid Fuzzy Sliding-Mode Control for a Class of Generalized, Under-Actuated and Uncertain Nonlinear Dynamic Systems

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Abstract—Due to the under-actuated feature, the reference signals using the combination of the system outputs, whose number is larger than that of reference signal, are designed so that the number of control inputs and sliding surfaces is the same, and that the uncontrollable mode is indirectly controlled. Under the uncertain environment, the sliding-mode under-actuated control (SMUC) with the satisfaction of suitable condition is designed to asymptotically track the reference signal. Otherwise, a bounded tracking result is obtained for the mild condition. In this situation, an on-line fuzzy modeling for the uncertainty is employed to design a fuzzy model-based sliding-mode under-actuated control (FSMUC) to improve the system performance; e.g., the bounded tracking result of SMUC becomes an asymptotical tracking. The proposed hybrid fuzzy sliding-mode under-actuated control (HFSMUC) combining SMUC and FSMUC with a transition can be applied to a class of generalized, under-actuated and uncertain nonlinear systems, e.g., the trajectory tracking control of a differential mobile robot (DMR). Finally, the simulations of the HFSMUC system are presented to confirm the efficiency and effectiveness of the proposed control.

Keywords—Under-actuated control system; Sliding-mode control; Fuzzy model-based adaptive control; Lyapunov stability theory;

I. INTRODUCTION

In recent years, the under-actuated systems are attractive due to these advantages: less energy consumption, less use of actuators, and less expense. The main feature for an under-actuated system is that the control objectives can be achieved merely using less actuators under certain constrained conditions. However, the controller design of an under-actuated system is a challenge problem. Furthermore, an under-actuated system in the presence of uncertainties is a difficult task. In this situation, a Hybrid Fuzzy Sliding-Mode Under-actuated Control (HFSMUC) is proposed to deal with a class of generalized, under-actuated and uncertain nonlinear dynamic systems.

In this paper, the reference signals for the controllable modes are designed to combine the signal of the uncontrollable modes so that the uncontrollable mode can be manipulated indirectly. As a result, the sliding surface

can be represented as the combination of these signals to become controllable. At the same time, the number of control inputs is the same with that of sliding surfaces; then the sliding-mode under-actuated control (SMUC) is first designed. Moreover, to improve system performance the learning compensation for the system uncertainties called fuzzy sliding-mode under-actuated control (FSMUC) is designed. However, to avoid possible transient phenomenon, a hybrid fuzzy sliding-mode under-actuated control (HFSMUC) combining SMUC and FSMUC with a transition is suggested.

The main contribution of this paper is depicted as follows. Utilizing the reference signals including the uncontrollable mode makes the number of sliding surfaces equal that of control inputs. Then an HFSMUC for a class of generalized, under-actuated and uncertain nonlinear dynamic system is designed.

II. RELATED WORKS

Over the past decades, related works about under-actuated systems was proposed massively. Firstly, Lin and Mon used hierarchical fuzzy sliding-mode (HFSM) to investigate the decoupling control of a double inverted pendulum system [1]. Raimondi and Melluso [2] presented a new fuzzy adaptive motion control system including on-line extended Kalman's filter (EKF) for wheeled under-actuated cars with non-holonomic constraints. Besdies, in [3], Wai et al. proposed a cascade adaptive fuzzy sliding-mode control (AFSMC) scheme including inner and outer control loops for the stabilizing and tracking control of a nonlinear two-axis inverted-pendulum servomechanism. In 2008, Márton et al. [4] presented LQR, subspace stabilization controller and combined error metric controller for robust control of under-actuated robots compared on the classic ball and beam system. Furthermore, a closed-loop method was presented by Elliot and Gans [5] achieving full-state trajectory tracking via input-output linearization and asymptotically stable internal dynamics for the two-wheeled driven cart. In [6], Refsnes et al. presented an observer backstepping technique for slender-

body underwater vehicles to track the desired pitch and heading angle. In addition, the papers of [7]-[9] presented a sliding mode control method respectively for a class of under-actuated mechanical systems and autonomous surface vessels. Cavdaroglu and Olgac [10] designed a desirable fixed control law to successfully accomplish the trajectory tracking of an under-actuated car-and-pendulum dynamics in the presence of multiple and independent time delays. Oryschuk et al. [11] implemented and tested the real-time control of a two-wheeled under-actuated mobile robot including test motions: translational, rotational, and uphill. In [12], neural networks and output feedback were used for a quadrotor unmanned aerial vehicle (UAV) with six DOFs which were controlled by only four control inputs.

III. PROBLEM FORMULATION

In the beginning, a class of generalized, under-actuated and uncertain nonlinear dynamic systems is expressed as follows:

$$\dot{x}_1(t) = f_1(x, t) \quad (1a)$$

$$\dot{x}_2(t) = f_2(x, t) + G(x, t)u(t) \quad (1b)$$

$$y_1(t) = H_1 x_1(t), y_2(t) = H_2 x_2(t) \quad (1c)$$

where $x = [x_1(t) \ x_2(t)]^T$, $x_1(t), y_1(t) \in \mathbb{R}^{n-m}$ and $x_2(t), y_2(t) \in \mathbb{R}^m$ are respectively denoted as the uncontrollable and controllable system state and system output, $u(t) \in \mathbb{R}^m$ is the control input, and H_1, H_2 are the output gain matrices. Furthermore, the system functions are described as follows:

$$f_i(x, t) = \bar{f}_i(x) + \Delta f_i(x, t), i = 1, 2 \quad (2)$$

$$G(x, t) = \bar{G}(x) + \Delta G(x, t) \quad (3)$$

where $\bar{f}_1(x) \in \mathbb{R}^{n-m}$, $\bar{f}_2(x) \in \mathbb{R}^m$ and $\bar{G}(x) \in \mathbb{R}^{n \times m}$ denote the nominal system functions, $\Delta f_i(x, t), i = 1, 2$ and $\Delta G(x, t)$ are uncertain system functions. Due to the under-actuated feature, the number of the control inputs is smaller than that of the system output, i.e., $m < n$. In summary, $y_1(t)$ and $y_2(t)$ are denoted as the uncontrollable modes and controllable modes of the system output, respectively. The tracking error is defined as follows:

$$e_i(t) = r_i(t) - y_i(t), i = 1, 2. \quad (4)$$

Our problem is to design the control input for the controlled system (1)~(3) such that the system output will track a reference input (refer to Fig. 1). First, a sliding-mode under-actuated control (SMUC) is designed for the uncertainty which is not huge. In this situation, the asymptotical tracking can be obtained for the smaller uncertainty; in general, a bounded tracking result is achieved. To enhance the tracking result, a fuzzy modeling is then employed to approximate the system uncertainty such that a real-time compensation of learning uncertainty is plugged into the fuzzy sliding-mode under-actuated control (FSMUC). Finally, a combination of SMUC and

FSMUC with a transition is called as hybrid fuzzy sliding-mode under-actuated control (HFSMUC). It is developed such that both advantages of SMUC and FSMUC (e.g., a better transient response of SMUC, an improved tracking performance of FSMUC) are accomplished, and that both of their disadvantages (e.g., poor tracking performance of SMUC, the transient response of FSMUC) are discarded.

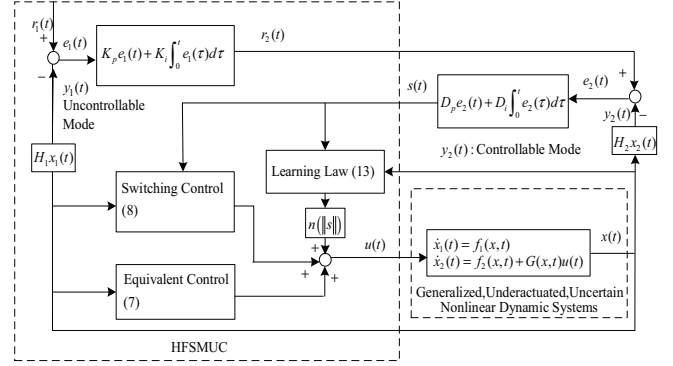


Fig. 1. Control block diagram of the overall system.

IV. FEATURES OF SLIDING SURFACE

Considering that proposed architecture is an under-actuated control system, the following sliding surface is first defined.

$$s(t) = D_p e_2(t) + D_i \int_0^t e_2(\tau) d\tau \quad (5)$$

where $D_p = \text{diag}\{d_{p1}, d_{p2}, \dots, d_{pm}\}$, $D_i = \text{diag}\{d_{i1}, d_{i2}, \dots, d_{im}\}$, $\in \mathbb{R}^{m \times m}$ are constant matrices, and $s(t) \in \mathbb{R}^m$. In addition, D_p and D_i are positive definite matrices, i.e., d_{pj} and $d_{ij} > 0$ for $j = 1, 2, \dots, m$. An effective reference input is assigned as follows:

$$r_2(t) = K_p e_1(t) + K_i \int_0^t e_1(\tau) d\tau \quad (6)$$

where K_p and $K_i \in \mathbb{R}^{(n-m) \times (n-m)}$, their entries are chosen as $(K_p)_{jl}$ and $(K_i)_{jl} > 0 \forall j, l = 1, 2, \dots, n-m$, such that $r_2(t)$ is a stable reference input. The reason to design the reference signal of (6) is that the uncontrolled mode $x_1(t)$ or $y_1(t)$ should be simultaneously included into the control mode, i.e., $x_2(t)$ or $y_2(t)$, so that they are indirectly manipulated by the controllable mode. In this paper, $r_1(t) = \dot{r}_1(t) = 0$ is considered and the corresponding bounded-input-bounded-output and asymptotical tracking properties can be referred to [13], [14]. For simplicity, it is omitted.

V. CONTROLLER DESIGNS

Three design steps for the proposed controller are given as follows.

A. Sliding-Mode Under-Actuated Control

The sliding-mode under-actuated control for the controlled system (1) and (2) is designed as follows:

$$u(t) = u_{eq}(t) + u_{sw}(t) \quad (6)$$

where

$$u_{eq}(t) = [D_p H_2 \bar{G}(x)]^{-1} \left\{ n(\|s(t)\|) \hat{W}^T(t) \Phi(z) + D_p [K_p (\dot{r}_1(t) - H_1 \bar{f}_1(x)) + K_i e_1(t) - H_2 \bar{f}_2(x)] + D_i e_2(t) \right\} \quad (7)$$

$$u_{sw}(t) = [D_p H_2 \bar{G}(x)]^{-1} \left\{ \Gamma_1 s(t) + \Gamma_2 s(t) / (\|s(t)\| + \varepsilon_b) \right\} / (1 - \lambda) \quad (8)$$

where $D_p H_2 \bar{G}(x)$ for all $x(t)$ is nonsingular; $n(\|s\|)$ is a scalar function and here it is assigned as zero; $\hat{W}^T(t) \Phi(z)$ represents the learning compensation for system uncertainties, which will be discussed in the subsection 5B; $\Gamma_1 = \text{diag}\{\gamma_{11}, \dots, \gamma_{1mm}\} > \delta I_m / 2 > 0$ (where I_m is the unit matrix of $\mathfrak{R}^m$ and δ is the exponential convergent rate to the sliding surface) and $\Gamma_2 = \text{diag}\{\gamma_{211}, \dots, \gamma_{2mm}\} > 0$ are the switching gain matrices; $\varepsilon_b \geq 0$ denotes the boundary layer of sliding surface; $\lambda > 0$ represents as the upper bound of uncertain control gain and satisfies the following inequality:

$$\left\| D_p H_2 \Delta G(x, t) (D_p H_2 \bar{G}(s))^{-1} \right\| \leq \lambda < 1, \quad \forall x(t), t. \quad (9)$$

It is assumed that the uncertain signal $K(x, u_{eq}, t)$ caused by $\Delta f_i(x, t), i=1, 2$ and $\Delta G(x, t)$ satisfying the following inequality:

$$\|K(x, u_{eq}, t)\| \leq \mu_0 + \mu_1 p(x), \quad \forall x(t), u_{eq}(t), t \quad (10)$$

where

$$K(x, u_{eq}, t) = -D_p K_p H_1 \Delta f_1(x, t) - D_p H_2 \Delta f_2(x, t) - D_p H_2 \Delta G(x, t) u_{eq}(t) \quad (11)$$

$p(x)$ is a known function, and $\mu_0 \geq 0, \mu_1 \geq 0$, whose values are constrained by the stability of the closed-loop system.

B. Fuzzy Model-Based Sliding-Mode Under-Actuated Control

If the system uncertainties (10) and (11), i.e., $K(x, u_{eq}, t)$ are huge, they must be learned to compensate their effect [15]. Based on the learning model, an FSMUC is then constructed to improve system performance. First, the extension of universal approximation theory using fuzzy modeling is stated as follows [16], [17]:

Theorem 1: Suppose $z(t) \in D(z)$ (a compact subset of $\mathfrak{R}^n$), $K(z, t): D \times \mathfrak{R}^+ \rightarrow \mathfrak{R}^m$ is a continuous function. For an

arbitrary constant $\varepsilon_0 \geq 0$, there exists an integer L (the number of membership functions) and real constant matrix $\bar{W} \in \mathfrak{R}^{L \times m}$, where $\|\bar{W}\| \leq W_M$, such that

$$K(z, t) = \bar{W}^T \Phi(z) + \varepsilon_K(z, t)$$

where $\Phi(z)$ denotes a known fuzzy basis function, $\|\varepsilon_K(z, t)\| \leq \varepsilon_0, \quad \forall z(t) \in D(z)$ and $\forall t \geq 0$.

Based on the result of *Theorem 1*, the system uncertainties in a compact subset $D(z)$ and $\forall t \geq 0$, are assumed to be continuous and approximated by the following fuzzy model:

$$K(x, u_{eq}, t) = \bar{W}^T \Phi(z) + \varepsilon_K(z, t) \quad (12)$$

where $z^T(t) = [x^T(t) \quad u_{eq}^T(t)]$, $\bar{W} \in \mathfrak{R}^{L \times m}$ is an unknown constant matrix which is not necessarily unique, $\|\varepsilon_K(z, t)\| \leq \varepsilon_0 \quad \forall z(t) \in D(z)$ and $t \geq 0$. In addition, the upper bound of $\bar{W}$ is known, i.e., $\|\bar{W}\|_F \leq \bar{W}_M$, where $\|\cdot\|_F$ is the Frobenius norm.

Then the following learning laws are considered.

$$\dot{\hat{W}}(t) = \Omega \Phi(z) s^T(t) - \Omega \Psi \hat{W}(t) \quad (13a)$$

where $\hat{W}(t) \in \mathfrak{R}^{L \times m}$ stands for the learning weight, $\Omega = \text{diag}\{\omega_1, \dots, \omega_L\} \in \mathfrak{R}^{L \times L} > 0$ is the learning matrix gain, $\Psi = \text{diag}\{\varphi_1, \dots, \varphi_L\} \in \mathfrak{R}^{L \times L} > 0$ is a matrix of e -modification rate to ensure the boundedness of the learning weight, $\Phi(z) \in \mathfrak{R}^{L \times 1}$ represents the fuzzy basis function:

$$\Phi(z) = [1 \quad \phi_2(z) \quad \dots \quad \phi_L(z)]^T \quad (13b)$$

where $\phi_j(z) = \exp\left[-\|z(t) - c_j\|^2 / \sigma_j^2\right]$, L, c_j, σ_j for $j = 2, 3, \dots$,

L are known, and the centers c_j for $j = 2, 3, \dots, L$ are chosen as normal distribution in the corresponding domain. Then the fuzzy model-based sliding-mode under-actuated control (FSMUC) is designed as the following equivalent control with $n(\|s(t)\|) = 1$ and the learning weight from (13).

C. Hybrid Fuzzy Sliding-Mode Under-Actuated Control

When the operating point is far away from the sliding surface, the learning law generally does not require so that the unnecessary instability and transience are avoided [15]. Although FSMUC is superior to SMUC, it is still a question when the learning law should be used. Important index for the sliding-mode control is the sliding surface. Due to the existence of system uncertainties, the operating point is always away from the sliding surface. Based on this knowledge, a scalar function $n(\|s\|)$ is designed as follows.

When the norm of the sliding surfaces is inside of a defined set, the learning law (13) start; similarly, as it is outside of the other set, the learning laws shut off. For reducing the

possibility of discontinuous control input, a transition between SMUC and FSMUC is designed. Hence, we design the following scalar function:

$$n(\|s\|) = \begin{cases} 0, & \text{as } \|s(t)\| > n_{s1} \\ 1, & \text{as } \|s(t)\| < n_{s2} \\ (n_{s1} - \|s(t)\|)/(n_{s1} - n_{s2}), & \text{otherwise} \end{cases} \quad (14)$$

where $n_{s1} > d_n > n_{s2} > c_n > 0$. The values of d_n and c_n are respectively the upper bounds of the convergent sets of $\|s(t)\|$ for the SMUC and FSMUC. If the system uncertainties are small enough, then the value of d_n is also small enough. If the value of d_n (or the system performance) satisfies the specification, then the value of n_{s1} approaches zero, i.e., the learning laws (13) can be shut down. On the other hand, the value of d_n (or the system performance) is large, then the value of n_{s2} is away from zero, i.e., the learning law (13) nearly executes all the time. As the value of $\|s(t)\|$ is between n_{s1} and n_{s2} , the linear combination of these two conditions is obtained.

Finally, the proposed HFSMUC is summarized as follows:

- Step 1: Obtain a nominal modeling of a generalized under-actuated and uncertain nonlinear dynamic system, i.e., $\bar{f}_i(x)$, $i = 1, 2$ and $\bar{G}(x)$.
- Step 2: Partition the uncontrollable mode (i.e., $y_1(t)$) and the controllable mode (i.e., $y_2(t)$), whose dimension is the same as that of control input $u(t)$ or sliding surface $s(t)$.
- Step 3: Estimate the system uncertainty $K(x, u_{eq}, t)$ such that $\mu_0 + \mu_1 p(x)$ in (10) is obtained.
- Step 4: Assign the suitable coefficients for the virtual reference $r_2(t)$, i.e., K_p and K_i , and the sliding surface, i.e., D_p and D_i . Then the SMUC with appropriate parameters of switching gains, i.e., Γ_1 and Γ_2 , is designed.
- Step 5: If the system performance is not satisfied, an on-line learning law (13) is constructed to learn the huge uncertainty, which is not tackled in a satisfactory manner by the previous SMUC. Then an FSMUC is described in (6)-(8) with $n(\|s\|) = 1$ and (13).
- Step 6: Besides an acceptable steady performance of an FSMUC, its transient phenomenon often occurs. In this situation, an HFSMUC is designed by the suitable selection of coefficients, i.e., n_{s1} and n_{s2} in (14) to avoid the possible transience and discontinuous control signals. Finally, the corresponding simulations are arranged to verify the effectiveness.

VI. ILLUSTRATIVE EXAMPLE

An illustrative example, namely, the trajectory tracking control of a DMR [8], is a two inputs and five states (or three outputs) under-actuated system as shown in Fig. 2 to demonstrate the validity of our proposed HFSMUC. Then the kinematic dynamic model of a DMR and the simulation results by HFSMUC are presented in the following two subsections.

A. Kinematic and Dynamic Model of a DMR

At beginning, the kinematic model of a DMR is expressed as follows:

$$\dot{x}(t) = v(t) \cos(\theta), \quad \dot{y}(t) = v(t) \sin(\theta), \quad \dot{\theta}(t) = \omega(t) \quad (15)$$

where the triple $(x(t), y(t), \theta(t))$ denotes the position and the orientation of the vehicle with respect to the world coordinate, and $v(t)$ and $\omega(t)$ are the linear and angular velocities of the DMR. The dynamic model is obtained by using the following relations:

$$M\dot{v}(t) - f_f(t, x) = f(t) \quad (16a)$$

$$I\dot{\omega}(t) - \tau_f(t, x) = \tau(t) \quad (16b)$$

where M is the mass of the DMR, I is the moment of inertia, $f(t) = [\tau_l(t) + \tau_r(t)]/r$ and $\tau(t) = L[\tau_l(t) - \tau_r(t)]/r$ with L being the distance between the center of mass and the wheel, $\tau_l(t)$ and $\tau_r(t)$ are the left and right wheel motor torques, and r is the radius of the rear wheel. To express the generalized form as equation (1), $x_1(t) = x(t)$, $x_2(t) = y(t)$, $x_3(t) = \theta(t)$, $x_4(t) = v(t)$ and $x_5(t) = \omega(t)$ are defined. Output gain matrices are selected as $H_1 = I_3$ and $H_2 = I_2$ where I_3 and I_2 respectively are 3×3 and 2×2 identity matrices. In addition, the frictional force $f_f(t, x)$ and frictional torque $\tau_f(t, x)$ are supposed to be

$$\begin{aligned} f_f(t, x) &= 3 \sin(tx_1) \cos(5t) + 2.1x_2 \sin(5tx_2) \\ &\quad - 0.3x_3 \cos(10tx_3) - 0.2 \sin(40tx_5) + 0.01 \sin(100tx_4) + 0.2 \\ \tau_f(t, x) &= 3x_2 \cos(tx_1) \sin(10tx_3) + 1.6x_2 \cos(5tx_3) \\ &\quad - 0.3x_2 \cos(10tx_2) - 0.1x_4 \sin(40tx_5) + 0.02x_3 \sin(100tx_4) - 0.3 \end{aligned} \quad (17)$$

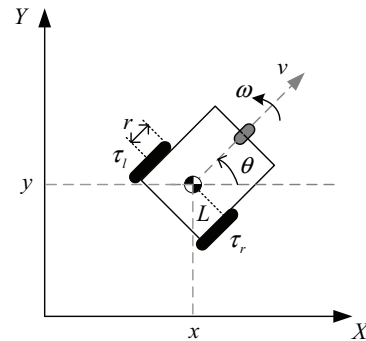


Fig. 2. Illustration of a DMR.

B. Simulation Results by HFSMUC

The simulation results by HFSMUC are considered as follows. The initial position and orientation of a DMR is selected as $\{X = 0.3m, Y = 0.1m, \theta = 1 \text{ rad}\}$ and the physical parameters of the DMR are chosen as $M = 6.0kg$, $I = 0.157kg \cdot m^2$, $r = 0.1m$, $L = 0.25m$. The control objective is to drive the DMR from the initial position back to the origin point of the world coordinate. The above frictional force and frictional torque (17) are also considered in the simulation. The corresponding responses of the simulation are shown in Figs. 3~8. It indicates that the proposed control is effective for an under-actuate controlled system. The uncertainty and its learning uncertainty are shown in Figs. 9 and 10, which are acceptable. Moreover, the corresponding proof will be undertaken via Lyapunov stability theory in the near future.

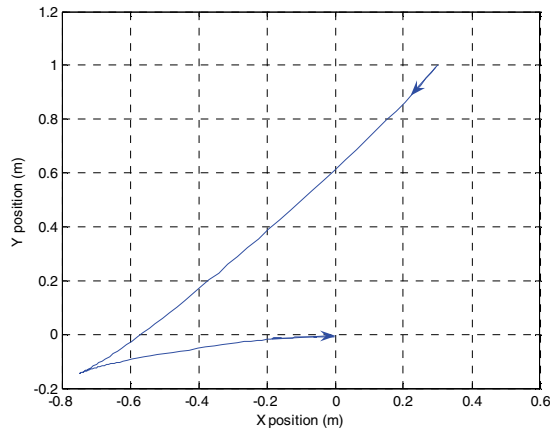


Fig. 3. The response of the DMR in the world coordinate (X, Y) .

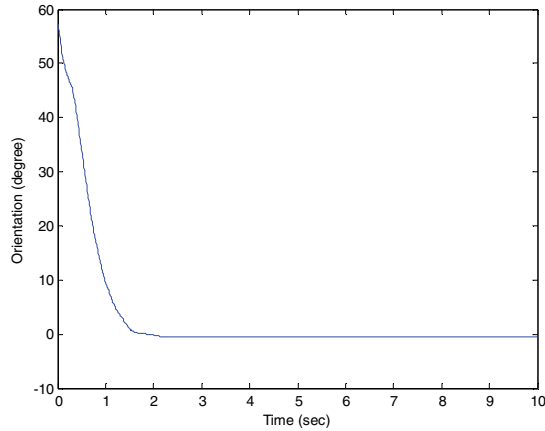


Fig. 4. The response of orientation $\theta(t)$.

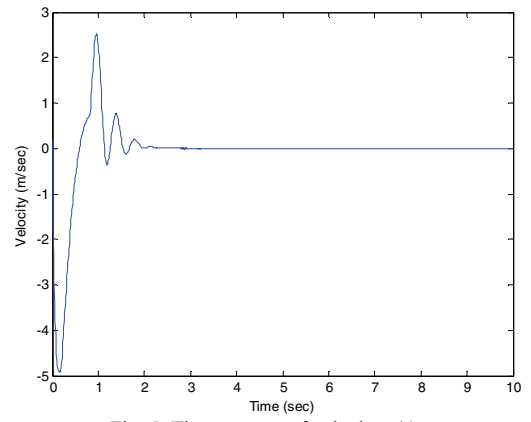


Fig. 5. The response of velocity $v(t)$.

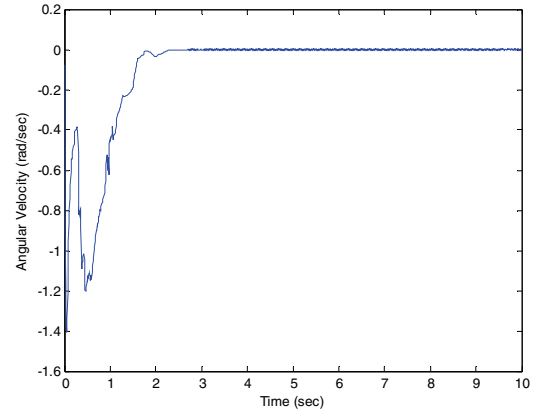


Fig. 6. The response of angular velocity $\omega(t)$.

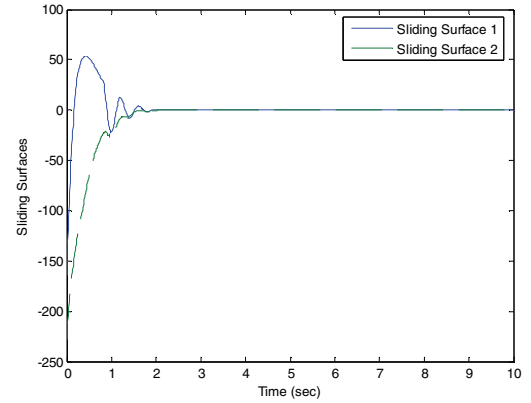


Fig. 7. The response of sliding surfaces $s(t)$.

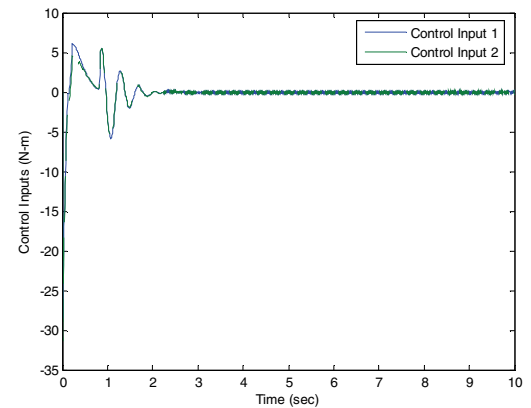


Fig. 8. The response of control inputs $\tau_l(t)$ and $\tau_r(t)$.

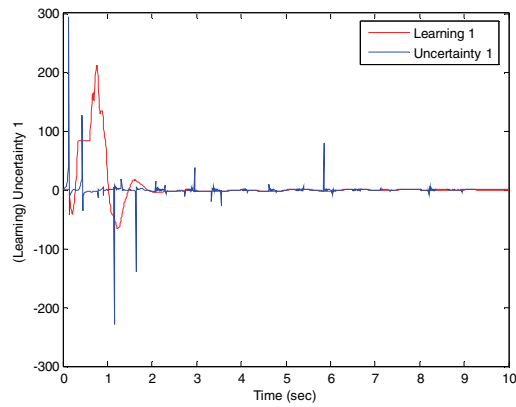


Fig. 9. Learning 1 & Uncertainty 1.

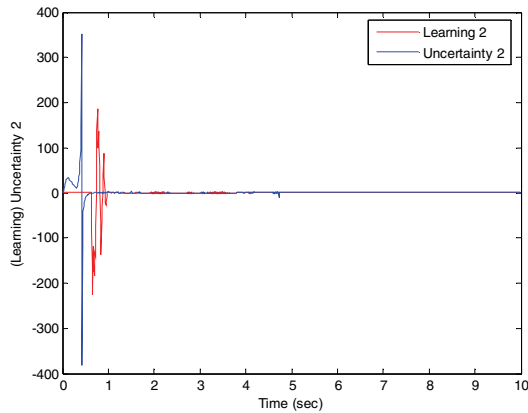


Fig. 10. Learning 2 & Uncertainty 2.

VII. CONCLUSIONS

An HFSMUC for a class of generalized under-actuated nonlinear uncertain dynamic systems is developed by the combination of SMUC and FSMUC with a transition. The proposed scheme presents both advantages of SMUC and FSMUC and simultaneously avoids the possible disadvantages coming from SMCU and FSMUC. First, the reference signals using the combination of the system outputs, whose number is larger than that of reference signal, are designed so that the number of control inputs and sliding surfaces is the same, and that the uncontrollable mode is indirectly controlled. Under the uncertain environment, the sliding-mode under-actuated control (SMUC) with the satisfaction of suitable condition is designed to asymptotically track the reference signal. Otherwise, a bounded tracking result is obtained for the mild condition. In this situation, an on-line fuzzy modeling of the uncertainty is employed to design a fuzzy sliding-mode under-actuated control (FSMUC) to improve the system performance; e.g., the bounded tracking result of previous sliding-mode control becomes an asymptotical tracking. Finally, an illustrative example: the trajectory tracking of a DMR, confirms the usefulness of the proposed HFSMUC. It is believed that the proposed control scheme can be applied to many under-actuated nonlinear uncertain dynamic systems.

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