

# T-S Fuzzy Tracking and Synchronous Control in a Gantry Stage

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**Abstract**—In this paper, we propose a T-S fuzzy model based controller for the H-type gantry stage. The synchronized error, which is caused by mismatch quality and friction, is an important control problem for the H-type gantry stage. Utilizing a Lagrangian equation, all states can be formulated as a state equation. Furthermore, the T-S fuzzy model based controller is designed to reduce velocity error, and the position error can also be obtained through internal transformation. Moreover, by utilizing additional transformations, the stabilization problem can be transformed into a linear matrix inequalities (LMIs) problem. Finally, numerical simulation and experimental results are illustrated to demonstrate that the control scheme can effectively reduce synchronized error and tracking error of a gantry stage.

**Keywords:** gantry stage; T-S fuzzy model; Lagrangian equation; Linear Matrix Inequalities (LMIs);

## I. INTRODUCTION

With the growth of industrial technology, motion systems with high speed and high precision have attracted increasing study and applications. As a coordinate system, the H-type gantry stage is widely applied in many fields, such as microelectronics, aerospace, solar cells, and flat panel manufacturing and inspection. Two motors of the stage are mounted on parallel slides that simultaneously push the third axis and motor, and a load can be installed on the third axis for applying in different systems. But motion of the load will affect behavior of the two parallel motors on the base, reducing precision and harming the machine's structure. Therefore, control of the synchronous error in the gantry configuration has become a challenge due to the increasing demands for high-speed and high-accuracy manufacturing and inspection.

Generally, the control problem of a gantry stage can be concluded as cascade control and parallel control. Cascade control contains two parallel axes, a master axis and a slave axis. For this control structure, control commands were sent to the master axis, and the response of master axis becomes a new command for slave axis. But the disadvantage of this structure is that it will cause state and input delays. For this reason, most studies adopt parallel control, which is a biaxial structure, can be independently designed, and can give same command simultaneously. Koren proposed cross-coupled biaxial computer control in 1980 [1], and variable-gain cross-coupling controller in 1991 [2]. Other studies utilize the optimum concept to decouple the structure [3-4]. In addition, other cross-coupled controllers have been investigated in parallel control structures [5]. However, these studies all discuss similar control structures. Teo first proposed a Lagrangian model to analyze the H-type gantry stage [6-7].

Fuzzy control technique has been widely used in many physical and engineering systems, especially for systems with incomplete plant information. In addition, fuzzy control has been widely applied to controller designs for nonlinear systems [8-14]. In most of these papers, a so-called Takagi-Sugeno (T-S) fuzzy model is adopted to approximate a nonlinear system, and then a fuzzy controller is designed via the parallel distributed compensation (PDC) scheme to stabilize the T-S fuzzy model. By examining, problems of stability and stabilization, LMI techniques have been adopted to find solutions for the T-S fuzzy model based control system.

In this paper, we propose a novel control scheme based on the Lagrangian model [6-7] and present a systematic design procedure to analyze this model. First we design a T-S fuzzy model based controller for velocity error. Secondly, through

state internal transformations, we obtain the controller gains of the velocity error and position error. Furthermore, by utilizing the structure, controller gains can be obtained through the LMIs and the velocity error and position error can be reduced. The main contributions of this paper include: i) first use of a T-S fuzzy model based control scheme for the H-type gantry stage, ii) propose a transformation method to transform the velocity error gains into position error gains, and iii) formulation all of stability and stabilization conditions into an LMI form.

This paper is organized as follows. In Section II, we discuss the development of dynamic Lagrangian model of gantry stage. In section III, the T-S fuzzy model based scheme is designed for this physical model utilizing the Lyapunov direct method, and stability/stabilization conditions are proposed to guarantee the system is asymptotic stable. Numerical simulation and experimental results are illustrated in Section IV. Finally, conclusions are given in Section V.

## II. DYNAMIC MODELING OF GANTRY SYSTEM

### A. Description of a gantry stage

A typical gantry mechanism and the corresponding coordinates are shown in Fig. 1. It can be considered as a three-degree-of-freedom servo mechanism. Two servo motors in y axis, considered in this paper are PMLSMs, carry a gantry on which a slider in x axis holding the inspection tool is mounted. Two motors on the y axis yield displacement  $d_{y1}$  and  $d_{y2}$  respectively. These two displacements will be equal when the two servo motors move synchronously, that implies  $d_{y1} = d_{y2}$ . But in practice, their movements will be different due to the different dynamics exhibited in the two servo motors and the time-varying loading presented by translation of the slider along the gantry. The central point C of the gantry is thus constrained to move along the central dashed line with one degree of freedom. Due to the deviation between  $d_{y1}$  and  $d_{y2}$ , the gantry may also rotate about the center C, and the rotated angle is indicated as  $\theta$ .

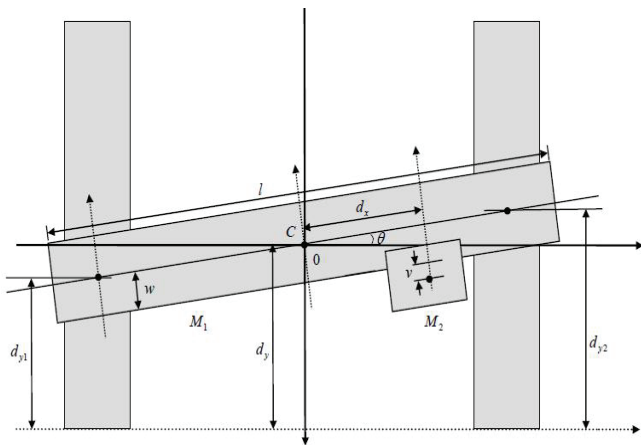


Figure 1. Three-degree-of-freedom scheme.

### B. Lagrangian-based three-degree-of-freedom dynamic model

Let  $M_1$  and  $M_2$  denote the mass of the gantry and the slider, respectively;  $l$  and  $2w$  denote the length and width of the gantry, respectively; and  $I_{M1}$  and  $I_{M2}$  denote the moment of inertia of the gantry and of the slider corresponding to the center point C, respectively.

$$I_{M1} = \frac{M_1}{12} (l^2 + (2w)^2) \quad (1)$$

$$I_{M2} = M_2 (d_x^2 + (w + v)^2) \quad (2)$$

Then the positions corresponding to the center of gravity of  $M_1$  and  $M_2$ ,  $\mathbf{p}_{M1}$  and  $\mathbf{p}_{M2}$  are represented as

$$\mathbf{p}_{M1} = \begin{bmatrix} 0 \\ d_y \end{bmatrix} \quad (3)$$

$$\mathbf{p}_{M2} = \begin{bmatrix} d_y - (w + v) \cos \theta + d_x \sin \theta \\ d_x \cos \theta + (w + v) \sin \theta \end{bmatrix} \quad (4)$$

where  $d_y = d_{y1} + (d_{y2} - d_{y1})/2$ . The velocities corresponding,  $\mathbf{v}_{M1}$  and  $\mathbf{v}_{M2}$ , to the center of gravity of  $M_1$  and  $M_2$  are represented as

$$\mathbf{v}_{M1} = \begin{bmatrix} 0 \\ \dot{d}_y \end{bmatrix} \quad (5)$$

$$\mathbf{v}_{M2} = \begin{bmatrix} \dot{d}_y + (w + v) \dot{\theta} \sin \theta + \dot{d}_x \sin \theta + d_x \dot{\theta} \cos \theta \\ \dot{d}_x \cos \theta - d_x \dot{\theta} \sin \theta + (w + v) \dot{\theta} \cos \theta \end{bmatrix} \quad (6)$$

Then the translational and rotational kinetic energy of the gantry and slider are shown as

$$K_{M1} = \frac{1}{2} M_1 \mathbf{v}_{M1}^T \mathbf{v}_{M1} + \frac{1}{2} I_1 \dot{\theta}^2 \quad (7)$$

$$K_{M2} = \frac{1}{2} M_2 \mathbf{v}_{M2}^T \mathbf{v}_{M2} + \frac{1}{2} I_2 \dot{\theta}^2 \quad (8)$$

Thus the total kinetic energy may be computed as

$$\begin{aligned} K &= K_{M1} + K_{M2} \\ &= \frac{1}{2} M_1 \mathbf{v}_{M1}^T \mathbf{v}_{M1} + \frac{1}{2} M_2 \mathbf{v}_{M2}^T \mathbf{v}_{M2} + \frac{1}{2} (I_1 + I_2) \dot{\theta}^2 \\ &= \frac{1}{2} (M_1 + M_2) \dot{d}_y^2 + \frac{1}{2} (I_1 + I_2 + M_2 d_x^2 + M_2 (w + v)^2) \dot{\theta}^2 \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} M_2 \dot{d}_x^2 + \dot{d}_y \dot{\theta} M_2 [(w+v) \sin \theta + d_x \cos \theta] \\
& + \dot{d}_y \dot{d}_x M_2 \sin \theta + \dot{\theta} \dot{d}_x M_2 (w+v)
\end{aligned} \quad (9)$$

which can be further written as

$$K = \frac{1}{2} \dot{\mathbf{X}}^T \mathbf{D} \dot{\mathbf{X}} \quad (10)$$

where  $\mathbf{X} = [d_y \quad \theta \quad d_x]^T$  and  $\mathbf{D}$  is the inertia matrix given by

$$\mathbf{D} = \begin{bmatrix} M_1 + M_2 & & \\ M_2(w+v) \sin \theta + M_2 d_x \cos \theta & & \\ M_2 \sin \theta & & \end{bmatrix}$$

$$\begin{bmatrix} M_2(w+v) \sin \theta + M_2 d_x \cos \theta & M_2 \sin \theta \\ I_1 + I_2 + M_2 d_x^2 + M_2(w+v)^2 & M_2(w+v) \\ M_2(w+v) & M_2 \end{bmatrix} \quad (11)$$

The government equation of the gantry mechanism can be derived from the Lagrange equation

$$\frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{\mathbf{X}}} \right] - \frac{\partial L}{\partial \mathbf{X}} = \mathbf{F} + \mathbf{U} \quad (12)$$

where  $L = K - V$ , in which  $K$  is the summation of kinetic energy,  $V$  is potential energy and  $\mathbf{U}$  is the driven force provided by motor and  $\mathbf{F}$  is the friction force. Next, derive the elements of the Coriolis and centrifugal matrix  $\mathbf{C}$  from

$$C_{ij} = \sum_{k=1}^3 (c_{ijk} \dot{q}_k) \quad (13)$$

where  $d_{ij}$  represents the element in the  $i$ th row and  $j$ th column of the inertia matrix  $\mathbf{D}$ . Substituting the inertia equations  $I_1$  and  $I_2$  into (11) and computing (13), the matrix  $\mathbf{C}$  can be obtained as

$$\mathbf{C} = M_2 \begin{bmatrix} 0 & & \\ -d_x \dot{\theta} \sin \theta + d \dot{\theta} \cos \theta + \dot{d}_x \cos \theta & & \\ \dot{\theta} \cos \theta & & \end{bmatrix}$$

$$\begin{bmatrix} -d_x \dot{\theta} \sin \theta + d \dot{\theta} \cos \theta + \dot{d}_x \cos \theta & \dot{\theta} \cos \theta \\ (-d_x \sin \theta + d \cos \theta) \dot{d}_y + 2d_x \dot{d}_x & 2d_x \dot{\theta} + \dot{d}_y \cos \theta \\ 2d_x \dot{\theta} + \dot{d}_y \cos \theta & 0 \end{bmatrix} \quad (14)$$

Finally, the dynamic model of the gantry position stage can be represented as

$$\mathbf{D} \ddot{\mathbf{X}} + \mathbf{C} \dot{\mathbf{X}} = \mathbf{E}(\mathbf{U} - \mathbf{F}) \quad (15)$$

where

$$\mathbf{E} = \begin{bmatrix} 1 & 1 & 0 \\ l \cos \theta & -l \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (16)$$

$$\mathbf{F} = [F_{y1}, F_{y2}, F_x]^T \quad (17)$$

$$\mathbf{U} = [u_{y1}, u_{y2}, u_x]^T \quad (18)$$

$F_{y1}$ ,  $F_{y2}$  and  $F_x$  are the frictional forces, and  $u_{y1}$ ,  $u_{y2}$  and  $u_x$  are the generated mechanical forces along  $d_{y1}$ ,  $d_{y2}$  and  $d_x$  respectively. The frictional forces are described as LuGre Model

$$\frac{dz}{dt} = v - \sigma_0 \frac{|v|}{g(v)} z \quad (19)$$

$$F_i = \sigma_0 z + \sigma_1 \frac{dz}{dt}, \quad i = y1, y2, x \quad (20)$$

The LuGre model proposed by C. Canudas de Wit [15], can capture most of the known frictional behaviors and is suitable for control systems. Here  $z$  is the average bending displacement of virtual bristles between the interface of two bodies,  $g(v)$  is a positive function of velocity, and where  $\sigma_0$  and  $\sigma_1$  are the stiffness and damping coefficients of average behavior of bristles.

### III. T-S FUZZY MODEL BASED CONTROLLER DESIGN

In this study, a controller based on the T-S fuzzy model, used to enhance the approximate ability of nonlinear system, is proposed to control the gantry position stage. Further rewrite the state equation (15) as

$$\ddot{\mathbf{X}}(t) = \mathbf{A} \dot{\mathbf{X}}(t) + \mathbf{B} \mathbf{U}(t) \quad (21)$$

where  $\mathbf{A} = -\mathbf{D}^{-1} \mathbf{C}$  and  $\mathbf{B} = \mathbf{D}^{-1} \mathbf{E}$ . We propose a novel control scheme based on the T-S fuzzy model, as shown in Fig. 2, to simultaneously achieve two control objectives: 1) position and velocity control of the gantry and slider, and 2) minimization of the synchronous position and velocity errors.

According to T-S fuzzy modeling method, the following equation can be obtained

$$\ddot{\mathbf{X}}(t) = \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) (\mathbf{A}_{ijk} \dot{\mathbf{X}}(t) + \mathbf{B}_{ijk} \mathbf{U}(t)) \quad (22)$$

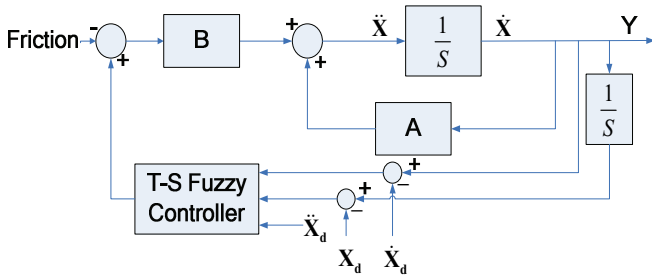


Figure 2. T-S fuzzy model based control scheme.

where  $h_i w_j p_k(z(t)) = h_i(z(t))w_j(z(t))p_k(z(t))$ . Two fuzzy rules of  $\dot{d}_y(t)$ ,  $\dot{\theta}(t)$  and  $\dot{d}_x(t)$  are adopted in this study. The boundary of  $\dot{d}_y(t)$  and  $\dot{d}_x(t)$  are chosen as the maximum speed of servo motor which is about  $\pm 0.5m/s$ , and the boundary of  $\dot{\theta}(t)$  is chosen as  $\pm 0.1rad/s$ , shown in Table. I and Fig. 3.

#### A. T-S fuzzy model based controller design

In general, the velocity tracking controller can be designed based on a T-S fuzzy model. That implies the objective is required to satisfy  $[\dot{\mathbf{X}}(t) - \dot{\mathbf{X}}_d(t)] \rightarrow 0$  as  $t \rightarrow \infty$ , where  $\dot{\mathbf{X}}(t) = [\dot{d}_y \ \dot{\theta} \ \dot{d}_x]^T$  and  $\dot{\mathbf{X}}_d(t)$  represent the velocity response and desired velocity command, respectively. Hence, the acceleration tracking error can be derived as

$$\begin{aligned} \ddot{\mathbf{X}}(t) &= \ddot{\mathbf{X}}(t) - \ddot{\mathbf{X}}_d(t) \\ &= \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) (\mathbf{A}_{ijk} \dot{\mathbf{X}}(t) + \mathbf{B}_{ijk} \mathbf{U}(t)) - \ddot{\mathbf{X}}_d(t) \\ &= \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) \mathbf{A}_{ijk} \dot{\mathbf{X}}(t) \\ &\quad - \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) \mathbf{A}_{ijk} \dot{\mathbf{X}}_d(t) \\ &\quad + \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) \mathbf{B}_{ijk} \mathbf{U}(t) \\ &\quad + \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) \mathbf{A}_{ijk} \dot{\mathbf{X}}_d(t) - \ddot{\mathbf{X}}_d(t) \end{aligned} \quad (23)$$

Furthermore, we design a reference control effort  $\tau(t)$  and it must be satisfied

$$\sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) \mathbf{B}_{ijk} \tau(t)$$

$$\begin{aligned} &= \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) \mathbf{B}_{ijk} \mathbf{U}(t) \\ &\quad + \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) \mathbf{A}_{ijk} \dot{\mathbf{X}}_d(t) - \ddot{\mathbf{X}}_d(t) \end{aligned} \quad (24)$$

Substituting (24) into (23), the following equation can be obtained

$$\begin{aligned} \ddot{\mathbf{X}}(t) &= \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) \mathbf{A}_{ijk} \dot{\mathbf{X}}(t) \\ &\quad + \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) \mathbf{B}_{ijk} \tau(t) \end{aligned} \quad (25)$$

According to each rule of  $h_i(z(t))$ ,  $w_j(z(t))$  and  $p_k(z(t))$ , the new controller is designed as

$$\tau(t) = -\mathbf{k}_{ijk} \dot{\mathbf{X}}(t) \quad (26)$$

Substituting (26) into (25), the closed loop form can be rewritten as

$$\begin{aligned} \ddot{\mathbf{X}}(t) &= \\ &\sum_{i=1}^2 \sum_{l=1}^2 \sum_{j=1}^2 \sum_{m=1}^2 \sum_{k=1}^2 \sum_{n=1}^2 h_i w_j p_k(z(t)) h_l w_m p_n(z(t)) (\mathbf{A}_{ijk} - \mathbf{B}_{ijk} \mathbf{k}_{lmn}) \dot{\mathbf{X}}(t) \end{aligned} \quad (27)$$

Let

$$\alpha(t) = \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) \mathbf{A}_{ijk}$$

$$\beta(t) = \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) \mathbf{B}_{ijk}$$

The relationship between actual and reference control efforts can be obtained from (24) as

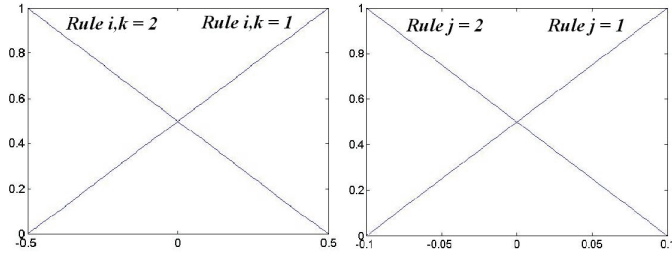
$$\beta(t)(\mathbf{U}(t) - \tau(t)) = -\alpha(t)\dot{\mathbf{X}}_d(t) + \ddot{\mathbf{X}}_d(t) \quad (28)$$

Then, substituting (28) into (25), the control effort  $\mathbf{U}(t)$  in the controller can be obtained as

$$\begin{aligned} \mathbf{U}(t) &= -\sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) \mathbf{k}_{ijk} \dot{\mathbf{X}}(t) \\ &\quad + \beta^{-1}(t)(\ddot{\mathbf{X}}_d(t) - \alpha(t)\dot{\mathbf{X}}_d(t)) \end{aligned} \quad (29)$$

TABLE I. FUZZY IF-THEN RULE

| Rule              |         | If                             | Then                           |
|-------------------|---------|--------------------------------|--------------------------------|
| $\dot{d}_y(t)$    | $i = 1$ | $\dot{d}_y(t) \approx 0.5$     | $h_1(z(t)) = 1, h_2(z(t)) = 0$ |
|                   | $i = 2$ | $\dot{d}_y(t) \approx -0.5$    | $h_1(z(t)) = 0, h_2(z(t)) = 1$ |
| $\dot{\theta}(t)$ | $j = 1$ | $\dot{\theta}(t) \approx 0.1$  | $w_1(z(t)) = 1, w_2(z(t)) = 0$ |
|                   | $j = 2$ | $\dot{\theta}(t) \approx -0.1$ | $w_1(z(t)) = 0, w_2(z(t)) = 1$ |
| $\dot{d}_x(t)$    | $k = 1$ | $\dot{d}_x(t) \approx 0.5$     | $p_1(z(t)) = 1, p_2(z(t)) = 0$ |
|                   | $k = 2$ | $\dot{d}_x(t) \approx -0.5$    | $p_1(z(t)) = 0, p_2(z(t)) = 1$ |

Figure 3. Membership function of rule  $i, j, k$ 

### B. T-S fuzzy model based controller with internal transformation

In the previous section, the controller is designed for velocity error, but real applications always demand high-precision position control. To design a new control gain of position error, first divide  $\mathbf{k}_{ijk}$  into  $\mathbf{k}_{ijk}^1$  and  $\mathbf{k}_{ijk}^2$  as

$$\mathbf{k}_{ijk} = \mathbf{k}_{ijk}^1 + \mathbf{k}_{ijk}^2 \quad (30)$$

To obtain the information of  $\tilde{\mathbf{X}}(t)$ , design a new feedback controller  $\mathbf{U}_{ijk}^2(t)$  to satisfy

$$\dot{\tilde{\mathbf{X}}}(t) = \mathbf{k}_{ijk}^{2^{-1}} (\tilde{\mathbf{X}}(t) + \mathbf{U}_{ijk}^2(t)) \quad (31)$$

where  $\mathbf{U}_{ijk}^2(t)$  is designed as (32), and the closed loop form can be obtained as (33)

$$\mathbf{U}_{ijk}^2(t) = -\mathbf{h}_{ijk} \tilde{\mathbf{X}}(t) \quad (32)$$

$$\dot{\tilde{\mathbf{X}}}(t) = \mathbf{k}_{ijk}^{2^{-1}} (\mathbf{I} - \mathbf{h}_{ijk}) \tilde{\mathbf{X}}(t) \quad (33)$$

Substituting (33) into (29),  $\mathbf{U}(t)$  can be rewritten as

$$\begin{aligned} \mathbf{U}(t) = & -\sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) (\mathbf{k}_{ijk}^1 \dot{\tilde{\mathbf{X}}}(t) + (\mathbf{I} - \mathbf{h}_{ijk}) \tilde{\mathbf{X}}(t)) \\ & + \beta^{-1}(t) (\ddot{\mathbf{X}}_d(t) - \alpha(t) \dot{\mathbf{X}}_d(t)) \end{aligned} \quad (34)$$

The T-S fuzzy model based controller with our proposed internal transformation is shown in Fig. 4.

### C. Lyapunov stability analysis

The augmented matrix of velocity and position tracking errors is given by

$$\Phi^T(t) = [\dot{\tilde{\mathbf{X}}}^T(t) \quad \tilde{\mathbf{X}}^T(t)]^T \quad (35)$$

After internal transformation, the following equation will be satisfied.

$$\dot{\Phi}(t) = \gamma \Phi(t) \quad (36)$$

where

$$\gamma = \begin{bmatrix} \alpha(t) - \beta(t) \mathbf{k}_{ijk} & 0 \\ 0 & \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 h_i w_j p_k(z(t)) (\mathbf{k}_{ijk}^{2^{-1}} (\mathbf{I} - \mathbf{h}_{ijk})) \end{bmatrix} \quad (37)$$

The Lyapunov function can be represented as  $V(\Phi(t)) = \Phi^T(t) \mathbf{P} \Phi(t)$ , where  $\mathbf{P}$  is chosen a positive matrix, and the differential of the Lyapunov function can be represented as

$$\begin{aligned} \dot{V}(\Phi(t)) &= \dot{\Phi}^T(t) \mathbf{P} \Phi(t) + \Phi^T(t) \mathbf{P} \dot{\Phi}(t) \\ &= \Phi^T(t) \gamma^T \mathbf{P} \Phi(t) + \Phi^T(t) \mathbf{P} \gamma \Phi(t) \\ &= \Phi^T(t) (\gamma^T \mathbf{P} + \mathbf{P} \gamma) \Phi(t) \end{aligned} \quad (38)$$

From (37),  $\mathbf{A} - \mathbf{B} \mathbf{k}_{ijk}$  and  $\mathbf{k}_{ijk}^{2^{-1}} (\mathbf{I} - \mathbf{h}_{ijk})$  can be separately designed by the separation theorem. In addition, the choice of  $\mathbf{k}_{ijk}^2$  must guarantee that  $\mathbf{k}_{ijk}^{2^{-1}}$  exists. Then we can obtain positive definite matrix  $\mathbf{P}_1, \mathbf{P}_2$  and feedback gains  $\mathbf{k}_{ijk}, \mathbf{h}_{ijk}$  by using iteration method to solve linear matrix inequalities (LMIs). Therefore, the condition of  $\dot{V}(\Phi(t)) < 0$  can be obtained because  $\gamma$  is negative definite, which implies the system will be asymptotic stable.

## IV. SIMULATION AND EXPERIMENTAL RESULTS

### A. Simulation Results

To verify the effectiveness of the proposed T-S fuzzy model based controller, the PID control which using three PID controllers for each independent axis is proposed in this study to compare the control performance. For the PID controller, time-domain tuning design is used to find the parameters, and MATLAB simulation is used for two cases. The measurement parameters of gantry system are selected as follows:  $M_1 = 2.88\text{kg}$ ,  $M_2 = 0.5\text{kg}$ ,  $l = 0.5\text{m}$ ,  $w = 0.0285\text{m}$ , and the LuGre friction parameters are  $\sigma_0 = 230.087$ ,

$\sigma_1 = 35.835$ ,  $F_c = 0.72$ ,  $F_s = 2.15$ ,  $v_s = 0.005$ . The high speed sine command (position, velocity) is depicted in Fig. 5.

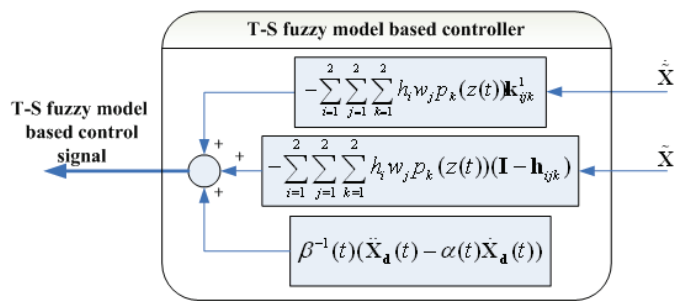


Figure 4. T-S fuzzy model based controller with control internal transformation.

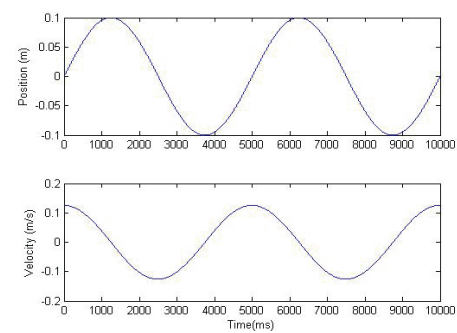


Figure 5. Desired high speed sine command.

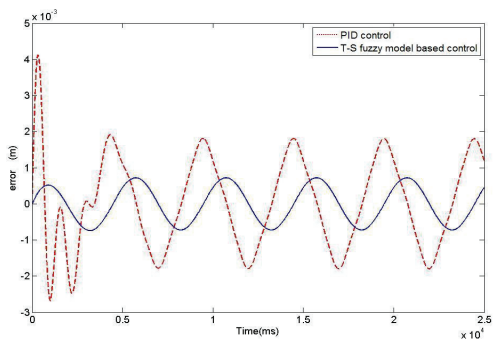


Figure 6. Tracking error of  $d_y$  (m).

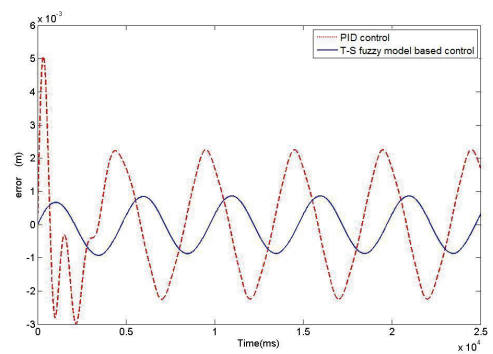


Figure 7. Tracking error of  $d_x$  (m).

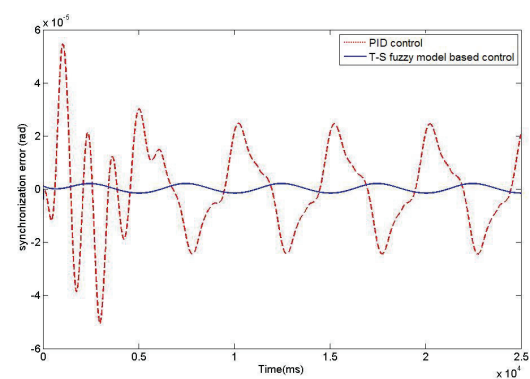


Figure 8. Synchronized error (rad).

TABLE II. SIMULATION RESULTS

| Control system<br>Performance index |     | PID control           | T-S fuzzy model<br>based control |
|-------------------------------------|-----|-----------------------|----------------------------------|
| Tracking error<br>of $d_y$<br>(m)   | Max | 0.0041                | 0.000742                         |
|                                     | IAE | 27.4033               | 11.1556                          |
| Tracking error<br>of $d_x$<br>(m)   | Max | 0.0051                | 0.000929                         |
|                                     | IAE | 35.3768               | 13.5032                          |
| synchronized<br>error<br>(rad)      | Max | $5.47 \times 10^{-5}$ | $3.79 \times 10^{-6}$            |
|                                     | IAE | 0.3389                | 0.0282                           |

The simulation results showing the error responses of  $d_y$ ,  $d_x$  are depicted in Fig. 6 and Fig. 7, while the synchronized error( $\theta$ ) is shown in Fig. 8. The high speed sine command initially rises fast, usually bringing with it a large error. From Fig. 6 and Fig. 7, the T-S fuzzy model based control outperforms the PID control, especially at the beginning. So when the velocity changes quickly, the proposed approach quickly yields significantly improved performance over PID control.

Fig. 8 shows that the proposed control algorithm provides cross-coupled control signal from state  $\theta$ , which can dramatically reduce the synchronized error, better than PID independence control. Furthermore, the  $E_{Max}$  and  $E_{IAE}$  from the Table. II illustrate that the proposed control algorithm can minimize the tracking error and synchronized error.

### B. Experimental Results

The block diagram of the DSP-based computer-controlled gantry position stage is shown in Fig. 9. A TMS320VC33 floating-point DSP is the core of the control computer. In addition, the control computer includes multi-channels of analog-to-digital converters (ADCs), digital-to-analog converters (DACs), parallel input/output (PIO) and encoder interface. The resolution of the linear scales for the PMLSM is  $1\mu m$ . Furthermore, the proposed control algorithms are

realized in the DSP using the “C” and “Assembly” languages. All the programs are developed under a Windows environment in a PC and then downloaded to the Flash ROM of the DSP.

The methodology proposed for the implementation of the real-time T-S fuzzy model based control system consists of a main program and one interrupt service routine (ISR). In the main program, parameters and input/output initialization are set first. Next, the interrupt interval for the ISR is set. After enabling the interrupt, the ISR with 1ms sampling rate is used for the encoder interface and DACs. The ISR first reads the positions of the three PMLSMs of the gantry position stage from the encoders and the velocities of the three PMLSMs of the gantry position stage are derived from the positions. Then the ISR calculates the tracking errors and generates the control Efforts  $U(t)$ , according to the proposed control algorithms. Finally, the calculated commands are sent to the  $x$ -axis,  $y1$ -axis and  $y2$ -axis motor servo drives via three DACs. Photos of the experimental setup, including the gantry position stage, PMLSM drives and DSP-based control computer are shown in Fig. 10. Moreover, the field-oriented control mechanism is implemented in the PMLSM drives.

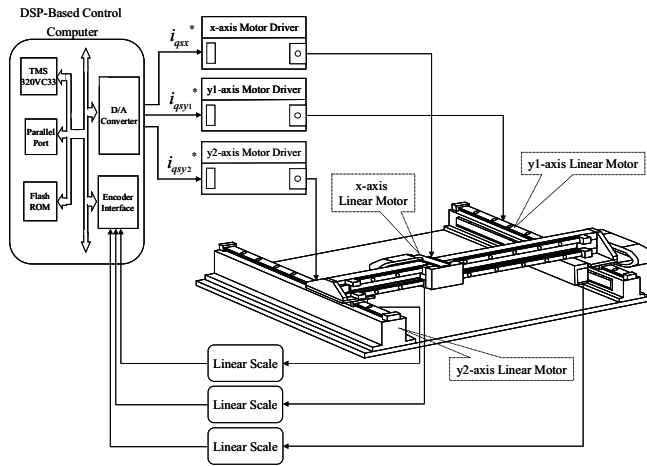


Figure 9. DSP-based computer-controlled gantry stage.

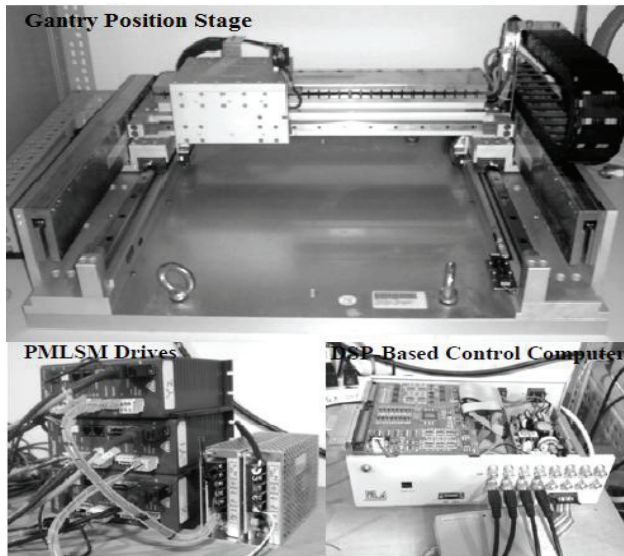


Figure 10. Experimental setup.

For PID-controlled implementation, where system identification is necessary, the closed velocity loop was excited by 1 volt step input as a test input and its output was recorded with a data acquisition system. From the experimental data, the approximate model parameters were obtained assuming the first order approximation as follows

$$G_i(s) = \frac{K_i}{\tau_i s + 1} (i = d_{y1}, d_{y2}, d_x) \quad (39)$$

The system Identification Toolbox in MATLAB can solve the parameters in (39), obtaining three transfer functions. Based on time-domain tuning design, the independent PID controllers are tuned as  $K_{py1} = 36.724$ ,  $K_{py2} = 35.883$ ,  $K_{px} = 33.671$ ,  $K_{ly1} = 1.386$ ,  $K_{ly2} = 1.353$ ,  $K_{lx} = 1$ ,  $K_{Dy1} = 0.173$ ,  $K_{Dy2} = 0.169$ ,  $K_{Dx} = 0.125$ .

Trajectory commands of the software simulation are used and the results are shown in Figs. 11–13. Fig. 11 and Fig. 12 are similar to the software simulations results, in steady state, performance of the T-S fuzzy model based controller is nearly the same as the PID controller, but it can effectively reduce the tracking error when the velocity is changing quickly. The  $E_{Max}$  and  $E_{IAE}$  from Table. III show that the T-S fuzzy model based controller is effective than the PID controller, especially in  $E_{Max}$ . In addition, Fig. 13 shows that the T-S fuzzy model based controller is able to minimize the synchronized error, whereas the PID controllers were only able to track individual trajectories independently. Table. III also shows the  $E_{Max}$  and  $E_{IAE}$  of synchronized error are reduced to 42.7% and 81.6 % respectively when using T-S fuzzy model based control.

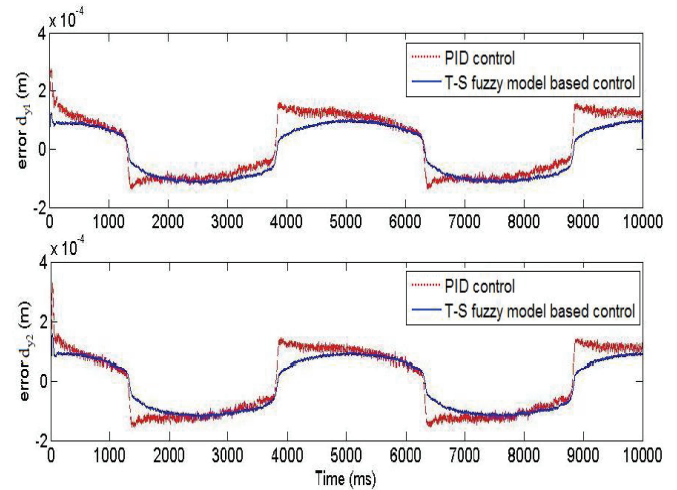


Figure 11. Tracking error of  $d_{y1}$ ,  $d_{y2}$  respectively (m).

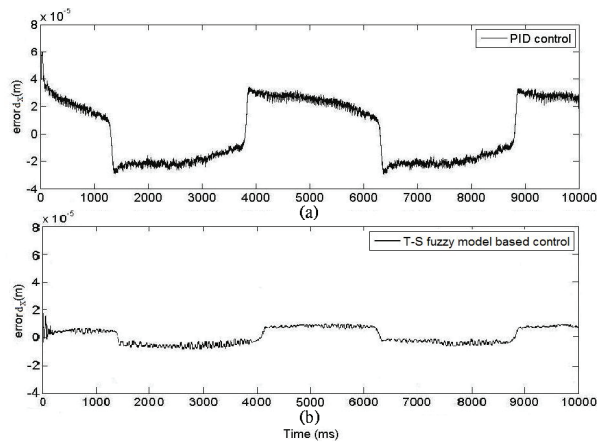


Figure 12. Tracking error of  $d_x$  using (a) PID control and (b) T-S fuzzy model based control.

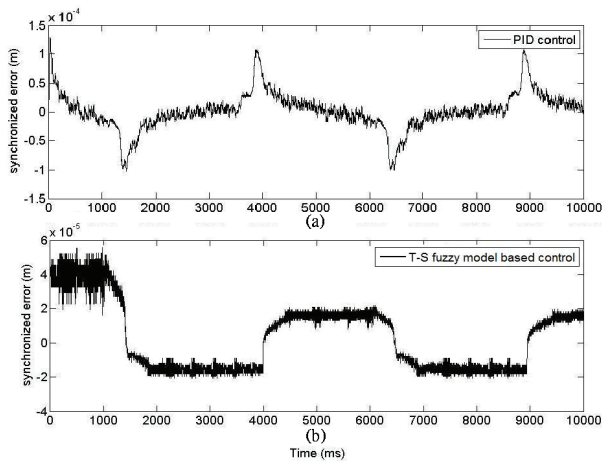


Figure 13. Synchronized error (m). Using (a) PID control, (b) T-S fuzzy model based control.

TABLE III. EXPERIMENTAL RESULTS

| <div>Control system</div> <div>Performance index</div> |     | PID control          | T-S fuzzy model based control |
|--------------------------------------------------------|-----|----------------------|-------------------------------|
| Tracking error of $d_y$ (m)                            | Max | 0.000269             | 0.000178                      |
|                                                        | IAE | 1.1577               | 0.91583                       |
| Tracking error of $d_x$ (m)                            | Max | $5.94\times10^{-5}$  | $1.88\times10^{-5}$           |
|                                                        | IAE | 0.23153              | 0.12323                       |
| synchronized error (m)                                 | Max | $1.293\times10^{-4}$ | $5.525\times10^{-5}$          |
|                                                        | IAE | 0.21389              | 0.17366                       |

## V. CONCLUSION

In this paper, we propose a T-S fuzzy model based controller for a gantry stage. Utilizing the Lagrangian equation, all states can be formulated as a state equation. Then a novel T-S fuzzy model based control scheme is proposed to achieve two control objectives together: 1) position and velocity control of the gantry and slider, and 2) minimization of the synchronous position and velocity errors. The simulation and experimental results show that this control system can dramatically decrease both the tracking error and synchronized error.

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