

Fuzzy Optimization Model Based Tolerance Approach to Timetable Rescheduling for High Speed Railway in China

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Abstract—A fuzzy optimization model based tolerance approach is proposed to handle timetable rescheduling in high speed railway during speed restriction period. As the limited speed and headway time are not crisp figures in practice especially when some natural hazards happen or some equipment failure, tolerance approach is introduced with the fuzzy membership functions of the original objective and soft constraints to find an new optimal objective with little slack of constraints. The original objective is treated in the same manner as the soft constraints, so the model is symmetric. The proposed fuzzy rescheduling model is simulated on the busiest part of a high speed railway line in China. The entire case study shows the significance of fuzzy optimization in case of speed restriction. The results shed light on how we could choose a better limited speed and headway time, so that the number of seriously impacted trains can be reduce greatly with little cost and risk.

Keywords- timetable rescheduling; fuzzy optimization; tolerance approach

I. INTRODUCTION

At present, China is extensively developing the infrastructure of high speed railway. The target is to cover its major economic areas with a high speed railway network, which consists of four horizontal and four vertical lines [1], in the following several years. The network scale is much larger than any existing ones in the world. Train timetable rescheduling is a focal point to improve the operation mode with the characteristics of high train speed, high train frequency and mixed train speed (HHM), which is of great theoretical and practical significance for safety and efficiency operation of China high speed railway network.

In China, train rescheduling is mostly manual developed by the operator with the suport of computer in the existing line or the high speed railway, which is depends on human experience to dominate only one line or some section in one line with fixed three hours period. In future China will establish five comprehensive operation centers of the railway network, each of which will dominate several lines that total length over thousands of kilometers [2]. The scale of rescheduling object is far larger and the relationship between lines is far more

complex than the existing situation, so the inefficient manually rescheduling, which seriously affect the use of the ability of high-speed rail and bring risk to railway safety, cannot support the advanced operation command mode.

Over the years, the research on the train operation automatic adjustment has been paid great attention and many different algorithms for optimal rescheduling have been put forward. The integer programming, such as branch and bound, linear programming and other traditional optimization method [3] are usually used for the simple transportation organization mode (only high speed or medium speed train running). In China, there are usually mixed high-speed and medium-speed trains running in one line, so there are great differences on the variable size, type of constraints and target values in the actual situation of the Chinese high-speed railway, and these methods cannot be directly applied to the train rescheduling problems in China.

Since the railway network is extensively developed and transportation organization mode is more complex in recent years, many heuristic methods of artificial intelligence, like DEDS-based simulation method [4], expert system [5], tabu search [6, 7], and some computational intelligence methods [8-14], like genetic algorithm, particle swarm optimization algorithm, are used to solve the large-scale combinatorial optimization problem by many scholars.

To some extent, these mathematical model and optimization algorithm have given a feasible solution for train operation automatic adjustment problem; actually there are many uncertainties of decision variables, parameters, constraints and goals for high speed railway timetable rescheduling optimization. For example, the running time at a section and the dwell time at a station of the train, which are typical uncertain variables, are easily impacted by railway and equipment status, technological level of crew and weather condition [15, 16]. So these methods that only consider the crisp parameters cannot adapt to the actual operation environment of high-speed railway under the great influence of uncertainty, and it can not achieve the purpose of automatic rescheduling, even play the role of decision support in practical applications. Therefore, the uncertainty character of high-speed

train operation automatic adjustment model should be considered in substance.

II. TIMETABLE RESCHEDULING PROBLEM

The aim of train rescheduling is to get a new timetable that adjusts the train movements to be consistent with the planned schedule as much as possible under some interference. The follow model focuses on minimizing the total delay as well as the number of seriously impacted trains.

A. Input Data

The numerical inputs are described as follows:

$inis_p, inie_p$: Initial start and end time of interval p according to original timetable.

z_p : The minimum size of interval p . If p is the possession of a station, d_p stands for the minimum dwell time of the train associated with p . Otherwise, it stands for the minimum running time of the train.

f_k : The minimum separation time of two intervals on station k , i.e. end time of an interval – start time of the subsequent interval.

hw_k : The headway of section k , i.e. start time of an interval – start time of the subsequent interval.

c_i^{delay} : The cost per time unit delay for train i .

w_i : Delay tolerance for train i .

W_i : Weight for objective i ;

$h_p = \begin{cases} 1, & \text{if interval } p \text{ is a planned stop,} \\ & \text{where } p \in Itv \\ 0, & \text{otherwise} \end{cases}$

M : A very big integer, e.g. 100000.

TABLE I. SETS DEFINED FOR MODELING

Set Name	Description	Size	Index of set
Trn	All the trains.	T	i
Seg	All the segments.	S	k
$Stn \subseteq Seg$	All the stations.	N	--
$Sec \subseteq Seg$	All the sections.	S-N	--
Itv	All the intervals where an interval is the possession of a segment by a train with specified start and end time.	--	p
$Itv_i \subseteq Itv$	The ordered set of intervals for train i according to the original timetable.	--	--
$Itv_i^{sm} \subseteq Itv_i$	The ordered set of intervals associated with stopping at stations for trains i .	--	--
$Itv_k \subseteq Itv$	The ordered set of intervals for segment k according to the original timetable.	--	--
Trk _k	All the tracks of station k	L _k	l

Remarks: For the ease of understanding, we use p as the index for every interval-related set (i.e. Itv , Itv_i , Itv_i^{sm}

and Itv_k). In each of the interval-related sets, $p+1$ indicates the first subsequent interval of p , and $\tilde{p}$ indicates all the subsequent intervals of p .

Some sets are defined in TABLE I for modeling. Note that the segment between two stations is called “section”, while the station segment is called “station” for short.

B. Decision variables

s_p, e_p : Start time and end time of interval p .

d_p : Delay of interval p , which is defined as the difference between the departure time after adjustment and the planned departure time in the original timetable.

$b_i = \begin{cases} 1, & \text{if train } i \text{ reaches its final considered stop} \\ & \text{with a delay larger than } w_i, \text{ where } i \in Trn \\ 0, & \text{otherwise} \end{cases}$

$\eta_{pl} = \begin{cases} 1, & \text{if interval } p \text{ uses track } l, \text{ where } p \in Itv_k, \\ & l \in Trk_k \text{ and } k \in Stn \\ 0, & \text{otherwise} \end{cases}$

$\alpha_{p\tilde{p}} = \begin{cases} 1, & \text{if interval } p \text{ occurs before } \tilde{p} \text{ as in the} \\ & \text{initial timetable, where } p, \tilde{p} \in Itv_k, k \in Seg \\ 0, & \text{otherwise} \end{cases}$

$\beta_{p\tilde{p}} = \begin{cases} 1, & \text{if interval } p \text{ is changed to occur after } \tilde{p}, \\ & \text{where } p, \tilde{p} \in Itv_k, k \in Seg \\ 0, & \text{otherwise} \end{cases}$

C. Objective functions

i) To minimize the delay cost:

$$\text{Minimize } \sum_{i \in Trn} \left(c_i^{delay} \cdot \sum_{p \in Itv_i^{sm}} d_p \right) \quad (1)$$

ii) To minimize the number of trains exceeding delay tolerance:

$$\text{Minimize } \sum_{i \in Trn} b_i \quad (2)$$

$$S = W_1 \sum_{i \in Trn} \left(c_i^{delay} \cdot \sum_{p \in Itv_i^{sm}} d_p \right) + W_2 \sum_{i \in Trn} b_i \quad (3)$$

D. Constraints

i) Interval restrictions

$$e_p - s_p \geq z_p, p \in Itv \quad (4)$$

The real departure time cannot be earlier than the original departure time:

$$e_p \geq inie_p, p \in Itv \text{ and } h_p = 1 \quad (5)$$

$$e_p - inie_p = d_p, p \in Itv \quad (6)$$

ii) Track restrictions

$$\sum_{l \in Trk_k} \eta_{pl} = 1, p \in Itv_k \text{ and } k \in Stn \quad (7)$$

iii) Connectivity restrictions

$$e_p = s_{p+1}, p \in Itv_i, p \neq last(Itv_i) \text{ and } i \in Trn \quad (8)$$

For each station, if two intervals use the same track, at least one of α and β is forced to be 1:

$$\eta_{pl} + \eta_{\tilde{p}l} - 1 \leq \alpha_{p\tilde{p}} + \beta_{p\tilde{p}}, \quad p, \tilde{p} \in Itv_k, p < \tilde{p}, l \in Trk_k \text{ and } k \in Stn \quad (9)$$

One train can enter a station after the preceding train, which uses the same track, leaves it:

$$s_{\tilde{p}} - e_p \geq f_k \alpha_{p\tilde{p}} - M(1 - \alpha_{p\tilde{p}}), \quad p, \tilde{p} \in Itv_k \text{ and } k \in Stn \quad (10)$$

$$s_p - e_{\tilde{p}} \geq f_k \beta_{p\tilde{p}} - M(1 - \beta_{p\tilde{p}}), \quad p, \tilde{p} \in Itv_k \text{ and } k \in Stn \quad (11)$$

For each section, at least one of α and β is forced to be 1 because there is only one track:

$$\alpha_{p\tilde{p}} + \beta_{p\tilde{p}} = 1, p, \tilde{p} \in Itv_k \text{ and } k \in Sec \quad (12)$$

Headways for each section:

$$s_{\tilde{p}} - s_p \geq hw_k \alpha_{p\tilde{p}} - M(1 - \alpha_{p\tilde{p}}), p, \tilde{p} \in Itv_k \text{ and } k \in Sec \quad (13)$$

$$s_p - s_{\tilde{p}} \geq hw_k \beta_{p\tilde{p}} - M(1 - \beta_{p\tilde{p}}), p, \tilde{p} \in Itv_k \text{ and } k \in Sec \quad (14)$$

iv) Auxiliary restrictions

$$d_{last(Itv_i)} - w_i \leq Mb_i \quad (15)$$

In practice, some constraints of the model are not satisfied strictly because of the inexact operation time. Two kinds of constraints are changed as below:

i) Interval restrictions

z_p stands for the minimum running time or the minimum dwell time of the train associated with p . Both of them may be impacted by some weather and human factors. The running time of the train at a section is formed by pure running time at the section and addition time for departure and arrival at a station, which is easily impacted by railway and equipment status, technological level of crew and weather condition. Since the average running speed of the train usually changes within a certain range, especially in speed restriction under some emergency, it is very important to find a safety and reasonable average speed that far below the limited speed to improve the optimization result greatly. Furthermore, many factors, like the train type, on and off time of passengers, preparation time for drivers and crew and station technology operation of the train may increase the uncertainty of the dwell time of the train at a station. So there should be a tolerance for z_p , and the constraint is changed as below:

$$g_1 = e_p - s_p \geq z_p, p \in Itv \quad (16)$$

ii) Headway restrictions

The headway time for each station is decided by the number of receiving-departure track, the operation time of turnout and holding time of the track, which is also an uncertainty constraint due to the factors of equipment status and human technological level. So there should be a tolerance for f_k . The headway constraint for each station is changed as below:

$$g_2 = s_{\tilde{p}} - e_p \geq f_k \alpha_{p\tilde{p}} - M(1 - \alpha_{p\tilde{p}}), \quad p, \tilde{p} \in Itv_k \text{ and } k \in Stn \quad (17)$$

$$g_3 = s_p - e_{\tilde{p}} \geq f_k \beta_{p\tilde{p}} - M(1 - \beta_{p\tilde{p}}), \quad p, \tilde{p} \in Itv_k \text{ and } k \in Stn \quad (18)$$

The headway time for each section is decided by the number and length of the block between two tracking trains and the speeds of the trains, which ranges from 2.4 minutes to 3 minutes according to [17]. The headway constraint for each section is changed as below:

$$g_4 = s_{\tilde{p}} - s_p \geq hw_k \alpha_{p\tilde{p}} - M(1 - \alpha_{p\tilde{p}}), p, \tilde{p} \in Itv_k \text{ and } k \in Sec \quad (19)$$

$$g_5 = s_p - s_{\tilde{p}} \geq hw_k \beta_{p\tilde{p}} - M(1 - \beta_{p\tilde{p}}), p, \tilde{p} \in Itv_k \text{ and } k \in Sec \quad (20)$$

For this condition, the model above turns meaningless in the sense of mathematical viewpoints. To construct a reasonable mathematical model under the uncertain environment, the tolerance approach based timetable rescheduling model will be introduced in the next section.

III. FUZZY OPTIMIZATION MODEL BASED TIMETABLE RESCHEDULING

In the paper, we use the fuzzy optimization based tolerance approach to solve the uncertainty program, some necessary backgrounds and notions of the approach are reviewed.

A. Fuzzy optimization based tolerance approach

Tolerances are indicated in any technical process, that is the admissible limit of variation around the object value and the deviations allowed from the specified parameters.

A general model of a fuzzy linear programming problem (FLP-programming) is presented by the following system.

$$\text{Minimize } \tilde{C}_1 x_1 \oplus \tilde{C}_2 x_2 \oplus \cdots \oplus \tilde{C}_n x_n \quad (21)$$

$$\text{Subject to } \tilde{A}_{i,1} x_1 \oplus \tilde{A}_{i,2} x_2 \oplus \cdots \oplus \tilde{A}_{i,n} x_n \geq \tilde{B}_i, \quad i = 1, \dots, m \quad (22)$$

$\tilde{A}_{i,j}, \tilde{B}_i, \tilde{C}_j$ ($i = 1, \dots, m; j = 1, \dots, n$) are fuzzy set in $\mathbb{R}$.

The symbol $\oplus$ represents the extended addition. Each real number a can be modeled as a fuzzy number.

$$\tilde{A} = \{(x, f_A(x)); x \in R\}$$

$$\text{with } f_A(x) = \begin{cases} 1 & x = a \\ 0 & x \neq a \end{cases} \quad (23)$$

Here we only discuss the special case where the objective function is crisp and some constraints have the soft form and the rest constraints are crisp.

$$\text{Minimize } z(x) = c_1x_1 + c_2x_2 + \dots + c_nx_n \quad (24)$$

Subject to

$$g_i(x) = a_{i,1}x_1 + a_{i,2}x_2 + \dots + a_{i,n}x_n \gtrsim \tilde{B}_i, \quad (25)$$

$$i = 1, \dots, m_1$$

$$a_{i,1}x_1 + a_{i,2}x_2 + \dots + a_{i,n}x_n \geq b_i, \quad (26)$$

$$i = m_1 + 1, \dots, m$$

$$x_1, \dots, x_n \geq 0$$

The m_1 soft constraints (25) may be described more precisely by the fuzzy set with the support $[b_i - d_i, b_i]$, and d_i is the tolerance according to b_i . Moreover the membership function $g_i(x)$ can be specified as the follow function.

$$\mu(g_i) = \begin{cases} 0, & g_i \leq b_i - d_i \\ \frac{g_i - (b_i - d_i)}{d_i}, & b_i - d_i < g_i \leq b_i, \\ 1, & b_i \leq g_i \end{cases} \quad (27)$$

Directly assigns a measure of the satisfaction of the i -th constraint to the solution $X = (x_1, x_2, \dots, x_n)$.

For the objective function $z(x)$, there is a fuzzy set $\tilde{Z} = \{(z, \mu_Z(z)) | z \in R\}$ Let

$$X_L = \{x \in R | a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \geq b_i - d_i, \quad (28)$$

$$\forall i = 1, 2, \dots, m_1 \text{ and } a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \geq b_i, \forall i = m_1 + 1, \dots, m\}$$

$$X_U = \{x \in R | a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \geq b_i, \quad (29)$$

$$\forall i = 1, 2, \dots, m\}$$

$$\bar{z} = \min_{x \in X_U} z(x) \quad (30)$$

and

$$\underline{z} = \min_{x \in X_L} z(x) \quad (31)$$

So the membership function $\mu_Z(z)$ of $\tilde{Z}$ is given by

$$\mu_Z(z) = \begin{cases} 0, & \bar{z} \leq z \\ 1 - \frac{z - \underline{z}}{\bar{z} - \underline{z}}, & \underline{z} < z \leq \bar{z} \\ 1, & z \leq \underline{z} \end{cases} \quad (32)$$

In order to determine a compromise solution, it is usually assumed in the literature that the total satisfaction of a decision maker may be described by

$$\lambda(x) = \min(\mu_Z(x), \mu_1(x), \dots, \mu_{m_1}(x)) \quad (33)$$

We treat the objective in the same manner as the soft constraints, so this is a symmetric model.

The optimization system is clearly equivalent to

$$\text{Maximize } \lambda \quad (34)$$

$$\text{Subject to } \mu_Z(x) \geq \lambda \quad (35)$$

$$\mu_i(x) \geq \lambda, \quad i = 1, \dots, m_1 \quad (36)$$

$$x \in X, \lambda \in [0, 1]$$

Using linear membership functions or piecewise linear, concave membership function, the system can easily be solved by well-known algorithms.

B. Fuzzy optimization model based tolerance approach to timetable rescheduling

In the rescheduling model mentioned above, three additional inputs are involved to describe the tolerances of minimum running time or dwell time, headway time for section and station respectively. An additional decision variable λ is involved to describe the fuzzy number for the objective and constraints. All the additional parameters are list as below:

c_{z_p} : Tolerance of z_p

c_{f_k} : Tolerance of f_k

c_{hw_k} : Tolerance of hw_k

λ : Decision variable for the fuzzy number

s^0 : The objective value of the original model according to z_p, f_k, hw_k .

s^* : The objective value of the original model according to $z_p - c_{z_p}, f_k - c_{f_k}, hw_k - c_{hw_k}$.

So fuzzy membership function of objective is given by

$$\mu_0 = \begin{cases} 1, & S < s^* \\ 1 - \frac{S - s^*}{s^0 - s^*}, & s^* \leq S \leq s^0 \\ 0, & S > s^0 \end{cases} \quad (37)$$

Fig. 1 shows the objective fuzzy membership function.

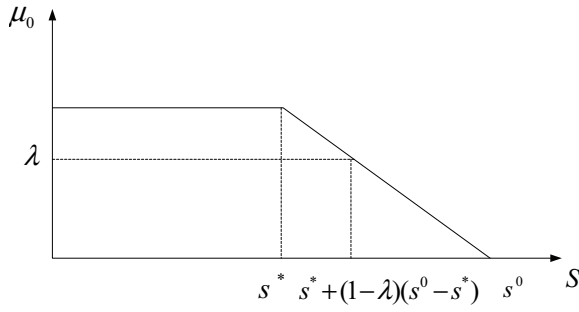


Figure 1. The objective fuzzy membership function

Fuzzy membership function of run time constraint is given by

$$\mu_1 = \begin{cases} 1, & g_1 > z_p \\ 1 - \frac{z_p - g_1}{c_{z_p}}, & z_p - c_{z_p} \leq g_1 \leq z_p \\ 0, & g_1 < z_p - c_{z_p} \end{cases} \quad (38)$$

Fig. 2 shows the fuzzy membership function of run time constraint.

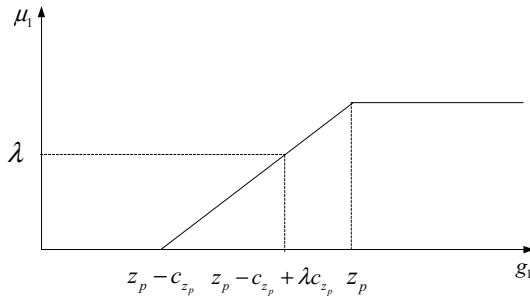


Figure 2. The run time constraint fuzzy membership function

Fuzzy membership functions of headway time constraints and their graphics are similar to figure 2.

$$\mu_2 = \begin{cases} 1, & g_2 > f_k \\ 1 - \frac{f_k - g_2}{c_{f_k}}, & f_k - c_{f_k} \leq g_2 \leq f_k \\ 0, & g_2 < f_k - c_{f_k} \end{cases} \quad (39)$$

$$\mu_3 = \begin{cases} 1, & g_3 > f_k \\ 1 - \frac{f_k - g_3}{c_{f_k}}, & f_k - c_{f_k} \leq g_3 \leq f_k \\ 0, & g_3 < f_k - c_{f_k} \end{cases} \quad (40)$$

$$\mu_4 = \begin{cases} 1, & g_4 > hw_k \\ 1 - \frac{hw_k - g_4}{c_{hw_k}}, & hw_k - c_{hw_k} \leq g_4 \leq hw_k \\ 0, & g_4 < hw_k - c_{hw_k} \end{cases} \quad (41)$$

$$\mu_5 = \begin{cases} 1, & g_5 > hw_k \\ 1 - \frac{hw_k - g_5}{c_{hw_k}}, & hw_k - c_{hw_k} \leq g_5 \leq hw_k \\ 0, & g_5 < hw_k - c_{hw_k} \end{cases} \quad (42)$$

So the model is changed as follow:

C. Objective functions

i) To maximize the fuzzy number:

$$\text{Maximize } \lambda \quad (43)$$

D. Constraints

i) Objective restrictions

$$W_1 \sum_{i \in Trn} \left(c_i^{delay} \cdot \sum_{p \in Itv_i^{tn}} d_p \right) + W_2 \sum_{i \in Trn} b_i \leq s^* + (1 - \lambda)(s^0 - s^*) \quad (44)$$

ii) Interval restrictions

$$e_p - s_p \geq z_p + (\lambda - 1)c_{z_p}, \quad p \in Itv \quad (45)$$

The real departure time cannot be earlier than the original departure time:

$$e_p \geq inie_p, \quad p \in Itv \text{ and } h_p = 1 \quad (46)$$

$$e_p - inie_p = d_p, \quad p \in Itv \quad (47)$$

iii) Track restrictions

$$\sum_{l \in Trk_k} \eta_{pl} = 1, \quad p \in Itv_k \text{ and } k \in Stn \quad (48)$$

iv) Connectivity restrictions

$$e_p = s_{p+1}, \quad p \in Itv_i, \quad p \neq last(Itv_i) \text{ and } i \in Trn \quad (49)$$

For each station, if two intervals use the same track, at least one of α and β is forced to be 1:

$$\eta_{pl} + \eta_{\tilde{p}l} - 1 \leq \alpha_{p\tilde{p}} + \beta_{p\tilde{p}}, \quad (50)$$

$$p, \tilde{p} \in Itv_k, \quad p < \tilde{p}, \quad l \in Trk_k \text{ and } k \in Stn$$

One train can enter a station after the preceding train, which uses the same track, leaves it:

$$s_{\tilde{p}} - e_p \geq (f_k + (\lambda - 1)c_{f_k})\alpha_{p\tilde{p}} - M(1 - \alpha_{p\tilde{p}}), \quad p, \tilde{p} \in Itv_k \text{ and } k \in Stn \quad (51)$$

$$s_p - e_{\tilde{p}} \geq (f_k + (\lambda - 1)c_{f_k})\beta_{p\tilde{p}} - M(1 - \beta_{p\tilde{p}}), \quad p, \tilde{p} \in Itv_k \text{ and } k \in Stn \quad (52)$$

For each section, at least one of α and β is forced to be 1 because there is only one track:

$$\alpha_{p\tilde{p}} + \beta_{p\tilde{p}} = 1, \quad p, \tilde{p} \in Itv_k \text{ and } k \in Sec \quad (53)$$

Headways for each section:

$$s_{\tilde{p}} - s_p \geq (hw_k + (\lambda - 1)c_{hw_k})\alpha_{p\tilde{p}} - M(1 - \alpha_{p\tilde{p}}), \quad p, \tilde{p} \in Itv_k \text{ and } k \in Sec \quad (54)$$

$$s_p - s_{\tilde{p}} \geq (hw_k + (\lambda - 1)c_{hw_k})\beta_{p\tilde{p}} - M(1 - \beta_{p\tilde{p}}), \quad p, \tilde{p} \in Itv_k \text{ and } k \in Sec \quad (55)$$

v) Auxiliary restrictions

$$d_{last(Itv_i)} - w_i \leq Mb_i \quad (56)$$

Since M is a very integer, the headway time constraints can be changed as below:

$$s_{\tilde{p}} - e_p \geq f_k + (\lambda - 1)c_{f_k} - M(1 - \alpha_{p\tilde{p}}) \quad (57)$$

$$s_p - e_{\tilde{p}} \geq f_k + (\lambda - 1)c_{f_k} - M(1 - \beta_{p\tilde{p}}) \quad (58)$$

$$s_{\tilde{p}} - s_p \geq hw_k + (\lambda - 1)c_{hw_k} - M(1 - \alpha_{p\tilde{p}}) \quad (59)$$

$$s_p - s_{\tilde{p}} \geq hw_k + (\lambda - 1)c_{hw_k} - M(1 - \beta_{p\tilde{p}}) \quad (60)$$

IV. CASE STUDY

This model is simulated on the busiest part of Jing-Hu high speed line, between Nanjing (Ning for short) and Shanghai (Hu for short). In the rest of this paper, “Hu-Ning Part” is used to represent this part of the Jing-Hu high speed line. Since Jing-Hu high speed line is double-track, we only consider the direction from Ning to Hu without loss of generality. All the models are solved by IBM ILOG CPLEX 12.2.

There are seven stations on the Hu-Ning Part, thereby 6 sections, whose lengths are 65110m, 61050m, 56400m, 26810m, 31350m and 43570m. Its daily service starts at 6:30am and ends at 11:30pm. As currently planned, there are 52 trains (14 high speed trains and 38 medium speed trains) from Jing-Hu line and 8 medium speed trains from Riverside line that go from Ning to Hu by the Hu-Ning part. The trains from Jing-Hu line are called self-line trains, while those from Riverside line are called cross-line trains. As in reality, self-line trains have higher priority than cross-line trains. As for self-line trains, the high speed trains have higher priority than the quasi-high speed trains. We did two studies as follows.

A. speed restriction in all section

When some natural hazards happened, like heavy storm and strong wind, railway will be greatly affected in a large area. In the simulation, we assume there is a speed restriction in all sections and the average limited speed ranges from 170km/h to 150km/h; the headway is set from 3 minutes to 2.5 minutes and the minimum separation time on track possession in each station is set to 1min.

First we use the original model to solve the problem, and get the follow results. When we set the speed as 150km/h and headway time as 3 minutes, the solution value is 2580587; there are 54 trains late for 30 minutes and 6 trains late for one hour; and the total delay time is 3052.95 minutes. When the

speed is 170km/h and headway time is 2.5 minutes, the Solution value is 1481413; there are 55 trains late for 30 minutes and no train late for one hour; and the total delay time is 2248.45 minutes. Although the latter result is very exciting, it is a significant risk to take the speed of 170km/h, as the highest speed must brings the high operation cost and may arouse some new delays. Then we using the above results as the inputs of the fuzzy model the paper described before and get the follow results. The fuzzy member λ is 0.689117, which means the average speed is 156km/h and headway time is 2.8445 minutes; there are 60 trains late for 30 minutes, no train late for one hour; and the total delay time is 2799.4 minutes. We can see that the fuzzy optimization result improved greatly with little slack of constraints that means little risk and operation cost. Fig. 3 is the rescheduling timetable by fuzzy model. The gray lines mean the initial timetable and blue lines mean the rescheduling timetable.

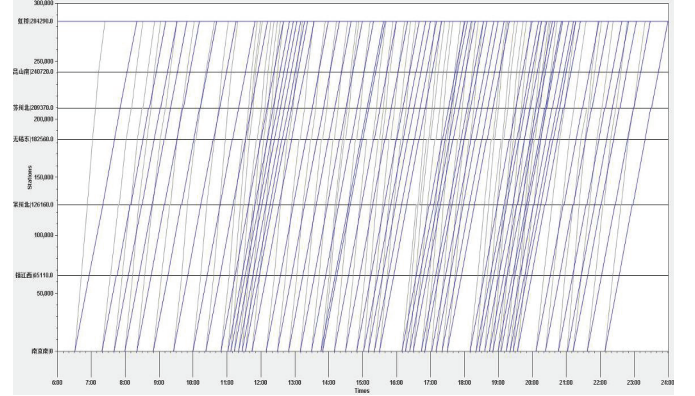


Figure 3. Rescheduling timetable with limited speed in all sections

B. speed restriction in one section

Sometimes equipment failure, like train signal failure, happens in some but not all the sections. In the simulation, we assume there is a speed restriction in first section and the limited speed ranges from 60km/h to 50km/h; the headway is also set from 3 minutes to 2.5 minutes; and the minimum separation time on track possession in each station is 1min.

First we use the original model to solve the problem and get the follow result. When the speed is 50km/h and headway time is 3 minutes, the solution value is 4040593; there are 52 trains late for 30 minutes and 8 trains late for one hour, and the total delay time is 3109.95 minutes. When the speed is 60km/h and headway time is 2.5 minutes, the solution value is 2518709; there are 56 trains late for 30 minutes and no train late for one hour, and the total delay time is 2328.45 minutes. Then we get the follow result using the fuzzy model the paper described before. The fuzzy member λ is 0.70606, which means the average speed is 53km/h and headway time is 2.85303 minutes; there are 60 trains late for 30 minutes, no train late for one hour, the total delay time is 2766.4 minutes. We can see that the optimization result also improved greatly with little slack of constraints. Fig. 4 is the rescheduling timetable with speed restriction in the first section.

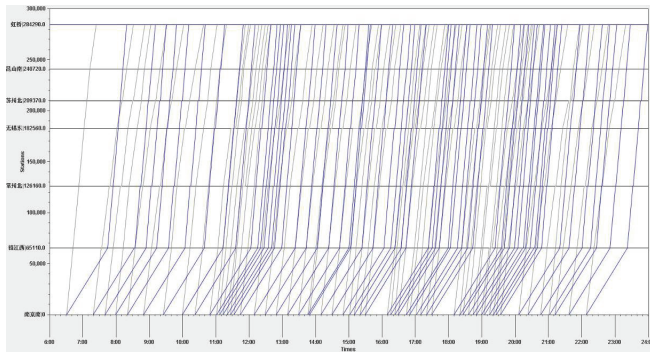


Figure 4. Rescheduling timetable with speed restriction in first section

These simulations are realized on the computer with Intel Core 2 Duo CPU E7500 and 2G Memory. Calculation time for the original model is about 4 seconds and it is less than 20 seconds for the fuzzy model.

V. CONCLUSION AND FUTURE WORK

This paper presents a fuzzy optimization model based tolerance approach for train rescheduling in speed restriction, which deal with the train running time at sections, train dwell time at stations, the headway time at sections and station as the fuzzy parameters. The simulation on Jing-Hu high speed line reveals that the fuzzy optimization result improved greatly with little slack of constraints, which means we can get a new timetable with fewer total delay time as well as the number of seriously impacted trains in safety and lower limited speed and enough headway time .

There remain many interesting areas to explore around membership functions of fuzzy parameters, which may be piecewise linear or more complex form in practice. In addition, there are also some stochastic factors in the timetable rescheduling problem. The more accurate the membership function is, the better result the uncertain optimization model gets. Our ultimate goal is to develop a real-time rescheduling system to significantly improve operation management and scheduling efficiency in the future.

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