

Generalized Intuitionistic Fuzzy Soft Set and its Application in Practical Medical Diagnosis Problem

M. Agarwal, M. Hanmandlu, K. K Biswas

Indian Institute of Technology

New Delhi, India

magarwal@cse.iitd.ac.in, mhmandlu@ee.iitd.ac.in, kkb@cse.iitd.ac.in

Abstract—In this paper, a generalized intuitionistic fuzzy soft set (GIFSS) is introduced and its various properties are presented. The generalized intuitionistic soft fuzzy relations on GIFSS have been defined and their properties are discussed. We have also devised a new scoring function to compare two intuitionistic fuzzy numbers. An application of GIFSS, generalized intuitionistic soft fuzzy relations on GIFSS and the novel score function are demonstrated through a practical example of a multi-criteria medical diagnosis problem.

Keywords—intuitionistic; fuzzy; soft; generalized; score function; multi criteria decision making; Sanchez diagnosis; medical diagnosis; relation

I. INTRODUCTION

The concept of intuitionistic fuzzy set characterized by a novel hesitancy function has been introduced by Atanassov [1, 2]. Molodtsov [3] has introduced the theory of soft sets characterized by parameterization tools for modelling the vagueness and uncertainties. The soft set theory finds wide range of applications in complex medical sciences, engineering, management, economics and social sciences, primarily due to its flexibility without restrictions on approximate description of the situation. Maji et al. [4-6] have extended the theory by proposing the concept of fuzzy soft sets and intuitionistic fuzzy soft sets, and their properties. Kong et al. [7-8] have demonstrated the application of soft set theory in decision making problems. Majumdar et al. [9-10] have explored the methods to find similarity measurement between the fuzzy soft sets, and proposed the notion of generalized fuzzy soft sets.

In this paper, we propose generalized intuitionistic fuzzy soft set (GIFSS). An intuitionistic degree is attached to the intuitionistic fuzzy soft set (IFSS) to indicate the degree of possibility of belongingness of the elements of the universal set in the IFSS. This adds to the second level of uncertainty in IFSS indicating the credibility of the values of IFSS. Relations and properties of GIFSS have been defined. A novel score function to compare intuitionistic fuzzy numbers (IFNs) has also been proposed. Finally Sanchez's method [11] of medical diagnosis is extended to demonstrate a practical medical application of GIFSS and novel score function.

The hesitancy parameter in Atanassov's [1] IFS is a feature of one's own perception, and hence itself ambiguous. It is insufficient to convey a unique meaning imperative for the correct medical diagnosis. For example, a patient might report the symptoms to a doctor as per his perceptions, and the real

information might get distorted in its reporting. Unless this misrepresentation is factored in by a doctor, it would lead to wrong diagnosis. The extra generalization parameter that we introduce normalizes an IFSS with an expert's opinion which indicates the credibility of the information presented to him. For example, a doctor reposes more confidence in the symptoms reported by an educated adult patient than that reported by a child.

The paper is organized in seven sections. In section II, a few preliminaries and useful definitions are given. In section III, GIFSS and its properties are introduced. In section IV, a novel score function is introduced, designed specifically for the medical diagnosis applications. In the next section, relations on GIFSS are defined and its properties are discussed. In section VI, an application of GIFSS is shown in the area of medical diagnosis extending Sanchez's approach. And finally in Section VII, conclusions are presented.

II. PRELIMINARIES

Before describing GIFSS, a few definitions regarding intuitionistic fuzzy sets (IFS) and fuzzy soft sets are presented here.

Definition 2.1 [1]: Let $X = \{x_1, x_2, \dots, x_n\}$ be a fixed finite non-empty set. Then a IFS $A = \{x_i, t_A(x_i), f_A(x_i) | x_i \in X\}$ where the functions $t_A: X \rightarrow [0, 1]$ and $f_A: X \rightarrow [0, 1]$ denote the degree of membership and the degree of non-membership of the element $x \in X$ to A respectively, such that for every $x \in X$: $0 \leq t_A(x) + f_A(x) \leq 1$. Also $h_A(x) = 1 - (t_A(x) + f_A(x))$ is called the degree of hesitation or indeterminacy of x to A . For convenience, we call $\tilde{a} = (t_{\tilde{a}}, f_{\tilde{a}})$ as an intuitionistic fuzzy number (IFN), where $t_{\tilde{a}} \in [0, 1]$, $f_{\tilde{a}} \in [0, 1]$, and $t_{\tilde{a}} + f_{\tilde{a}} \leq 1$.

Definition 2.2 ([3]): Let U denote an initial universal set and E be a set of parameters. Let $P(U)$ be the power set of U . Then pair (F, E) is called a soft set over U iff F is a mapping given by $F: E \rightarrow P(U)$.

Definition 2.3 ([4]): Let U denote an initial universal set and E be a set of parameters. Let I^U denote the power set of all

fuzzy subsets of U . Let $A \subset E$. A pair (F, E) is called a fuzzy soft set over U , where F is a mapping given by $F : A \rightarrow I^U$.

Definition 2.4 ([4]) Let U denote an initial universal set and E be a set of parameters. Let IF^U denote the collection of all intuitionistic fuzzy subsets of U . Let $A \subseteq E$. A pair (F, A) is called intuitionistic fuzzy soft set (IFSS) over U , where F is a mapping given by $\tilde{F} : A \rightarrow IF^U$.

Example 2.1: Let $U = \{c_1, c_2, c_3, c_4\}$ be the set of popular bicycles and $E = \{\text{cost } (e_1), \text{colour } (e_2), \text{weight } (e_3)\}$ be the set of parameters. Let $A \subseteq E$ denotes the list of select parameters that would determine Tom's buying decision. Let $A = \{e_1, e_3\} \subset E$.

Then, $(C, A) = [C(e_1) = \{c_1, c_3, c_4\}, C(e_3) = \{c_2, c_3\}]$ is the crisp soft set over U that describes the bicycles which Tom may buy.

Instead, if we have a fuzzy mapping, $F : A \rightarrow I^U$ with each criteria having a fuzzy membership for each sample of U , then an example of fuzzy soft set, (F, A) , could be $[F(e_1) = \{c_1|0.8, c_2|0.2, c_3|0.5, c_4|0.4\}, F(e_3) = \{c_1|0.7, c_2|0.5, c_3|0.3, c_4|0.8\}]$.

(F, A) is the fuzzy soft set over U that describes the cycles which Tom may buy.

If we have a mapping $\tilde{F} : A \rightarrow IF^U$ with each criteria having an intuitionistic fuzzy membership for each sample of U , then an example of IFSS, (I, A) , could be $[I(e_1) = \{c_1|(0.8, 0.1), c_2|(0.2, 0.6), c_3|(0.5, 0.3), c_4|(0.4, 0.5)\}, I(e_3) = \{c_1|(0.7, 0.1), c_2|(0.5, 0.2), c_3|(0.3, 0.6), c_4|(0.8, 0.0)\}]$.

(I, A) is the IFSS over U that describes the cycles which Tom may buy.

Definition 2.5 ([1]): If A and B are two IFSSs of the set E of form $\{x_i, t(x_i), f(x_i) \mid x_i \in X\}$, then

$$A \subset B \text{ iff } \forall x \in E, [t_A(x) \leq t_B(x) \text{ and } f_A(x) \geq f_B(x)] \quad (1)$$

$$A \supset B \text{ iff } B \subset A \quad (2)$$

$$A = B \text{ iff } \forall x \in E, [t_A(x) = t_B(x) \text{ and } f_A(x) = f_B(x)] \quad (3)$$

$$\bar{A} = \{x, f_A(x), t_A(x) \mid x \in E\} \quad (4)$$

$$A \cup B = \{(x, \max(t_A(x), t_B(x)), \min(f_A(x), f_B(x))) \mid x \in E\} \quad (5)$$

$$A \cap B = \{(x, \min(t_A(x), t_B(x)), \max(f_A(x), f_B(x))) \mid x \in E\} \quad (6)$$

Definition 2.6 ([1]): An intuitionistic fuzzy relation (IFR) R from two IFSSs X to Y is an IFS of $X \times Y$ characterized by the membership function t_R and non-membership function f_R , and denoted as $R (X \rightarrow Y)$

Definition 2.7 ([11]): Let A be IFS in X . Let $R (X \rightarrow Y)$ be an IFR from X to Y . Then max-min-max composition of the IFR $R (X \rightarrow Y)$ with A would result in another IFS B of Y ,

denoted by $B = R \circ A$. The membership function of B is defined as

$$t_{R \circ A}(y) = \bigvee_x [t_A(x) \wedge t_R(x, y)] \quad (7)$$

Similarly, the non-membership function is defined as

$$f_{R \circ A}(y) = \bigwedge_x [f_A(x) \vee f_R(x, y)] \quad (8)$$

$\forall y \in Y$ (where $\bigvee = \max, \bigwedge = \min$).

III. GENERALIZED INTUITIONISTIC FUZZY SOFT SET (GIFSS)

We hereby present a generalized version of IFSS with an extra parameter, indicating an expert's opinion regarding the authenticity of the IFSS values.

A. Basic Definitions

A few basic definitions of GIFSS are defined, and illustrated through examples.

Definition 3.1: Let U denote the universal set of elements and $E = \{e_1, e_2, \dots, e_m\}$ be the universal set of parameters. The pair (U, E) is called a soft universe. Let $A \subseteq E$ and $\tilde{F} : A \rightarrow IF^U$ and $\tilde{\alpha}$ be an intuitionistic fuzzy subset of E , i.e., $\tilde{\alpha} \rightarrow IF = (t_{\tilde{\alpha}}, f_{\tilde{\alpha}})$, where $t_{\tilde{\alpha}} \in [0, 1], f_{\tilde{\alpha}} \in [0, 1], t_{\tilde{\alpha}} + f_{\tilde{\alpha}} \leq 1$, and IF^U is the collection of all intuitionistic fuzzy subsets of U . Mapping $\tilde{F}_{\tilde{\alpha}} : A \rightarrow IF^U \times IF$ is defined as follows:

$$\tilde{F}_{\tilde{\alpha}}(e) = (\tilde{F}(e), \tilde{\alpha}(e)), \quad \text{where } \tilde{F}(e) \in IF^U, \text{ and } \tilde{\alpha}(e) \in IF \quad (9)$$

$\tilde{F}_{\tilde{\alpha}}$ is called generalized intuitionistic fuzzy soft set (GIFSS in short) over the soft universe (U, E) , where $\tilde{F}(e)$ refers to the elements of U in the IFSS, and the extra generalization parameter $\tilde{\alpha}(e)$ indicates degree of possibility of belongingness of the elements of U in $\tilde{F}(e_i)$.

Example 3.1: Let $U = \{c_1, c_2, c_3, c_4\}$ be the set of popular cycles and $E = \{\text{cost } (e_1), \text{colour } (e_2), \text{weight } (e_3)\}$ be the set of parameters. Let $A \subseteq E$ denote the list of select parameters that would determine Tom's buying decision. Let $A = \{e_1, e_3\} \subset E$.

Let $\tilde{\alpha} : E \rightarrow IF$ be defined as follows:

$$\tilde{\alpha}(e_1) = (0.6, 0.3), \tilde{\alpha}(e_3) = (0.3, 0.5)$$

So, if we assume the values of a member of IF^U , the function

$\tilde{F}_{\tilde{\alpha}} : A \rightarrow IF^U \times IF$ is defined as follows:-

$$\tilde{F}_{\tilde{\alpha}}(e_1) = \{ \{c_1|(0.8, 0.1), c_2|(0.2, 0.6), c_3|(0.5, 0.3), c_4|(0.4, 0.5)\}, (0.6, 0.3) \}$$

$$\tilde{F}_{\tilde{\alpha}}(e_3) = \{ \{c_1|(0.7, 0.1), c_2|(0.5, 0.2), c_3|(0.3, 0.6),$$

$$c_4|(0.8, 0.0)\}, (0.3, 0.5)\}$$

Here, $\tilde{F}_{\tilde{\alpha}}$ is a GIFSS over (U, E). Here the value $\tilde{\alpha}(e)$ indicates an expert's confidence in the values of IFSS with regards to their correctness.

Definition 3.2: Let $\tilde{F}_{\tilde{\alpha}}$ and $\tilde{G}_{\tilde{\delta}}$ be two GIFSS over soft universe (U, E). Now, $\tilde{F}_{\tilde{\alpha}}$ would be called as generalized intuitionistic fuzzy soft subset of $\tilde{G}_{\tilde{\delta}}$, i.e., $\tilde{F}_{\tilde{\alpha}} \subseteq \tilde{G}_{\tilde{\delta}}$, if

- i) $\tilde{\alpha}$ is an intuitionistic fuzzy subset of $\tilde{\delta}$.
- ii) $\tilde{F}(e)$ is a intuitionistic fuzzy subset of $\tilde{G}(e)$, $\forall e \in E$.

Example 3.2: Let us consider the GIFSS, $\tilde{F}_{\tilde{\alpha}}$ as in Example 3.1. Let $\tilde{G}_{\tilde{\delta}}$ be another GIFSS over (U, E) defined as follows:

$$\begin{aligned}\tilde{G}_{\tilde{\delta}}(e_1) &= \{\{c_1|(0.3, 0.5), c_2|(0.2, 0.7), c_3|(0.3, 0.5), \\ &\quad c_4|(0.2, 0.6)\}, (0.5, 0.4)\} \\ \tilde{G}_{\tilde{\delta}}(e_3) &= \{\{c_1|(0.6, 0.3), c_2|(0.4, 0.4), c_3|(0.1, 0.8), \\ &\quad c_4|(0.5, 0.3)\}, (0.1, 0.8)\}\end{aligned}$$

In the light of definition 3.2, the GIFSS, $\tilde{G}_{\tilde{\delta}}$, is a intuitionistic fuzzy subset of $\tilde{F}_{\tilde{\alpha}}$.

Definition 3.3: Let $\tilde{F}_{\tilde{\alpha}}$ be a GIFSS over soft universe (U, E). Then the inverse of $\tilde{F}_{\tilde{\alpha}}$, $\tilde{F}_{\tilde{\alpha}}^i = \tilde{G}_{\tilde{\delta}}$ where

- i) $\tilde{\delta}(e) = \tilde{\alpha}^i(e)$
- ii) $\tilde{G}(e) = \tilde{F}^i(e)$, $\forall e \in E$.

Example 3.3: Find the inverse of the GIFSS, $\tilde{F}_{\tilde{\alpha}}(e_1)$ as in Example 3.1.

$$\begin{aligned}\tilde{F}_{\tilde{\alpha}}^i(e_1) &= \{\{c_1|(0.1, 0.8), c_2|(0.6, 0.2), c_3|(0.3, 0.5), \\ &\quad c_4|(0.5, 0.4)\}, (0.3, 0.6)\}\end{aligned}$$

Definition 3.4: Let $\tilde{F}_{\tilde{\alpha}}$ be a GIFSS over soft universe (U, E). Then the complement of $\tilde{F}_{\tilde{\alpha}}$ is defined as

$$\tilde{F}_{\tilde{\alpha}}^c = \tilde{G}_{\tilde{\delta}}$$

where

- i) $\tilde{\delta}(e) = \tilde{\alpha}^c(e)$
- ii) $\tilde{G}(e) = \tilde{F}^c(e)$, $\forall e \in E$.

Definition 3.5: Let $\tilde{F}_{\tilde{\alpha}}$ and $\tilde{G}_{\tilde{\delta}}$ be two GIFSS over soft universe (U, E). Union of $\tilde{F}_{\tilde{\alpha}}$ and $\tilde{G}_{\tilde{\delta}}$, denoted by $\tilde{F}_{\tilde{\alpha}} \tilde{\cup} \tilde{G}_{\tilde{\delta}}$, would yield GIFSS $\tilde{H}_{\tilde{\gamma}} : E \rightarrow IF^U \times IF$, where

- i) $\tilde{H}_{\tilde{\gamma}}(e) = (\tilde{H}(e), \tilde{\gamma}(e))$, where $\tilde{H}(e) \in IF^U$, $\tilde{\gamma}(e) \in IF$ $\tilde{F}(e)$
- ii) $\tilde{H}(e) = \tilde{F}(e) \circ \tilde{G}(e)$

- iii) $\tilde{\gamma}(e) = \tilde{\alpha}(e) \circ \tilde{\delta}(e)$
- iv) $\circ$ is a suitable t-conorm operator
- v) $\tilde{\cup}$ is the union operator adapted for GIFSS.

Definition 3.6: Let $\tilde{F}_{\tilde{\alpha}}$ and $\tilde{G}_{\tilde{\delta}}$ be two GIFSS over soft universe (U, E). Then the intersection of $\tilde{F}_{\tilde{\alpha}}$ and $\tilde{G}_{\tilde{\delta}}$, denoted by $\tilde{F}_{\tilde{\alpha}} \tilde{\cap} \tilde{G}_{\tilde{\delta}}$, would yield GIFSS $\tilde{K}_{\tilde{\gamma}} : E \rightarrow IF^U \times IF$, where

- i) $\tilde{K}_{\tilde{\gamma}}(e) = (\tilde{K}(e), \tilde{\gamma}(e))$, $\tilde{K}(e) \in IF^U$, $\tilde{\gamma}(e) \in IF$
- ii) $\tilde{K}(e) = \tilde{F}(e) * \tilde{G}(e)$
- iii) $\tilde{\gamma}(e) = \tilde{\alpha}(e) * \tilde{\delta}(e)$
- iv) $*$ is a suitable t-norm operator
- v) $\tilde{\cap}$ is the intersection operator adapted for GIFSS.

Example 3.4: Find the union of the two GIFSS, specified in examples 3.1 and 3.2. The t-conorm ($\circ$) operator is the standard union operator, as defined in (5).

Let $\tilde{F}_{\tilde{\alpha}}(e) \tilde{\cup} \tilde{G}_{\tilde{\delta}}(e) = \tilde{H}_{\tilde{\gamma}}(e)$, $\forall e \in E$. Then,

$$\begin{aligned}\tilde{H}_{\tilde{\gamma}}(e_1) &= \{\{c_1|(0.8, 0.1), c_2|(0.2, 0.6), c_3|(0.5, 0.3), \\ &\quad c_4|(0.4, 0.5)\}, (0.6, 0.3)\} \\ \tilde{H}_{\tilde{\gamma}}(e_3) &= \{\{c_1|(0.7, 0.1), c_2|(0.5, 0.2), c_3|(0.3, 0.6), \\ &\quad c_4|(0.8, 0.0)\}, (0.3, 0.5)\}\end{aligned}$$

Example 3.5: Find the intersection of the two GIFSS, as taken in examples 3.1 and 3.2. The t-norm ($*$) operator is the standard intersection operator, as defined in (6).

Let $\tilde{F}_{\tilde{\alpha}}(e) \tilde{\cap} \tilde{G}_{\tilde{\delta}}(e) = \tilde{K}_{\tilde{\gamma}}(e)$, $\forall e \in E$. Then,

$$\begin{aligned}\tilde{K}_{\tilde{\gamma}}(e_1) &= \{\{c_1|(0.3, 0.5), c_2|(0.2, 0.7), c_3|(0.3, 0.5), \\ &\quad c_4|(0.2, 0.6)\}, (0.5, 0.4)\} \\ \tilde{K}_{\tilde{\gamma}}(e_3) &= \{\{c_1|(0.6, 0.3), c_2|(0.4, 0.4), c_3|(0.1, 0.8), \\ &\quad c_4|(0.5, 0.3)\}, (0.1, 0.8)\}\end{aligned}$$

Definition 3.7: A GIFSS is said to be a generalized intuitionistic null fuzzy soft set, $\tilde{\Phi}_{\tilde{\theta}} : E \rightarrow IF^U \times IF$, if

$$\begin{aligned}\tilde{\Phi}_{\tilde{\theta}}(e) &= (\tilde{F}(e), \tilde{\theta}(e)), \text{ where } \tilde{F}(e) = (0.0, 1.0), \\ &\text{ and } \tilde{\theta}(e) = (0.0, 1.0), \forall e \in E.\end{aligned}$$

Definition 3.8: A GIFSS is said to be a generalized intuitionistic absolute fuzzy soft set, $\tilde{A}_{\tilde{\alpha}} : E \rightarrow IF^U \times IF$, if

$$\begin{aligned}\tilde{A}_{\tilde{\alpha}}(e) &= (\tilde{F}(e), \tilde{\alpha}(e)), \text{ where } \tilde{F}(e) = (1.0, 0.0), \\ &\text{ and } \tilde{\alpha}(e) = (1.0, 0.0), \forall e \in E.\end{aligned}$$

B. Properties of GIFSS

A few properties of GIFSS are stated in the following theorems.

Theorem 3.1: Let $\tilde{F}_{\gamma}$ be a GIFSS over (U, E), then the following holds:

- i) $\tilde{F}_{\gamma}$ is a subset of $\tilde{F}_{\gamma} \cup \tilde{F}_{\gamma}$
- ii) $\tilde{F}_{\gamma} \cap \tilde{F}_{\gamma}$ is a subset of $\tilde{F}_{\gamma}$
- iii) $\tilde{F}_{\gamma} \cup \tilde{\Phi}_{\theta} = \tilde{F}_{\gamma}$
- iv) $\tilde{F}_{\gamma} \cap \tilde{\Phi}_{\theta} = \tilde{\Phi}_{\theta}$
- v) $\tilde{F}_{\gamma} \cup \tilde{A}_{\alpha} = \tilde{A}_{\alpha}$
- vi) $\tilde{F}_{\gamma} \cap \tilde{A}_{\alpha} = \tilde{F}_{\gamma}$
- vii) $\tilde{F}_{\gamma} \cup \tilde{F}_{\gamma}^c = \tilde{A}_{\alpha}$
- viii) $\tilde{F}_{\gamma} \cup \tilde{F}_{\gamma}^c = \tilde{\Phi}_{\theta}$

Proof: The results follow trivially from definitions 3.1 to 3.7.

Theorem 3.2: Let $\tilde{F}_{\gamma}$, $\tilde{G}_{\delta}$ and $\tilde{H}_{\nu}$ be any three GIFSS over (U, E), then the commutative and associative properties hold:-

$$i) \tilde{F}_{\gamma} \cup \tilde{G}_{\delta} = \tilde{G}_{\delta} \cup \tilde{F}_{\gamma} \quad (10)$$

$$ii) \tilde{F}_{\gamma} \cap \tilde{G}_{\delta} = \tilde{G}_{\delta} \cap \tilde{F}_{\gamma} \quad (11)$$

$$iii) \tilde{F}_{\gamma} \cup (\tilde{G}_{\delta} \cap \tilde{H}_{\nu}) = (\tilde{F}_{\gamma} \cup \tilde{G}_{\delta}) \cap \tilde{H}_{\nu} \quad (12)$$

$$iv) \tilde{F}_{\gamma} \cap (\tilde{G}_{\delta} \cup \tilde{H}_{\nu}) = (\tilde{F}_{\gamma} \cap \tilde{G}_{\delta}) \cup \tilde{H}_{\nu} \quad (13)$$

Proof: The properties of commutativity and associativity of $\cup$ and $\cap$ trivially follows from the definitions.

Theorem 3.3: Let $\tilde{F}_{\gamma}$, $\tilde{G}_{\delta}$ and $\tilde{H}_{\nu}$ be any three GIFSS over (U, E), and the t-conorm ($\cup$) and t-norm ($\cap$) operators are the standard union (max) and intersection operators (min) (as defined in (5) and (6) respectively), then the distributive property holds:-

$$i) \tilde{F}_{\gamma} \cup (\tilde{G}_{\delta} \cap \tilde{H}_{\nu}) = (\tilde{F}_{\gamma} \cup \tilde{G}_{\delta}) \cap (\tilde{F}_{\gamma} \cup \tilde{H}_{\nu}) \quad (14)$$

$$ii) \tilde{F}_{\gamma} \cap (\tilde{G}_{\delta} \cup \tilde{H}_{\nu}) = (\tilde{F}_{\gamma} \cap \tilde{G}_{\delta}) \cup (\tilde{F}_{\gamma} \cap \tilde{H}_{\nu}) \quad (15)$$

Proof: Since the standard $\cup$ (max/union) and $\cap$ (min/intersection) operators as defined in (5) and (6) are commutative and associative, the above properties hold good, when these standard operators are used for t-conorm and t-norm.

Note: - If the t-conorm and t-norm operators are different from the standard union and intersection operators, the above rules may not hold good.

Theorem 3.4: Let $\tilde{F}_{\gamma}$, $\tilde{G}_{\delta}$ be any two GIFSS over (U, E), then the following holds:-

$$i) (\tilde{F}_{\gamma} \cup \tilde{G}_{\delta})^c = \tilde{F}_{\gamma}^c \cap \tilde{G}_{\delta}^c \quad (16)$$

$$ii) (\tilde{F}_{\gamma} \cap \tilde{G}_{\delta})^c = \tilde{F}_{\gamma}^c \cup \tilde{G}_{\delta}^c \quad (17)$$

Proof: The above results follow from definitions 3.3, 3.4 and 3.5.

Theorem 3.5: Let $\tilde{F}_{\gamma}$ be any GIFSS over (U, E) with $\tilde{G}_{\delta}$ being its generalized intuitionistic fuzzy soft subset, and the t-conorm ($\cup$) and t-norm ($\cap$) operators are the standard union (max) and intersection operators (min) (as defined in (5) and (6) respectively), then the following holds:-

$$i) \tilde{F}_{\gamma} \cup \tilde{G}_{\delta} = \tilde{F}_{\gamma} \quad (18)$$

$$ii) \tilde{F}_{\gamma} \cap \tilde{G}_{\delta} = \tilde{G}_{\delta} \quad (19)$$

Proof: The above results follow trivially from definitions 3.4 and 3.5, and (5) and (6).

IV. NOVEL SCORE FUNCTION

A novel score function is devised here to compare two intuitionistic fuzzy numbers (IFNs) by emphasizing the membership value over the non-membership value. Xu [12] has developed a ranking method based on the score function value and accuracy function value of interval-valued intuitionistic fuzzy numbers. Ye [13] has ranked the interval-valued intuitionistic fuzzy numbers based on a novel accuracy function. However, in particular applications, the proposed techniques using a score function and an accuracy function are not suitable to compare IFNs. Therefore, it is necessary to devise application specific score functions

Definition 4.1: Let $\tilde{F} = (t_f, f_f)$, $\tilde{G} = (t_g, f_g)$ be two IFNs. Then scores of $\tilde{F}$ and $\tilde{G}$ are given as $\Delta\tilde{F} = t_f^2 - f_f^2$, and $\Delta\tilde{G} = t_g^2 - f_g^2$ respectively. The accuracy degrees of $\tilde{F}$ and $\tilde{G}$ are given as $H_f = t_f^2 + f_f^2$, and $H_g = t_g^2 + f_g^2$ respectively.

- If $\Delta\tilde{F} < \Delta\tilde{G}$, then $\tilde{F} < \tilde{G}$;
- If $\Delta\tilde{F} = \Delta\tilde{G}$, then
 - If $H_f = H_g$, then $\tilde{F} = \tilde{G}$;
 - If $H_f < H_g$, then $\tilde{F} < \tilde{G}$; (20)

V. RELATION ON GIFSS

The notion of intuitionistic fuzzy soft relation is defined and generalized in the context of GIFSS.

Definition 5.1 : An intuitionistic fuzzy soft relation (IFSR) $\tilde{R}$ between two IFSS, $\tilde{F}$ and $\tilde{G}$ over soft universe (U, E) and (U, F) respectively, is defined as $\tilde{R}(e, f) = \tilde{F}(e) \cap \tilde{G}(f)$, $\forall e \in E$ and $\forall f \in F$, where $\tilde{R}: K \rightarrow IF^U$ is an IFSS over (U, K), where $K \subseteq E \times F$.

Definition 5.2: A generalized intuitionistic fuzzy soft relation (GIFSR) $\tilde{R}_{\tilde{\tau}}$ between two GIFSS, $\tilde{F}_{\tilde{\alpha}}$ and $\tilde{G}_{\tilde{\delta}}$ over soft universe (U, E) and (U, F) respectively, is defined as:

$$\begin{aligned}\tilde{R}_{\tilde{\tau}}(e, f) &= \tilde{F}_{\tilde{\alpha}}(e) \tilde{\cap} \tilde{G}_{\tilde{\delta}}(f), \forall e \in E \text{ and } \forall f \in F, \\ \text{where, } \tilde{R}_{\tilde{\tau}}: K &\rightarrow IF^U \times IF \text{ is a GIFSS over } (U, K), \\ K &\subseteq E \times F.\end{aligned}$$

Definition 5.3: Let $\tilde{R}_1$ be GIFSR between two GIFSS, $\tilde{F}_{\tilde{\alpha}}$ and $\tilde{G}_{\tilde{\delta}}$ over soft universe (U, E) and (U, K) respectively. Let $\tilde{R}_2$ be GIFSR between GIFSS, $\tilde{G}_{\tilde{\delta}}$ and $\tilde{H}_{\tilde{\kappa}}$ over soft universe (U, K) and (U, L) respectively. Then composition of $\tilde{R}_1$ and $\tilde{R}_2$ is defined as:-

$$\begin{aligned}(\tilde{R}_1 \text{ composition } \tilde{R}_2)(e, l) &= \tilde{R}_1(e, k) \tilde{\cap} \tilde{R}_2(k, l) \\ \text{where, } \tilde{R}_1: E \times K &\rightarrow IF^U \times IF \\ \tilde{R}_2: K \times L &\rightarrow IF^U \times IF\end{aligned}\quad (21)$$

Definition 5.4: Let $\tilde{R}_1$ and $\tilde{R}_2$ be two GIFSRs between two GIFSS, $\tilde{F}_{\tilde{\alpha}}$ and $\tilde{G}_{\tilde{\delta}}$ over soft universe (U, A) and (U, B) respectively. Then the standard operations of union, intersection and inverse are defined as follows:

$$(\tilde{R}_1 \cup \tilde{R}_2)(a, b) = \max \{\tilde{R}_1(a, b), \tilde{R}_2(a, b)\} \quad (22)$$

$$(\tilde{R}_1 \cap \tilde{R}_2)(a, b) = \min \{\tilde{R}_1(a, b), \tilde{R}_2(a, b)\} \quad (23)$$

$$\tilde{R}_1^{-1}(a, b) = \tilde{R}_1^{-1}(b, a), \forall (a, b) \in A \times B \quad (24)$$

$$\begin{aligned}\text{where, } \tilde{R}_1: A \times B &\rightarrow IF^U \times IF \\ \tilde{R}_2: A \times B &\rightarrow IF^U \times IF\end{aligned}$$

VI. AN APPLICATION OF GIFSS IN MEDICAL DIAGNOSIS

We present a practical example of an expert system to diagnose the disease of multiple patients among multiple diseases, deploying the proposed GIFSS and GIFSR and the score function. A medical expert system with intuitionistic medical knowledge as a GIFSR from S (set of symptoms) to D (set of diagnosis) as $S \times D$ is defined.

A. Case-Study 6.1

Tom, Bob, Ram and Edwin are four class-mates, and display the symptoms: fever, headache, body-pain, cough, and loss-of-appetite. The possible diseases could be Typhoid, Malaria, Flu, Jaundice or Throat infection.

Step 1: The symptoms of patients are noted in first five columns of table I. The last column is the newly proposed generalization parameter provided by a moderator indicating his trust in the fuzzy data furnished by the patients.

Step 2: Table II indicates standard medical knowledge relating symptoms with the set of diseases under consideration. The rows indicate the symptoms while the columns correspond to various diseases.

Step 3: In Table III, we obtain patients' diagnosis applying Sanchez's approach as per (7) and (8) to each row-column pair of the two matrices in order to arrive at patients' diagnosis. For ex., Tom's symptoms are composed with Typhoid symptoms as below:-

For membership function: $(0.5 \wedge 0.3) \vee (0.8 \wedge 0.4) \vee (0.8 \wedge 0.8) \vee (0.1 \wedge 0.1) \vee (0.6 \wedge 0.9) \vee (0.6 \wedge 1.0) = 0.8$

For non-membership function: $(0.4 \vee 0.7) \wedge (0.2 \vee 0.5) \wedge (0.1 \vee 0.1) \wedge (0.8 \vee 0.9) \wedge (0.2 \vee 0.1) \wedge (0.3 \vee 0.0) = 0.1$

Thus, the value obtained, (0.8, 0.1) gives the possibility of Tom, having Typhoid. Similarly other values of table III are obtained.

Step 4: The IFNs in Table III are compared, applying score function, as per (20), so as to find the maximum number that indicates the most probable diagnosis of the patients. The score values are populated in Table IV.

Result: The maximum score in a row indicates the diagnosis for each patient. Hence Tom (with score 0.63) and Ram (with score 0.63) suffer from Typhoid, Bob (with score 0.63) and Edwin (0.32) suffer from Malaria.

B. Validation of Results

A few additional steps, as given below, may also be performed so as to be more certain about the results obtained above.

Step 5: Obtain the inverse of the entries of Table II, and populate the values in Table V.

Step 6: Perform the composition of Table I and Table V, as explained in Step 3.

Step 7: Calculate the score of each number in the Table VI, and populate the values in Table VII.

Step 8: Calculate the difference between entries in Table IV and Table VII, to arrive at the net score, as given in Table

VIII. The higher the score, higher is the possibility of the patient affected with the respective disease.

Result: Again, it can be concluded that Tom and Ram suffers from Typhoid, Bob and Edwin suffer from Malaria.

C. Discussion of the Result

The new generalization parameter, indicating an expert's level of trust in the credibility of information, significantly makes the approach quite close to real-life situations. This helps in removing the individual bias from the input data and brings more credibility to the final decision arrived at.

The generalization parameter facilitates the decision making process by including the degree of possibility of correctness of the information presented. In the above example, the doctor appears to be more confident about the reporting by Tom, Ram and Bob (as seen by high values of the generalization parameter), while he is least sure about Edwin's reporting.

For standard medical knowledge, used as the basic rule engine, the generalization parameter is taken as (1, 0), indicating the absolute belief. The results obtained are also validated by performing the composition of non-symptoms of diseases with patients' symptoms so as to obtain the score that indicates non-possibility of a particular disease. The two scores are merged to arrive at the final diagnosis.

TABLE I. PATIENTS AND THEIR SYMPTOMS

Patients	Fever	Headache	Bodypain	Cough	Loss-of-Appetite	Degree of Belonging
Tom	(0.5, 0.4)	(0.8,0.2)	(0.8,0.1)	(0.1, 0.8)	(0.6, 0.2)	(0.6, 0.3)
Bob	(0.8, 0.1)	(0.9, 0.1)	(0.7, 0.2)	(0.1, 0.9)	(0.2, 0.7)	(0.4, 0.2)
Ram	(0.1, 0.8)	(0.3, 0.7)	(0.8, 0.1)	(0.6, 0.2)	(0.1, 0.8)	(0.5, 0.1)
Edwin	(0.6, 0.2)	(0.6, 0.2)	(0.5, 0.3)	(0.1)	(0.2, 0.7)	(0.2, 0.6)

TABLE II. SYMPTOMS AND RELATED DISEASES

Symptoms	Typhoid	Malaria	Flu	Jaundice	Throat-Infection
Fever	(0.3, 0.7)	(0.8,0.1)	(0.5,0.4)	(0.3, 0.5)	(0.5, 0.2)
Headache	(0.4, 0.5)	(0.6, 0.3)	(0.5, 0.2)	(0.2, 0.7)	(0.2, 0.7)
Bodypain	(0.8, 0.1)	(0.7, 0.2)	(0.2, 0.7)	(0.3, 0.5)	(0.4, 0.4)
Cough	(0.1, 0.9)	(0.3, 0.6)	(0.9, 0.1)	(0.2,0.7)	(0.8, 0.1)
Loss-of-Appetite	(0.9, 0.1)	(0.4, 0.5)	(0.1, 0.8)	(0.8,0.2)	(0.2, 0.7)
Degree of Belonging	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)

TABLE III. PATIENTS AND THEIR DIAGNOSIS (TABLE I o TABLE II)

Patients	Typhoid	Malaria	Flu	Jaundice	Throat-Infection
Tom	(0.8, 0.1)	(0.7,0.2)	(0.5,0.2)	(0.6, 0.2)	(0.5, 0.3)
Bob	(0.7, 0.2)	(0.8, 0.1)	(0.5, 0.2)	(0.4, 0.2)	(0.5, 0.2)
Ram	(0.8, 0.1)	(0.7, 0.1)	(0.6, 0.1)	(0.5, 0.1)	(0.6, 0.1)
Edwin	(0.5, 0.3)	(0.6, 0.2)	(0.5, 0.2)	(0.3,0.5)	(0.5, 0.2)

TABLE IV. SCORE OF DIAGNOSIS OF PATIENTS

Score _R	Typhoid	Malaria	Flu	Jaundice	Throat-Infection
Tom	0.63	0.45	0.21	0.32	0.16
Bob	0.45	0.63	0.21	0.12	0.21
Ram	0.63	0.48	0.35	0.24	0.35
Edwin	0.16	0.32	0.21	- 0.16	0.21

TABLE V. NON-SYMPTOMS AND DISEASES

	Typhoid	Malaria	Flu	Jaundice	Throat-Infection
Fever	(0.7, 0.3)	(0.1,0.8)	(0.4,0.5)	(0.5, 0.3)	(0.2, 0.5)
Headache	(0.5, 0.4)	(0.3, 0.6)	(0.2, 0.5)	(0.7, 0.2)	(0.7, 0.2)
Bodypain	(0.1, 0.8)	(0.2, 0.7)	(0.7, 0.2)	(0.5, 0.3)	(0.4, 0.4)
Cough	(0.9, 0.1)	(0.6, 0.3)	(0.1, 0.9)	(0.7,0.2)	(0.1, 0.8)
Loss-of-Apetite	(0.1, 0.9)	(0.5, 0.4)	(0.8, 0.1)	(0.2,0.8)	(0.7, 0.2)
Degree of Belonging	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)

TABLE VI. PATIENTS AND THEIR NON-DIAGNOSIS (TABLE I o TABLE V)

	Typhoid	Malaria	Flu	Jaundice	Throat-Infection
Tom	(0.5, 0.3)	(0.5,0.3)	(0.7,0.2)	(0.7, 0.2)	(0.7, 0.2)
Bob	(0.7, 0.3)	(0.3, 0.7)	(0.7, 0.2)	(0.7, 0.2)	(0.7, 0.2)
Ram	(0.6, 0.2)	(0.6, 0.3)	(0.7, 0.2)	(0.6, 0.2)	(0.4, 0.4)
Edwin	(0.6, 0.3)	(0.3, 0.3)	(0.5, 0.3)	(0.6,0.2)	(0.6, 0.2)

TABLE VII. SCORE OF NON-DIAGNOSIS VALUES FOR PATIENTS

Score_R	Typhoid	Malaria	Flu	Jaundice	Throat-Infection
Tom	0.16	0.16	0.45	0.45	0.45
Bob	0.40	- 0.40	0.45	0.45	0.45
Ram	0.32	0.27	0.45	0.32	0
Edwin	0.27	0	0.16	0.32	0.32

TABLE VIII. FINAL SCORE FOR PATIENTS

Score_R	Typhoid	Malaria	Flu	Jaundice	Throat-Infection
Tom	0.47	0.29	- 0.24	- 0.13	- 0.29
Bob	0.05	1.03	- 0.24	- 0.33	- 0.24
Ram	0.31	0.21	- 0.10	- 0.08	0.35
Edwin	- 0.11	0.32	0.05	- 0.48	- 0.11

VII. CONCLUSIONS

Information often gets distorted from the reality during its representation. This may happen due to lack of domain knowledge, vagueness of information, or similar reasons. In order to address the problem, we have introduced an extra generalization parameter in IFSS. This is another IFS, which is the perception of an expert, indicating the credibility of the information presented. The GIFSS is defined and its properties are studied in detail. Various relations on GIFSS are given and its properties are discussed. A novel score function is also devised specifically for medical diagnosis applications, strengthening the effect of the membership function and limiting that of the non-membership function.

Finally a new approach is developed applying the GIFSS and the novel score function with a view to evolve an expert system for the diagnosis of multiple patients. This approach takes the existing medical diagnosis methods using fuzzy sets to a higher level, being empowered with normalizing factor that indicates the quality of the input information. The approach could be the basis for development of an expert system for medical diagnosis that takes as input the symptoms of patient, and the doctor's opinion about the quality of the input furnished by the patients.

REFERENCES

- [1] K. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, August 1986.
- [2] K. Atanassov, *Intuitionistic Fuzzy Sets: Theory and Applications*, Physica-Verlag, Heidelberg, 1999.
- [3] D. Molodtsov, "Soft set theory--First results," *Computers & Mathematics with Applications*, vol. 37, no. 4-5, pp. 19-31, Feb – Mar 1999.
- [4] P.K Maji, R. Biswas, and A. R. Roy, "Fuzzy soft-sets," *Journal of Fuzzy Mathematics*, vol. 9, no. 3, pp. 589-602, 2001.
- [5] P.K Maji, A. R. Roy, and R. Biswas, "An application of soft sets in a decision making problem," *Computers & Mathematics with Applications*, vol. 44, pp. 1077-1083, 2002.
- [6] P.K Maji, R. Biswas, and A. R. Roy, "Soft set theory," *Computers & Mathematics with Applications*, vol. 45, pp. 555-562, 2003.
- [7] Z. Kong, L. Gao, L. Wang and S. Li, "The normal parameter reduction of soft sets and its algorithm," *Computers & Mathematics with Applications*, vol. 56, no. 12, pp. 3029-3037, Dec. 2008.
- [8] Z. Kong, L. Gao and L. Wang, "Comment on 'A fuzzy soft set theoretic approach to decision making problems'," *Computers & Mathematics with Applications*, vol. 223, no. 2, pp. 540-542, Jan. 2009.
- [9] P. Majumdar, S.K. Samanta, "Similarity measure of soft sets," *New Mathematics and Natural Computation*, vol. 4, no. 1, pp. 1-12, 2008.
- [10] P. Majumdar, S.K. Samanta, "Generalised fuzzy soft sets," *Computers & Mathematics with Applications*, vol. 59, no. 4, pp. 1425-1432, 2010.
- [11] E. Sanchez, "Inverses of fuzzy relations, Application to possibility distributions and medical diagnosis," *Fuzzy Sets and Systems*, vol. 2, no. 1, pp. 75-86, 1979.
- [12] Z. S. Xu, "Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making," *Control and Decision*, vol. 22, no. 2, pp. 215-219, 2007.
- [13] J. Ye, "Multicriteria fuzzy decision-making method based on a novel accuracy function under interval-valued intuitionistic fuzzy environment," *Expert Systems with Applications*, vol. 36, no. 3, pp. 6899-6902, April 2009.