

A Study on Atanassov's Intuitionistic Fuzzy Graphs

M.G.Karunambigai
Department of Mathematics
Sri Vasavi College
Erode, Tamilnadu
India
Email: gkaruns@yahoo.co.in

R. Parvathi
Department of Mathematics
Vellalar College for Women
Erode, Tamilnadu
India
Email: paarvathis@rediffmail.com

O.K.Kalaivani
Department of Mathematics
T.John Institute of Technology
Bangalore,Karnataka
India
Email: kalaivani83@gmail.com

Abstract—Atanassov [1][9][10] defined Intuitionistic Fuzzy Graphs(IFGs) using five types of Cartesian products. In this paper IFGs so defined are named. Homomorphism, weak isomorphism and co-weak isomorphism of minmax IFGs are defined with suitable illustrations. Some of the properties of isomorphism on IFG and isomorphism on strong IFG are also analyzed.

Index Terms— μ -product IFG, ν -product IFG, minmax IFG, maxmin IFG, homomorphism, weak isomorphism and co-weak isomorphism.

I. INTRODUCTION

The motivation for introducing IFGs is due to [1][9][10]. The first definition and concept of Intuitionistic Fuzzy Graph(IFG) was introduced by Krassimir T. Atanassov 1994[1]. R.Parvathi and M.G.Karunambigai analyzed the properties of minmax IFGs in [3][8]. Shannon and Atanassov[1][9][10] defined intuitionistic fuzzy graphs using five types of Cartesian products. In this paper IFGs so defined are named. Some isomorphic properties on IFGs are discussed. Nagoor Gani, et al., discussed the properties of isomorphism on fuzzy graphs and properties of isomorphism on strong fuzzy graphs in [4][5][6] which motivated us to develop the same on IFGs and on strong IFGs.

The main aim of this study is to build basic definitions of an IFG which will be useful for the researchers for their future study in IFGs. Since Atanassov[1][9][10] is the pioneer of our study, the title of the paper is given so. For graph theoretical definitions see[2] and throughout this paper all the properties are analyzed for simple minmax IFG.

II. ATANASSOV'S IFG

Definition 2.1: [1] Let E_1 and E_2 be two universes and let $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in E_1\}$
 $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in E_2\}$
be two IFGs; A –over E_1 and B –over E_2 . Then define
 $A \otimes_1 B = \{\langle \langle x, y \rangle, \mu_A(x) \bullet \mu_B(y), \nu_A(x) \bullet \nu_B(y) \rangle / \langle x, y \rangle \in E_1 \times E_2\}$,
 $A \otimes_2 B = \{\langle \langle x, y \rangle, \mu_A(x) + \mu_B(y) - \mu_A(x) \bullet \mu_B(y), \nu_A(x) \bullet \nu_B(y) \rangle / \langle x, y \rangle \in E_1 \times E_2\}$,
 $A \otimes_3 B = \{\langle \langle x, y \rangle, \mu_A(x) \bullet \mu_B(y), \nu_A(x) + \nu_B(y) - \nu_A(x) \bullet \nu_B(y) \rangle / \langle x, y \rangle \in E_1 \times E_2\}$,
 $A \otimes_4 B = \{\langle \langle x, y \rangle, \min(\mu_A(x), \mu_B(y)), \max(\nu_A(x), \nu_B(y)) \rangle / \langle x, y \rangle \in E_1 \times E_2\}$,
 $A \otimes_5 B = \{\langle \langle x, y \rangle, \max(\mu_A(x), \mu_B(y)), \min(\nu_A(x), \nu_B(y)) \rangle / \langle x, y \rangle \in E_1 \times E_2\}$.

We must note that $A \times_i B$ is an IFS, but it is an IFS over the universe $E_1 \times E_2$, where $\otimes_i$ is one of the five Cartesian products above, $\times$ is the classical Cartesian product on ordinary sets and $\bullet$ is a crisp product. An Intuitionistic Fuzzy Graph (IFG) is of the form $G = (V, E)$ where

- (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\nu_1 : V \rightarrow [0, 1]$, denotes the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$, for every $v_i \in V, (i = 1, 2, \dots, n)$,
- (ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\nu_2 : V \times V \rightarrow [0, 1]$, are such that
 $\mu_2(v_i, v_j) \leq (\mu_1(v_i) \circ \mu_1(v_j))$
 $\nu_2(v_i, v_j) \leq (\nu_1(v_i) \circ \nu_1(v_j))$,
denotes the degree of membership and non-membership of an edge $(v_i, v_j) \in E$ respectively, where, $\circ \in \{\otimes_1, \otimes_2, \otimes_3, \otimes_4, \otimes_5\}$ and $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$, for every $(v_i, v_j) \in E$.

Notations 2.1: The triple $(v_i, \mu_{1i}, \nu_{1i})$ denotes the degree of membership and non-membership of the vertex v_i . The triple $(e_{ij}, \mu_{2ij}, \nu_{2ij})$ denotes the degree of membership and non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on V .

Note 2.1: (1) When $\mu_{2ij} = 0, \nu_{2ij} = 0$ for some i and j , then there is no edge between v_i and v_j .

- (2) When either one of the following is true, then there is an edge between v_i and v_j
 $\diamond \mu_{2ij} = 0$ or $\nu_{2ij} > 0$
 $\diamond \mu_{2ij} > 0$ or $\nu_{2ij} = 0$

The IFGs defined by Atanassov[1] by using five Cartesian products and those IFGs are defined and named here as follows:

A. Product IFG

An Intuitionistic Fuzzy Graph (IFG) $G = (V, E)$ is said to be a product IFG if

- (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\nu_1 : V \rightarrow [0, 1]$, denotes the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$, for every $v_i \in V, (i = 1, 2, \dots, n)$,
- (ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\nu_2 : V \times V \rightarrow [0, 1]$, are such that
 $\mu_2(v_i, v_j) \leq (\mu_1(v_i) \bullet \mu_1(v_j))$
 $\nu_2(v_i, v_j) \leq (\nu_1(v_i) \bullet \nu_1(v_j))$,
denotes the degree of membership and non-membership

of an edge $(v_i, v_j) \in E$ respectively, where, $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$, for every $(v_i, v_j) \in E$.

B. μ -Product IFG

An Intuitionistic Fuzzy Graph (IFG) $G = (V, E)$ is said to be a μ -product IFG if

- (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\nu_1 : V \rightarrow [0, 1]$, denotes the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$, for every $v_i \in V$, $(i = 1, 2, \dots, n)$,
- (ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\nu_2 : V \times V \rightarrow [0, 1]$, are such that $\mu_2(v_i, v_j) \leq (\mu_1(v_i) \bullet \mu_1(v_j))$
 $\nu_2(v_i, v_j) \leq (\nu_1(v_i) + \nu_1(v_j) - \nu_1(v_i) \bullet \nu_1(v_j))$,
denotes the degree of membership and non-membership of an edge $(v_i, v_j) \in E$ respectively, where, $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$, for every $(v_i, v_j) \in E$.

C. ν -Product IFG

An Intuitionistic Fuzzy Graph (IFG) $G = (V, E)$ is said to be a ν -product IFG if

- (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\nu_1 : V \rightarrow [0, 1]$, denotes the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$, for every $v_i \in V$, $(i = 1, 2, \dots, n)$,
- (ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\nu_2 : V \times V \rightarrow [0, 1]$, are such that $\mu_2(v_i, v_j) \leq (\mu_1(v_i) + \mu_1(v_j) - \mu_1(v_i) \bullet \mu_1(v_j))$
 $\nu_2(v_i, v_j) \leq (\nu_1(v_i) \bullet \nu_1(v_j))$,
denotes the degree of membership and non-membership of an edge $(v_i, v_j) \in E$ respectively, where, $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$, for every $(v_i, v_j) \in E$.

D. Minmax IFG

An Intuitionistic Fuzzy Graph (IFG) $G = (V, E)$ is said to be a minmax IFG if

- (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\nu_1 : V \rightarrow [0, 1]$, denotes the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$, for every $v_i \in V$, $(i = 1, 2, \dots, n)$,
- (ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\nu_2 : V \times V \rightarrow [0, 1]$, are such that $\mu_2(v_i, v_j) \leq \min(\mu_1(v_i), \mu_1(v_j))$
 $\nu_2(v_i, v_j) \leq \max(\nu_1(v_i), \nu_1(v_j))$,
denotes the degree of membership and non-membership of an edge $(v_i, v_j) \in E$ respectively, where, $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$, for every $(v_i, v_j) \in E$.

E. Maxmin IFG

An Intuitionistic Fuzzy Graph (IFG) $G = (V, E)$ is said to be a maxmin IFG if

- (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\nu_1 : V \rightarrow [0, 1]$, denotes the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$, for every $v_i \in V$, $(i = 1, 2, \dots, n)$,

- (ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\nu_2 : V \times V \rightarrow [0, 1]$, are such that $\mu_2(v_i, v_j) \leq \max(\mu_1(v_i), \mu_1(v_j))$
 $\nu_2(v_i, v_j) \leq \min(\nu_1(v_i), \nu_1(v_j))$,
denotes the degree of membership and non-membership of an edge $(v_i, v_j) \in E$ respectively, where, $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$, for every $(v_i, v_j) \in E$.

F. Type n IFG

An Intuitionistic Fuzzy Graph (IFG) $G = (V, E)$ is said to be a type n IFG if

- (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\nu_1 : V \rightarrow [0, 1]$, denotes the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$, for every $v_i \in V$, $(i = 1, 2, \dots, n)$,
- (ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\nu_2 : V \times V \rightarrow [0, 1]$, are such that $\mu_2(v_i, v_j) \leq (\mu_1^n(v_i) \bullet \mu_1^n(v_j))$
 $\nu_2(v_i, v_j) \leq (\nu_1^n(v_i) \bullet \nu_1^n(v_j))$,
denotes the degree of membership and non-membership of an edge $(v_i, v_j) \in E$ respectively, where, $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$, for every $(v_i, v_j) \in E$.

Note 2.2: If $n = 1$, then the IFG is a type 1 IFG which is the product IFG, if $n = 2$, then the IFG is called a type 2 IFG, if $n = 3$, then the IFG is called a type 3 IFG and so on .

III. PRELIMINARIES

Definition 3.1: [3] An IFG $G : (V, E)$ is said to be a *strong* IFG if $\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j))$ and $\nu_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j))$, $\forall (v_i, v_j) \in E$.

Definition 3.2: [3] An IFG $G : (V, E)$ is said to be a *complete* IFG if $\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j))$ and $\nu_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j))$, $\forall v_i, v_j \in V$.

Definition 3.3: [3] A *path* P in an IFG is a sequence of distinct vertices $v_1, v_2, \dots, v_n$ such that either one of the following condition is satisfied.

- (i) $\mu_{2ij} > 0$ and $\nu_{2ij} = 0$ for some i and j .
- (ii) $\mu_{2ij} = 0$ and $\nu_{2ij} > 0$ for some i and j .

Definition 3.4: [3] The μ -strength of a path $P = v_1, v_2, \dots, v_n$ is defined as $\min(\mu_2(v_i, v_j))$ for all i and j and is denoted by S_μ .

The ν -strength of a path $P = v_1, v_2, \dots, v_n$ is defined as $\max(\nu_2(v_i, v_j))$ for all i and j and is denoted by S_ν .

Note 3.1: If same edge possesses the value of both μ -strength and ν -strength, then it is the strength of the path P . In other words, the strength of a path is defined to be the weight of the weakest edge of the path. That is the strength of a path is denoted by (S_μ, S_ν) .

Definition 3.5: Let $G : (V, E)$ be an IFG. The μ -strength of the paths connecting any two vertices v_i, v_j is defined as $\max(S_\mu)$ and is denoted by μ_{2ij}^∞ . The ν -strength of the paths

connecting any two vertices v_i, v_j is defined as $\min(S_\nu)$ and is denoted by ν_{2ij}^∞ .

Note 3.2: If same edge possesses both the μ -strength and ν -strength values, then it is the strength of the strongest path P and it is denoted by $S_P = (\mu_{2ij}^\infty, \nu_{2ij}^\infty)$ for all $i, j = 1, 2, \dots, n$.

Definition 3.6: An edge (v_i, v_j) is said to be a *strong edge* in an IFG, $G = (V, E)$, if $\mu_{2ij} \geq \mu_{2ij}^\infty, \nu_{2ij} \leq \nu_{2ij}^\infty$.

Definition 3.7: A path is said to be a *strong path* in $G : (V, E)$ if it consists of only strong edges.

Definition 3.8: [7] The *order* of an IFG $G = (V, E)$ is defined to be $O(G) = (O_\mu(G), O_\nu(G))$ where

$$O_\mu(G) = \sum_{v_i \in V} \mu_1(v_i)$$

$$O_\nu(G) = \sum_{v_i \in V} \nu_1(v_i)$$

Definition 3.9: [7] The *size* of an IFG is defined to be $S(G) = (S_\mu(G), S_\nu(G))$

$$S_\mu(G) = \sum_{v_i, v_j \in V} \mu_2(v_i, v_j)$$

$$S_\nu(G) = \sum_{v_i, v_j \in V} \nu_2(v_i, v_j)$$

Definition 3.10: [7] The *degree* of a vertex v_i is defined by $d(v_i) = (d_\mu(v_i), d_\nu(v_i))$ where

$$d_\mu(v_i) = \sum_{v_i \neq v_j} \mu_2(v_i, v_j)$$

$$d_\nu(v_i) = \sum_{v_i \neq v_j} \nu_2(v_i, v_j)$$

Definition 3.11: [3] The *complement* of an IFG $G = (V, E)$ is an IFG $\bar{G} = (\bar{V}, \bar{E})$, where

- (i) $\bar{V} = V$
- (ii) $\bar{\mu}_{1i} = \mu_{1i}$ and $\bar{\nu}_{1i} = \nu_{1i}$, for all $i = 1, 2, \dots, n$.
- (iii) $\bar{\mu}_{2ij} = \min(\mu_{1i}, \mu_{1j}) - \mu_{2ij}$ and $\bar{\nu}_{2ij} = \max(\nu_{1i}, \nu_{1j}) - \nu_{2ij}$, for all $i, j = 1, 2, \dots, n$.

Definition 3.12: [3] Consider the two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. An *isomorphism* between two IFGs G_1 and G_2 , denoted by $G_1 \cong G_2$, is a bijective map $I : V_1 \rightarrow V_2$ which satisfies

$$\begin{aligned} I(v_i) &= v'_i, v_i \in V \\ \mu_1(v_i) &= \mu'_1(I(v_i)), \nu_1(v_i) = \nu'_1(I(v_i)), \forall v_i \in V \\ \mu_2(v_i, v_j) &= \mu'_2(I(v_i), I(v_j)) \text{ and} \\ \nu_2(v_i, v_j) &= \nu'_2(I(v_i), I(v_j)), \forall v_i, v_j \in V. \end{aligned}$$

Definition 3.13: A *homomorphism* between two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is defined as $H : V_1 \rightarrow V_2$ is a map which satisfies

$$\begin{aligned} H(v_i) &= v'_i, v_i \in V \\ \mu_1(v_i) &\leq \mu'_1(H(v_i)), \nu_1(v_i) \geq \nu'_1(H(v_i)), \forall v_i \in V_1 \\ \mu_2(v_i, v_j) &\leq \mu'_2(H(v_i), H(v_j)) \text{ and} \\ \nu_2(v_i, v_j) &\geq \nu'_2(H(v_i), H(v_j)), \forall v_i, v_j \in V. \end{aligned}$$

Definition 3.14: A *weak isomorphism* between two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is defined as $I : V_1 \rightarrow V_2$ is a bijective homomorphism that satisfies

$$\mu_1(v_i) = \mu'_1(I(v_i)), \nu_1(v_i) = \nu'_1(I(v_i)), \forall v_i \in V.$$

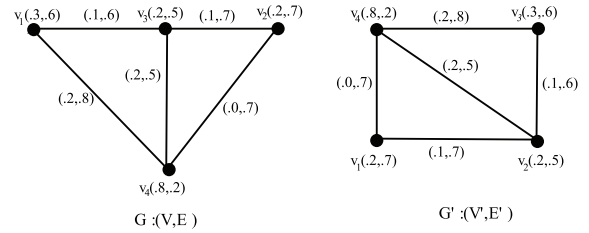


Fig. 1. G_1 is isomorphic to G_2

Definition 3.15: A *co-weak isomorphism* between two IFGs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is defined as $I : V_1 \rightarrow V_2$ is a bijective homomorphism that satisfies

$$\begin{aligned} \mu_2(v_i, v_j) &= \mu'_2(I(v_i), I(v_j)) \text{ and} \\ \nu_2(v_i, v_j) &= \nu'_2(I(v_i), I(v_j)), \forall v_i, v_j \in V_1 \end{aligned}$$

Definition 3.16: An *automorphism* of an intuitionistic fuzzy graph $G : (V, E)$ is an isomorphism of $G : (V, E)$ to itself.

IV. PROPERTIES OF ISOMORPHISM ON INTUITIONISTIC FUZZY GRAPHS

Theorem 4.1: For any two isomorphic IFG their order and size are same.

Proof: If $h : G \rightarrow G'$ is an isomorphism between the IFG G and G' with the underlying sets V and V' respectively then $\mu_1(v_i) = \mu'_1(h(v_i))$ and $\nu_1(v_i) = \nu'_1(h(v_i)), \forall v_i \in V$. $\mu_2(v_i, v_j) = \mu'_2(h(v_i), h(v_j))$ and $\nu_2(v_i, v_j) = \nu'_2(h(v_i), h(v_j)), \forall v_i, v_j \in V$. we know that

$$\begin{aligned} \text{order}(G_\mu) &= \sum_{v_i \in V} \mu_1(v_i) = \sum_{v_i \in V} \mu'_1(h(v_i)) = \text{order}(G'_\mu) \\ \text{order}(G_\nu) &= \sum_{v_i \in V} \nu_1(v_i) = \sum_{v_i \in V} \nu'_1(h(v_i)) = \text{order}(G'_\nu) \end{aligned}$$

$$\begin{aligned}
size(G_\mu) &= \sum_{v_i, v_j \in V} \mu_2(v_i, v_j) \\
&= \sum_{v_i, v_j \in V} \mu'_2(h(v_i), h(v_j)) \\
&= size(G'_\mu)
\end{aligned}$$

$$\begin{aligned}
size(G_\nu) &= \sum_{v_i, v_j \in V} \nu_2(v_i, v_j) \\
&= \sum_{v_i, v_j \in V} \nu'_2(h(v_i), h(v_j)) \\
&= size(G'_\nu)
\end{aligned}$$

Corollary 4.1: Converse of the above theorem need not be true.

Example 4.1: Here, $O_\mu(G) = 1.2$, $O_\nu(G) = 1.8$, $S_\mu(G) = .4$, $S_\nu(G) = 1.5$, $O_\mu(G') = 1.2$, $O_\nu(G') = 1.8$, $S_\mu(G') = .4$, $S_\nu(G') = 1.5$.

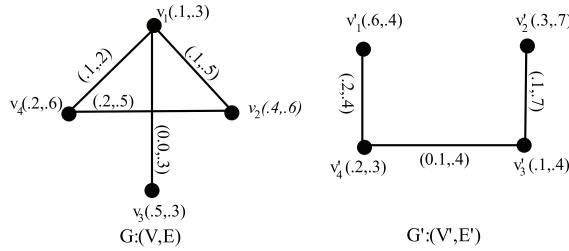


Fig. 2. Graphs of same order and size but not isomorphic

Remark 4.1: If the IFGs are weak isomorphic then their order are same. But the IFGs are of same order need not be weak isomorphic.

Example 4.2:

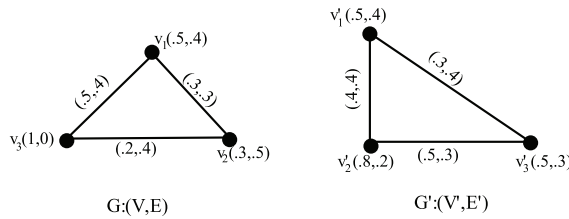


Fig. 3. Graphs of same order but not weak isomorphic

Remark 4.2: If the IFGs are co-weak isomorphic then their size are same. But the IFGs are of same size need not be co-weak isomorphic.

Example 4.3:

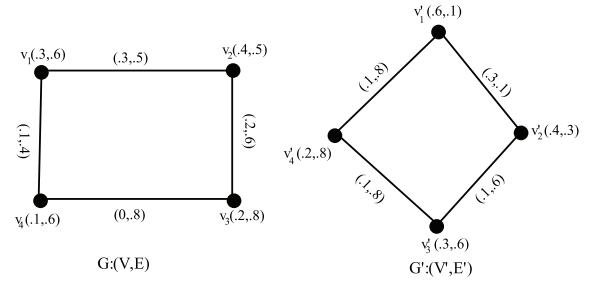


Fig. 4. Graphs of same size but not co-weak isomorphic

Theorem 4.2: If G and G' are isomorphic IFGs then the degrees of their vertices are preserved.

Proof: Let $h : V \rightarrow V'$ be an isomorphism of G and G' . By the definition of an isomorphism, $\mu_2(v_i, v_j) = \mu'_2(h(v_i), h(v_j)), \forall v_i, v_j \in V$. $\nu_2(v_i, v_j) = \nu'_2(h(v_i), h(v_j)), \forall v_i, v_j \in V$.

$$\begin{aligned}
d_\mu(v_i) &= \sum_{v_i \neq v_j} \mu_2(v_i, v_j) \\
&= \sum_{v_i \neq v_j} \mu'_2(h(v_i), h(v_j)) \\
&= d_\mu[h(v_i)] \\
d_\nu(v_i) &= \sum_{v_i \neq v_j} \nu_2(v_i, v_j) \\
&= \sum_{v_i \neq v_j} \nu'_2(h(v_i), h(v_j)) \\
&= d_\nu(h(v_i))
\end{aligned}$$

Corollary 4.2: Converse of the above theorem need not be true.

Example 4.4: Consider the two graphs $G : (V, E)$ and $G' : (V', E')$ which preserving the degree of vertices but G and G' are not isomorphic.

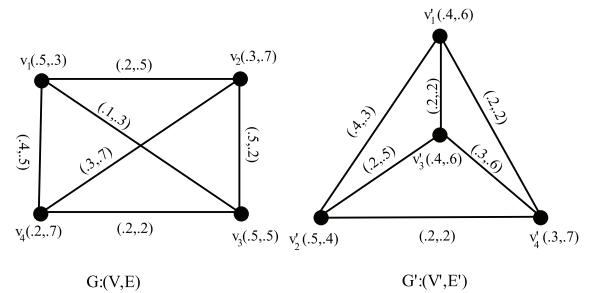


Fig. 5. Graphs preserving the degree of the vertices but not isomorphic

Theorem 4.3: Isomorphism between IFG is an equivalence relation.

Proof: Let $G : (V, E)$, $G' : (V', E')$, $G'' : (V'', E'')$ be IFGs.

(i) Reflexive:

Consider the identity map $h : V \rightarrow V$, such that

$$h(v_i) = v_i, \forall v_i \in V \quad (1)$$

This h is bijective map satisfying

$$\mu_1(v_i) = \mu_1(h(v_i)) \quad (2)$$

$$\nu_1(v_i) = \nu_1(h(v_i)), \forall v_i \in V \quad (3)$$

$$\mu_2(v_i, v_j) = \mu_2(h(v_i), h(v_j)) \quad (4)$$

$$\nu_2(v_i, v_j) = \nu_2(h(v_i), h(v_j)), \forall v_i, v_j \in V \quad (5)$$

Hence h is an isomorphism of an IFG onto itself.

(ii) Symmetric:

Let $h : V \rightarrow V'$,

$$h(v_i) = v'_i, v_i \in V \quad (6)$$

be an isomorphism of G onto G' satisfying,

$$\mu_1(v_i) = \mu'_1(h(v_i)), \nu_1(v_i) = \nu'_1(h(v_i)), \forall v_i \in V \quad (7)$$

$$\mu_2(v_i, v_j) = \mu'_2(h(v_i), h(v_j)) \text{ and} \quad (8)$$

$$\nu_2(v_i, v_j) = \nu'_2(h(v_i), h(v_j)), \forall v_i, v_j \in V \quad (9)$$

from equation (6) we have $h^{-1}(v'_i) = v_i, \forall v'_i \in V'$,
Therefore,

$$\mu_1(h^{-1}(v'_i)) = \mu'_1(v'_i), \nu_1(h^{-1}(v'_i)) = \nu'_1(v'_i), \forall v'_i \in V' \quad (10)$$

$$\mu_2(h^{-1}(v'_i), h^{-1}(v'_j)) = \mu'_2(v'_i, v'_j) \text{ and} \quad (11)$$

$$\nu_2(h^{-1}(v'_i), h^{-1}(v'_j)) = \nu'_2(v'_i, v'_j), \forall v'_i, v'_j \in V' \quad (12)$$

which implies $h^{-1} : V' \rightarrow V$ is a bijective map and $G' \cong G$.

(iii) Transitive:

Let $h : V \rightarrow V'$ and $g : V' \rightarrow V''$ be two isomorphisms of the IFGs G onto G' and G' onto G'' respectively.

Then we have to prove that $g \circ h$ is a one – one onto map from V to V'' ,

where $(g \circ h)(v_i) = g(h(v_i)), \forall v_i \in V$.

As h is an isomorphism, we have, $h(v_i) = v'_i, \forall v_i \in V$.

$$\mu_1(v_i) = \mu'_1(h(v_i)), \nu_1(v_i) = \nu'_1(h(v_i)), \forall v_i \in V \quad (13)$$

$$\mu_2(v_i, v_j) = \mu'_2(h(v_i), h(v_j)) \text{ and} \quad (14)$$

$$\nu_2(v_i, v_j) = \nu'_2(h(v_i), h(v_j)), \forall v_i, v_j \in V \quad (15)$$

using equation (6)

$$\mu_1(v_i) = \mu'_1(v'_i), \nu_1(v_i) = \nu'_1(v'_i) \quad (16)$$

$$\mu_2(v_i, v_j) = \mu'_2(v'_i, v'_j) \text{ and} \quad (17)$$

$$\nu_2(v_i, v_j) = \nu'_2(v'_i, v'_j) \quad (18)$$

As g is an isomorphism from V' to V'' , we have,
 $g(v'_i) = v''_i, v'_i \in V'$ and

$$\mu'_1(v'_i) = \mu''_1(g(v'_i)), \nu'_1(v'_i) = \nu''_1(g(v'_i)), \forall v'_i \in V' \quad (19)$$

$$\mu'_2(v'_i, v'_j) = \mu''_2(g(v'_i), g(v'_j)) \text{ and} \quad (20)$$

$$\nu'_2(v'_i, v'_j) = \nu''_2(g(v'_i), g(v'_j)), \forall v'_i, v'_j \in V' \quad (21)$$

from equations (16) and (19) and using $h(v_i) = v'_i, v_i \in V$, we get

$$\mu_1(v_i) = \mu'_1(v'_i) = \mu''_1(g(v'_i)),$$

$$\mu_1(v_i) = \mu''_1(g(h(v_i))), \forall v_i \in V$$

$$\nu_1(v_i) = \nu'_1(v'_i) = \nu''_1(g(v'_i)),$$

$$\nu_1(v_i) = \nu''_1(g(h(v_i))), \forall v_i \in V$$

from equations (17), (18), (20) and (21), we get

$$\mu_2(v_i, v_j) = \mu'_2((v'_i), (v'_j))$$

$$= \mu''_2(g(v'_i), g(v'_j)), \forall v'_i, v'_j \in V'$$

$$= \mu''_2(g(h(v_i)), g(h(v_j))), \forall v_i, v_j \in V$$

$$\nu_2(v_i, v_j) = \nu'_2((v'_i), (v'_j))$$

$$= \nu''_2(g(v'_i), g(v'_j)), \forall v'_i, v'_j \in V'$$

$$= \nu''_2(g(h(v_i)), g(h(v_j))), \forall v_i, v_j \in V$$

Therefore $g \circ h$ is an isomorphism between G and G'' .

Hence isomorphism between IFGs is an equivalence relation. ■

V. PROPERTIES OF ISOMORPHISM ON STRONG INTUITIONISTIC FUZZY GRAPHS

Theorem 5.1: If $G \cong G'$ then G is a strong IFG if and only if G' is also a strong IFG .

Proof: Let $G = (V, E)$ and $G' = (V', E')$ be an IFG. If $G \cong G'$ then there exists an isomorphism $I : V \rightarrow V'$, which is bijective that satisfies

$$\mu_1(v_i) = \mu'_1(I(v_i)), \nu_1(v_i) = \nu'_1(I(v_i)), \forall v_i \in V \quad (22)$$

$$\mu_2(v_i, v_j) = \mu'_2(I(v_i), I(v_j)) \text{ and} \quad (23)$$

$$\nu_2(v_i, v_j) = \nu'_2(I(v_i), I(v_j)), \forall v_i, v_j \in V \quad (24)$$

G is a strong IFG if and only if

$$\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j)),$$

$$\nu_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j)), \forall (v_i, v_j) \in E$$

if and only if

$$\mu'_2(I(v_i), I(v_j)) = \min(\mu'_1(I(v_i)), \mu'_1(I(v_j)))$$

$$\nu'_2(I(v_i), I(v_j)) = \max(\nu'_1(I(v_i)), \nu'_1(I(v_j))), \forall (v_i, v_j) \in E$$

if and only if

$$\mu'_2(v'_i, v'_j) = \min(\mu'_1(v'_i), \mu'_1(v'_j))$$

$$\nu'_2(v'_i, v'_j) = \max(\nu'_1(v'_i), \nu'_1(v'_j)), \forall (v'_i, v'_j) \in E'$$

if and only if G' is also a strong IFG. ■

Theorem 5.2: If G is co-weak isomorphic with a strong IFG G' , then G is also a strong IFG.

Proof: If G is co-weak isomorphic with G' , then there exists a co-weak isomorphism $I : V \rightarrow V'$ which is bijective that satisfies

$$\mu_1(v_i) \leq \mu'_1(I(v_i)), \nu_1(v_i) \geq \nu'_1(I(v_i)), \forall v_i \in V \quad (25)$$

$$\mu_2(v_i, v_j) = \mu'_2(I(v_i), I(v_j)) \text{ and} \quad (26)$$

$$\nu_2(v_i, v_j) = \nu'_2(I(v_i), I(v_j)), \forall v_i, v_j \in V \quad (27)$$

Since G_2 is a strong IFG, we've

$$\begin{aligned}\mu'_2(I(v_i), I(v_j)) &= \min(\mu'_1(I(v_i)), \mu'_1(I(v_j))) \\ \nu'_2(I(v_i), I(v_j)) &= \max(\nu'_1(I(v_i)), \nu'_1(I(v_j))), \forall v_i, v_j \in V\end{aligned}$$

Now we consider,

$$\begin{aligned}\min(\mu'_1(I(v_i)), \mu'_1(I(v_j))) &= \mu'_2(I(v_i), I(v_j)) \\ &= \mu_2(v_i, v_j) \\ &\leq \min(\mu_{1i}, \mu_{1j}) \\ &\leq \min(\mu'_1(I(v_i)), \mu'_1(I(v_j))), \forall (v_i, v_j) \in E \\ \Rightarrow \mu_2(v_i, v_j) &= \min(\mu_{1i}, \mu_{1j}), \forall (v_i, v_j) \in E\end{aligned}$$

Now, we consider,

$$\begin{aligned}\max(\nu'_1(I(v_i)), \nu'_1(I(v_j))) &= \nu'_2(I(v_i), I(v_j)) \\ &= \nu_2(v_i, v_j) \\ &\leq \max(\nu_{1i}, \nu_{1j}) \\ &\leq \max(\nu'_1(I(v_i)), \nu'_1(I(v_j))), \forall (v_i, v_j) \in E \\ \Rightarrow \nu_2(v_i, v_j) &= \max(\nu_{1i}, \nu_{1j}), \forall (v_i, v_j) \in E\end{aligned}$$

Hence G is a strong IFG. $\blacksquare$

Theorem 5.3: $G = (V, E)$ is a strong IFG if and only if $\bar{G} = (\bar{V}, \bar{E})$ is also a strong IFG

Proof: G is a strong IFG if and only if

$$\begin{aligned}\mu_2(v_i, v_j) &= \min(\mu_1(v_i), \mu_1(v_j)), \\ \nu_2(v_i, v_j) &= \max(\nu_1(v_i), \nu_1(v_j)), \forall (v_i, v_j) \in E\end{aligned}$$

if and only if

$$\begin{aligned}\bar{\mu}_2(v_i, v_j) &= \begin{cases} 0, & \forall (v_i, v_j) \in E \\ \min(\mu_{1i}, \mu_{1j}), & \forall (v_i, v_j) \notin E \end{cases} \\ \bar{\nu}_2(v_i, v_j) &= \begin{cases} 0, & \forall (v_i, v_j) \in E \\ \max(\nu_{1i}, \nu_{1j}), & \forall (v_i, v_j) \notin E \end{cases}\end{aligned}$$

Hence,

$$\begin{aligned}\bar{\mu}_2(v_i, v_j) &= \min(\mu_1(v_i), \mu_1(v_j)), \\ \bar{\nu}_2(v_i, v_j) &= \max(\nu_1(v_i), \nu_1(v_j)), \forall (v_i, v_j) \in \bar{E}\end{aligned}$$

if and only if $\bar{G} = (\bar{V}, \bar{E})$ is a strong IFG. $\blacksquare$

Theorem 5.4: If $G = (V, E)$ is an IFG then

$$\begin{aligned}S_\mu(G) + S_\mu(\bar{G}) &\leq 2 \sum_{v_i \neq v_j} \mu_2(v_i, v_j) \\ S_\nu(G) + S_\nu(\bar{G}) &\leq 2 \sum_{v_i \neq v_j} \nu_2(v_i, v_j).\end{aligned}$$

Proof: By definition, order of G = order of $\bar{G}$. That is

$$\begin{aligned}O_\mu(G) &= O_\mu(\bar{G}), \quad O_\nu(G) = O_\nu(\bar{G}) \\ \mu_2(v_i, v_j) &\leq \min(\mu_1(v_i), \mu_1(v_j)),\end{aligned}\tag{28}$$

$$\nu_2(v_i, v_j) \leq \max(\nu_1(v_i), \nu_1(v_j)), \forall (v_i, v_j) \in E\tag{29}$$

$$\bar{\mu}_2(v_i, v_j) = \min(\mu_{1i}, \mu_{1j}) - \mu_{2ij}, \forall v_i, v_j \in V\tag{30}$$

$$\mu_2(v_i, v_j) + \bar{\mu}_2(v_i, v_j) = \min(\mu_{1i}, \mu_{1j})\tag{31}$$

Also,

$$\bar{\mu}_2(v_i, v_j) \leq \min(\mu_{1i}, \mu_{1j})\tag{32}$$

From equation(28),(31) and (32), we get

$$\begin{aligned}\min(\mu_{1i}, \mu_{1j}) &= \mu_2(v_i, v_j) + \bar{\mu}_2(v_i, v_j) \\ &\leq 2 \min(\mu_{1i}, \mu_{1j}), \forall v_i, v_j \in V \\ \mu_2(v_i, v_j) + \bar{\mu}_2(v_i, v_j) &\leq 2 \min(\mu_{1i}, \mu_{1j}), \forall v_i, v_j \in V\end{aligned}$$

Taking summation on both sides, we get

$$\begin{aligned}\sum_{v_i, v_j \in V} \mu_2(v_i, v_j) + \sum_{v_i, v_j \in V} \bar{\mu}_2(v_i, v_j) \\ \leq 2 \sum_{v_i, v_j \in V} \min(\mu_{1i}, \mu_{1j})\end{aligned}$$

By the definition of size, we get

$$\begin{aligned}S_\mu(G) + S_\mu(\bar{G}) &\leq 2 \sum_{v_i, v_j \in V} \min(\mu_{1i}, \mu_{1j}) \\ &= 2 \sum_{v_i, v_j \in V} \mu_2(v_i, v_j)\end{aligned}$$

Now,

$$\bar{\nu}_2(v_i, v_j) = \max(\nu_{1i}, \nu_{1j}) - \nu_{2ij}, \forall v_i, v_j \in V\tag{33}$$

$$\nu_2(v_i, v_j) + \bar{\nu}_2(v_i, v_j) = \max(\mu_{1i}, \mu_{1j})\tag{34}$$

Also,

$$\bar{\nu}_2(v_i, v_j) \leq \max(\nu_{1i}, \nu_{1j})\tag{35}$$

From equation(29),(34) and (35), we get

$$\begin{aligned}\max(\nu_{1i}, \nu_{1j}) &= \nu_2(v_i, v_j) + \bar{\nu}_2(v_i, v_j) \\ &\leq 2 \max(\nu_{1i}, \nu_{1j}), \forall v_i, v_j \in V \\ \nu_2(v_i, v_j) + \bar{\nu}_2(v_i, v_j) &\leq 2 \max(\nu_{1i}, \nu_{1j}), \forall v_i, v_j \in V\end{aligned}$$

Taking summation on both sides, we get

$$\begin{aligned}\sum_{v_i, v_j \in V} \nu_2(v_i, v_j) + \sum_{v_i, v_j \in V} \bar{\nu}_2(v_i, v_j) \\ \leq 2 \sum_{v_i, v_j \in V} \max(\mu_{1i}, \mu_{1j})\end{aligned}$$

By the definition of size, we get

$$\begin{aligned}S_\nu(G) + S_\nu(\bar{G}) &\leq 2 \sum_{v_i, v_j \in V} \max(\mu_{1i}, \mu_{1j}) \\ &= 2 \sum_{v_i, v_j \in V} \nu_2(v_i, v_j).\end{aligned}$$

VI. CONCLUSION

This paper presents the basic definitions and some properties of isomorphism on strong IFG. We have shown that if $G : (V, E)$ is a strong IFG then its complement is also strong IFG and vice-versa. We've also given the boundary conditions for μ -size and ν -size of an IFG. The main motivation of this paper is to encourage IFG researchers to pursue similar ideas in the field in order to advance the use of IFGs in all other fields.

ACKNOWLEDGMENT

The authors would like to thank the University Grants Commission, New Delhi, India for its financial support to Minor Research Project F. MRP - 3365/ 10 (MRP/UGC - SERO) dated 23rd July 2010 and to UGC Research Award No. F. 30 - 1/ 2009 (SA - II) dated 2nd July 2009.

REFERENCES

- [1] K. Atanassov, *Intuitionistic Fuzzy Sets: Theory and Applications*, Springer Physica-Verlag, Berlin, 1999.
- [2] Bondy, J.A and Murthy, U.S.R. *Graph Theory with Applications*, American Elsevier Publishing Co., New York, 1976.
- [3] M.G. Karunambigai and R. Parvathi, *Intuitionistic Fuzzy Graphs*, Journal of Computational Intelligence, Theory and Applications, Vol. 20, 139-150, 2006.
- [4] A. Nagoor Gani and V.T. Chandrasekaran, *A First look at Fuzzy Graph Theory*, by the Allied Publishers Pvt.Ltd, 2010.
- [5] A. Nagoor Gani and J. Malarvizhi, *Isomorphism Properties on Strong Fuzzy Graphs*, International Journal of Algorithms, Computing and Mathematics, Vol 2, Number 1, 2009.
- [6] A. Nagoor Gani and J. Malarvizhi, *Isomorphism on Fuzzy Graphs*, International Journal of Computational and Mathematical Sciences, 2:4 2008
- [7] A. Nagoor Gani and S. Shajitha Begum, *Degree, Order and Size in Intuitionistic Fuzzy Graphs*, International Journal of Algorithms, Computing and Mathematics, Vol 3, Number 3, 2010.
- [8] R. Parvathi and M.G. Karunambigai, *Operations on Intuitionistic Fuzzy Graphs*, Proceedings of FUZZ-IEEE 2009, South Korea, 1396-1401, 2009.
- [9] A. Shannon , K. Atanassov, *A First Step to a Theory of the Intuitionistic Fuzzy Graphs*, Proc. of the First Workshop on Fuzzy Based Expert Systems (D. Atanassov, Ed.), Sofia, 59-61, 1994.
- [10] Shannon A., Atanassov K., *Intuitionistic Fuzzy Graphs from α, β and $(\alpha; \beta)$ - levels*, Notes on Intuitionistic Fuzzy Sets, Vol. 1, No. 1, 32-35, 1995.