

Interactive Fuzzy Programming through Possibility Measures and Probability Maximization for Two-Level Linear Programming Problems Involving Fuzzy Random Variable Coefficients

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Abstract—In this research, focusing on two-level linear programming problems involving fuzzy random variables, we propose a new decision making model through possibility measures. Taking into account vagueness of judgments of decision makers for each objective function, fuzzy goals are introduced. Noting that the degree of the possibility fluctuates stochastically, we propose a probability maximization model in stochastic programming. Through probability maximization, the transformed stochastic two-level programming problem can be reduced to a deterministic one. Interactive fuzzy programming to derive a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relation between decision makers is presented.

Index Terms—two-level programming; possibility; probability maximization; interactive fuzzy decision making

I. INTRODUCTION

In the real world, the coefficients of objectives or constraints in problems are rather uncertain than constant. In order to deal with such uncertainty, stochastic programming [3] and fuzzy programming [21] were considered. They are useful tools for the decision making under a stochastic environment or a fuzzy environment, respectively. Most studies in respect to mathematical programming take account of either fuzziness or randomness. However, in practice, decision makers are faced with the situations where both fuzziness and randomness exist. In the case where some expert estimates coefficients of objective functions or constraints with uncertainty, they do not always represent the coefficients as random variables or fuzzy sets but as the values including both fuzziness and randomness. In such a case, it is important to realize that simultaneous considerations of both fuzziness and randomness would be required. A fuzzy random variable [7] is one of the mathematical concepts dealing with fuzziness and randomness simultaneously. Let us consider the situation that a profit per unit crop acreage is dependent on weathers, i. e., fine, cloudy, rain, etc., which are considered as scenarios that occur randomly. When a realized profit under each scenario is estimated as a fuzzy set or fuzzy number, it turns out

that the profit is expressed by a fuzzy random variable. Studies on linear programming problems with fuzzy random variable coefficients, called fuzzy random linear programming problems, were initiated by Wang and Qiao [19], Qiao, Zhang and Wang [11] as seeking the probability distribution of the optimal solution and optimal value. A brief survey of major fuzzy stochastic programming models was found in the paper by Luhandjula [9].

Meanwhile, we often encounter situations where there are two or more decision makers in an organization with a hierarchical structure, and they make decisions in turn or at the same time so as to optimize their objective functions. In particular, consider a case where there are two decision makers; one of the decision makers first makes a decision, and then the other who knows the decision of the opponent makes a decision. Such a situation is formulated as a two-level programming problem [13]. In the context of two-level programming, the decision maker at the upper level first specifies a strategy, and then the decision maker at the lower level specifies a strategy so as to optimize the objective with full knowledge of the action of the decision maker at the upper level. In conventional multi-level mathematical programming models employing the solution concept of Stackelberg equilibrium, it is assumed that there is no communication among decision makers, or they do not make any binding agreement even if there exists such communication [1], [16], [17]. Compared with this, for decision making problems in such as decentralized large firms with divisional independence, it is quite natural to suppose that there exists communication and some cooperative relationship among the decision makers [13].

Assuming that the decision makers essentially cooperate with each other, Sakawa et al. [14], [15] have proposed interactive fuzzy programming for two-level or multi-level linear programming problems to obtain satisfactory solution for decision makers by eliminating the fuzzy goals for the decision variables.

Under these circumstances, in this research, assuming coop-

erative behavior of the decision makers, we consider solution methods for decision making problems in hierarchical organizations under fuzzy random environments. To deal with the formulated two-level linear programming problems involving fuzzy random variables, possibility measures are introduced and a stochastic two-level linear programming problem is defined for guaranteeing the degree of realization of the problem. Taking into account vagueness of judgments of decision makers, fuzzy goals are introduced and the stochastic two-level linear programming problem is transformed into the problem to maximize the satisfaction degree for each fuzzy goal. Following probability maximization, the transformed stochastic two-level programming problem can be reduced to a deterministic one. Interactive fuzzy programming to obtain a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relation between decision makers is presented. It is shown that all of the problems to be solved in the proposed interactive fuzzy programming can be easily solved by the simplex method or the combined use of the bisection method and the simplex method.

II. FUZZY RANDOM TWO-LEVEL LINEAR PROGRAMMING PROBLEM

Fuzzy random variables, first introduced by Kwakernaak [7], have been defined in various ways [7], [10], [6], [8]. For example, as a special case of fuzzy random variables given by Kwakernaak, Kruse and Meyer [6] defined a fuzzy random variable as follows.

Definition 1 (Fuzzy random variable): Let (Ω, B, P) be a probability space, $F(\mathcal{R})$ the set of fuzzy numbers with compact supports and X a measurable mapping $\Omega \rightarrow F(\mathcal{R})$. Then X is a fuzzy random variable if and only if given $\omega \in \Omega$, $X_\alpha(\omega)$ is a random interval for any $\alpha \in (0, 1]$, where $X_\alpha(\omega)$ is an α -level set of the fuzzy set $X(\omega)$.

Although there exist some minor differences in several definitions of fuzzy random variables, fuzzy random variables are considered to be random variables whose observed values are fuzzy sets.

In this paper, we deal with two-level linear programming problems involving fuzzy random variable coefficients in objective functions formulated as:

$$\left. \begin{array}{l} \text{minimize}_{\text{for DM1}} \quad z_1(\mathbf{x}_1, \mathbf{x}_2) = \tilde{\tilde{C}}_{11}\mathbf{x}_1 + \tilde{\tilde{C}}_{12}\mathbf{x}_2 \\ \text{minimize}_{\text{for DM2}} \quad z_2(\mathbf{x}_1, \mathbf{x}_2) = \tilde{\tilde{C}}_{21}\mathbf{x}_1 + \tilde{\tilde{C}}_{22}\mathbf{x}_2 \\ \text{subject to} \quad A_1\mathbf{x}_1 + A_2\mathbf{x}_2 \leq \mathbf{b} \\ \quad \quad \quad \mathbf{x}_1 \geq \mathbf{0}, \mathbf{x}_2 \geq \mathbf{0} \end{array} \right\} \quad (1)$$

where $\mathbf{x}_1$ is an n_1 dimensional decision variable column vector for the DM at the upper level (DM1), $\mathbf{x}_2$ is an n_2 dimensional decision variable column vector for the DM at the lower level (DM2), $z_1(\mathbf{x}_1, \mathbf{x}_2)$ is the objective function for DM1 and $z_2(\mathbf{x}_1, \mathbf{x}_2)$ is the objective function for DM2.

It should be emphasized here that randomness and fuzziness of the coefficients are denoted by the “dash above” and “wave above” i.e., “-” and “~”, respectively. In this formulation, $\mathbf{x}_1$ is an n_1 dimensional decision variable column vector for

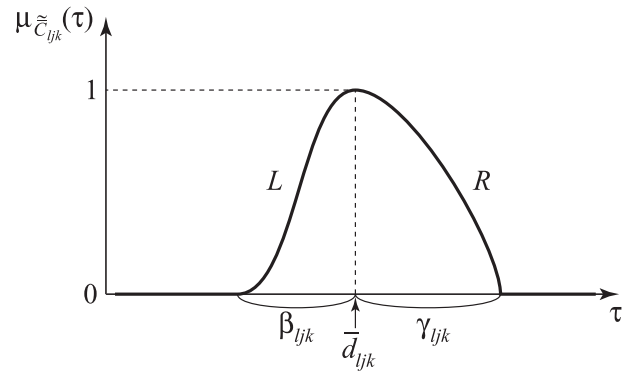


Fig. 1. An example of the membership function $\mu_{\tilde{\tilde{C}}_{ljk}}(\cdot)$ of a fuzzy random variable $\tilde{\tilde{C}}_{ljk}$.

the DM at the upper level (DM1), $\mathbf{x}_2$ is an n_2 dimensional decision variable column vector for the DM at the lower level (DM2), $z_1(\mathbf{x}_1, \mathbf{x}_2)$ is the objective function for DM1 and $z_2(\mathbf{x}_1, \mathbf{x}_2)$ is the objective function for DM2. In (1), $\tilde{\tilde{C}}_{lj}$, $l = 1, 2$, $j = 1, 2$ are vectors whose elements $\tilde{\tilde{C}}_{ljk}$, $k = 1, 2, \dots, n_j$ are fuzzy random variables characterized by the following membership function:

$$\mu_{\tilde{\tilde{C}}_{ljk}}(\tau) = \begin{cases} L\left(\frac{\bar{d}_{ljk} - \tau}{\beta_{ljk}}\right) & , \quad \text{if } \tau \leq \bar{d}_{ljk} \\ R\left(\frac{\tau - \bar{d}_{ljk}}{\gamma_{ljk}}\right) & , \quad \text{otherwise} \end{cases}$$

where $L : [0, \infty) \rightarrow [0, 1]$ is a monotone decreasing function defined as $L(t) = \max\{0, \lambda(t)\}$ and $R : [0, \infty) \rightarrow [0, 1]$ is also a monotone decreasing function defined as $R(t) = \max\{0, \rho(t)\}$. Here, $\lambda(t)$ and $\rho(t)$ are monotone decreasing functions which satisfy $\lambda(0) = 1$ and $\rho(0) = 1$, respectively. Furthermore, parameters $\bar{d}_{ljk}$, β_{ljk} and γ_{ljk} represent a mean value, the left spread and the right spread of $\tilde{\tilde{C}}_{ljk}$, respectively. In this paper, parameters $\bar{d}_{ljk}$ are random variables defined as $\bar{d}_{ljk} = d_{ljk}^1 + \bar{t}_l d_{ljk}^2$, using random variables $\bar{t}_l$ with mean M_l , $l = 1, 2$. This definition of random variables is one of the simplest randomization modeling of coefficients using dilation and translation of random variables, as discussed by Stancu-Minasian [18] and Katagiri et al. [5].

Fig. 1 illustrates an example of the membership function of a fuzzy random variable $\tilde{\tilde{C}}_{ljk}$.

Fuzzy random two-level linear programming problems formulated as (1) are often seen in actual decision making situations. For example, consider a supply chain planning [12] where the distribution center (DM1) and the production part (DM2) hope to minimize the distribution cost and the production cost respectively. Since coefficients of these objective functions are often affected by the economic conditions varying at random, they can be regarded as random variables. In addition, since observed values of them are often ambiguous and estimated by fuzzy numbers, they are expressed by fuzzy random variables. Then, the supply chain planning problem

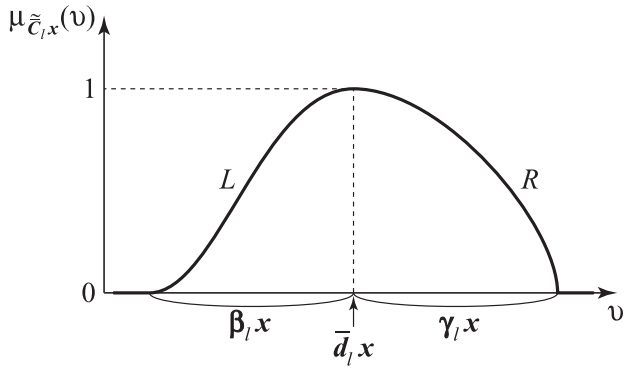


Fig. 2. An example of the membership function $\mu_{\tilde{C}_l x}(\cdot)$ of the objective function of DML.

can be formulated as a two-level linear programming problem involving fuzzy random variable coefficients like (1).

Since each coefficient $\tilde{C}_{ljk}$ is a fuzzy random variable defined as a random variable whose observed values are L - R fuzzy numbers, each objective function $\tilde{C}_l x = \tilde{C}_{l1}x_1 + \tilde{C}_{l2}x_2$ is also a fuzzy random variable whose observed values are fuzzy numbers characterized by the following membership function.

$$\mu_{\tilde{C}_l x}(v) = \begin{cases} L\left(\frac{\bar{d}_l x - v}{\beta_l x}\right) & , \text{ if } v \leq \bar{d}_l x \\ R\left(\frac{v - \bar{d}_l x}{\gamma_l x}\right) & , \text{ otherwise} \end{cases}$$

An example of the membership function of the objective function of DML is shown in Fig. 2.

III. PROBABILITY MAXIMIZATION MODEL THROUGH POSSIBILITY MEASURES

Problem (1) is not a well-defined problem due to both fuzziness and randomness of the coefficients, and it cannot be minimized in the sense of deterministic mathematical programming. Therefore, it is necessary to interpret the problem from some point of view and to transform the problem into the deterministic equivalent one. Katagiri and Ishii [4] proposed a fuzzy random programming model using a possibility measure [20] for a linear programming problem where the right-hand side of a constraint is a fuzzy random variable. In this research, assuming that the decision makers are interested in the probability that each objective function attains a goal value rather than the expectation or variance of each membership function, we consider the case where the decision maker prefers to maximize the degree of possibility that the objective function value satisfies the fuzzy goal.

A. Fuzzy goal

A fuzzy goal for the objective is characterized by the following linear membership function:

$$\mu_{\tilde{G}_l}(y) = \begin{cases} 1 & (y < g_l^1) \\ g_l(y) & (g_l^1 \leq y \leq g_l^0) \\ 0 & (y > g_l^0) \end{cases}$$

where $g_l(y)$ is a monotone decreasing function.

B. Possibility and stochastic programming approach

Using the concept of a possibility measure, we define a degree of possibility that the objective function value $\tilde{Y}$ satisfies a fuzzy goal $\tilde{G}$ by

$$\Pi_{\tilde{Y}_l}(\tilde{G}_l) = \sup_y \min\{\mu_{\tilde{Y}_l}(y), \mu_{\tilde{G}_l}(y)\}$$

where Π denotes a possibility measure. Then, if the decision maker prefers to maximize the degree of possibility, problem (1) is reformulated as

$$\left. \begin{array}{ll} \text{maximize} & \Pi_{\tilde{C}_1 x}(\tilde{G}_1) \\ \text{for DM1} & \\ \text{maximize} & \Pi_{\tilde{C}_2 x}(\tilde{G}_2) \\ \text{for DM2} & \\ \text{subject to} & A_1 x_1 + A_2 x_2 \leq b \\ & x_1 \geq 0, x_2 \geq 0 \end{array} \right\}. \quad (2)$$

The degree of possibility is expressed as the crossing point of two membership functions.

Since $\Pi_{\tilde{C}_l x}(\tilde{G}_l), l = 1, 2$ varies randomly due to the randomness of $\tilde{C}_j$, problem (2) can be regarded as a stochastic linear programming problem. In stochastic programming, there are some major models [3]: an expectation optimization model, a variance minimization model and a probability maximization model.

In this research, we focus on the probability maximization model, which is to maximize the probability that the profit is greater than or equal to a certain permissible level and formulate the following problem:

$$\left. \begin{array}{ll} \text{maximize} & \Pr[\Pi_{\tilde{C}_1 x}(\tilde{G}_1) \geq h_1] \\ \text{for DM1} & \\ \text{maximize} & \Pr[\Pi_{\tilde{C}_2 x}(\tilde{G}_2) \geq h_2] \\ \text{for DM2} & \\ \text{subject to} & A_1 x_1 + A_2 x_2 \leq b \\ & x_1 \geq 0, x_2 \geq 0 \end{array} \right\} \quad (3)$$

where $\Pr[\cdot]$ denotes probability and h_l is a goal value, called a permissible level, for the membership function $\mu_l(\cdot)$.

Since $\mu_l(\cdot), l = 1, 2$ are monotone decreasing, (3) can be rewritten as:

$$\left. \begin{array}{ll} \text{maximize} & \Pr[\Pi_{\tilde{C}_1 x}(\tilde{G}_1) \leq \mu_{\tilde{G}_1}^*(h_1)] \\ \text{for DM1} & \\ \text{maximize} & \Pr[\Pi_{\tilde{C}_2 x}(\tilde{G}_2) \leq \mu_{\tilde{G}_2}^*(h_2)] \\ \text{for DM2} & \\ \text{subject to} & A_1 x_1 + A_2 x_2 \leq b \\ & x_1 \geq 0, x_2 \geq 0 \end{array} \right\} \quad (4)$$

where $\mu_{\tilde{G}_l}^*(\cdot)$ is a pseudo-inverse function of $\mu_{\tilde{G}_l}(\cdot)$ defined by $\mu_{\tilde{G}_l}^*(h_l) = \sup\{r | \mu_{\tilde{G}_l}(r) \geq h_l\}$ for $h_l \in (0, 1]$.

Let $L^*(\cdot)$ be a pseudo-inverse function of $L(\cdot)$ defined by $L^*(h_l) = \sup\{r | L(r) \geq h_l, r \geq 0\}$. Then the left side of the first and second constraint in (4) can be expressed as:

$$\Pr[\Pi_{\tilde{C}_l x}(\tilde{G}_l) \leq \mu_{\tilde{G}_l}^*(h_l)] = \Pr[\{\bar{d}_l - L^*(h_l)\beta_l\}x \leq \mu_{\tilde{G}_l}^*(h_l)].$$

From the assumption, $\bar{d}_l$ is a random variable vector defined as $\bar{d}_l = d_l^1 + \bar{t}_l \cdot d_l^2$ using a random variable $\bar{t}_l$ with mean M_l .

Provided that $\bar{d}_l^2 x > 0$, $l = 1, 2$,

$$\begin{aligned}
& \Pr[\Pi_{\tilde{C}_l}(\tilde{G}_l) \geq h_l] \\
&= \Pr[\{\bar{d}_l - L^*(h_l)\beta_l\}x \leq \mu_{\tilde{G}_l}^*(h_l)] \\
&= \Pr[\{(d_l^1 + \bar{t}_l d_l^2) - L^*(h_l)(\beta_l^1 + \bar{t}_l \beta_l^2)\}x \leq \mu_{\tilde{G}_l}^*(h_l)] \\
&= \Pr\left[\bar{t}_l \leq \frac{\{L^*(h_l)\beta_l^1 - d_l^1\}x + \mu_{\tilde{G}_l}^*(h_l)}{\{d_l^2 - L^*(h_l)\beta_l^2\}x}\right] \\
&= T_l\left(\frac{\{L^*(h_l)\beta_l^1 - d_l^1\}x + \mu_{\tilde{G}_l}^*(h_l)}{\{d_l^2 - L^*(h_l)\beta_l^2\}x}\right) \\
&= T_l\left(\frac{\sum_{j=1}^2 \sum_{k=1}^{n_j} L^*(h_l)\{\beta_{lj}^1 - d_{lj}^1\}x_{jk} + \mu_{\tilde{G}_l}^*(h_l)}{\sum_{j=1}^2 \sum_{k=1}^{n_j} \{d_{lj}^2 - L^*(h_l)\beta_{lj}^2\}x_{jk}}\right)
\end{aligned}$$

where $T_l(\cdot)$ is a probability distribution of $\bar{t}_l$ for DM1.

In this way, (4) can be transformed into the following problem:

$$\begin{aligned}
& \text{maximize}_{\text{for DM1}} p_1(x_1, x_2) = \\
& T_1\left(\frac{\sum_{j=1}^2 \sum_{k=1}^{n_j} L^*(h_1)\{\beta_{1j}^1 - d_{1j}^1\}x_{jk} + \mu_{\tilde{G}_1}^*(h_1)}{\sum_{j=1}^2 \sum_{k=1}^{n_j} \{d_{1j}^2 - L^*(h_1)\beta_{1j}^2\}x_{jk}}\right) \\
& \text{maximize}_{\text{for DM2}} p_2(x_1, x_2) = \\
& T_2\left(\frac{\sum_{j=1}^2 \sum_{k=1}^{n_j} L^*(h_2)\{\beta_{2j}^1 - d_{2j}^1\}x_{jk} + \mu_{\tilde{G}_2}^*(h_2)}{\sum_{j=1}^2 \sum_{k=1}^{n_j} \{d_{2j}^2 - L^*(h_2)\beta_{2j}^2\}x_{jk}}\right) \\
& \text{subject to } A_1 x_1 + A_2 x_2 \leq b \\
& \quad x_1 \geq 0, x_2 \geq 0
\end{aligned} \quad (5)$$

IV. INTERACTIVE FUZZY PROGRAMMING

Observing the transformed problem (5) is a deterministic two-level programming problem, we can now construct the interactive algorithm to derive a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relationships between DM1 and DM2.

Interactive fuzzy programming

Step 1: In order to calculate the individual minimum of $E\{z_l(x_1, x_2)\}$, solve the following problems:

$$\begin{aligned}
& \text{minimize } E\{z_l(x_1, x_2)\} = (d_l^1 + m_l \cdot d_l^2) x \\
& \text{subject to } A_1 x_1 + A_2 x_2 \leq b \\
& \quad x_1 \geq 0, x_2 \geq 0
\end{aligned} \quad \left. \vphantom{\begin{aligned} \text{minimize} \\ \text{subject to} \end{aligned}} \right\} \quad l = 1, 2. \quad (6)$$

Let $x_{\min}^l$ and $z_{l,\min}^E$ be optimal solutions to (6) and minimal objective function values to (6), respectively. Observing that (6) are linear programming problems, they can be easily solved by some linear programming technique like the simplex method.

Step 2: Ask DMs to specify the membership functions $\mu_l(\cdot)$, $l = 1, 2$ by considering the obtained values of $z_{l,\min}^E$, $l = 1, 2$.

Step 3: Ask DM1 to determine the permissible range $[\Delta_{\min}, \Delta_{\max}]$ and the initial value of permissible levels h_l , $l = 1, 2$.

Step 4: For the specified values of h_l , $l = 1, 2$, the following maximin problem is solved:

$$\begin{aligned}
& \text{maximize}_{l=1,2} \min \left\{ T_l\left(\frac{L^*(h_l)\{\beta_l^1 - d_l^1\}x + \mu_{\tilde{G}_l}^*(h_l)}{\{d_l^2 - L^*(h_l)\beta_l^2\}x}\right) \right\} \\
& \text{subject to } A_1 x_1 + A_2 x_2 \leq b \\
& \quad x_1 \geq 0, x_2 \geq 0
\end{aligned} \quad (7)$$

In view of (5), this problem is rewritten as:

$$\begin{aligned}
& \text{maximize } v \\
& \text{subject to} \\
& T_1\left(\frac{(-d_1^1 + L^*(h_1)\beta_1)x + \mu_{\tilde{G}_1}^*(h_1)}{(d_1^2 - L^*(h_1)\beta_1^2)x}\right) \geq \mu_1^*(v) \\
& T_2\left(\frac{(-d_2^1 + L^*(h_2)\beta_2)x + \mu_{\tilde{G}_2}^*(h_2)}{(d_2^2 - L^*(h_2)\beta_2^2)x}\right) \geq \mu_2^*(v) \\
& A_1 x_1 + A_2 x_2 \leq b \\
& x_1 \geq 0, x_2 \geq 0
\end{aligned} \quad (8)$$

equivalently,

$$\begin{aligned}
& \text{maximize } v \\
& \text{subject to} \\
& \frac{(-d_1^1 + L^*(h_1)\beta_1)x + \mu_1^*(h_1)}{(d_1^2 - L^*(h_1)\beta_1^2)x} \geq T_1^*(\mu_1^*(v)) \\
& \frac{(-d_2^1 + L^*(h_2)\beta_2)x + \mu_2^*(h_2)}{(d_2^2 - L^*(h_2)\beta_2^2)x} \geq T_2^*(\mu_2^*(v)) \\
& A_1 x_1 + A_2 x_2 \leq b \\
& x_1 \geq 0, x_2 \geq 0
\end{aligned} \quad (9)$$

where $T_l^*(\cdot)$ is a pseudo-inverse function of $T_l(\cdot)$. Obtaining the optimal value of v to this problem is equivalent to finding the maximum of v so that the set of feasible solutions to (9) is not empty. Although this problem is a nonlinear programming problem, we can easily find the maximum of v by the following algorithm based on the bisection method and the simplex method since the constraints of (9) are linear if v is fixed.

The combined use of the bisection method and the simplex method

4-1 Set $l := 0$ and $v := 0$. Test whether the set of feasible solutions to (9) for $v = 0$ is empty or not using the phase one of the simplex method. If it is empty, DMs must reassess

membership functions, α or h_l . Otherwise, let $v_{\text{feasible}} := v$ and go to 4-2

- 4-2 Set $v := 1$. Test whether the set of feasible solutions to (9) for $v = 1$ is empty or not using the phase one of the simplex method. If it is not empty, $v = 1$ is the optimal value v^* to (9) and the algorithm is terminated. Otherwise, the maximum of v so that the set of feasible solutions to (9) is not empty exists between 0 and 1. Let $v_{\text{infeasible}} := v$ and go to 4-3.
- 4-3 Set $v := (v_{\text{feasible}} + v_{\text{infeasible}})/2$, $l := l + 1$ and go to 4-4.
- 4-4 Test whether the set of feasible solutions to (9) for v determined in 4-3 is empty or not using the phase one of the simplex method. Observe that the sensitivity analysis technique can be applied for this test. If it is not empty and $(1/2)^l \leq \varepsilon$, the current value of v is regarded as the optimal value v^* to (9) and the algorithm is terminated. If it is not empty and $(1/2)^l > \varepsilon$, let $v_{\text{feasible}} := v$ and return to 4-3. On the other hand, if it is empty, let $v_{\text{infeasible}} := v$ and return to 4-3.

Then, for the obtained optimal value v^* , we can determine the corresponding optimal value $\mathbf{x}^*$ by solving the following linear fractional programming problem:

$$\left. \begin{array}{ll} \text{maximize} & p_1(\mathbf{x}_1, \mathbf{x}_2) \\ \text{subject to} & p_2(\mathbf{x}_1, \mathbf{x}_2) \geq \mu_2^*(v^*) \\ & A_1 \mathbf{x}_1 + A_2 \mathbf{x}_2 \leq \mathbf{b} \\ & \mathbf{x}_1 \geq \mathbf{0}, \mathbf{x}_2 \geq \mathbf{0} \end{array} \right\} \quad (10)$$

It should be noted here that linear fractional programming problems can be reduced to equivalent linear programming problems through the variable transformation method by Charnes and Cooper [2]. Therefore, we can obtain an optimal solution to (10) by some linear programming technique.

- Step 5: The DM1 is supplied with the current values of $\mu_1(p_1(\mathbf{x}_1^*, \mathbf{x}_2^*))$ and $\mu_2(p_2(\mathbf{x}_1^*, \mathbf{x}_2^*))$ for the optimal solution $\mathbf{x}^*$ calculated in step 4. If DM1 is satisfied with the current membership function values, the interaction process is terminated. If DM1 is not satisfied and desires to update h_l , $l = 1, 2$, ask DM1 to update h_l and return to step 4. Otherwise, ask DM1 to specify the minimal satisfactory level $\hat{\delta}$ for $\mu_1(p_1(\mathbf{x}_1, \mathbf{x}_2))$ and the permissible range $[\Delta_{\min}, \Delta_{\max}]$ of the ratio of membership functions $\Delta = \mu_2(p_2(\mathbf{x}_1, \mathbf{x}_2))/\mu_1(p_1(\mathbf{x}_1, \mathbf{x}_2))$.

Observe that the larger the minimal satisfactory level is assessed, the smaller the DM2's satisfactory degree becomes. Consequently, in order to take account of the overall satisfactory balance between both decision makers, DM1 needs to compromise with DM2 on DM1's own minimal satisfactory level. To do so,

the permissible range of the ratio of the satisfactory degree of DM2 to that of DM1 is helpful.

- Step 6: For the specified value of $\hat{\delta}$, solve the following problem to maximize the membership function $\mu_2(p_2(\mathbf{x}_1, \mathbf{x}_2))$ of DM2 considering the constraint that the membership function $\mu_1(p_1(\mathbf{x}_1, \mathbf{x}_2))$ of DM1 must be greater than or equal to $\hat{\delta}$.

$$\left. \begin{array}{ll} \text{maximize} & \mu_2(p_2(\mathbf{x}_1, \mathbf{x}_2)) \\ \text{subject to} & \mu_1(p_1(\mathbf{x}_1, \mathbf{x}_2)) \geq \hat{\delta} \\ & A_1 \mathbf{x}_1 + A_2 \mathbf{x}_2 \leq \mathbf{b} \\ & \mathbf{x}_1 \geq \mathbf{0}, \mathbf{x}_2 \geq \mathbf{0} \end{array} \right\} \quad (11)$$

This problem can be rewritten as:

$$\left. \begin{array}{ll} \text{maximize} & p_2(\mathbf{x}_1, \mathbf{x}_2) \\ \text{subject to} & p_1(\mathbf{x}_1, \mathbf{x}_2) \geq \mu_1^*(\hat{\delta}) \\ & A_1 \mathbf{x}_1 + A_2 \mathbf{x}_2 \leq \mathbf{b} \\ & \mathbf{x}_1 \geq \mathbf{0}, \mathbf{x}_2 \geq \mathbf{0} \end{array} \right\} \quad (12)$$

Observing that problem (12) is a linear fractional programming problem, it can be easily solved by some linear programming technique through the variable transformation method by Charnes and Cooper [2] like (10). For the optimal solution $\mathbf{x}^*$ to (11), calculate $p_1(\mathbf{x}_1^*, \mathbf{x}_2^*)$, $p_2(\mathbf{x}_1^*, \mathbf{x}_2^*)$ and Δ .

- Step 7: DM1 is supplied with the current values of $p_1(\mathbf{x}_1^*, \mathbf{x}_2^*)$, $p_2(\mathbf{x}_1^*, \mathbf{x}_2^*)$ and Δ calculated in step 6. If $\Delta \in [\Delta_{\min}, \Delta_{\max}]$ and DM1 is satisfied with the current membership function values for the optimal solution $\mathbf{x}^*$, the interaction process is terminated. Otherwise, ask DM1 to update the permissible level h_l , $l = 1, 2$ or the minimal satisfactory level $\hat{\delta}$, and return to step 6.

In the proposed algorithm, $\Delta_{\min}$ and $\Delta_{\max}$ are usually set to be less than 1 since $p_1(\mathbf{x}_1^*, \mathbf{x}_2^*)$ should be greater than $p_2(\mathbf{x}_1^*, \mathbf{x}_2^*)$ because of the priority of DM1. In Step 6, if $\Delta < \Delta_{\min}$, i.e., $p_1(\mathbf{x}_1^*, \mathbf{x}_2^*)$ is much greater than $p_2(\mathbf{x}_1^*, \mathbf{x}_2^*)$, DM1 will decrease $\hat{\delta}$ to improve $p_2(\mathbf{x}_1^*, \mathbf{x}_2^*)$ and increase Δ . Conversely, if $\Delta_{\max} < \Delta$, i.e., $p_1(\mathbf{x}_1^*, \mathbf{x}_2^*)$ is slightly greater or less than $p_2(\mathbf{x}_1^*, \mathbf{x}_2^*)$, DM1 will increase $\hat{\delta}$ to improve $p_1(\mathbf{x}_1^*, \mathbf{x}_2^*)$ and decrease Δ . On the other hand, if DM1 decreases (increases) h_l , $l = 1, 2$, both $p_1(\mathbf{x}_1^*, \mathbf{x}_2^*)$ and $p_2(\mathbf{x}_1^*, \mathbf{x}_2^*)$ would increase (decrease). With this observation, it can be expected that desirable values of $p_1(\mathbf{x}_1^*, \mathbf{x}_2^*)$, $p_2(\mathbf{x}_1^*, \mathbf{x}_2^*)$ and Δ will be obtained through a series of update procedures of $\hat{\delta}$, h_l , $l = 1, 2$ with DM1.

V. CONCLUSION

In this research, assuming cooperative behavior of the decision makers, interactive decision making methods in hierarchical organizations under fuzzy random environments were considered. For the formulated fuzzy random two-level linear programming problems, possibility measures were introduced and a stochastic two-level linear programming problem was defined for guaranteeing the degree of realization of the problem. Considering the vague natures of decision makers' judgments, fuzzy goals were introduced and the stochastic

two-level linear programming problem was transformed into the problem to maximize the satisfaction degree for each fuzzy goal. Through the possibility maximization model, the transformed stochastic two-level programming problem was reduced to a deterministic one. Interactive fuzzy programming to obtain a satisfactory solution for the decision maker at the upper level in consideration of the cooperative relation between decision makers was presented. In near future, we will report the result of the application of the proposed method to a numerical example to derive a satisfactory solution.

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