

Linguistic Description of Adult Skeletal Age-at-Death Estimations from Fuzzy Integral Acquired Fuzzy Sets

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Abstract—Previously, we introduced a novel method to estimate adult skeletal age-at-death using the Sugeno fuzzy integral (FI). Specifically, we took a multi-hypothesis testing approach to make the classical FI yield a fuzzy set (FS)-valued result, which is not guaranteed to be normal or convex, based on interval-valued sources of information (aging methods). We showed quantitative results for summarizing the FS and comparing the single decision to a known age-at-death. In this article, we extend our prior work and present formulas to measure the uncertainty in the resultant FSs. We generate linguistic descriptions to establish domain standardization for the goal of assisting forensic and biological anthropologists. Specifically, we introduce fuzzy class definitions for age-at-death FSs and we present an OWA contrast approach to measure the degree of specificity in age-at-death FSs.

Keywords; *Sugeno fuzzy integral; linguistic description; skeletal age-at-death; anthropology*

I. INTRODUCTION

Age-at-death estimation of an individual skeleton is important to forensic and biological anthropologists for identification and demographic analysis. It has been shown that current individual aging methods are often unreliable because of skeletal variation and taphonomic factors. In [1], we demonstrated that the fuzzy integral (FI) can be used for age-at-death estimation. Our algorithm produces a decision regarding age-at-death using multiple interval-valued aging methods while taking into account the numeric accuracies of the methods and the condition of the bone being examined. Advantages of this FI approach are that it does not require a population, it can (quantitatively) produce a single age-at-death estimate and it (qualitative) produces a fuzzy set (FS).

Forensic and biological anthropology is rooted heavily in definitions, methods and standardization. It is important that techniques are well established and that their findings be reproducible and understandable by different scientists. Our prior work is based on a set of accepted aging methods, their documented reliabilities and the weathering or quality of the skeletal remains. However, while our quantitative procedure is well established in [1], the qualitative component, i.e. interpretation of the FSs, is currently based on a set of crisp rules. Herein, we assert that the (linguistic) anthropological desired results are not crisp but instead fuzzy. Therefore, we require a mathematical (fuzzy) way to model these concepts

and their associated uncertainties. We focus on the FSs and we introduce a systematic way to interpret the discoveries.

This article is important to anthropology because it makes the qualitative FS analysis procedure explicit and researchers can therefore better understand how to interpret results acquired from different investigators on different types of skeletal remains. This article is important to the FS community because it demonstrates a real-world application of the classical number-valued FI to construct (potentially non-normal and non-convex) FSs. In addition, we present a feature-based method to determine different age-at-death FS anthropological classes and the degree of FS specificity. This work is a case in which it is useful to maintain as much uncertainty as possible throughout the entire analysis procedure, exemplifying Marr's Principle of Least Commitment [2]. That is, one should not stop with the production of a single quantitative result. Instead, we show that it is possible to generate linguistic descriptions of FSs, and that analysis of this information results in different knowledge than what the quantitative approach yields.

II. BACKGROUND

A. Related Work in the FS Community

In [1], we show how the classical FI with a number-valued integrand and number-valued fuzzy measure (FM) can be used to produce FS-valued results based on interval-valued sources of information. Our approach, which is summarized later in this article, is one of multiple hypotheses testing. Grabisch et al. [3] previously demonstrated a *literal* extension of the number-valued FI to FS-valued integrands and number-valued FMs. Grabisch's FI, for a FS-valued integrand and number-valued FM, is comprised of a series of interval-valued integrand and number-valued FM calculations (based on the work of Dubois [4]), which in return is decomposed into the classical FI for a number-valued integrand and FM. While it might initially appear to be possible to cast our anthropological problem into one of a FI with a FS-valued integrand and number-valued measure, the problem is that Grabisch's approach requires that the FSs be normal and convex. As we discuss later, while our FSs are usually convex they are generally not normal.

Table 1. Morphological Stages and Age Ranges for Pubic Symphysis, Auricular Surface, and Cranial Suture Closure

Pubic Symphysis ¹		Auricular Surface ²		Ectocranial Lateral-Anterior Suture Closure ³		Ectocranial Vault Suture Closure ³	
Stage	Age Range (yrs)	Stage	Age Range (yrs)	Composite Score	Age Range (yrs)	Composite Score	Age Range (yrs)
1	18-19	1	20-24	0	<50	0	<49
2	20-21	2	25-29	1	19-48	1,2	18-45
3	22-24	3	30-34	2	25-49	3,4,5,6	22-48
4	25-26	4	35-39	3,4,5	23-68	7,8,9,10,11	24-60
5	27-30	5	40-44	6	23-63	12,13,14,15	24-75
6	30-35	6	45-49	7,8	32-65	16,17,18	30-71
7	35-39	7	50-59	9,10	33-76	19,20	23-76
8	39-44	8	60+	11,12,13,14	34-68	21	Closed (40-)
9	45-50			15	Closed		
10	50+						

¹Todd 1920²Lovejoy et al. 1985b³Meindl and Lovejoy 1985

Yager and Klir [5, 6], to name two, have previously presented various specificity measures for FSs. To the best of our knowledge, our approach to linguistically measuring the specificity of an age-at-death FS, based on the OWA [7] and contrast in different regions of a FS is new.

B. Aging Methods

Forensic investigators rely on the accuracy of age indicators for personal identification purposes, whereas biological anthropologists generally have a broader area of applicability for accurate age estimation, such as paleodemographic studies [8]. In general, all skeletal remains are not guaranteed to be present in a forensic case. Different aging methods have been established in the field in order to approximate age-at-death based on the analysis of different skeletal remains. The aging methods used in our prior work and herein include the pubic symphysis morphology [9], auricular surface morphology [10], and ectocranial suture closure [11] used for adult age estimation. The intervals that these techniques produce are shown in table 1. For the methods used here, researchers compare the skeletal remains to a discrete set of stages, or use a composite score and lookup the age-range (interval) in a table.

C. Uncertainty in Age-at-Death Estimation and Prior Work

It is well known that there is uncertainty in age-at-death estimation [12, 13, 14, 15]. Causes of uncertainty come from actual death age, the amount of exposure to various unknown elements, and cause and timing of death. Even more specifically, we know: (1) skeletally immature and young adult individuals are likely to be aged more precisely than older adult individuals, (2) bones are in a continual process of change, usually degenerative and influenced by the habitual activities and health of the individual as well as the age of the individual, (3) skeletal age assessment produces (crisp) age ranges instead of a specific age or fuzzy intervals, and (4) aging methods have varying accuracies.

Due to the inherent uncertainty in this domain, some have developed procedures to aggregate information acquired from multiple aging methods in order to improve age-at-death estimation. Prior approaches in the field are all statistical and are generally based on populations. Existing multifactorial aging methods include Acsádi and Nemeskéri, [16]; Lovejoy et al. [17]; Boldsen et al. [18]. Our method, [1], however can incorporate more information. In addition to the (objective)

strength in a hypothesis, we also take into account the variable (subjective) accuracies associated with each method and the weathering or quality of remains.

D. Accuracy and Quality Indices

Because of the various stated uncertainties, it is important to include as much known or observable information as possible about the skeleton so that all factors available can be taken into consideration when determining skeletal age-at-death. The accuracy of the methods and condition or quality of the skeleton becomes important because they are related to the overall assessment of the skeleton.

Accuracy index. Accuracy in age estimation is defined as how well the estimated or biological age conforms to the real chronological age, and relies on how strongly correlated the biological and chronological ages are to each other. We use the correlation coefficient of the chronological and biological age indicator that is associated with each adult aging method (Katz and Suchey [18]; Meindl and Lovejoy [11]; Meindl et al. [15]).

Quality. Bone quality is an additional piece of information that can be taken into consideration when analyzing a skeleton. Because the condition of the skeleton is dependent on environmental factors, the quality of a skeleton affects determining age-at-death. Previously, we evaluated and scored the bone quality using a modified version of Behrensmeier [19] weathering stages provided in Standards [20], but with a stage 6 added to take into account the absence of an age indicator. Quality values are assigned, based on the weathering stages, and are associated specifically with the part of the skeleton for which the aging methods are being applied. If the bone is deteriorated but can still be used, the aging method will still be applied but the decision reached will respectively be associated with a lower support value. The quality indices are ordinal and arbitrarily assigned values. They must be decreasing values and no greater than one and no less than 0. They were assigned in this manner to indicate the least weathering as producing the best results (1) and the most weathering as producing poor or no results (0). A score of 0 is assigned if the bone is not present. Previously, we assigned Stage 0 a quality value of 1.0, Stage 1 a value of 0.8, Stage 2 is 0.6, Stage 3 is 0.4, Stage 4 is 0.2, Stage 5 is 0.1, and Stage 6 is 0. However, the data used in this study were collected on museum specimens and therefore the quality index does not vary considerably.

E. Sugeno Fuzzy Integral

The fusion of information using the fuzzy integral (Sugeno or Choquet) has a rich history. Much of the theory and several applications can be found in [21, 3, 22]. With respect to this problem, we consider a finite set of sources of information, $X = \{x_1, x_2, \dots, x_E\}$ and a function that maps X into some domain (initially $[0,1]$) that represents the partial support of a hypothesis from the standpoint of each source of information. Depending on the problem domain, X can be a set of experts, sensors, features, pattern recognition algorithms, etc. The hypothesis is usually thought of as an alternative in a decision process or a class label in pattern recognition. Both Sugeno and Choquet integrals take partial support for the hypothesis from the standpoint of each source of information and fuse it with the (perhaps subjective) worth of each subset of X in a non-linear fashion. This worth is encoded into a fuzzy measure [23]. Initially, the function $h: X \rightarrow [0,1]$, and the measure $g: 2^X \rightarrow [0,1]$ take real number values in $[0,1]$. Certainly, the output range for both function and measure can be (and have been) defined more generally, but it is convenient to think of them in the unit interval for confidence fusion.

More formally, for a finite set $X = \{x_1, x_2, \dots, x_E\}$, a (numeric) fuzzy measure is a function $g: 2^X \rightarrow [0,1]$, (a real valued set function) such that

1. $g(\emptyset) = 0$ and $g(X) = 1$;
2. If $A, B \subseteq X$ with $A \subseteq B$, then $g(A) \leq g(B)$.

Note that if X is an infinite set, a third condition guaranteeing continuity is required, but it is a moot point for finite X . In this article, we use the Sugeno λ -fuzzy measure [23]. The Sugeno λ -fuzzy measure builds the measure, all $2^N - 1$ values, given a set of densities (the measure for all singletons). Alternatively, for a few number of sources, one can define the fuzzy measure by hand or learn it from training data.

Given a finite set X , a fuzzy measure $g: 2^X \rightarrow [0,1]$ and a function $h: X \rightarrow [0,1]$, the (numeric) Sugeno integral of h with respect to g is given by

$$\int h \circ g = \bigvee_{i=1}^E (h(x_{(i)}) \wedge g(\{x_{(1)}, \dots, x_{(i)}\})),$$

where $X = \{x_{(1)}, \dots, x_{(E)}\}$ has been sorted so that $h(x_{(1)}) \geq h(x_{(2)}) \geq \dots \geq h(x_{(E)})$. This finite realization of the actual definition highlights the fact that the Sugeno integral represents the best pessimistic agreement between the objective evidence in support of a hypothesis (h function) and the (possibly subjective) worth of the supporting evidence as provided by the fuzzy measure g .

F. Sugeno FI for Adult Skeletal Age-at-Death Estimation

After the age, accuracy, and quality indices are collected, a hypothesis is tested. The hypothesis is “the skeleton is age $_$ (a specific age, not range)”. The Sugeno integral is repeatedly applied, once for each possible age using the respective accuracy, range and quality information. Every age, in discrete

one year increments from 1 to 110 is tested. The age indicators provide input based on if the age tested is in their respective interval. The $h(x_i)$ values are a function (t-norm) of the quality, a $[0,1]$ value, and if the aging method indicates that the age tested falls in the age interval, either a 0 for false or 1 for true. The result of this procedure is a collection of (age tested, FI result) pairs, which is a FS defined over the age domain. In this respect, we are able to address the case of sub-normal FSs.

G. Prior Method to Classify Age-at-Death FSs

As shown in figures 1 and 4, at least four different potential FS classes are possible; specific, figure 1(a), interval, figure 1(b), reconsideration, figure 1(c), and inconclusive, figure 1(d).

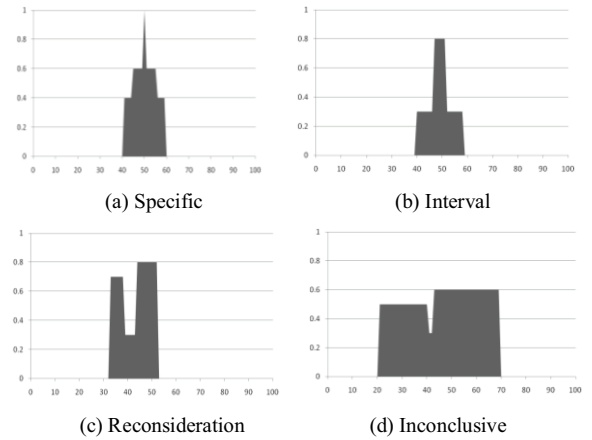


Fig. 1. Illustration of age-at-death FS classes for the FI results.

In [1], ‘Specific’ means that the procedure has identified an exact age-at-death, e.g. 25 years old. Class ‘Interval’ indicates that the procedure has identified a range (interval) of possible ages, e.g. 25-30. Class ‘Reconsideration’ means that there are multiple similar age-at-death ranges (intervals). Therefore, a researcher needs to decide how to resolve the conflict. Class ‘Inconclusive’ means that there is too little solid evidence to indicate any age-at-death.

Initially, an age-at-death FS was classified into one of the four just mentioned classes according to the characteristics of its intervals. A plateau is defined here to be an interval such that the interval directly before it and the interval directly after it (if they exist) have smaller FI values. For example, figure 1(a) has one plateau while figure 1(c) has two plateaus. The following tests, analyzed in the order specified below, were used in [1] to determine the type of FS. Each test is a yes or no question.

- **Test 1:** Is the length of the maximum plateau greater than or equal to 30 years? If the answer is yes, then the FS is placed in the category inconclusive, if not, then the next test is conducted.
- **Test 2:** Is there a single plateau and is its length equal to one? If yes, the result is specific age.

- *Test 3*: Is there a single plateau and is its length 29 or less years? If yes, the result is age interval.

In the case that the FS does not pass one of the above three tests then it is labeled as type reconsideration.

III. FUZZY SET APPROACH TO CLASS DETERMINATION

A shortcoming of [1] is its crisp class definitions. Initially, this set of definitions appeared to be suitable. However, after investigating different results, it was discovered that these initial definitions do not capture the inherent fuzziness that exists in age-at-death FS classes. Figure 2 contains example age-at-death FSs illustrating the inherent fuzziness of the classes.

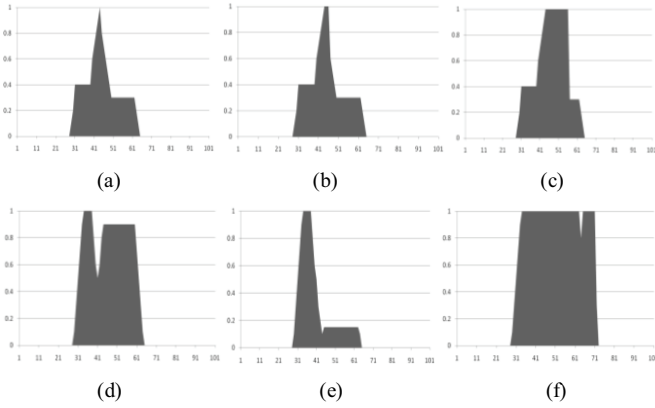


Fig. 2. FSs showing the fuzziness inherent to defining age-at-death FS classes.

We begin this discussion with the comparison of figure 2(a) to figure 2(b). In figure 2(a), the maximum plateau has a width of 1. In figure 2(b), the maximum plateau has a width of 3. The old definitions classify figure 2(b) as type ‘Interval’. As it turns out, figure 2(a) is an interval, just with a small (length 1) width. We assert that there is little difference between figures 2(a) and 2(b). Figure 2(b) should instead have the highest membership in ‘Specific’ instead of ‘Interval’. In comparison, figure 2(c) should have the highest membership in the class ‘Interval’. The question is to what degree do these examples belong to these concepts?

Next, consider figures 2(d) and 2(e). The prior definitions in [1] classify these FSs as type ‘Reconsideration’. However, the second (smaller) plateau in figure 2(e) has a very low (0.15) membership degree. Should these two FSs have the same membership degree in class ‘Reconsideration’? Should figure 2(e) not have a higher degree in ‘Interval’ or ‘Specific’?

On a final note, figure 2(f) is classified as ‘Inconclusive’ by the old set of definitions. The maximum plateau has a width of 30. Should this FS have the same membership degree as a FS with a maximum plateau with width 35, 40, or higher? What about a FS with a maximum plateau with width of 28? The recurring theme between these different examples is that the definitions provided prior do not accurately take into account the fuzziness that exists in determining the class of age-at-death FSs.

What is needed is a set of measurements of age-at-death FSs and FS class definitions. In this respect, a baseline for analysis can be established and a common understanding of age-at-death FS interpretation can take place.

First, we extract all plateau, $I = \{I_1, \dots, I_N\}$, where $I_n = [I_{n,start}, I_{n,end}]$ and $I_{n,start} \leq I_{n,end}$. Herein, we restrict our analysis to a discrete set of ages, $I_{n,start}, I_{n,end} \in \{1, 2, \dots, 110\}$. Each interval is also associated with a membership degree, $\mu_{FI,n}$, which is the output of our FI procedure.

The first feature is the width, $|I_n|$, of the plateau that has the largest $\mu_{FI,n}$ value. The width of the maximum plateau is important for our understanding of how well the FI procedure singled out an age-at-death. The linguistic variable ‘Maximum Plateau Width’ (MPW) is defined and has the terms ‘little’, ‘some’, and ‘big’ (reported in table 2). The FSs used herein are trapezoidal membership functions (MFs), which are defined with respect to four ordered points, $[a \ b \ c \ d]$. These four numbers specify an interval in which the membership is value 1, i.e. between $[b \ c]$, a linearly increasing interval, i.e. $[a \ b]$, a linearly decreasing interval, i.e. $[c \ d]$, and any point that is less than a , or greater than d , has a membership value of zero.

Table 2. Linguistic variable and terms for ‘Maximum Plateau Width’.

	little	moderate	big
MF	[0 1 4 6]	[1 5 29 35]	[25 30 110 110]

The next feature is the distance (in terms of membership value) between the maximum plateau and the second largest plateau. The calculation is

$$\beta = \begin{cases} 1 & \text{if } |I| = 1 \\ (f_{\max 1} - f_{\max 2}) & \text{else} \end{cases},$$

where

$$f_{\max 1} = \max_n \mu_{FI,n},$$

$$f_{\max 2} = \max_{k \neq n} \mu_{FI,k}.$$

Table 3 reports the set of terms for the linguistic variable ‘Plateau Separation’ (PS), defined on the domain β .

Table 3. Linguistic variable and terms for ‘Plateau Separation’.

	little	moderate	big
MF	[0 0 .3 .5]	[.3 .5 .7 .9]	[.5 .7 1 1]

Based on these two features, we are now able to define the four age-at-death FS classes. The definitions are no longer crisp. The confidence (membership degree) that an age-at-death FS belongs to the category (concept) ‘Specific’ is

$$\mu_{\text{Specific}} = \mu_{\text{little}}^{\text{MPW}} * \mu_{\text{big}}^{\text{PS}},$$

‘Interval’ is

$$\mu_{\text{Interval}} = \mu_{\text{moderate}}^{\text{MPW}} * \mu_{\text{big}}^{\text{PS}},$$

‘Inconclusive’ is

$$\mu_{\text{Inconclusive}} = \mu_{\text{big}}^{\text{MPW}},$$

and ‘Reconsideration’ is

$$\mu_{\text{Reconsideration}} = \mu_{\text{small}}^{\text{PS}}.$$

For the example FSs in figure 2, the membership values for the above four terms are provided in table 4.

Table 4. FS class memberships for the FSs in figure 2.

	μ_{Specific}	μ_{Interval}	$\mu_{\text{Inconclusive}}$	$\mu_{\text{Reconsideration}}$
2(a)	$1*1=1$	$0*1=0$	0	0
2(b)	$1*1=1$	$0.5*1=0.5$	0	0
2(c)	$0*1=0$	$1*1=1$	0	0
2(d)	$0.5*0=0.5$	$1*0=0$	0	1
2(e)	$0.5*1=0.5$	$1*1=1$	0	0
2(f)	$0*0=0$	$0.83*0=0$	1	0

Table 4 shows what we expect. Figure 2(a) is ‘Specific’, figure 2(b) is more ‘Specific’ than ‘Interval’, figure 2(c) is ‘Interval’, figure 2(d) is somewhat ‘Specific’ but more of type ‘Reconsideration’, figure 2(e) is somewhat ‘Specific’, but mostly ‘Interval’, and figure 2(f) is ‘Inconclusive’.

IV. MEASURING AGE-AT-DEATH FS SPECIFICITY

While the last section detailed a feature and FS-based approach to determining the membership degree of an age-at-death FS in the four anthropological classes, a question still remains regarding how *specific* the FS is with respect to being able to narrow the age-at-death to a single value? For example, even if the method indicates a single age-at-death, i.e. class ‘Specific’, how specific are the findings in terms of there being evidence that the skeletal remains might be a different age? Figure 3 and figure 2(a) illustrate this concept.

Figure 3 illustrates what we already know. The smaller the area in the age-at-death FS, the greater our confidence is that the aging methods agreed on a specific age-at-death. That is, the less fuzzy the age-at-death FS, the more *specific* the age. In reality, figure 3(b) is unlikely because aging methods produce intervals and not a single age-at-death. Because a specific age is not expected to ever be encountered using current crisp aging methods, it is important that we include a measure of age-at-death FS specificity. The first thought was to use the standard FS scalar-cardinality to measure age-at-death specificity,

$$\text{sc}(A) = \sum_{i=1}^{110} A(x_i).$$

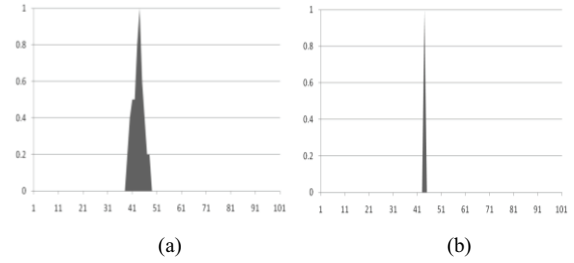


Fig. 3. Differences in age-at-death *specificity* for age-at-death FSs. Compared to figure 2(a), figure 3(a) is more specific, while figure 3(b) is the most specific.

However, it turns out that we need something tailored specifically to age-at-death FSs. If the goal is to have a low $\text{sc}(A)$, namely a FS with value 1 at one domain location and zero elsewhere, i.e. figure 3(b), then in return we open the door for other types of age-at-death FSs that could acquire a low $\text{sc}(A)$ values. Namely, this can occur because our FI procedure can yield FSs that are not normal. For example, if there is significant disagreement between all of the aging methods, then $\text{sc}(A)$ can have a near zero value.

Instead, we address age-at-death FS specificity using an ordered weighted average (OWA) approach [7]. The OWA begins by sorting (in descending order) the membership values. The OWA also takes a set of weights that generally sum to one (or the calculation is weighted by the sum of the weights). The first weight is for the largest sorted value, the second weight is for the second largest value, etc. The OWA is capable of producing many familiar order statistics, i.e. the weighted average, maximum, minimum, etc. For example,

$$\text{OWA}\left(0, \underbrace{\frac{1}{10}, \dots, \frac{1}{10}}_{10}, \underbrace{0, \dots, 0}_{99}\right)$$

is an OWA operator with a weight of 0 for the maximum value, the average of the next 10 largest values and weights of 0 for all the remaining values.

We use different OWAs to calculate the membership of the age-at-death specificity for the terms ‘Exact’, ‘High’, ‘Moderate’ and ‘Low’. The membership of ‘Exact’ is

$$\mu_{\text{Exact}} = \text{OWA}\left(1, \underbrace{0, \dots, 0}_{109}\right) - \text{OWA}\left(0, \underbrace{\frac{1}{10}, \dots, \frac{1}{10}}_{10}, \underbrace{0, \dots, 0}_{99}\right),$$

‘High’ is

$$\mu_{\text{High}} = \text{OWA}\left(1, \underbrace{0, \dots, 0}_{109}\right) *$$

$$\left(\frac{\left(\text{OWA}\left(0, \underbrace{\frac{1}{10}, \dots, \frac{1}{10}}_{10}, \underbrace{0, \dots, 0}_{99}\right) \right) - \left(\text{OWA}\left(0, \dots, 0, \underbrace{\frac{1}{20}, \dots, \frac{1}{20}}_{20}, \underbrace{0, \dots, 0}_{79}\right) \right)}{\left(\text{OWA}\left(0, \underbrace{\frac{1}{10}, \dots, \frac{1}{10}}_{10}, \underbrace{0, \dots, 0}_{99}\right) \right)} \right),$$

‘Moderate’ is

$$\mu_{\text{Moderate}} = \text{OWA}(1, \underbrace{0, \dots, 0}_{109}) *$$

$$\left(\frac{\left(\text{OWA}(\underbrace{0, \dots, 0}_{20}, \underbrace{\frac{1}{20}, \dots, \frac{1}{20}}_{20}, \underbrace{0, \dots, 0}_{70}) \right) - \left(\text{OWA}(\underbrace{0, \dots, 0}_{30}, \underbrace{\frac{1}{20}, \dots, \frac{1}{20}}_{30}, \underbrace{0, \dots, 0}_{50}) \right)}{\left(\text{OWA}(\underbrace{0, \dots, 0}_{20}, \underbrace{\frac{1}{20}, \dots, \frac{1}{20}}_{20}, \underbrace{0, \dots, 0}_{70}) \right)} \right),$$

and ‘Low’ is

$$\mu_{\text{Low}} = \text{OWA}(1, \underbrace{0, \dots, 0}_{109}) * \left(\text{OWA}(\underbrace{0, \dots, 0}_{30}, \underbrace{\frac{1}{10}, \dots, \frac{1}{10}}_{10}, \underbrace{0, \dots, 0}_{70}) \right).$$

These formulas take into the account the maximum membership value and ‘contrast’ between different parts of the FS. For the example FSs in figures 2 and 3, the membership values for the above four terms are provided in table 5.

Table 5. Age-at-death specificity value for the FSs in Figure 2.

	μ_{Exact}	μ_{High}	μ_{Moderate}	μ_{Low}
2(a)	0.36	0.6	0.78	0
2(b)	0.31	0.63	0.78	0
2(c)	0	0.34	0.81	0
2(d)	0.06	0.39	0.85	0
2(e)	0.2	0.84	0.75	0
2(f)	0	0.05	0.63	.89
3(a)	0.62	1	0	0
3(b)	1	0	0	0

Again, table 5 reports what we expect. Figure 2(a) is mostly ‘Moderate’ but also somewhat ‘High’, figure 2(b) is essentially the same as figure 2(a), figure 2(c) is more ‘Moderate’ and less ‘High’ than figure 2(a-b), figure 2(e) has a higher specificity than figure 2(d), figure 2(f) has a very low specificity, while figure 3(b) is the most specific.

V. CONFIDENCE

Formulas have now been presented for age-at-death FS classification and determination of the amount of specificity. The final component of this article is interpretation of the FI output degree. No new procedure is presented here in terms of measurements. The importance is establishing a vocabulary for

the concept ‘Confidence in Age-at-Death’. Table 6 reports the terms and their values.

Table 6. Linguistic variable and terms for ‘Confidence in Age-at-Death’.

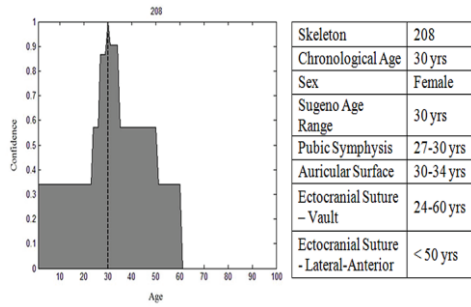
	High	moderate	low	no
MF	[.8 .9 1 1]	[.4 .6 .8 1]	[0 .2 .4 .6]	[0 0 .1 .2]

VI. DATA SET AND EXPERIMENTS

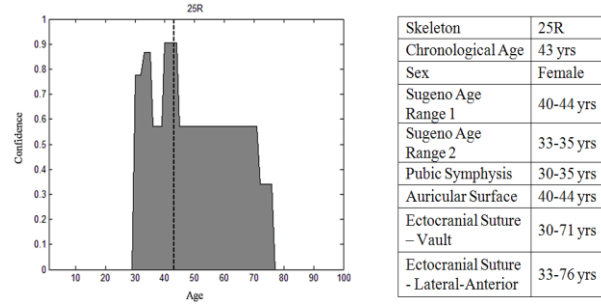
Most of the skeletal aging methods are based on collections with known aged individuals, such as the Terry Anatomical Collection [24, 25]. This collection contains mainly black and white adult skeletons with dates of birth ranging from 1822-1943. It contains 1,728 individuals with the majority between 20 to 80 years of age. The data used here, which is the same data used in our prior FI article, was collected as part of another study by L. Konigsberg, N. Herrmann, and D. Wescott. It is available at <http://konig.la.utk.edu/paleod.htm>. Skeletal remains of known age from the Terry Collection were scored for pubic symphysis morphology [9], auricular surface morphology [10] and ectocranial suture closure [11]. Our data set is comprised of 735 individuals, specifically 323 females and 412 males. There are too many skeletons to report all findings, so a select set are shown to emphasize the proposed method. We use the same examples from our prior article [1]. The first four results in figure 4, (a-d), are the findings of the procedure introduced in this article for four skeletal cases from the Terry Anatomical Collection.

The Terry collection only contains skeletal remains with a quality value of 1. If any wearing is present, it is most likely due to researchers working on the skeletal remains. Because of this, in [1] we introduced 4 hypothetical cases (HCs) based on individuals from the Terry collection in which the quality value was varied and additional methods were simulated to test scenarios outside of the data set analyzed (figure 4 e-h). These experiments are summarized here, while the full information is contained in [1].

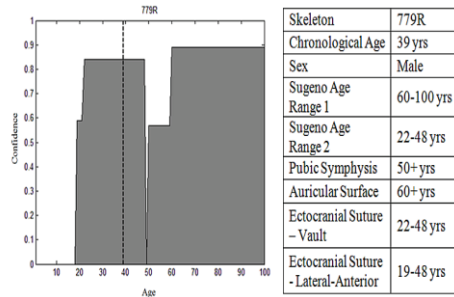
Figure 4 tells the following story. Skeleton 208 (4a) is clearly of type specific, has a low specificity, and there is a high FI confidence. This reinforces the class label that we found prior in [1]. Discoveries for skeletons 25R and 635 also reinforce what we found earlier (figures 4b and 4d). However, for skeleton 779R (4c), we now discover two class labels with high membership value, inconclusive and reconsideration. According to our definitions, it is inconclusive due to the length of the maximum plateau, however, there is also very little difference (in confidence) between the maximum plateau and the second largest. Even if one could reduce the width of the interval for the maximum plateau, there would still remain substantial disagreement between it and the second largest.



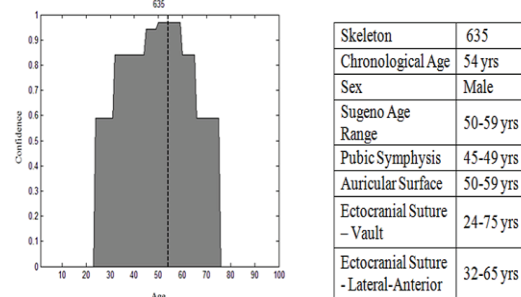
a: class=(1,0,0,0), specificity=(0,0.17,0.38,0.57) and conf=(1,0,0,0)



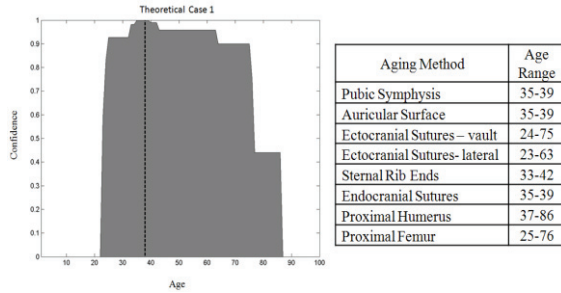
b: class=(0,0,0,1), specificity=(0.05,0.3,0.45,0.31) and conf=(1,0.48,0,0)



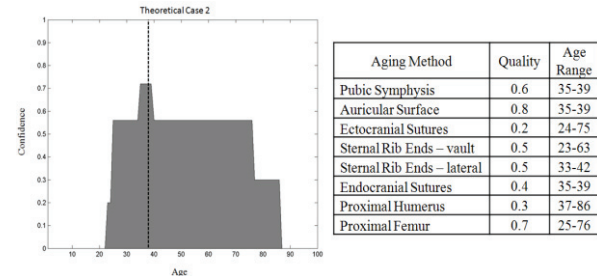
c: class=(0,0,1,1), specificity=(0,0,0.03,0.79) and conf=(0.89,0.55,0,0)



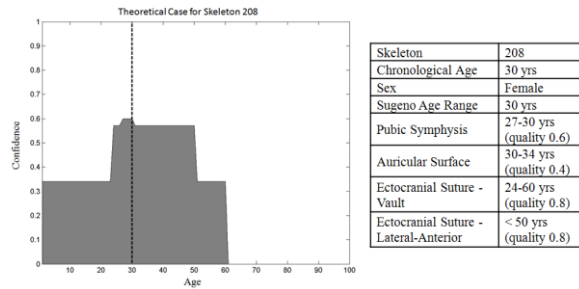
d: class=(0,1,0,0), specificity=(0,0.17,0.38,0.57) and conf=(1,0.15,0,0)



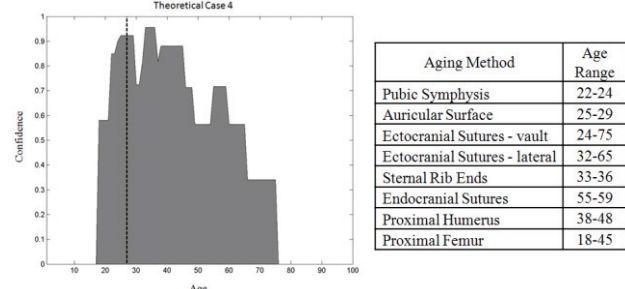
e: class=(0.5,1,0,0), specificity=(0.01,0.04,0.16,0.90) and conf=(1,0,0,0)



f: class=(0.5,1,0,0), specificity=(0.1,0.07,0.1,0.40) and conf=(0,1,0,0)



g: class=(0.5,0.38,0,0), specificity=(0.02,0.12,0.13,0.21) and conf=(0,1,0,0)



h: class=(0,0,0,1), specificity=(0.03,0.21,0.3,0.52) and conf=(1,0.22,0,0)

Fig. 4. Four skeletal examples from the Terry Anatomical Collection (a-d) and four hypothetical examples from [1] (e-h). For each age-at-death FS, we present the membership values in (1) the type of FS, (Specific, Interval, Inconclusive, Reconsideration), (2) the specificity, (Exact, High, Moderate, Low), and (3) the confidence interpretation, (High, Moderate, Low, No).

In HC1 (figure 4e), numerous (8) aging methods are simulated and the result is a single plateau of FI value 1 between ages 37-39. The highest membership in 'Interval', but there is also support in 'Specific'. In HC2 (figure 4f), the qualities of the skeletal remains were varied and the result was a single plateau of FI value 0.7 and age between ages 35 to 39. We see the same basic trend for HC2 as seen in HC1. In HC3 (figure 4g), the quality of the skeleton 208 (figure 4a) were

varied. The result is a value of 0.6 for the ages 27 to 30. For HC3, there is little support for any class label. The measurements do not match our models to any high degree. There is more support in 'Specific', however almost the same confidence amount in 'Interval'. Additionally, the specificity degree has decreased between HCs 2 and 3. In part, this is because our OWA approach takes into account the maximum

FI value. Thus, as the maximum FI value decreases, so does our confidence in the amount of attainable specificity of the FS.

Lastly, for HC4 (figure 4h), we showed a case in which the quality value is 1 but the aging methods were in extreme disagreement. Multiple plateaus are discovered. The maximum plateau has a FI value of 0.95 for the ages 33 to 36 and the second largest is of FI value 0.91 for ages 25 to 29. In this case, there is a distinct class label of ‘Reconsideration’ with a high confidence and indication of ‘Low’ FS specificity.

VII. CONCLUSIONS

In this article, we extended our previous anthropological FI-based adult skeletal age-at-death estimation research [1]. This work is important to anthropology because it makes the prior qualitative procedure of manually analyzing age-at-death FSs now explicit. Therefore, researchers can better understand how to interpret results acquired from different investigators on different types of skeletal remains. This work is important to the fuzzy set community because we show how the FI for a number-valued integrand and number-valued measure can produce FS-valued results. We also demonstrate a feature-based method of defining and measuring classes of different FSs in the context of anthropology. Lastly, we demonstrate an OWA-based contrast approach to measure the amount of specificity in an age-at-death FS. The method proposed in this article both reinforces many findings from our earlier work as well as shedding light on new discoveries. The proposed method makes the previous process of qualitatively analyzing the FI produced FSs explicit and understandable by others.

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