

An Observer Based Adaptive Iterative Learning Control for Robotic Systems

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Abstract—In this paper, an observer based adaptive iterative learning control is proposed for robotic systems. Due to the joint velocities are assumed to be not measurable, a state observer is introduced to design the iterative learning controller. We first derive an observation error model based on an tracking error observer. Then we apply an averaging filter to design the ILC algorithm. A fuzzy neural learning component using a filtered fuzzy neural network is presented to solve the problem of unknown nonlinearities. A robust learning component using sliding-mode like design is used to overcome the uncertainties, including fuzzy neural approximation error and the error induced by using state estimation errors. We show that all the adjustable parameters as well as internal signals remain bounded for all iterations. Finally, the norm of output tracking error will asymptotically converge to a tunable residual set as iteration goes to infinity.

Index Terms—adaptive iterative learning control; observer; robotic systems; filtered fuzzy neural network; averaging filter approach

I. INTRODUCTION

Due to the repeatability of operation for robotic systems, it is suitable to perform the control task by using the technique of iterative learning control (ILC). The ILC is a non-model based learning approach which is very effective in dealing with repetitive control tasks. Based on the contraction mapping theory and a certain priori knowledge, the ILC algorithms for robotic systems were initially developed in [1]. In general, the most popular designs of the ILC algorithms for robotic system can be roughly classified into D-type ILC [1], [2] and P-type ILC [3], [4]. In D-type ILC, the acceleration error of joint variable is used to construct the learning controller. In order to guarantee the convergence of the learning systems, it is necessary to assume that the nonlinearity of the plant is Lipschitz in states. Although the condition is mild, the acceleration measurement is unfortunately not realizable in practice and becomes the major disadvantage of D-type ILC. On the other hand, P-type ILC uses only the velocity error for the design of updated learning function. Although the requirement of acceleration measurement is removed, more strict conditions on the plant are needed for technical analysis. Recently, adaptive iterative learning control (AILC) have been widely studied [5], [6] for robotic systems. In order to relax the restrict Lipschitz condition on the plant's nonlinearity, the Lyapunov-like approach is applied to analyze the stability and

convergence instead of contraction mapping theory. However, both PID-type ILCs and AILCs require at least the joint velocities to develop the ILC algorithms. If only the measurement of joint positions is available, an observer is one of the possible choice to design the ILC.

Actually, observer based ILCs were developed in [7], [8] for a class of nonlinear systems. Same as the case of the most ILC works, the plant nonlinearities must be known or satisfy Lipschitz continuous condition in [7], [8]. In this paper, to solve the relative degree problem caused by the condition that joint velocities are not measurable, an s -domain transfer function technique is adopted to derive an output error model based on a tracking error observer. An averaging filter is then proposed for the controller design. Under the derived error model, a filtered FNN using state estimation vector as the network input is used to design a fuzzy neural learning component for approximation of the unknown nonlinearities. A robust learning component is also constructed to compensate for the uncertainties from approximation error and state estimation error. Based on a Lyapunov like analysis, we show that all the adjustable parameters and internal signals remain bounded during the learning process. Furthermore, the norm of output tracking error will asymptotically converge to a tunable residual set if iteration number is large enough.

In the subsequent discussions, the following notations will be used.

- (1) $|\cdot|$ denotes the absolute value of a scalar or the Euclidean or any other consistent norm of a vector or matrix.
- (2) $\|(\cdot)_t\|_\infty = \sup_{\tau \leq t} |(\cdot)(\tau)|$ denotes the truncated L_∞ norm of the argument function or vector [10].
- (3) $\|P(s)\|_\infty$ denotes the H_∞ norm of the transfer function $P(s)$.
- (4) $G(s)[u]$ denotes the filtered version of u with any proper or strictly proper transfer function $G(s)$.

II. PROBLEM FORMULATION

In this paper, we consider an uncertain robotic system with n rigid bodies which can perform a given task repeatedly over a finite time interval $[0, T]$ as follows:

$$D(q^j(t))\ddot{q}^j(t) + B(q^j(t), \dot{q}^j(t))\dot{q}^j(t) + F(q^j(t), \dot{q}^j(t)) = u^j(t) \quad (1)$$

where $j \in Z_+$ denotes the index of iteration number and $t \in [0, T]$ denotes the time index¹. The signals $q^j, \dot{q}^j, \ddot{q}^j \in R^n$ are respectively the generalized joint position, joint velocity and joint acceleration vectors. $D(q^j) \in R^{n \times n}$ is the inertia matrix, $B(q^j, \dot{q}^j) \in R^{n \times n}$ is the centripetal plus Coriolis force vector, $F(q^j, \dot{q}^j) \in R^n$ is the gravitational plus frictional forces and $u^j \in R^n$ is the joint torque vector. It is noted the inertia matrix $D(q^j)$ is assumed to be positive definite and bounded for all $t \in [0, T]$ and iteration $j \geq 1$ as

$$0 < m_1 I \leq D(q^j) \leq m_2 I$$

where $m_1, m_2 > 0$ and I is an $n \times n$ identity matrix. Since the inverse of inertia matrix exists for all joint variables, the dynamic formulation of the robotic system can be written as follows,

$$\ddot{q}^j = -D^{-1}(q^j) [B(q^j, \dot{q}^j)\dot{q}^j + F(q^j, \dot{q}^j)] + D^{-1}(q^j)u^j \quad (2)$$

Let $f(q^j, \dot{q}^j) = D^{-1}(q^j)[B(q^j, \dot{q}^j)\dot{q}^j + F(q^j, \dot{q}^j)] \in R^n$ and $b(q^j) = D^{-1}(q^j) \in R^{n \times n}$ and choose the output variable as $y^j = q^j \in R^n$, state variable as $X^j = [x_1^{j\top}, x_2^{j\top}]^\top = [y^{j\top}, \dot{y}^{j\top}]^\top = [q^{j\top}, \dot{q}^{j\top}]^\top \in R^{2n}$, then we have

$$\begin{aligned} \dot{x}_1^j &= x_2^j \\ \dot{x}_2^j &= -f(X^j) + b(X^j)u^j \\ y^j &= x_1^j \end{aligned} \quad (3)$$

or equivalently in the following state-space form :

$$\begin{aligned} \dot{X}^j &= AX^j + B[-f(X^j) + b(X^j)u^j] \\ y^j &= C^\top X^j \end{aligned} \quad (4)$$

where

$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad C = \begin{bmatrix} I \\ 0 \end{bmatrix},$$

Here $0 < b(X^j) \leq m_1^{-1}I$. In this paper, we assume that only joint position y^j is measurable for controller design. Let the desired state trajectory be defined as $X_d = [y_d^\top, \dot{y}_d^\top]^\top$. Now, given a specified desired output trajectory $y_d, t \in [0, T]$ and a possible initial resetting error $y_d(0) \neq y^j(0)$ for all $j \geq 1$, the control objective for the robotic systems doing a repeatable task is to force the system output y^j to follow y_d as close as possible.

III. DERIVATIONS OF ERROR MODEL AND CONTROLLER

A. Derive the Error Model

Define the output error as $e^j = y_d - y^j$ and the state tracking error vector as $E^j = X_d - X^j = [e^{j\top}, \dot{e}^{j\top}]^\top \in R^{2n}$, then we can derive $\ddot{e}^j$ as follows,

$$\begin{aligned} \ddot{e}^j &= \ddot{y}_d + f(X^j) - b(X^j)u^j \\ &= -K_c^\top E^j + f(X^j) + \ddot{y}_d + K_c^\top E^j - b(X^j)u^j \end{aligned} \quad (5)$$

¹the time index t will be omitted unless otherwise specified.

where $K_c = [k_2^o I, k_1^o I]^\top \in R^{2n \times n}$ is the feedback gain matrix such that the characteristic polynomial of $A_c = A - BK_c^\top$ is Hurwitz. Therefore, the tracking error dynamics will satisfy

$$\begin{aligned} \dot{E}^j &= A_c E^j + B(f(X^j) + \ddot{y}_d + K_c^\top E^j - b(X^j)u^j) \\ &= A_c E^j + B(f(X^j) + \ddot{y}_d + K_c^\top X_d - K_c^\top X^j \\ &\quad - u^j + (1 - b(X^j))u^j) \\ &= A_c E^j + B(h(X^j) + \alpha - u^j + (1 - b(X^j))u^j) \\ e^j &= C^\top E^j \end{aligned} \quad (6)$$

where $h(X^j) \equiv f(X^j) - K_c^\top X^j$, $\alpha \equiv \ddot{y}_d + K_c^\top X_d$. Note that the state tracking error vector E^j in the tracking error dynamics (6) is not assumed to be available for measurement. Hence, it is necessary to construct a state estimation vector $\hat{X}^j = [\hat{x}_1^{j\top}, \hat{x}_2^{j\top}]^\top \in R^{2n} \times [0, T]$ for estimation of the state vector. In order to construct a state estimation vector, we define a state tracking error estimation vector $\hat{E}^j$ as

$$\hat{E}^j = X_d - \hat{X}^j = [\hat{e}^{j\top}, \dot{\hat{e}}^{j\top}]^\top \in R^{2n} \times [0, T]$$

The tracking error observer is designed as

$$\begin{aligned} \dot{\hat{E}}^j &= A_o \hat{E}^j + K_o(e^j - \hat{e}^j) \\ \hat{e}^j &= C^\top \hat{E}^j \end{aligned} \quad (7)$$

where $K_o = [k_2^o I, k_1^o I]^\top \in R^{2n \times n}$ is the observer gain vector such that the characteristic polynomial of $A_o = A_c - K_o C^\top$ is Hurwitz. If we further define the observation error vector as $\tilde{E}^j = E^j - \hat{E}^j$, the observation error dynamics can be derived by using (6) and (7) as follows

$$\begin{aligned} \dot{\tilde{E}}^j &= A_o \tilde{E}^j + B(h(X^j) + \alpha - u^j + (1 - b(X^j))u^j) \\ \tilde{e}^j &= C^\top \tilde{E}^j \end{aligned} \quad (8)$$

Based on the universal approximation theorem, we know that the nonlinear function $h(X^j)$ can be approximated by an FNN [9] $O^{(4)}(X^j, W^j) = W^{j\top} O^{(3)}(X^j)$. Here $O^{(3)}(X^j) \in R^M$ is the basis function vector with M being the number of rule nodes and W is the weight matrix of the output layer. According to the universal approximation theorem, there will exist an optimal weight matrix W^* such that $h(X^j) = O^{(4)}(X^j, W^*) + \epsilon^j = W^{*\top} O^{(3)}(X^j) + \epsilon^j$, where ϵ^j is the approximation error satisfying $|\epsilon^j| \leq \epsilon^*$ in a certain compact set. This implies that (8) can be rewritten as

$$\begin{aligned} \dot{\tilde{E}}^j &= A_o \tilde{E}^j + B(W^{*\top} O^{(3)}(X^j) + \epsilon^j + \alpha - u^j \\ &\quad + (1 - b(X^j))u^j) \\ &= A_o \tilde{E}^j + B(W^{*\top} O^{(3)}(\hat{X}^j) - u^j + \delta^j \\ &\quad + (1 - b(X^j))u^j) \\ \tilde{e}^j &= C^\top \tilde{E}^j \end{aligned} \quad (9)$$

where $\delta^j = W^{*\top} (O^{(3)}(X^j) - O^{(3)}(\hat{X}^j)) + \epsilon^j + \alpha$ is the lumped uncertainty term which includes the difference between

FNN network with input using state and estimated state, the network approximation error and the desired state trajectory. It is easily proven that the lumped uncertainty term satisfying $|\delta^j| \leq \delta^*$. However, it is noted that the value of the unknown constant δ^* might be large.

To see how to design the iterative learning controller u^j to achieve the control objective, we now adopt the mixed use of a time signal and a Laplace transfer function to obtain the explicit expression of $\tilde{e}^j$ in (9) in time domain with a filtered version as follows,

$$\tilde{e}^j = H(s) \left[W^{*\top} O^{(3)}(\hat{X}^j) - u^j + \delta^j + (1 - b(X^j))u^j \right] \quad (10)$$

where $H(s) = C^\top (sI - A_o)^{-1} B = \frac{1}{s^2 + (k_1^c + k_1^o)s + (k_1^c k_1^o + k_2^c k_2^o)}$. The observer gain vector $K_o = [k_1^o I, k_2^o I]^\top$ is chosen such that $H(s) = \frac{1}{\ell(s)L(s)}$ with $\ell(s) = s + \lambda$ and $L(s)$ being any Hurwitz polynomial of order 1. Then (10) can be rewritten as

$$\begin{aligned} \tilde{e}^j &= H(s)L(s) \left[W^{*\top} \frac{1}{L(s)} [O^{(3)}(\hat{X}^j)] - \frac{1}{L(s)} [u^j] \right. \\ &\quad \left. + \frac{1}{L(s)} [(1 - b(X^j))u^j + \delta^j] \right] \\ &= \frac{1}{\ell(s)} \left[W^{*\top} \xi^j - \frac{1}{L(s)} [u^j] + \delta_L^j \right] \end{aligned} \quad (11)$$

where $\xi^j = \frac{1}{L(s)} [O^{(3)}(\hat{X}^j)]$ and $\delta_L^j = \frac{1}{L(s)} [(1 - b(X^j))u^j + \delta^j]$.

B. Construct Some Useful Signals

According to the filtered version of output tracking error model (11), we define an augmented signal with filtered version as

$$y_a^j = \frac{1}{\ell(s)} \left[v^j - \frac{1}{L(s)} [u^j] \right], \quad y_a^j(0) = 0 \quad (12)$$

where v^j is an auxiliary input to be designed later. Then, design an auxiliary error signal as

$$e_a^j = \tilde{e}^j - y_a^j \quad (13)$$

The initial condition $e_a^j(0)$ will satisfy $|e_a^j(0)| = |\tilde{e}^j(0) - y_a^j(0)| = |\tilde{e}^j(0)| \equiv \varepsilon^j$. Substituting (11) and (12) into (13), we can find that

$$e_a^j = \tilde{e}^j - y_a^j = \frac{1}{\ell(s)} \left[W^{*\top} \xi^j - v^j + \delta_L^j \right] \quad (14)$$

It is noted that we apply the Laplace operation to derive the tracking error model (14) for technical analysis later as that used in the traditional model reference adaptive control [10]. The equivalent time-domain state space representation of (14) is

$$\dot{e}_a^j = -\lambda e_a^j + W^{*\top} \xi^j - v^j + \delta_L^j \quad (15)$$

In order to overcome the uncertainty from initial output tracking error, a new signal e_ϕ^j is introduced as follows,

$$e_\phi^j = e_a^j - \phi^j \text{sat} \left(\frac{e_a^j}{\phi^j} \right), \quad \phi^j = \varepsilon^j e^{-\lambda t} \quad (16)$$

where $\text{sat}(\frac{e_a^j}{\phi^j}) = [\text{sat}(\frac{e_{a,1}^j}{\phi^j}), \dots, (\frac{e_{a,n}^j}{\phi^j})]^\top \in R^n$ and

$$\text{sat} \left(\frac{e_{a,i}^j}{\phi^j} \right) = \begin{cases} 1 & \text{if } e_{a,i}^j > \phi^j \\ \frac{e_{a,i}^j}{\phi^j} & \text{if } |e_{a,i}^j| \leq \phi^j, i = 1, \dots, n \\ -1 & \text{if } e_{a,i}^j < -\phi^j \end{cases}$$

and ϕ^j is the width of boundary layer. Note that ϕ^j is designed to decrease along time axis with the initial condition chosen as $\phi^j(0) = \varepsilon^j$ for j th iteration and $0 < \varepsilon^j e^{-\lambda T} \leq \phi^j \leq \varepsilon^j, \forall t \in [0, T], j \geq 1$. According to (16), it is easy to show that $e_\phi^j(0) = 0, \forall j \geq 1$. Now let us differentiate $\frac{1}{2} e_\phi^{j\top} e_\phi^j$ as follows,

$$\begin{aligned} \frac{1}{2} \frac{d}{dt} e_\phi^{j\top} e_\phi^j &= e_\phi^{j\top} \dot{e}_\phi^j \\ &= e_\phi^{j\top} (\dot{e}_a^j - \text{sgn}(e_\phi^j) \dot{\phi}^j) \\ &= e_\phi^{j\top} \left\{ -\lambda e_a^j - \text{sgn}(e_\phi^j) \dot{\phi}^j \right\} \\ &\quad + e_\phi^{j\top} \left\{ W^{*\top} \xi^j - v^j + \delta_L^j \right\} \\ &= -\lambda e_\phi^{j\top} e_\phi^j + e_\phi^{j\top} \left\{ W^{*\top} \xi^j - v^j + \delta_L^j \right\} \end{aligned} \quad (17)$$

where $\text{sgn}(e_\phi^j) = [\text{sgn}(e_{\phi,1}^j), \dots, \text{sgn}(e_{\phi,n}^j)]^\top$ is the typical signum function vector. In the following, we will show that the uncertainty term δ_L^j in (11) can be bounded in a linearly parameterized form if the following normalization signal m^j [11] is utilized

$$m^j = \frac{\delta_2}{s + \delta_1} [1 + |u^j|], \quad m^j(0) > \frac{\delta_2}{\delta_1} \quad (18)$$

where $\delta_1, \delta_2 > 0$ and $\delta_1 < \delta^*$. Here δ^* is the least positive constant such that $\frac{1}{L(s-\delta^*)}$ is a stable system. In practical, δ_1 can be chosen as small as possible. Note that $1 - b(X^j)$, α and δ^j are bounded and $\frac{1}{L(s)}$ is a strictly proper stable transfer function. This implies that

$$|\delta_L^j| \leq \theta_1^* m^j + \theta_2^* = [\theta_1^*, \theta_2^*] \begin{bmatrix} m^j \\ 1 \end{bmatrix} \equiv \theta^{*\top} Y^j \quad (19)$$

for some positive constants θ_1^*, θ_2^* .

C. Design the Iterative Learning Controller Using Filtered FNN

Based on the derived error model and the useful signals, we design u^j and v^j as follows

$$u^j = \frac{L(s)}{F^{n-1}(\tau s)} [v^j] \quad (20)$$

$$v^j = W^j \theta^j \xi^j + \text{sat} \left(\frac{e_a^j}{\phi^j} \right) \theta^{j\top} Y^j + e_\phi^j Y^{j\top} Y^j \quad (21)$$

where W^j is the weight matrix of the network and θ^j is control parameter vector which are used to compensate for the unknown W^* and θ^* respectively. Furthermore, $F(\tau s) = (\tau s + 1)^2$ with $\tau > 0$ being a small constant. In the literature, $\frac{1}{F(\tau s)}$ is referred to as an *averaging filter*, which is obviously a low-pass filter whose bandwidth can be arbitrarily enlarged

as τ approaches 0. If we define the parameter error as $\widetilde{W} = W^j - W^*$ and $\widetilde{\theta}^j = \theta^j - \theta^*$ and substitute (21) into (17), we have the following error dynamics for technical analysis later,

$$\begin{aligned} & \frac{1}{2} \frac{d}{dt} e_\phi^{j\top} e_\phi^j \\ &= -\lambda(e_\phi^j)^2 + e_\phi^j \left\{ -\widetilde{W}^{j\top} \xi^j - \text{sat} \left(\frac{e_a^j}{\phi^j} \right) \theta^{j\top} Y^j \right. \\ & \quad \left. - e_\phi^j Y^{j\top} Y^j + \delta_L^j \right\} \\ &\leq -\lambda(e_\phi^j)^2 - e_\phi^j \widetilde{W}^{j\top} \xi^j - \left| e_\phi^j \right| \widetilde{\theta}^{j\top} Y^j - (e_\phi^j)^2 Y^{j\top} Y^j \end{aligned} \quad (22)$$

A set of stable adaptive laws is necessary to tune the control parameters. The adaptive laws combining time domain and iteration domain adaptation without knowledge of known bounds on optimal parameters or deadzone mechanism are proposed as follows,

$$(1 - \gamma_1) \dot{W}^j = -\gamma_1 W^j + \gamma_1 W^{j-1} + \beta_1 e_\phi^j \xi^j \quad (23)$$

$$(1 - \gamma_2) \dot{\theta}^j = -\gamma_2 \theta^j + \gamma_2 \theta^{j-1} + \beta_2 |e_\phi^j| Y^j \quad (24)$$

with $W^j(0) = W^{j-1}(T)$, $\theta^j(0) = \theta^{j-1}(T)$ for $j \geq 1$, and $0 < \gamma_1, \gamma_2 < 1$, $\beta_1, \beta_2 > 0$. In these adaptive laws, γ_1, γ_2 and β_1, β_2 are defined as the weighting gains and adaptation gains, respectively. The setting of initial network parameter vector W^0 and θ^0 are specified in [12].

IV. ANALYSIS OF STABILITY AND CONVERGENCE

To prove the stability and convergence of the proposed learning system, we first give the following two facts.

• **Fact 1** : Choose a Lyapunov-like positive function as

$$V_a^j = \frac{1}{2} e_\phi^{j\top} e_\phi^j + \frac{(1 - \gamma_1)}{2\beta_1} \text{tr} \left\{ \widetilde{W}^{j\top} \widetilde{W}^j \right\} + \frac{(1 - \gamma_2)}{2\beta_2} \widetilde{\theta}^{j\top} \widetilde{\theta}^j$$

and compute its derivative with respect to time t , we have the following result by using similar technique in [12] as

$$\dot{V}_a^j \leq -k V_a^j + \bar{k}^{j-1} \quad (25)$$

where $k = \min \{ 2\bar{\lambda}, \frac{\gamma_1}{1-\gamma_1}, \frac{\gamma_2}{1-\gamma_2} \}$, and $\bar{k}^{j-1} = \frac{\gamma_1}{2\beta_1} \text{tr} \left\{ \widetilde{W}^{j-1\top} \widetilde{W}^{j-1} \right\} + \frac{\gamma_2}{2\beta_2} \widetilde{\theta}^{j-1\top} \widetilde{\theta}^{j-1}$. It is noted that $\widetilde{W}^0 = W^0 - W^* = W^{ini} - W^* \equiv \bar{W}^0$ and $\widetilde{\theta}^0 = \theta^0 - \theta^* = \theta^{ini} - \theta^* \equiv \bar{\theta}^0$ are bounded for all $t \in [0, T]$ so that if $j = 1$, (25) can be rewritten as

$$\begin{aligned} \dot{V}_a^1 &\leq -k V_a^1 + \bar{k}^0 \\ &= -k V_a^1 + \frac{\gamma_1}{2\beta_1} \text{tr} \left\{ \bar{W}^{0\top} \bar{W}^0 \right\} + \frac{\gamma_2}{2\beta_2} \bar{\theta}^{0\top} \bar{\theta}^0 \end{aligned} \quad (26)$$

Note that the initial value $V_a^1(0)$ is bounded since $e_\phi^1(0) = 0$, $\widetilde{W}^1(0) = W^1(0) - W^* = W^0(T) - W^* = \bar{W}^0$, and $\widetilde{\theta}^1(0) = \theta^1(0) - \theta^* = \theta^0(T) - \theta^* = \bar{\theta}^0$. Together with the result of (26), it readily implies $V_a^1, e_\phi^1, \widetilde{W}^1, \widetilde{\theta}^1, W^1, \theta^1 \in L_{\infty}[0, T]$. Since the basis function vector $O^{(3)j}$ is bounded for all $j \geq 1$, we conclude that e_a^1 (by (16)) and $\dot{W}^1$ (by 23) $\in L_{\infty}[0, T]$.

• **Fact 2** : Define a positive function $V^j(T)$ as

$$\begin{aligned} V^j(T) &= \int_0^T \left[\frac{\gamma_1}{2\beta_1} \widetilde{W}^{j\top} \widetilde{W}^j + \frac{\gamma_2}{2\beta_2} \widetilde{\theta}^{j\top} \widetilde{\theta}^j \right] dt \\ &\quad + \frac{(1 - \gamma_1)}{2\beta_1} \text{tr} \left\{ \widetilde{W}^j(T) \widetilde{W}^j(T) \right\} \\ &\quad + \frac{(1 - \gamma_2)}{2\beta_2} \widetilde{\theta}^j(T) \widetilde{\theta}^j(T) \end{aligned} \quad (27)$$

Using the technique of integration by parts, we can show that

$$\begin{aligned} V^j(T) - V^{j-1}(T) &\leq \int_0^T \left(e_\phi^{j\top} \widetilde{W}^{j\top} \xi^j + |e_\phi^j| \widetilde{\theta}^{j\top} Y^j \right) dt \end{aligned} \quad (28)$$

Integrating both side of (22) from 0 to T gives

$$\begin{aligned} &\int_0^T \left(e_\phi^{j\top} \widetilde{W}^{j\top} \xi^j + |e_\phi^j| \widetilde{\theta}^{j\top} Y^j \right) dt \\ &\leq - \int_0^T \left(\lambda e_\phi^{j\top} e_\phi^j + e_\phi^{j\top} e_\phi^j Y^{j\top} Y^j \right) dt \\ &\quad - \frac{1}{2} e_\phi^{j\top}(T) e_\phi^j(T) \end{aligned} \quad (29)$$

where we use the property of $\frac{1}{2} e_\phi^{j\top}(0) e_\phi^j(0) = 0$. Substituting (29) into (28), it yields

$$\begin{aligned} V^j(T) - V^{j-1}(T) &\leq -\frac{1}{2} e_\phi^{j\top}(T) e_\phi^j(T) - \int_0^T \left(\lambda e_\phi^{j\top} e_\phi^j + e_\phi^{j\top} e_\phi^j Y^{j\top} Y^j \right) dt \end{aligned} \quad (30)$$

Since $V^1(T)$ is bounded by **Fact 1** and $V^j(T)$ is positive and monotonically decreasing, we conclude by the result of (30) that $V^j(T)$ is bounded for all $j \geq 1$ and will converge as j approaches infinity to some limit value V_T which is independent of j . Furthermore, (30) also implies

$$\int_0^T e_\phi^{j\top} e_\phi^j Y^{j\top} Y^j dt \leq V^{j-1}(T) - V^j(T) \leq V^1(T) \quad (31)$$

$$\int_0^T \lambda e_\phi^{j\top} e_\phi^j dt \leq V^{j-1}(T) - V^j(T) \leq V^1(T) \quad (32)$$

$$\frac{1}{2} e_\phi^{j\top}(T) e_\phi^j(T) \leq V^{j-1}(T) - V^j(T) \leq V^1(T) \quad (33)$$

It follows that $\widetilde{W}^j(T)$, $\widetilde{\theta}^j(T)$ and $e_\phi^{j\top}(T) e_\phi^j(T)$ are bounded for all $j \geq 1$. Furthermore, $\lim_{j \rightarrow \infty} \int_0^T e_\phi^{j\top} e_\phi^j dt = 0$, and $\lim_{j \rightarrow \infty} e_\phi^{j\top}(T) e_\phi^j(T) = 0$.

Lemma 1 : Consider the robotic system (1) performing a repetitive control task. If we apply the output based fuzzy neural adaptive iterative learning controller (12), (13), (16), (18), (20), and (21) with adaptation laws (23) and (24), then we ensure that all the internal signals are bounded, i.e., $e_\phi^j, e_a^j, \tilde{e}^j, y_a^j, \tilde{e}^j, e^j, \tilde{E}^j, \hat{E}^j, E^j, W^j, \theta^j, u^j, \dot{W}^j, \dot{\theta}^j \in L_{\infty}[0, T]$.

Proof : Integrating (25) from 0 to t , we have

$$\begin{aligned} V_a^j &\leq V_a^j(0) + \int_0^t \bar{k}^{j-1}(z) dz \\ &\leq V_a^j(0) + \int_0^T \bar{k}^{j-1}(t) dt \end{aligned} \quad (34)$$

Then we can conclude that $V_a^j, e_\phi^j, W^j, \theta^j, e_a^j, \dot{W}^j \in L_{\infty}[0, T]$ by using Fact 2.

However, the boundedness of $V_a^j, e_\phi^j, \tilde{W}^j, \tilde{\theta}^j, e_a^j, \dot{W}^j$ can not guarantee the boundedness of Y^j (or equivalently m^j) and input u^j . In order to show the boundedness of Y^j for all $t \in [0, T]$, we first note that the boundedness of $\int_0^T \gamma \tilde{e}_\phi^{j\top} \tilde{e}_\phi^j Y^{1\top} Y^1 dt$ are then established from (31). Furthermore, since $\int_0^t e_\phi^{j\top} e_\phi^j Y^{j\top} Y^j dz \leq \int_0^T e_\phi^{j\top} e_\phi^j Y^{j\top} Y^j dz$, it implies that $\int_0^t e_\phi^{j\top} e_\phi^j Y^{j\top} Y^j dz \in L_{\infty}[0, T]$. But the boundedness of e_ϕ^j and $\int_0^t e_\phi^{j\top} e_\phi^j Y^{j\top} Y^j dz$ only ensure that Y^j is bounded everywhere except on a set of measure zero. Now we adopt some techniques given in chapter 2 of [10]. Consider u^j in (20),

$$\begin{aligned} u^j &= \frac{L(s)}{F^{n-1}(\tau s)} [v^j] \\ &= \frac{L(s)}{F^{n-1}(\tau s)} \left[W^{j\top} \xi^j + \text{sat} \left(\frac{e_a^j}{\phi^j} \right) \theta^{j\top} Y^j \right] \\ &\quad + \frac{sL(s)}{F^{n-1}(\tau s)} \left[\int_0^t e_\phi^j Y^{j\top} Y^j dz \right] \end{aligned} \quad (35)$$

Since W^j, ξ^j, θ^j and $\int_0^t e_\phi^j Y^{j\top} Y^j dz$ are bounded for $t \in [0, T]$, and $\frac{L(s)}{F^{n-1}(\tau s)}, \frac{sL(s)}{F^{n-1}(\tau s)}$ are strictly proper stable transfer functions, (35) implies that u^j will satisfy

$$|u^j| \leq k_1 \|(Y^j)_t\|_\infty + k_1 \leq k_2 \|(m^j)_t\|_\infty + k_2 \quad (36)$$

for some $k_1, k_2 > 0$ by lemma 2.6 (output is bounded by truncated L_∞ norm of input for a stable linear system) in [10]. Now we construct an extended dynamic equation by using (??), (8) and (18) as follows :

$$\begin{aligned} \begin{bmatrix} \dot{E}^j \\ \dot{m}^j \end{bmatrix} &= \begin{bmatrix} A_c & 0 \\ 0 & -\delta_1 \end{bmatrix} \begin{bmatrix} E^j \\ m^j \end{bmatrix} \\ &\quad + \begin{bmatrix} B \\ 0 \end{bmatrix} \left(-b(X^j)u^j + W^{*\top} O^{(3)}(\hat{X}^j) + \delta^j \right) \\ &\quad + \begin{bmatrix} 0 \\ \delta_2 (1 + |u^j|) \end{bmatrix} \end{aligned} \quad (37)$$

Let X_a^j be the state vector of the extended dynamic equation (37). Taking norms on (37) will yield

$$\|\dot{X}_a^j\| \leq k_3 \|X_a^j\| + k_3 |u^j| + k_3 \leq k_4 \|(X_a^j)_t\|_\infty + k_4 \quad (38)$$

for some $k_3, k_4 > 0$. This implies that X_a^j is regular [10] so that X_a^j and hence, E^j, m^j can grow at most exponentially fast and no finite time escape during a finite time interval $[0, T]$. This condition guarantees a certain degree of smoothness of signal m^j . Therefore, we show that Y^j is a certain degree of smooth signal. Together with the result of $\int_0^t e_\phi^j Y^{j\top} Y^j dz \in L_{\infty}[0, T]$, we can conclude that $Y^j \in L_{\infty}[0, T]$. Due to $W^j, \xi^j, \theta^j, Y^j, e_\phi^j \in L_{\infty}[0, T]$, this also implies that $v^j \in L_{\infty}[0, T]$ (by (21)). Because $e_\phi^j, W^j, \theta^j, \xi^j, Y^j \in L_{\infty}[0, T]$, we conclude that $\dot{W}^j(t)$ (by 23)), $\dot{\theta}^j(t)$ (by 24) $\in L_{\infty}[0, T]$. Since $v^j \in L_{\infty}[0, T]$ and $\frac{L(s)}{F^{n-1}(\tau s)}$ is a proper stable transfer

function, this implies that $u^j \in L_{\infty}[0, T]$ (by (20)). Since $v^j, u^j \in L_{\infty}[0, T]$, $\frac{1}{L(s)}$ and $\frac{1}{\ell(s)}$ are strictly proper stable transfer functions, this implies that $y_a^j \in L_{\infty}[0, T]$ (by (12)). As noted above, $e_a^j, y_a^j \in L_{\infty}[0, T]$, we have $\tilde{e}^j \in L_{\infty}[0, T]$ (by (13)). Moreover, because A_c is a Hurwitz matrix and $\tilde{e}^j \in L_{\infty}[0, T]$, this implies that $\hat{e}^j, \hat{E}^j \in L_{\infty}[0, T]$ (by (7)). Finally, since A_c and A_o are Hurwitz matrices and $-b(X^j)u^j + W^{*\top} O^{(3)}(\hat{X}^j) + \delta^j \in L_{\infty}[0, T]$, we have $\tilde{e}^j, \tilde{E}^j \in L_{\infty}[0, T]$ (by (8)) and $e^j, E^j \in L_{\infty}[0, T]$ (by (6)). This completes the proof. Q.E.D.

Based on **Fact 1**, **Fact 2** and **Lemma 1**, we now get the main result of this learning system.

Theorem 1 : Consider the system set-up in Lemma 1. The proposed adaptive iterative learning controller guarantees the tracking performance and system stability as follows :

- (T1) $\lim_{j \rightarrow \infty} e_\phi^j(t)^\top e_\phi^j(t) = 0$, for all $t \in [0, T]$.
- (T2) $\lim_{j \rightarrow \infty} |e_a^j(t)| \leq e^{-\lambda t} \varepsilon^\infty$, for all $t \in [0, T]$.
- (T3) $\lim_{j \rightarrow \infty} |e^j(t)| \leq e^{-\lambda t} \varepsilon^\infty + \tau k_6$, for all $t \in [0, T]$.
- (T4) Let δ be the positive constant such that $\Delta(D - \delta)$ is still a Hurwitz polynomial, then $\lim_{j \rightarrow \infty} |\tilde{e}^j(t)| = |\hat{e}^\infty(t)| \leq m_2 \left(\varepsilon^\infty \frac{e^{-\lambda t} - e^{-\delta t}}{\delta - \lambda} + \tau k_6 \frac{1 - e^{-\delta t}}{\delta} \right)$, for all $t \in [0, T]$.
- (T5) $\lim_{j \rightarrow \infty} |e^j(t)| = |e^\infty(t)| \leq e^{-\lambda t} \varepsilon^\infty + \tau k_6 + m_2 \left(\varepsilon^\infty \frac{e^{-\lambda t} - e^{-\delta t}}{\delta - \lambda} + \tau k_6 \frac{1 - e^{-\delta t}}{\delta} \right)$, for all $t \in [0, T]$.

Proof : Based on Fact 1, Fact 2 and Lemma 1, then we can conclude the result of (T1) and (T2). For (T3), Substituting (20) into (12), we can find that e_a^j actually satisfies

$$\begin{aligned} e_a^j &= \tilde{e}^j - y_a^j \\ &= \tilde{e}^j - \frac{1}{\ell(s)} \left(1 - \frac{1}{F^{n-1}(\tau s)} \right) [v^j] \\ &\triangleq \tilde{e}^j - R^j \end{aligned} \quad (39)$$

Since v^j is bounded and the H_∞ norm of $\left\| \frac{1}{s} \left(1 - \frac{1}{F^{n-1}(\tau s)} \right) \right\|_\infty = 2(n-1)\tau$ and $\left\| \frac{s}{\ell(s)} \right\|_\infty = k_5$ for some $k_5 > 0$, we can conclude that

$$|R^j| \leq \left\| \frac{1}{s} \left(1 - \frac{1}{F^{n-1}(\tau s)} \right) \right\|_\infty \left\| \frac{s}{\ell(s)} \right\|_\infty \|(v^j)_t\|_\infty \leq \tau k_6$$

for some $k_6 > 0$. Taking norms on (39), we find that

$$|\tilde{e}^j| \leq |e_a^j| + |R^j| \leq |e_a^j| + \tau k_6$$

As iteration goes to infinity,

$$\lim_{j \rightarrow \infty} |\tilde{e}^j| \leq e^{-\lambda t} \varepsilon^\infty + \tau k_6 \leq \phi^\infty + \tau k_6$$

Finally, we apply a similar technique in [13], (T4) and (T5) are achieved.

V. SIMULATION EXAMPLE

In this example, we apply the observer based adaptive iterative learning controller to robotic system [14]

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1^j \\ \ddot{q}_2^j \end{bmatrix} + \begin{bmatrix} -h\ddot{q}_2^j & -h(\dot{q}_1^j + \dot{q}_2^j) \\ h\dot{q}_1^j & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1^j \\ \dot{q}_2^j \end{bmatrix} = \begin{bmatrix} u_1^j \\ u_2^j \end{bmatrix}$$

where $D_{11} = m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2} \cos(q_2^j)) + I_1 + I_2$, $D_{12} = D_{21} = m_2 l_1 l_{c2} \cos(q_2^j) + m_2 l_{c2}^2 + I_2$, $H_{22} = m_2 l_{c2}^2 + I_2$, $h = m_2 l_1 l_{c2} \sin(q_2^j)$. Here m_i , I_i , l_i and l_{ci} represent mass, inertia, length of link i , and the distance from the previous joint to the center of mass of link i , respectively. In this simulation, $m_1 = 10\text{kg}$, $m_2 = 5\text{kg}$, $l_1 = 1\text{m}$, $l_2 = 0.5\text{m}$, $l_{c1} = 0.5\text{m}$, $l_{c2} = 0.25\text{m}$, $I_1 = 0.83\text{kg-m}^2$ and $I_2 = 0.3\text{kg-m}^2$. The control objective is to let $q^j = [q_1^j, q_2^j]^T$ track the desired trajectory $q_d = [q_{d1}, q_{d2}]^T = [\sin(t), \cos(t)]^T$ as close as possible over a finite time interval $[0, 15]$ when only the angular position q^j is measurable. In this simulation, we set $\lambda = 10$, $\gamma_1 = \gamma_2 = 0.5$ and $\beta_1 = \beta_2 = 10$. In this simulation, we assume that the initial output for the first five iterations : $q^j(0) = [0.05, 0.3]^T, [0.25, 0.8]^T, [0.14, 0.6]^T, [0.09, 0.4]^T, [-0.1, 0.5]^T$. Finally, we show the learning performance even there exist initial output error in Fig. 1. It is clear that the effectiveness of the proposed controller can be obtained by the simulation results.

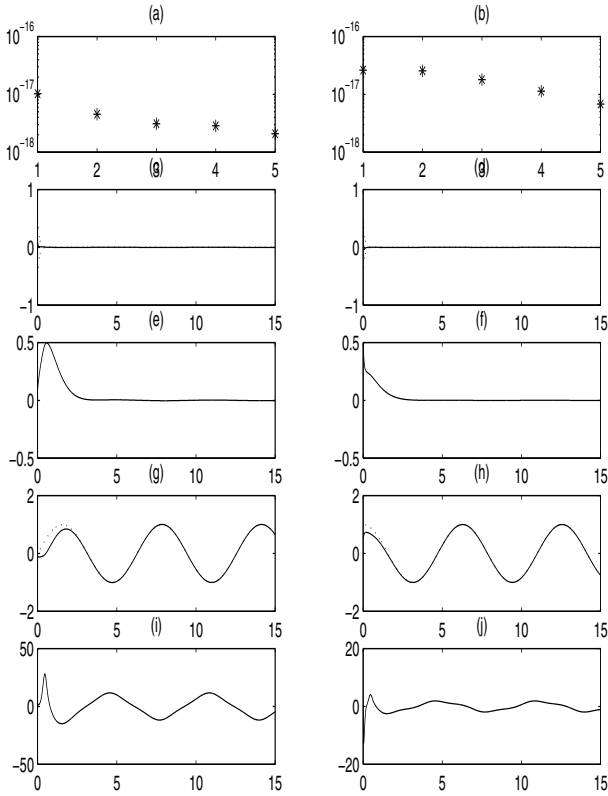


Fig. 1. (a) $\sup_{t \in [0,15]} |e_{\phi_1}^j(t)|$ versus iteration j ; (b) $\sup_{t \in [0,15]} |e_{\phi_2}^j(t)|$ versus iteration j ; (c) $e_{a1}^5(t)$ (solid line) and $\pm \phi^5(t)$ (dotted lines) versus time t ; (d) $e_{a2}^5(t)$ (solid line) and $\pm \phi^5(t)$ (dotted lines) versus time t ; (e) $e_1^5(t)$ (solid line) versus time t ; (f) $e_2^5(t)$ (solid line) versus time t ; (g) $q_1^5(t)$ (solid line) and $q_{d1}(t)$ (dotted line) versus time t ; (h) $q_2^5(t)$ (solid line) and $q_{d2}(t)$ (dotted line) versus time t ; (i) $u_1^5(t)$ versus time t ; (j) $u_2^5(t)$ versus time t .

VI. CONCLUSION

An observer based AILC for repeated tracking control of uncertain robotic systems is proposed in this paper. Only joint positions are assumed to be measurable so that a tracking error observer is designed to estimate the unknown joint variables. An error observation dynamic based on the tracking error observer is derived for the design of the AILC. Using a technique of averaging filter, a fuzzy neural learning component based on a filtered FNN is used to approximate the unknown nonlinear function and a robust learning component is designed to compensate for the other uncertainties. Based on a Lyapunov like analysis, we show that all adjustable parameters and the internal signals inside the closed-loop system remain bounded and the output tracking error asymptotically converge to a tunable residual set as iteration goes to infinity.

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