

A Perceptual Computer Based Method for Supplier Selection Problem

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Abstract—The supplier selection is one of the most important components of production and logistics management for many companies. The supplier selection problem involves different kinds of words in which all of the criteria are weighted using one kind of words and the performance evaluations for all sub-criteria are another kind of words. How to aggregate these words without losing information is a very daunting task using a type-1 fuzzy set (T1 FS) approach. This paper applies a new methodology—Perceptual Computer (Per-C)—to help solve this hierarchical group multi-criteria decision making problem. The Per-C has three components: encoder, computing with words (CWW) engine and decoder. First, the interval approach (IA) is used to obtain interval type-2 fuzzy set (IT2 FS) word models for the words in a pre-specified vocabulary. Second, a linguistic weighted average (LWA) is used to aggregate all the words modeled by IT2 FSs. Finally, a centroid-based ranking method is used to rank the supplier selections, and a similarity measure is used to obtain similarities of the supplier selections. A committee of judges organized with P decision makers and K experts decides the winning supplier selection as the one with the highest ranking and least similarity to other suppliers.

Keywords—interval type-2 fuzzy set (IT2 FS); perceptual computer (Per-C); supplier selection.

I. INTRODUCTION

In today's highly competitive environment, outsourcing has become a main stream practice in global business operations [1]; hence, supplier selection is one of the most important components of production and logistics management for many companies. The selection of appropriate suppliers can help companies significantly enhance competitiveness in the market and increase customer satisfaction.

Supplier Selection is a multiple-criteria decision-making (MCDM) problem which is affected by several conflicting factors. Many methods have been proposed and used for supplier evaluation and selection [1-20].

In classical MCDM methods, the ratings and the weights of the criteria are known precisely [3-5]. In fact, human judgments including preferences are often vague, so that a judge can not estimate his preference with an exact numerical

value. A more realistic approach may be to use linguistic assessments instead of numerical values to describe the desired value and important weights of criteria.

Because words can mean different things to different people, it is important to use a fuzzy set (FS) model for a word that lets us capture word uncertainty. At present, there are two possible choices, a type-1(T1) FS or an interval type-2(IT2) FS [21-23]. A large number of papers have solved the supplier selection problem using T1 FSs [2,6-20]. That using IT2 FSs as models for words is scientifically correct, whereas using a T1 FS model for a word is scientifically incorrect, follows from the following reasoning [22,24]: (1) A T1 fuzzy set A for a word is well-defined by its MF $\mu_A(x)$ ($x \in X$) that is totally certain once all of its parameters are specified; (2) Words mean different things to different people, and so are uncertain; and, therefore, (3) it is contradiction to say that something certain can model something that is uncertain.

Lotfi Zadeh [25-27], the father of Fuzzy Logic, coined the phrase “computing with words” (CWW). He also stated he was interested in developing a computational theory of perceptions—the development of machinery for computing and reasoning with perceptions [28]. Although perceptual computing is a sub-set of CWW, it has a much longer history than CWW. Perceptual Computing used in this paper is associated with the machinery for computing and reasoning with perceptions and data. The architecture for perceptual computing is called a Perceptual Computer (Per-C), and was first used by Mendel [22, 29-30].

This paper is about how the Per-C theory can be applied to the supplier selection problem. The rest of this paper is organized as follows. Section II provides a complete description of the supplier selection problem. Section III introduces a Per-C approach for supplier selections. Section IV gives an example and obtains results using the Per-C approach. Section V draws conclusions.

II. SUPPLIER SELECTION PROBLEM STATEMENT

To achieve a company's purchasing objective, the supplier selection problem may involve n candidate suppliers, each denoted by S_i where $i = 1, \dots, n$, from which the best supplier is chosen. A committee is organized with $P + K$ judges, where P decision makers are from cross-functional divisions within the company, each denoted by D_p where $p = 1, \dots, P$, and where K experts are outsiders (i.e., consultants), each denoted by E_k where $k = 1, \dots, K$. These P decision-makers come up with and decide on a list of M main criteria and l sub-criteria

under each main criterion for supplier selection through brainstorming sessions, and give the weights of all criteria, respectively. The K experts are used to evaluate the performance scores of the sub-criteria for each supplier. The supplier evaluation problem is summarized in Fig. 1. The evaluation criteria of the problem are adopted from [10] on the basis of our literature review. Table I summarizes and defines the abbreviation that are used in Fig. 1.

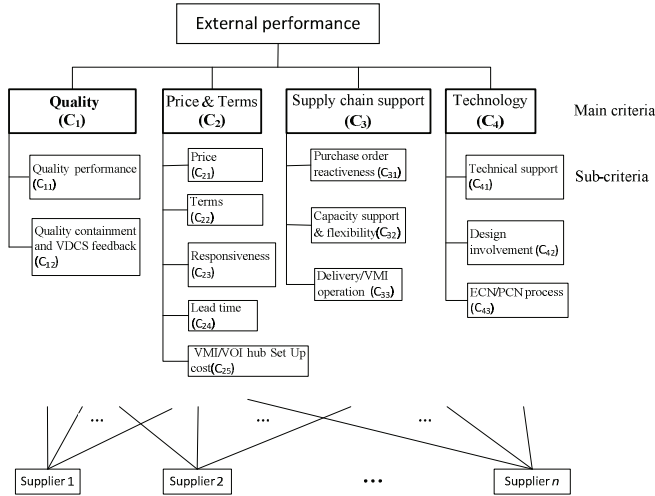


Figure 1. Hierarchical evaluating structure for supplier selections.

It is very clear from this figure that this is a multi-criteria and two-level decision making problem. At the first level each of the n suppliers is evaluated for its performance on four main criteria: Quality (C1), Price & Terms (C2), Supply chain support (C3) and Technology (C4). There are 1 sub-criteria ($C_{m1}, C_{m2}, \dots, C_{ms}, \dots, C_{ml}$) under main criterion m , where $1 \leq m \leq 4$, $1 \leq s \leq l$, and l equals 2, 5, 3 and 3 for C1, C2, C3 and C4, respectively (see Fig. 1). The second level in this hierarchical decision making problem involves a weighted aggregation of the four main criteria for each of n suppliers.

The all sub-criteria are defined using linguistic terms represented by seven words: very poor, poor, medium poor, fair, medium good, good, and very good. The importance weights for all criteria are also defined using linguistic terms represented by seven words: very low, low, medium low, medium, medium high, high, and very high.

This paper provides a new solution method for this group multi-criteria hierarchical supplier selection problem using Perceptual Computing theory and IT2 FSs.

TABLE I. ABBREVIATIONS AND DEFINITIONS

Abbreviations	Definitions
VDCS	Vender defect correcting sheet
VMI	Vendor Managed Inventory
VOI	Vendor Owned Inventory
ECN	Engineering Change Notice
PCN	Process Change Notice

III. PER-C FOR SUPPLIERS EVALUATION

The Per-C has three components: encoder, computing with words (CWW) engine and decoder. It is depicted in Fig. 2.

The encoder transforms words into IT2 FSs that activate the CWW engine. The decoder maps the output of the CWW engine into a recommendation and some accompanying data. The CWW engine maps IT2 FSs into IT2 FSs. When perceptual computing is used for the supplier evaluation problem, each of three components must be considered. Table II summarizes and defines the symbols that are used throughout the rest of this paper.

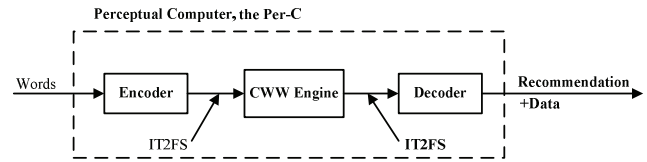


Figure 2. The Per-C that uses IT2 FS models for words [24, 30].

TABLE II. SYMBOLS AND DEFINITIONS

Symbol	Definitions
s	
$\tilde{X}_{ms}^k(S_i)$	Assessment of sub-criterion s for major criterion m provided by expert k about S_i (supplier i)
$\tilde{X}_{ms}(S_i)$	Aggregation of $\tilde{X}_{ms}^k(S_i)$ across all K experts at S_i
$\tilde{Y}_m(S_i)$	Aggregation of $\tilde{X}_{ms}(S_i)$ for major criterion m across all of its sub-criteria, at S_i
$\tilde{Y}(S_i)$	Aggregation of $\tilde{Y}_m(S_i)$, at S_i
$\tilde{W}_m^p$	Linguistic weight assigned by decision maker p to major criterion m
$\tilde{W}_{ms}^p$	Linguistic weight assigned by decision maker p to sub-criterion s for major criterion m
$\tilde{W}_{ms}$	Aggregated weight across the P decision makers for sub-criterion s associated with major criterion m
$\tilde{W}_m$	Aggregated weight across the P decision makers for major criterion m

A. Encoder

Two codebooks are needed, one for the words that will be used by the experts and another for the weights that will be used by the decision makers. The codebook contains the words and their IT2 FS models.

1) *Word FOU*s: The interval approach (IA) [31] can be used to model a word using an IT2 FS. It is based on collecting interval end-point data from a group of subjects and does not require any a priori knowledge about FSs; hence, it can be used by anyone. It is a very practical method for mapping subject's data intervals into a footprint of uncertainty (FOU) for a word. The resulting FOU are either interior, left-shoulder or right-shoulder FOU (as in Fig. 3), and, it is the data that establishes which FOU models a word.

The IA consists of two parts, a Data Part and a Fuzzy Set (FS) Part. In the Data Part, data intervals (about where a word lies on a pre-specified scale) that have been collected from a group of subjects are pre-processed, after which data statistics are computed for the surviving intervals. In the FS part, FS uncertainty measures are established for a pre-specified triangle T1 MF [always beginning with the assumption that the FOU is an interior FOU, and, if need be, later switching to a shoulder FOU (as in Fig. 3)]. Then the parameters of the

triangle T1 MF are determined using the data statistics, and the derived T1 MFs are aggregated using union leading to an FOU for a word, and finally to a mathematical model for the FOU.

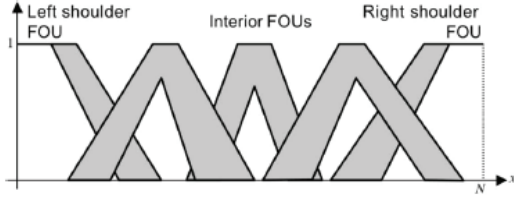


Figure 3. Left-shoulder, right-shoulder and interior FOUs, all of whose LMFs and UMFs are piecewise linear [24,31].

A word that is modeled by an interior FOU has an upper membership function (UMF) that is a trapezoid and a lower membership function (LMF) that is a triangle, but in general neither the trapezoid nor the triangle are symmetrical. A word that is modeled as a left- or right-shoulder FOU has trapezoidal upper and lower MFs; however, the legs of the respective two trapezoids are not necessarily parallel.

2) *Word codebook used by experts*: This codebook contains the seven words (*very poor, poor, medium poor, fair, medium good, good, very good*) and their FOU that are used to assess some sub-criteria.

Some sub-criteria have a linguistic negative connotation, and for them, antonyms [32-34] (e. g. , “low” is the antonym of “high”) are used instead of the raw words, i. e.

$$\mu_{10-\tilde{A}} = \mu_{\tilde{A}}(10-x) \quad (1)$$

where $10-\tilde{A}$ is the antonym of the IT2 FS $\tilde{A}$. Because an IT2 FS is completely characterized by its LMF ($\underline{\tilde{A}}$) and UMF ($\overline{\tilde{A}}$), each of which is a T1 FS, $\mu_{10-\tilde{A}}(x)$ above is obtained by applying antonyms of T1 FS to $\underline{\tilde{A}}$ and $\overline{\tilde{A}}$. In this paper the following sub-criteria are modeled using antonyms: trade variables, labor cost, inland transportation cost, land cost, law on investment and investment restrictions.

3) *Codebook for the weights*: This codebook contains the seven words (*very low, low, medium low, medium, medium high, high, very high*) and their FOUs are generated from interval data collected from domain managers and experts.

B. CWW Engine

The Linguistic Weighted Average (LWA) is used for the CWW Engine, because all the weights and ratings are modeled by IT2 FSs.

The $P+K$ judges are collected together as a whole committee, so that they can interact and exchange ideas with each other. First, the evaluations of each sub-criterion are aggregated by K experts in the committee to get a single evaluation for it; the weights of each criterion assigned by P decision makers are aggregated to get a single weight for each of them. Then the aggregated sub-criteria evaluations are aggregated with the aggregated sub-criteria weights to get the four aggregated main criteria evaluations for each supplier. Finally, the aggregated main criteria evaluations are further aggregated with the aggregated weights to get the overall

evaluation for each supplier.

Next, we perform four aggregations, as follows (see Table II for the definitions of symbols):

(1) Aggregation of $\tilde{X}_{ms}(S_i)$ across the K experts at S_i (supplier i):

$$\tilde{X}_{ms}(S_i) = \frac{\sum_{k=1}^K \tilde{X}_{ms}^k(S_i)}{K} \quad (2)$$

(2) Aggregation of linguistic weights $\tilde{W}_{ms}$ and $\tilde{W}_m$ ($m=1,2,\dots,4$) across the P decision makers:

$$\tilde{W}_{ms} = \frac{\sum_{p=1}^P \tilde{W}_{ms}^p}{P} \quad (3)$$

$$\tilde{W}_m = \frac{\sum_{p=1}^P \tilde{W}_m^p}{P} \quad (4)$$

(3) Aggregation of $\tilde{Y}_m(S_i)$ ($m=1,2,3,4$):

$$\tilde{Y}_m(S_i) = \frac{\sum_{s=1}^I \tilde{X}_{ms}(S_i) \tilde{W}_{ms}}{\sum_{s=1}^I \tilde{W}_{ms}} \quad (5)$$

(4) Aggregation of the overall performance $\tilde{Y}(S_i)$

($i=1,2,\dots,n$):

$$\tilde{Y}(S_i) = \frac{\sum_{m=1}^4 \tilde{Y}_m(S_i) \tilde{W}_m}{\sum_{m=1}^4 \tilde{W}_m} \quad (6)$$

C. Decoder

The decoder maps the CWW Engine output FOU into a recommendation. It computes similarity, centroid and rank. The supplier selection with the largest ranking, and with the least similarity to other suppliers will be the best suitable supplier.

1) *Similarity computation*: Similarity assesses the agreement between FSs. There are six similarity measures for IT2 FSs, but each of them has their drawbacks for CWW, so this paper adopts the similarity measure described in [24, 35], which is an extension of Jaccard similarity measure. In this paper, similarity, $SI[\tilde{Y}(S_i), \tilde{Y}(S_j)]$, is computed only for the supplier's overall performances $\tilde{Y}(S_i)$ ($i=1,2,\dots,n$), so that a judge can observe how similar the overall performances are. More specifically, $SI[\tilde{Y}(S_i), \tilde{Y}(S_j)]$ is computed, as:

$$SI[\tilde{Y}(S_i), \tilde{Y}(S_j)] = \frac{\sum_{i=1}^N \min(\bar{\mu}_{\tilde{Y}(S_i)}(x_i), \bar{\mu}_{\tilde{Y}(S_j)}(x_i)) + \sum_{i=1}^N \min(\underline{\mu}_{\tilde{Y}(S_i)}(x_i), \underline{\mu}_{\tilde{Y}(S_j)}(x_i))}{\sum_{i=1}^N \max(\bar{\mu}_{\tilde{Y}(S_i)}(x_i), \bar{\mu}_{\tilde{Y}(S_j)}(x_i)) + \sum_{i=1}^N \max(\underline{\mu}_{\tilde{Y}(S_i)}(x_i), \underline{\mu}_{\tilde{Y}(S_j)}(x_i))} \quad (7)$$

where $\bar{\mu}_{\tilde{Y}(S_i)}(x_i)$, $\underline{\mu}_{\tilde{Y}(S_i)}(x_i)$ are the MF values on the UMF($\tilde{Y}(S_i)$) and LMF($\tilde{Y}(S_i)$), respectively.

2) *Centroid computation*: The centroid of an IT2 FS provides a measure of the uncertainty of such a fuzzy set [32]. Centroids, $C_{\tilde{Y}(S_i)}$, are only computed for the supplier's overall performances $\tilde{Y}(S_i)$ ($i=1,2,\dots,n$) and provide a measure of uncertainty for each overall ranking, since all $\tilde{Y}(S_i)$ have propagated both numerical and linguistic uncertainties through

their computations. $C_{\tilde{Y}(S_i)}$ is the union of the centroids of all of the embedded T1 FSs of $\tilde{Y}(S_i)$; $C_l(S_i)$ and $C_r(S_i)$ are the smallest and largest element of $C_{\tilde{Y}(S_i)}$, and $C_{\tilde{Y}_M(S_i)}$ is the mean of $C_{\tilde{Y}(S_i)}$, i. e.

$$C_{\tilde{Y}(S_i)} = [C_l(S_i), C_r(S_i)] \quad (8)$$

$$C_l(S_i) = \frac{\sum_{j=1}^L x_i \mu_{\tilde{A}}(x_i) + \sum_{j=L+1}^N x_i \mu_{\tilde{A}}(x_i)}{\sum_{j=1}^L \mu_{\tilde{A}}(x_i) + \sum_{j=L+1}^N \mu_{\tilde{A}}(x_i)} \quad (9)$$

$$C_r(S_i) = \frac{\sum_{j=1}^R x_i \mu_{\tilde{A}}(x_i) + \sum_{j=R+1}^N x_i \mu_{\tilde{A}}(x_i)}{\sum_{j=1}^R \mu_{\tilde{A}}(x_i) + \sum_{j=R+1}^N \mu_{\tilde{A}}(x_i)} \quad (10)$$

$$C_{\tilde{Y}_M(S_i)} = \frac{C_l(S_i) + C_r(S_i)}{2} \quad (11)$$

In equations (9) and (10), L and R are called “switch points”, that are determined by KM Algorithms. Details on KM Algorithms can be found in [21, 24, 37-39].

3) *Rank computation*: $\tilde{Y}(S_i)$ ($i=1,2,\dots,n$) in (6) are IT2 FSs; hence, the decoder must rank such IT2 FSs to find the best supplier. According to [24, 35], there are only two ranking methods for IT2 FSs; however both methods have their limitations; hence, this paper uses a simple ranking method based on the centroids of IT2 FSs proposed in [35].

First, the average centroid $C_{\tilde{Y}_M(S_i)}$ is computed for each supplier's overall performance $\tilde{Y}(S_i)$ ($i=1,2,\dots,n$), using (8)-(11), and then $C_{\tilde{Y}_M(S_i)}$ are sorted to obtain the rank of $\tilde{Y}(S_i)$. Additionally, ranking bands of $\tilde{Y}(S_i)$, $r_{\tilde{Y}(S_i)}$, are computed, i. e.,

$$r_{\tilde{Y}(S_i)} = C_{\tilde{Y}_M(S_i)} \pm \delta_{\tilde{Y}(S_i)} \quad (12)$$

where $\delta_{\tilde{Y}(S_i)}$ are the half-lengths of each centroid $C_{\tilde{Y}_M(S_i)}$ ($n=1,2,\dots,N$), i. e.

$$\delta_{\tilde{Y}(S_i)} = \frac{C_r(S_i) - C_l(S_i)}{2} \quad (13)$$

IV. EXAMPLE

In this section, we use an example of a supplier selection decision to demonstrate that the Per-C approach is more appropriate than type-1 fuzzy set (T1 FS) approach. A high technology manufacturing company desires to select a suitable material supplier from three alternative suppliers (S_1 , S_2 , and S_3) to purchase the key components of new products. Suppose a committee of seven judges, i. e., three decision makers (D_1, D_2 and D_3) and four experts (E_1, E_2, E_3 and E_4), has been formed to determine the best of three suppliers. Table III and Table IV contain the data that will be needed in the example.

Referring to both Table III and Table IV, observe that:

(1) Quality has two sub-criteria, Price & Terms have five sub-criteria, Supply chain support has three sub-criteria, and Technology has three sub-criteria; hence, there are 13 sub-criteria all of which were established ahead of time by three decision makers.

(2) Every major criterion has three weights (in bold-face) all of which were assigned by the three decision makers using words.

TABLE III. CRITERIA WITH THEIR WEIGHTS TABLE ^a

Item	Weighting		
	D_1	D_2	D_3
Criterion1: Quality (C₁)	H	VH	H
1. Quality performance (C ₁₁)	MH	MH	VH
2. Quality containment and VDCS feedback (C ₁₂)	ML	M	MH
Criterion2: Price & Terms (C₂)	MH	H	H
1. Price (C ₂₁)	H	M	MH
2. Terms (C ₂₂)	MH	H	MH
3. Responsiveness (C ₂₃)	H	VH	VH
4. Lead time (C ₂₄)	H	M	H
5. VMI/VOI hub Set Up cost (C ₂₅)	M	MH	H
Criterion3: Supply chain support (C₃)	M	H	VH
1. Purchase order reactivity (C ₃₁)	MH	M	MH
2. Capacity support & flexibility (C ₃₂)	M	MH	M
3. Delivery/VMI operation (C ₃₃)	MH	M	H
Criterion4: Technology (C₄)	ML	M	ML
1. Technical support (C ₄₁)	L	M	M
2. Design involvement (C ₄₂)	M	M	MH
3. ECN/PCN process (C ₄₃)	VL	ML	L

^a In the 2nd and 4th column: VL=Very Low, L=Low, ML=Medium Low, M=Medium, MH=Medium High, H=High, VH=Very High (see Fig. 5)

TABLE IV. SUB-CRITERIA PERFORMANCE EVALUATION TABLE ^a

Item	Supplier 1 (S ₁)				Supplier 2 (S ₂)				Supplier 3 (S ₃)			
	E ₁	E ₂	E ₃	E ₄	E ₃	E ₂	E ₃	E ₄	E ₁	E ₂	E ₃	E ₄
Criterion1: Quality (C₁)												
1. Quality performance (C ₁₁)	G	G	VG	MG	MG	G	G	F	VC	G	MG	G
2. Quality containment and VDCS feedback (C ₁₂)	G	P	F	MP	F	G	MP	MG	MP	F	MP	MG
Criterion2: Price & Terms (C₂)												
1. Price (C ₂₁)												
2. Terms (C ₂₂)	MG	F	MP	F	G	G	MG	G	MG	G	MP	F
3. Responsiveness (C ₂₃)	MP	G	F	G	MG	MP	MP	F	F	F	MG	F
4. Lead time (C ₂₄)	G	MG	F	F	MG	F	MP	MG	F	G	MP	F
5. VMI/VOI hub Set Up cost (C ₂₅)	F	MP	MG	G	MP	G	F	MP	MG	F	MP	G
	F	MP	MG	G	MG	F	MP	F	F	G	F	MG
Criterion3: Supply chain support (C₃)												
1. Purchase order reactivity (C ₃₁)	G	MG	G	MP	F	MG	G	MG	P	MP	F	MP
2. Capacity support & flexibility (C ₃₂)	G	MG	VG	G	MP	F	F	MP	G	VG	G	G
3. Delivery/VMI operation (C ₃₃)	G	F	MG	G	F	G	MG	G	F	MP	MP	F
Criterion4: Technology (C₄)												
1. Technical support (C ₄₁)	G	F	MG	G	G	F	G	MG	VG	F	MG	G
2. Design involvement (C ₄₂)	MP	MG	F	MP	MG	F	G	MG	MP	G	F	MG
3. ECN/PCN process (C ₄₃)	F	MP	MG	P	G	MG	VG	MG	MG	F	F	G

^a In the 2nd -4th columns: VP=Very Poor, P=Poor, MP=Medium Poor, F=Fair, MG=Medium Good, G=Good, VG=Very Good (see Fig. 4).

(3) Every sub-criterion has three weights all of which were assigned by the three decision makers using words.

(4) Every sub-criterion has four performance evaluations, which are shown for all 13 sub-criteria for the three suppliers, and, are words. The word evaluations were obtained from the four experts.

How to aggregate all of these words, although seemingly very difficult, is accomplished by using (2)-(6).

Next, we provide the details for the Encoder, CWW Engine and Decoder and how the outputs from the decoder are used to decide the final winning supplier.

Step 1: Encoder

As noted in Section III. A, two codebooks are needed, one for the words that will be used by the experts and the other for the weights used by the decision maker. Two codebooks contain the words and their IT2 FS models.

(1) Codebook used by experts

Seven words were used in this sub-codebook, namely *very poor*, *poor*, *medium poor*, *fair*, *medium good*, *good*, *very good*. FOUs for these words are depicted in Fig. 4. They should be obtained by collecting data from a pool of domain managers and experts and their data intervals should be mapped into word FOU's using the IA whose details can be found in [31,36]. Because this application is being used only to illustrate how a Per-C can be used for supplier evaluation, and we did not have access to domain experts, interval end-point data were not collected for these words. Instead, we adopted the codebook from [31]. Unfortunately, none of the seven words that are actually used in this paper appear in that codebook; so, each word was mapped into a word that was felt to be a synonym for it. The relationships between the terms of the sub-criteria codebook here and that of the codebook in [31] are: *very poor*=*tiny*, *poor*=*low amount*, *medium poor*=*fair amount*, *fair*=*modest amount*, *medium good*=*good amount*, *good*=*a lot*, *very good*=*huge amount*.

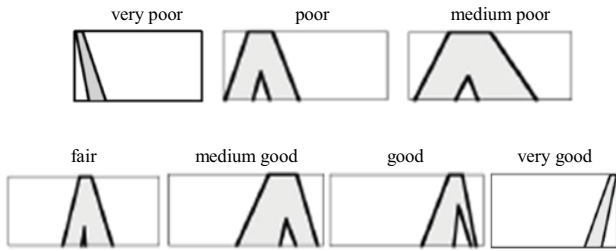


Figure 4. FOU's of the sub-criteria codebook.

For nine sub-criteria with a negative connotation [Quality containment and VDCS feedback (C_{12}), Price (C_{21}), Terms (C_{22}), Responsiveness (C_{23}), Lead time (C_{24}), VMI/VOI hub set up cost (C_{25}), Purchase order reactivity (C_{31}), Delivery/VMI operation (C_{33}), ECN/PCN process(C_{43})], antonyms were used instead of the raw words, according to (1). (2) Codebook for the weights used by the decision makers

The codebook for the weights contains the seven words *very low*, *low*, *medium low*, *medium*, *medium high*, *high*, *very high*, and their FOU's that are depicted in Fig. 5. As above, we adopted each word's synonym from the codebook in [31]. i. e. *very low*=*very little*, *low*=*little*, *medium low*=*some*, *medium*=*medium*, *medium high*=*good amount*, *high*=*large*, *very high*=*very large*.

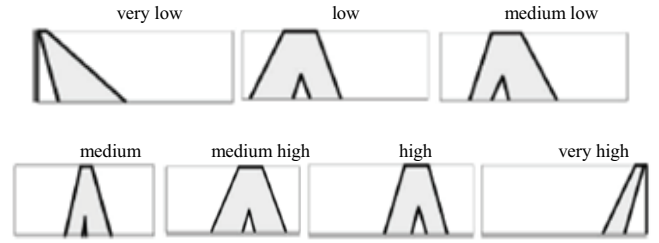


Figure 5. FOU's of the weights codebook.

Step 2: CWW Engine

The linguistic Weighted Average (LWA) was used for the CWW Engine. $\tilde{X}_{ms}^k(S_i)$ ($k=1,2,3,4$) were aggregated by a LWA to obtain $\tilde{X}_{ms}(S_i)$ ($m=1,2,3,4$; $i=1,2,3$), using (2); $\tilde{W}_{ms}^p$ and $\tilde{W}_m^p$ ($p=1,2,3$) were aggregated by LWAs to obtain $\tilde{W}_{ms}$ and $\tilde{W}_m$ ($m=1,2,3,4$), using (3) and (4), respectively. Each of them can be represented with 9 real numbers.

Each of the four major criteria has a LWA $\tilde{Y}_m(S_i)$ ($m=1,2,3,4$; $i=1,2,3$) computed for it using (5). These four LWAs are then aggregated by another LWA to obtain the overall performance for each supplier, $\tilde{Y}(S_i)$ ($i=1,2,3$), using (6).

Step 3: Decoder

The decoder computes the centroid, similarity and rank of the supplier's overall performance $\tilde{Y}(S_i)$ ($i=1,2,3$).

(1) Centroid computation

Centroids, $C_{\tilde{Y}(S_i)}$, and the mean of the centroid, $C_{\tilde{Y}_M(S_i)}$, were computed for the three supplier's overall performances $\tilde{Y}(S_i)$ ($i=1,2,3$) using (8)-(11).

(2) Similarity computation

Similarities, $SI(\tilde{Y}(S_i), \tilde{Y}(S_j))$ ($i=1,2,3$; $j=1,2,3$), between the three supplier's overall performances $\tilde{Y}(S_i)$ ($i=1,2,3$) were computed using (7).

(3) Rank computation

Rankings, $r_{\tilde{Y}(S_i)}$ ($i=1,2,3$), of the three suppliers were obtained from (12) and (13).

Step 4: Compute the evaluation results and decide the final winner

This section illustrates the supplier evaluation results for the three suppliers. In all figures, Supplier 1 is represented by the solid curve, Supplier 2 is represented by the dashed curve, and Supplier 3 is represented by the dotted curve. FOU's for Quality, Price & Terms, Supply chain support and Technology are depicted in Fig. 6 (a)-(d), respectively. FOU's for Overall Performance are depicted in Fig. 6 (e). Observe from Fig. 6(e) that not only is FOU ($\tilde{Y}(S_3)$) visually well to the right of the other two FOU's, but its average centroid (which is on the horizontal axis) is also well to the right of those for Suppliers 1 and 2. FOU ($\tilde{Y}(S_1)$) is almost the same as FOU ($\tilde{Y}(S_2)$). So, based on ranking alone, Supplier 3 would be declared the

winner. This happens because Supplier 3 ranks first in Quality and Supply chain support by a significant amount.

Although Supplier 3 ranks last for Technology, its FOU is close to those of Suppliers 1 and 2, whereas FOU of Supplier 3 for Price & Terms(C_2) is also very close to those of Suppliers 1 and 2.

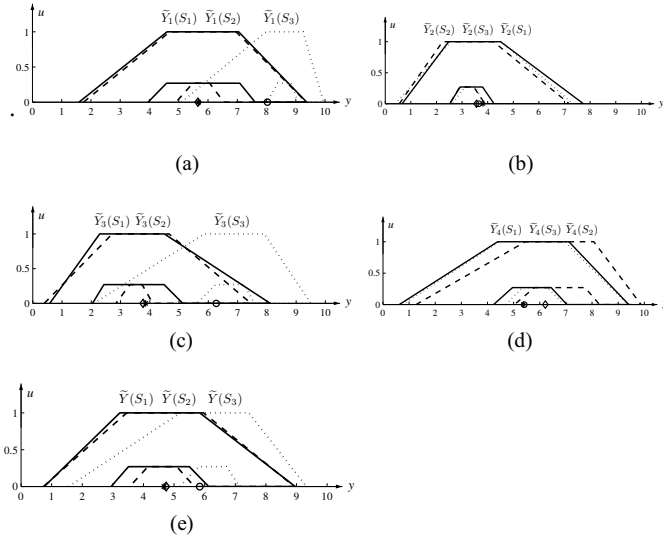


Figure 6. Aggregation results for (a) Criterion 1: Quality; (b) Criterion 2: Price & Terms; (c) Criterion 3: Supply chain support; (d) Criterion 4: Technology; and (e) Overall performances of the three suppliers. The average centroids for Suppliers A, B and C are shown in all figures by $*$, $\diamond$, and $\circ$, respectively.

Table V summarizes the similarities between $\tilde{Y}(S_1)$, $\tilde{Y}(S_2)$ and $\tilde{Y}(S_3)$. Observe that $\tilde{Y}(S_3)$ is not very similar to either $\tilde{Y}(S_1)$ or $\tilde{Y}(S_2)$, and $\tilde{Y}(S_1)$ is very similar to $\tilde{Y}(S_2)$, so choosing Supplier 3 as the winner is further reinforced.

Table VI summarizes the centroids, average centroids, half-lengths and rankings of $\tilde{Y}(S_1)$, $\tilde{Y}(S_2)$ and $\tilde{Y}(S_3)$. Observe that supplier 3 has the largest ranking, while it doesn't have the smallest uncertainty band about that ranking, but the uncertainty band about ranking of it is close to that of the Supplier 1 which has the smallest uncertainty band.

TABLE V. SIMILARITIES OF $\tilde{Y}$ FOR THREE COMPANIES

Supplier	$\tilde{Y}(S_1)$	$\tilde{Y}(S_2)$	$\tilde{Y}(S_3)$
$\tilde{Y}(S_1)$	1.00	0.93	0.56
$\tilde{Y}(S_2)$	0.93	1.00	0.58
$\tilde{Y}(S_3)$	0.56	0.58	1.00

TABLE VI. CENTROIDS, CENTERS OF CENTROID AND RANKING BANDS

Supplier	$\tilde{Y}(S_1)$	$\tilde{Y}(S_2)$	$\tilde{Y}(S_3)$
$C_{\tilde{Y}(S_1)}$	[3.17, 6.17]	[296, 6.52]	[4.25, 7.43]
$C_{\tilde{Y}_M(S_1)}$	4.67	4.74	5.84
$r_{\tilde{Y}(S_1)}$	4.67 ± 1.50	4.74 ± 1.78	5.84 ± 1.59

The seven judges ultimately decide supplier 3 is the winner by comparing the three suppliers' overall performances and similarities between them, because supplier 3's overall performance not only has the largest ranking but it also is not similar to the other two suppliers, although the similarity is greater than 0.5 that implies more similar to than less similar to, and one may be less certain about choosing Supplier 3 as the winner.

In summary, the Per-C can consider scenarios with different numbers of judges; hence, one can evaluate the robustness of the final decision.

All the computations were implemented using Matlab software under MS Windows platform. Some basic routines can be found in the personal website of Mendel (<http://sipi.usc.edu/~mendel/software>).

V. CONCLUSIONS

In order to enhance competitiveness in the market and increase customer satisfaction, the supplier selection problem for companies is very important. This paper applies a new methodology —Perceptual Computer (Per-C)—to help solve the supplier selection problem. This problem is a hierarchical group multi-criteria decision making problem in which a committee of seven (or more) judges has to decide which of three (or more) suppliers will be chosen. The Per-C has three components: encoder, CWW engine and decoder. During its design the concepts of an antonym and codebooks were needed. Without the LWA it would not have been possible to do what the Per-C has been able to do. The Per-C used the IA to obtain an IT2 FS FOU for a word; it used the LWA to aggregate all the words modeled by IT2 FSs; it used a centroid-based ranking method to rank competing suppliers; and, it also used a Jaccard similarity measure to obtain similarities of the computing suppliers.

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