

A Robust Fuzzy Trajectory Estimation Design of High Speed Reentry Vehicles

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Abstract—An H_∞ fuzzy trajectory estimation is proposed for robust tracking guidance and early warning in antitactical ballistic missile systems with model uncertainties and external disturbances. In order to avoid solving the nonlinear Hamilton-Jacobi-Isaac (H-J-I) partial differential equation for H_∞ trajectory estimation of the nonlinear reentry vehicle (RV) dynamic equation, the Takagi and Sugeno fuzzy linear model is employed to interpolate piecewise to approximate the RV nonlinear dynamic equation. Then, from the fuzzy linear model, a fuzzy H_∞ state estimator is developed for robust trajectory estimation of RV with optimal attenuation of the worst-case effect of system uncertainties such as approximation error, external disturbance, maneuver and unpredictable external forces in flight and other sources. The proposed method parameterizes the H_∞ trajectory estimation problem in terms of an eigenvalue problem (EVP), so that the worst-case effect of system uncertainties on the trajectory estimation error is minimized in the robust trajectory estimation design subject to certain linear matrix inequality (LMI) constraints. Convex optimization techniques are employed to solve the EVP of the H_∞ fuzzy trajectory estimation in antitactical ballistic missile systems. Simulation results indicate that the proposed method possesses a satisfactory performance.
Keywords—Tactical ballistic missile; H_∞ Trajectory estimation

I. INTRODUCTION

In antitactical ballistic missile (ATBM) systems, trajectory estimation of a reentry vehicle from a single radar plays an important role in radar tracking, missile guidance, and early-warning systems. Trajectory estimation of the reentry vehicle in the terminal phase is typically investigated in postflight analysis to identify state variables and key parameters from available flight data measured by radar, satellite, and on-board sensor [1]-[4]. Since the ATBM system is uncertain and nonlinear, one of the major topics behind trajectory estimation problems is related to model uncertainties, which are normally induced by simplification of assumptions, parameter uncertainty, maneuver and unpredictable external forces in flight, and other sources.

In general, an on-line optimal trajectory estimation in ATBM warfare is more difficult because one needs to solve a nonlinear partial differential Hamilton Jacobi (H-J) equation and all mathematical model quantities to be identified must

be measurable by radar [1]. Therefore, a simplified model is desired, but this may induce a more serious model uncertainty problem. Klein et al. have introduced an important stepwise regression method in the determination of air plane model structure with both model orders and control derivatives [5]. However, it is too complex to be implemented in on-line trajectory estimations. Furthermore, the on-line trajectory estimation only operates well with a simplified model and needs to maintain a low sensitivity in order to avoid uncertainties.

In recent years, trajectory estimation of a RV in antitactical missile systems has received more attention. A filtering algorithm for state and parameter estimation has been developed by Chang et al. for a maneuvering RV subject to a simplified dynamic model with maneuver forces in terms of related coefficients like drag force [7]. They proposed an augmented state vector which contains position, velocity, and corresponding parameters to drag and maneuver forces in their filter. In order to avoid solving the nonlinear estimation problem by H-J equation, the extended Kalman filter was employed to perform the trajectory estimation work. The trajectory estimation performance, however, is degraded if the maneuvering forces are absent. Furthermore, the simplified dynamic model also confines the applications to maneuvering cases and is rarely applied to real flight. Manohar and Krishnan have proposed a trajectory reconstruction algorithm for a rocket system by using a differential correction method [8]. The dynamic equations for trajectory construction contain an unmodeled acceleration term to take care of all the forces acting on the vehicle other than those from aerodynamic, propulsive, and gravitational sources. In their trajectory algorithm, trajectory states and unmodeled acceleration values are estimated simultaneously. However, the required data are beyond what a simple radar can provide.

In the past two decades, input estimation methods have been successfully applied to tracking problems and initial leveling of strapdown inertial navigation system. A trajectory estimation algorithm using input estimation technique has recently been proposed for a RV via an extended Kalman filter to provide accuracy and fast trajectory estimation with low sensitivities. Since the estimation algorithm is developed from the linearized dynamic model on the present estimated state vector, the estimation errors may propagate with increasing

estimation steps. Furthermore, input estimation needs a large amount of computations and precise statistic knowledge of external noises, which may confine its practical applications. H_∞ filter has been employed for state estimation of nonlinear systems with uncertain external noises by minimizing the worst-case effect of external noises. However, a nonlinear partial H-J-I equation needs to be solved. At present, an effective method for treating the H-J-I equation has not been found.

Recently, fuzzy techniques have been effectively used to approximate nonlinear dynamics and successfully applied to control system design. In this study, a fuzzy nonlinear H_∞ filter based on Takagi-Sugeno fuzzy is proposed. At first, the three-dimensional (3D) RV dynamic system is approximated by a fuzzy system motivated by a Takagi-Sugeno fuzzy model, which is obtained by interpolating several linearized systems at different operation points through the smoothing of fuzzy basis functions. Then, the nonlinear H_∞ estimation problem of RV systems is solved by fuzzy interpolation filtering method via an eigenvalue technique. For the convenience of design, the nonlinear H_∞ trajectory estimation problem via the fuzzy filtering approach is characterized in terms of an EVP, i.e., the worst-case effect of external disturbances on the estimation error is minimized subject to certain LMI constraints. The EVP or LMIP is to be solved via convex optimization techniques with the help of LMI optimization toolbox of Matlab. Finally, a simulation example is given to illustrate the design procedure and to confirm the estimation performance of our proposed method.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Consider the RV along a ballistic trajectory. For trajectory estimation and analysis, a mathematical model of ballistic missile system must be constructed first with some simplified assumptions to describe its motion. Fig. 1 shows a vehicle in the reentry phase over a flat, nonrotating earth with constant gravity. Since the RV has flown over several hundred kilometers along a ballistic trajectory above the rotating earth, the distance that the RV travels in this phase is shorter than the distance traveled before reentry. The RV can be considered as a point mass with constant weight following a ballistic trajectory in which two types of significant forces, i.e., drag and gravity, act on the RV. Extra forces induced by model uncertainties or disturbances must be considered when assumptions on the system are violated or the RV undertakes a maneuver. These effects can be grouped as uncertain accelerations in each axis. Suppose the RV system model in radar coordinates (O, X, Y, Z) centered at radar site can be expressed as,

$$\begin{aligned}\dot{v}_x(t) &= -\frac{\rho v^2(t)}{2\beta} g \cos \gamma_1(t) \sin \gamma_2(t) + a_x(t) \\ \dot{v}_y(t) &= -\frac{\rho v^2(t)}{2\beta} g \cos \gamma_1(t) \cos \gamma_2(t) + a_y(t) \\ \dot{v}_z(t) &= -\frac{\rho v^2(t)}{2\beta} g \sin \gamma_1(t) - g + a_z(t)\end{aligned}\quad (1)$$

Where v_x , v_y , and v_z denote the velocity components along X , Y , and Z , respectively, $a_x(t)$, $a_y(t)$ and $a_z(t)$ are uncertain accelerations due to model uncertainties and RV's maneuvering along X , Y , and Z , respectively, ρ stands for the air density. γ_1 , γ_2 and the ballistic coefficient β are defined as,

$$\begin{aligned}\gamma_1(t) &= \tan^{-1}\left(-\frac{v_z(t)}{\sqrt{v_x^2(t) + v_y^2(t) + v_z^2(t)}}\right) \\ \gamma_2(t) &= \tan^{-1}\left(\frac{v_x(t)}{v_y(t)}\right) \\ \beta &= \frac{W}{sC_{D0}}\end{aligned}\quad (2)$$

where s , W , and C_{D0} represent the reference area, weight, and zero-lift drag coefficient, respectively. (1) can be translated into the following equations,

$$\begin{aligned}\dot{v}_x(t) &= -\frac{\rho}{2\beta} g \left(\sqrt{v_x^2(t) + v_y^2(t) + v_z^2(t)} \cdot v_x(t) \right) + a_x(t) \\ \dot{v}_y(t) &= -\frac{\rho}{2\beta} g \left(\sqrt{v_x^2(t) + v_y^2(t) + v_z^2(t)} \cdot v_y(t) \right) + a_y(t) \\ \dot{v}_z(t) &= -\frac{\rho}{2\beta} g \left(\sqrt{v_x^2(t) + v_y^2(t) + v_z^2(t)} \cdot v_z(t) \right) - g + a_z(t)\end{aligned}\quad (3)$$

Let us denote the state vector as,

$$\begin{aligned}x(t) &= [x_1(t) x_2(t) x_3(t) x_4(t) x_5(t) x_6(t)]^T \\ &= [x_m(t) \ y_m(t) \ z_m(t) \ v_x(t) \ v_y(t) \ v_z(t)]^T\end{aligned}\quad (4)$$

where $x_m(t)$, $y_m(t)$, and $z_m(t)$ are the RV positions in X , Y , and Z , respectively; $v_x(t)$, $v_y(t)$ and $v_z(t)$ are the RV velocities along X , Y , and Z , respectively. Then, we get the nonlinear state equation for RV trajectories as follows:

$$\dot{x}(t) = F(x(t)) + Dw(t) \quad (5)$$

where,

$$\begin{aligned}F(x(t)) &= \begin{bmatrix} x_4(t) \\ x_5(t) \\ x_6(t) \\ -\left(\frac{\rho}{2\beta}\right) \left(\sqrt{x_4^2(t) + x_5^2(t) + x_6^2(t)} \cdot x_4(t) \right) \cdot g \\ -\left(\frac{\rho}{2\beta}\right) \left(\sqrt{x_4^2(t) + x_5^2(t) + x_6^2(t)} \cdot x_5(t) \right) \cdot g \\ -\left(\frac{\rho}{2\beta}\right) \left(\sqrt{x_4^2(t) + x_5^2(t) + x_6^2(t)} \cdot x_6(t) \right) \cdot g - g \end{bmatrix} \\ D &= \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix} \\ w(t) &= \begin{bmatrix} 0 & 0 & 0 & a_x(t) & a_y(t) & a_z(t) \end{bmatrix}\end{aligned}\quad (6)$$

The air density ρ in (3) is a function of altitude and the altitude dramatically changes as RV flies over 44 km. An

approximate model of air density ρ in g/cm³ is given as follows:

$$\rho = 0.12492((1 - 0.0000225577 \cdot z_m(t))^{4.2561}) \quad (7)$$

for $z_m(t) < 44332m$

For conventional air defense missile systems, the ground radar is the major instrument for detecting the RV. The measurement equation is then,

$$y(t) = Cx(t) + n(t) \quad (8)$$

where $n(t)$ denotes the measurement noise vector, and $C = I_{6 \times 6}$ denotes the response distribution matrix.

Suppose the following fuzzy system is employed to approximate the nonlinear RV system in (5) and (8), the i th rule of this fuzzy model for the nonlinear RV system (5) is of the following form;

System Rule i :

If $a_1(t)$ is G_{i1} and ...and $a_g(t)$ is G_{ig} ,

Then $\dot{x}(t) = A_i x(t) + Dw(t)$

$$y(t) = Cx(t) + n(t), \text{ for } i = 1, 2, \dots, l \quad (9)$$

where, G_{ij} is the fuzzy set, $A_i \in R^{6 \times 6}$, $D \in R^{6 \times 6}$, l is the number of If-Then rules, and $a_1(t), \dots, a_g(t)$ are the premise variables.

The overall fuzzy RV system is inferred as follows:

$$\dot{x}(t) = \frac{\sum_{i=1}^l \mu_i(a(t))(A_i x(t))}{\sum_{i=1}^l \mu_i(a(t))} + Dw(t) \quad (10)$$

$$y(t) = Cx(t) + n(t)$$

or,

$$\dot{x}(t) = \sum_{i=1}^l h_i(a(t))(A_i x(t) + Dw(t)) \quad (11)$$

$$y(t) = Cx(t) + n(t)$$

where,

$$\mu_i(a(t)) = \prod_{j=1}^g G_{ij}(a_j(t))$$

$$h_i(a(t)) = \frac{\mu_i(a(t))}{\sum_{i=1}^l \mu_i(a(t))}$$

$$a(t) = [a_1(t), a_2(t), \dots, a_g(t)]$$

and $G_{ij}(a_j(t))$ is the grade of membership of $a_j(t)$ in G_{ij} .

In this study, we assume

$$\mu_i(a(t)) \geq 0, \text{ for } i = 1, 2, \dots, l$$

and

$$\sum_{i=1}^l \mu_i(a(t)) > 0$$

for all t . Therefore, we get the certainty functions,

$$h_i(a(t)) \geq 0$$

(12)

for $i = 1, 2, \dots, l$ and

$$\sum_{i=1}^l h_i(a(t)) > 1$$

(13)

The physical meaning of the fuzzy system in (11) is that the local linearized systems $\dot{x}(t) = A_i x(t) + Dw(t)$ at different operation regions (different fuzzy set G_{ij}) are interpolated via the fuzzy basis functions $h_i(a(t))$ to approximate the nonlinear RV system in (6).

Therefore, from (9), we get,

$$\begin{aligned} \dot{x}(t) &= F(x(t)) + Dw(t) \\ &= \sum_{i=1}^l h_i(a(t))(A_i x(t) + Dw(t)) \\ &\quad + \left\{ F(x(t)) - \sum_{i=1}^l h_i(a(t))(A_i x(t)) \right\} \end{aligned} \quad (14)$$

where $\left\{ F(x(t)) - \sum_{i=1}^l h_i(a(t))(A_i x(t)) \right\}$ and $h_i(a(t)) \geq 0$

denotes the approximation error and interpolation function between the nonlinear RV system (6) and the fuzzy RV model (9). Let us denote the sum of approximation error and external disturbance as the following uncertain disturbance,

$$\xi(t) = \left[F(x(t)) - \sum_{i=1}^l h_i(a(t))(A_i x(t)) \right] + Dw(t)$$

Then, we get the fuzzy RV system in the following equivalent form,

$$\dot{x}(t) = \sum_{i=1}^l h_i(a(t))(A_i x(t)) + \xi(t) \quad (15)$$

$$y(t) = Cx(t) + n(t)$$

This study proposes using the following fuzzy trajectory estimator to deal with the state estimation of nonlinear RV system in (6) and (8) or the fuzzy equivalent system in (11), on Rule i :

If $a_1(t)$ is G_{i1} and ...and $a_g(t)$ is G_{ig}

$$\text{Then } \dot{\hat{x}}(t) = A_i \hat{x}(t) + L_i (y(t) - \hat{y}(t)) \quad (16)$$

where L_i is the estimator parameter to be designed for $i=1, \dots, l$. The overall fuzzy trajectory estimator in (16) is equivalent to,

$$\dot{\hat{x}}(t) = \sum_{i=1}^l h_i(a(t)) \{A_i \hat{x}(t) + L_i (y(t) - C \hat{x}(t))\} \quad (17)$$

Now let us denote the estimation error as

$$\tilde{x}(t) = x(t) - \hat{x}(t) \quad (18)$$

By differentiating (18), and from (11) and (17), it is seen,

$$\dot{\tilde{x}}(t) = \sum_{i=1}^l h_i(a(t)) \{ (A_i - L_i C) \tilde{x}(t) - L_i n(t) \} + \xi(t) \quad (19)$$

After some manipulations, (19) can be expressed in a compact form:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^l h_i(a(t)) \{ \tilde{A}_i \tilde{x}(t) + \tilde{E}_i \tilde{w}(t) \} \quad (20)$$

Where

$$\tilde{A}_i = [A_i - L_i C], \tilde{E}_i = [-L_i \quad I], \tilde{w}(t) = \begin{bmatrix} n(t) \\ \xi(t) \end{bmatrix} \quad (21)$$

In order to attenuate the effect of uncertain disturbance $\tilde{w}(t)$ resulting from the uncertain target accelerations, approximation error and measurement noise $n(t)$ on the performance of the above estimation error dynamics, the H_∞ estimation performance is proposed to attenuate the worst-case effect of $\tilde{w}(t)$ on $\tilde{x}(t)$ as follows;

$$\frac{\int_0^{t_f} \tilde{x}^T(t) Q \tilde{x}(t) dt}{\int_0^{t_f} \tilde{w}^T(t) \tilde{w}(t) dt} \leq \gamma^2, \quad \forall \tilde{w}(t) \in L_2[0, t_f] \quad (22)$$

where Q is a weighting matrix with $Q^T = Q \geq 0$ and γ^2 denotes a prescribed attenuation level which is always chosen to be less than 1. The above equation is equivalent to

$$\int_0^{t_f} \tilde{x}^T(t) Q \tilde{x}(t) dt \leq \gamma^2 \int_0^{t_f} \tilde{w}^T(t) \tilde{w}(t) dt, \quad \forall \tilde{w}(t) \in L_2[0, t_f]$$

Theorem 1: Suppose the fuzzy estimator in (17) is employed in the nonlinear trajectory estimation of RV dynamic system in (6) and (8), and $P = P^T > 0$ is the common solution of the following Riccati-like inequalities,

$$\tilde{A}_i^T P + P \tilde{A}_i + P \left(\frac{1}{\gamma^2} \tilde{E}_i \tilde{E}_i^T \right) P + Q < 0, \text{ for } i = 1, 2, \dots, l \quad (23)$$

the H_∞ estimation performance in (22) is guaranteed for the fuzzy trajectory estimator in (17) with a prescribed γ^2 in presence of external uncertain disturbance $\tilde{w}(t)$.

III. SIMULATION RESULTS

To demonstrate the performance robustness of the proposed estimation method, the following case of initial condition is considered in the simulation example.

Case1 : the reentry target is with $\beta = 2400$ (kg/m²) and maneuvering accelerations of $4g$ and $-4g$ in X and Y , respectively, during $14(\text{sec}) \leq t \leq 18(\text{sec})$

and $7(\text{sec}) \leq t \leq 9(\text{sec})$ as figure 2, respectively as Fig. 2. The following is the initial condition of the high speed reentry target:

$$\begin{aligned} x_m(0) &= 4\text{km}, & y_m(0) &= 4\text{km}, & z_m(0) &= 55\text{km}, \\ v_x(0) &= -550\text{m/s}, & v_y(0) &= -650\text{m/s}, & v_z(0) &= -1800\text{m/s}. \end{aligned}$$

and the initial condition for the estimation system is set up as follows:

$$\begin{aligned} \hat{x}_m(0) &= 6\text{km}, & \hat{y}_m(0) &= 6\text{km}, & \hat{z}_m(0) &= 70\text{km}, \\ \hat{v}_x(0) &= -900\text{m/s}, & \hat{v}_y(0) &= -800\text{m/s}, & \hat{v}_z(0) &= -1900\text{m/s}. \end{aligned}$$

As seen in Figs 3-6, the proposed nonlinear H_∞ fuzzy filter yields satisfactory estimation results under ballistic coefficient $\beta = 2400$ (kg/m²) and the strong effect of a maneuvering acceleration: step maneuver as shown in Fig. 2. Figs. 3 show the 3D profiles of the true trajectory of RV systems and their estimation result by the proposed trajectory estimation method corresponding to the above high differentiation in initial conditions between high speed reentry target and the estimator. These results imply that the proposed method is robust in overriding the effect of modeling uncertainties: modeling uncertainties of reentry targets in our design procedures include 1. model approximation errors, 2. unpredictable accelerations of targets and 3. measurement noise. Since, the proposed method has minimized the worst-case effect of modeling uncertainties in advance; the superior performance of proposed fuzzy H_∞ filter is predictable. This is reasonable because several linearized models around different operation points of the nonlinear system were first obtained by the ANFIS algorithm, hence, the interpolation model of the local linear models by the fuzzy basis function are very similar to that of the nonlinear RV system, i.e., the nonlinear RV systems are approximated piecewise by the fuzzy-interpolated local linear models with small approximation errors, and then the worst-case effect of the proposed method has been eliminated with a systematic analysis. From simulation results, it is obvious that the state trajectories of the RV system can be efficiently estimated by the proposed fuzzy H_∞ filter.

IV. CONCLUSIONS

In this study, a nonlinear fuzzy H_∞ filter is successfully developed for the state estimation of high speed reentry vehicles. In general, it needs to solve the Hamilton-Jacobi-Isaac equation for obtaining a robust filter with noise reduction and modeling uncertainty elimination ability. To

avoid solving the complicated Hamilton-Jacobi-Isaac equation in the conventional H_∞ filter design, the robust fuzzy H_∞ filter design problem is first transformed to an eigenvalue problem with the fuzzy approximation technology and then formulates it as a solvable LMI so that the trajectory estimation problem of reentry vehicles could be efficiently solved with the available computer software. The simulation results reveal that the proposed method yield accurate and satisfactory estimation ability. Simulation results also show that the proposed H_∞ filter offers the potential application in the precise trajectory estimation of RV systems.

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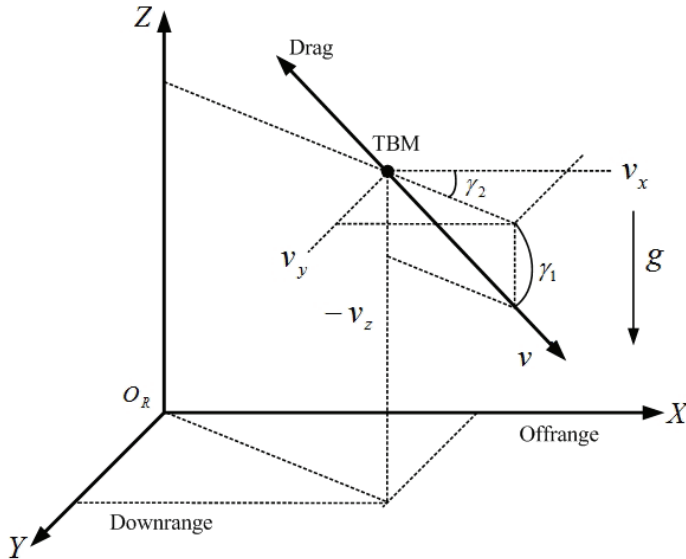


Fig.1 Tactical ballistic missile geometry

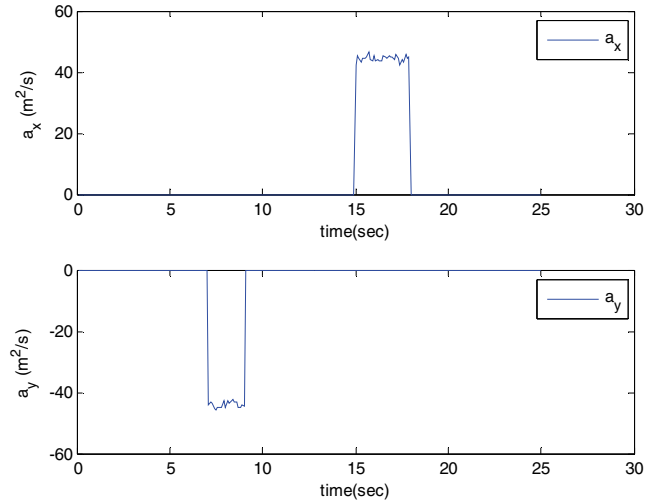


Fig.2 TBM's random maneuver in X and Y axis

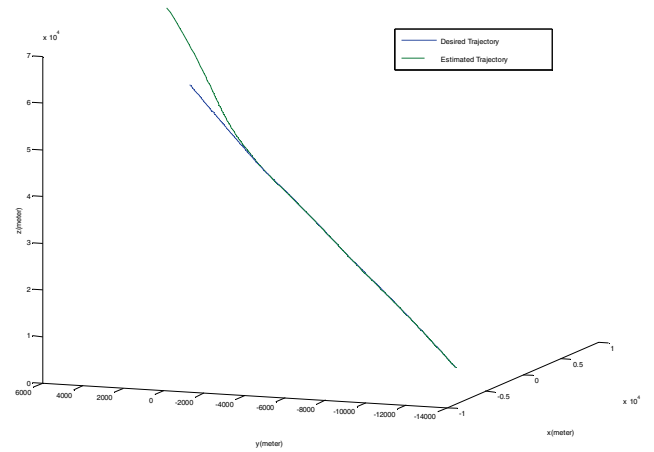


Fig.3 Three dimensional profiles of the trajectory estimations corresponding to the initial conditions of Case 1

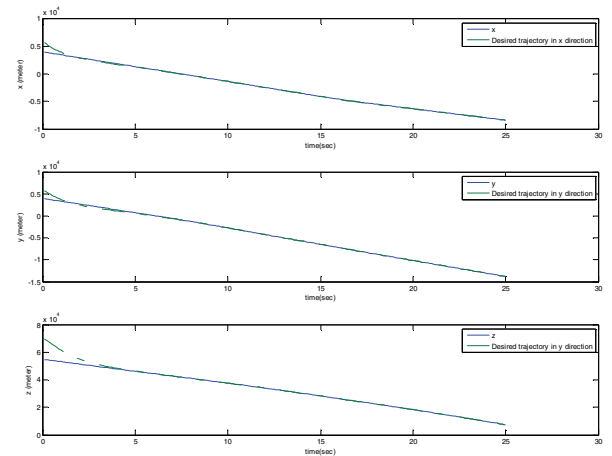


Fig.4 Estimation results of altitude x_m , downrange y_m and offrange z_m

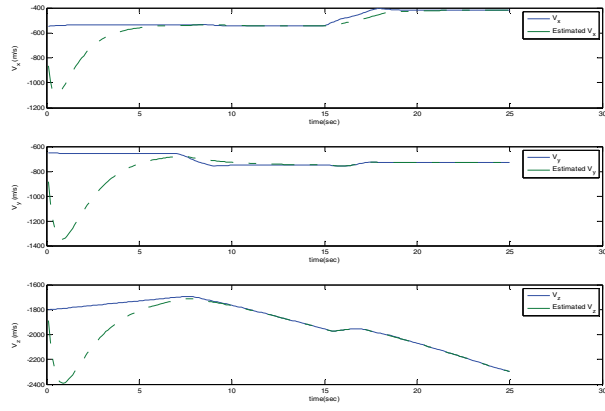


Fig.5 Estimation results of altitude v_x , downrange v_y and offrange v_z

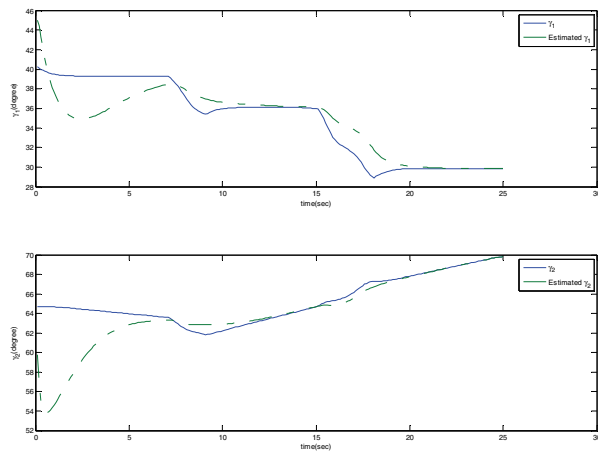


Fig.6 Estimation results of Azimuth γ_1 and elevation angle γ_2