

# An Interactive Satisficing Method for Multiobjective Random Fuzzy Programming Problems through the Possibility-Based Probability Model

Hideki Katagiri

Graduate School of Engineering  
Hiroshima University

1-4-1 Kagamiyama, Higashi-Hiroshima, 739-8527 Japan  
Email: katagiri-h@hiroshima-u.ac.jp

Masatoshi Sakawa

Graduate School of Engineering  
Hiroshima University

1-4-1 Kagamiyama, Higashi-Hiroshima, 739-8527 Japan  
Email: sakawa@hiroshima-u.ac.jp

Takeshi Matsui

Graduate School of Engineering  
Hiroshima University

1-4-1 Kagamiyama, Higashi-Hiroshima, 739-8527 Japan  
Email: tak-matsui@hiroshima-u.ac.jp

**Abstract**—This paper considers multiobjective linear programming problems where each coefficient of the objective functions is expressed by a random fuzzy variable. A new decision making model is proposed in order to maximize both of possibility and probability with respect to the objective function values. An interactive algorithm is constructed to obtain a satisficing solution for a decision maker from among a set of Pareto optimal solutions.

**Index Terms**—random fuzzy variable; multiobjective programming; interactive algorithm; possibility measure; probability maximization

## I. INTRODUCTION

In the real-world, we are often faced with decision making problems under uncertainty. Recently, many researchers focus on mathematical programming problems including not only randomness but fuzziness. One of mathematical concepts dealing with both fuzziness and randomness is a *fuzzy random variable* [1], [2], and its applications are discussed [3], [4], [5], [6], [7], [8].

On the other hand, very recently, a random fuzzy variable [11] draws attention as a new tool for decision making under uncertainty. For example, parameters such as mean and variance of random variables may not be estimated as constants. In this case, random variables with fuzzy mean are expressed as random fuzzy variables.

Some studies focus on applications of random fuzzy variable [9], [10], [11], [12], [13]. These previous studies discussed a single objective case. However, when considering the real-world decision making situation that there are several conflicting objective functions, it needs to tackle a multiobjective case under random fuzzy environments. Therefore, this article develops a new multiobjective random fuzzy programming model which obtains a satisficing solution from among a Pareto optimal solution for a decision maker.

## II. RANDOM FUZZY VARIABLE

In the framework of stochastic programming [14], [15], [16], it is implicitly assumed that the uncertain parameter which well represents the stochastic factor of real systems can be definitely expressed as a single random variable. However, from the expert's experimental point of view, the experts may think of a collection of random variables to be appropriate to express stochastic factors rather than only a single random variables. In this case, reflecting the expert's conviction degree that each of random variables properly represents the stochastic factor, it would be quite reasonable to assign the different degrees of possibility to each of random variables. For handling such an uncertain parameter, a random fuzzy variable was defined by [11] as a function from a possibility space to a collection of random variables, which is considered to be an extended concept of fuzzy variable [17]. Considering that the fuzzy variables can be viewed as another way of dealing with the imprecision which was originally represented by fuzzy sets, the random fuzzy variable can be defined as an extension of fuzzy sets using the concept of membership function, instead of fuzzy variables. In this paper, we define the random fuzzy variables along the fusion line of membership functions and random variables.

**Definition 1 (Random fuzzy variable)** Let  $\Gamma$  be a collection of random variables. Then, a random fuzzy variable  $\tilde{C}$  is defined by its membership function

$$\mu_{\tilde{C}} : \Gamma \rightarrow [0, 1]. \quad (1)$$

In Definition 1, the membership function  $\mu_{\tilde{C}}$  assigns each random variable  $\bar{\gamma} \in \Gamma$  to a real number  $\mu_{\tilde{C}}(\bar{\gamma})$ . It should be noted here that if  $\Gamma$  is defined as  $\mathbb{R}$ , then (1) becomes equivalent to the membership function of an ordinary fuzzy set. In this sense, a random fuzzy variable can be regarded as

an extended concept of fuzzy sets. On the other hand, if  $\Gamma$  is defined as a singleton  $\Gamma = \{\bar{\gamma}\}$  and  $\mu_{\bar{C}}(\bar{\gamma}) = 1$ , then the corresponding random fuzzy variable  $\bar{C}$  can be viewed as an ordinary random variable.

When taking account of the imprecise nature of the realized values of random variables, it would be appropriate to employ the concept of fuzzy random variables. However, it should be emphasized here that if mean and/or variance of random variables are specified by the expert as a set of real values or fuzzy sets, such uncertain parameters can be represented by not fuzzy random variables but random fuzzy variables.

As a simple example of random fuzzy variables, we consider a Gaussian random variable whose mean value is not definitely specified as a constant. For example, when some random parameter  $\bar{\gamma}$  is represented by the Gaussian random variable  $N(s_i, 10^2)$  where the expert identifies a set  $\{s_1, s_2, s_3\}$  of possible mean values as  $(s_1, s_2, s_3) = (90, 100, 110)$ , if the membership function  $\mu_{\bar{C}}$  is defined by

$$\mu_{\bar{C}}(\bar{\gamma}) = \begin{cases} 0.5 & \text{if } \bar{\gamma} \sim N(90, 10^2) \\ 0.7 & \text{if } \bar{\gamma} \sim N(100, 10^2) \\ 0.3 & \text{if } \bar{\gamma} \sim N(110, 10^2) \\ 0 & \text{otherwise,} \end{cases}$$

then  $\bar{C}$  is a random fuzzy variable. More generally, when the mean values are expressed as fuzzy sets or fuzzy numbers, the corresponding random variable with the fuzzy mean is represented by a random fuzzy variable, as will be discussed in the following sections.

### III. PROBLEM FORMULATION

Most of the real-world decision making problems usually involve multiple, noncommensurable, and conflicting objectives which should be considered simultaneously. In this paper, we consider the following multiobjective programming problem in which random fuzzy variables are involved in the objective functions:

$$\left. \begin{array}{l} \text{minimize } \bar{C}_1 \mathbf{x} \\ \vdots \\ \text{minimize } \bar{C}_k \mathbf{x} \\ \text{subject to } A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}, \end{array} \right\} \quad (2)$$

where  $\mathbf{x}$  is an  $n$ -dimensional decision variable column vector,  $A$  is an  $m \times n$  matrix,  $\mathbf{b}$  is an  $m \times 1$  constant column vector, and  $\bar{C}_l = (\bar{C}_{l1}, \dots, \bar{C}_{ln})$  is a random fuzzy variable coefficient vector. Here, assume that  $\bar{C}_{lj}$  is a Gaussian random variable whose mean value is a fuzzy number  $\bar{M}_{lj}$  characterized by the membership function

$$\mu_{\bar{M}_{lj}}(\tau) = \begin{cases} L\left(\frac{m_{lj} - \tau}{\alpha_{lj}}\right) & \text{if } m_{lj} \geq \tau \\ R\left(\frac{\tau - m_{lj}}{\beta_{lj}}\right) & \text{if } m_{lj} < \tau, \end{cases} \quad (3)$$

where  $L$  and  $R$  are nonincreasing continuous functions from  $[0, \infty)$  to  $[0, 1]$ .

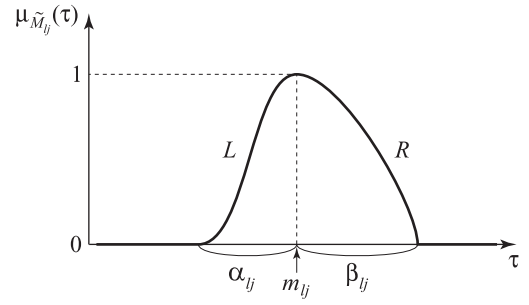


Fig. 1. Membership function  $\mu_{\bar{M}_{lj}}$ .

Let  $\Gamma$  be a collection of all possible Gaussian random variables  $N(s, \sigma^2)$  where  $s \in (-\infty, \infty)$  and  $\sigma^2 \in (0, \infty)$ . Then, the membership function of  $\bar{C}_{lj}$  is expressed as

$$\mu_{\bar{C}_{lj}}(\bar{\gamma}_{lj}) = \{\mu_{\bar{M}_{lj}}(s_{lj}) | \bar{\gamma}_{lj} \sim N(s_{lj}, \sigma_{lj}^2)\}, \quad \forall \bar{\gamma}_{lj} \in \Gamma. \quad (4)$$

Using the Zadeh's extension principle, each objective function  $\bar{C}_l \mathbf{x}$  is expressed as a single random fuzzy variable characterized by the membership function

$$\mu_{\bar{C}_l \mathbf{x}}(\bar{u}_l) \triangleq \sup_{\bar{\gamma}_l} \left\{ \min_{1 \leq j \leq n} \mu_{\bar{C}_{lj}}(\bar{\gamma}_{lj}) \mid \bar{u}_l = \sum_{j=1}^n \bar{\gamma}_{lj} x_j \right\}, \quad \forall \bar{u}_l \in \Gamma, \quad (5)$$

where  $\bar{\gamma}_l = (\bar{\gamma}_{l1}, \dots, \bar{\gamma}_{ln})$ .

By substituting (4) into (5), the membership function of a random fuzzy variable corresponding to the objective function  $\bar{C}_l \mathbf{x}$  in (2) is rewritten as

$$\begin{aligned} & \mu_{\bar{C}_l \mathbf{x}}(\bar{u}_l) \\ &= \sup_{\mathbf{s}_l} \left\{ \min_{1 \leq j \leq n} \mu_{\bar{M}_{lj}}(s_{lj}) \mid \bar{u}_l \sim N\left(\sum_{j=1}^n s_{lj} x_j, \sum_{j=1}^n \sigma_{lj}^2 x_j^2\right) \right\}, \end{aligned} \quad (6)$$

where  $\mathbf{s}_l = (s_{l1}, \dots, s_{ln})$ .

### IV. POSSIBILITY-BASED PROBABILITY MODEL

Assuming that the DM concerns about the probability that each of the objective function values  $\bar{C}_l \mathbf{x}$  is smaller than or equal to a certain target values  $f_l$ ,  $l = 1, \dots, k$ , we introduce the probability  $P(\omega \mid \bar{C}_l(\omega) \mathbf{x} \leq f_l)$  which is expressed as a fuzzy set  $\bar{P}_l$  with the membership function

$$\mu_{\bar{P}_l}(p_l) = \sup_{\bar{u}_l} \left\{ \mu_{\bar{C}_l \mathbf{x}}(\bar{u}_l) \mid p_l = P(\omega \mid u_l(\omega) \leq f_l) \right\}, \quad (7)$$

where  $f_l$ ,  $l = 1, \dots, k$  are specified by the DM as constants. In order to consider the imprecise nature of the DM's judgments for the probability with respect to the  $l$ th objective function, we introduce the fuzzy goal  $\bar{G}_l$  with the nondecreasing continuous membership function  $\mu_{\bar{G}_l}$ .

Recalling that the membership function  $\mu_{\bar{P}_l}(p_l)$  can be viewed as a possibility distribution on the basis of the concept of possibility measure [18], [19], the degree of possibility that

the fuzzy goal  $\tilde{G}_l$  for the probability  $\tilde{P}_l$  is satisfied is expressed by

$$\Pi_{\tilde{P}_l}(\tilde{G}_l) \triangleq \sup_{p_l} \min\{\mu_{\tilde{P}_l}(p_l), \mu_{\tilde{G}_l}(p_l)\}. \quad (8)$$

Then, we consider the possibility-based probability model for multiobjective random fuzzy programming problems formulated as

$$\left. \begin{array}{l} \text{maximize } \Pi_{\tilde{P}_1}(\tilde{G}_1) \\ \dots\dots\dots \\ \text{maximize } \Pi_{\tilde{P}_k}(\tilde{G}_k) \\ \text{subject to } A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0} \end{array} \right\} \quad (9)$$

or equivalently

$$\left. \begin{array}{l} \text{maximize } h_1 \\ \vdots \\ \text{maximize } h_k \\ \text{subject to } \Pi_{\tilde{P}_l}(\tilde{G}_l) \geq h_l, 0 \leq h_l \leq 1, l = 1, \dots, k \\ A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}. \end{array} \right\} \quad (10)$$

It should be noted here that (10) is regarded as a multiobjective decision making problem, and that there rarely exists a complete optimal solution that simultaneously optimizes all the objective functions. In this case, it is reasonable to find a Pareto optimal solution or a non-dominated solution.

## V. INTERACTIVE SATISFICING METHOD

By introducing an aggregation function  $f_D(\mathbf{x}, \mathbf{h})$  for the  $k$  objective functions in (10), where  $\mathbf{h} = (h_1, \dots, h_k)$ , the multiobjective programming problem (10) can be generally rewritten as

$$\left. \begin{array}{l} \text{maximize } f_D(\mathbf{x}, \mathbf{h}) \\ \text{subject to } (\mathbf{x}, \mathbf{h}) \in X, \end{array} \right\} \quad (11)$$

where  $X$  is a feasible solution of (10).

The most crucial problem in (11) is probably the identification of an appropriate aggregation function which well represents the DM's preference. If  $f_D(\mathbf{x}, \mathbf{h})$  can be explicitly identified, then (11) can be reduced to a standard mathematical programming problem, and the corresponding optimal solution is regarded as a Pareto optimal solution which is most desirable for the decision maker. However, the success in identifying such an aggregation function rarely happens, and then as an alternative way, it is recommended for the DM to examine multiple possible solutions in the set of Pareto optimal solutions through an interactive solution procedure to find a satisficing solution to (11).

In such an interactive satisficing method, to generate a candidate for a satisficing solution which is also Pareto optimal, the DM is asked to specify the aspiration levels of achievement for all the objective function values, called the reference levels.

For the DM's reference levels  $\hat{h}_l$ ,  $l = 1, \dots, k$ , the corresponding Pareto optimal solution, which is the nearest to the vector of the given reference levels in the minimax sense or

better than it if the reference levels are attainable, is obtained by solving the minimax problem

$$\left. \begin{array}{l} \text{minimize } \max_{l=1, \dots, k} \{\hat{h}_l - h_l\} \\ \text{subject to } \Pi_{\tilde{P}_l}(\tilde{G}_l) \geq h_l, l = 1, \dots, k \\ A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}. \end{array} \right\} \quad (12)$$

Here, it should be stressed that if an optimal solution  $(\mathbf{x}^*, \mathbf{h}^*)$  to (12) is not unique, then Pareto optimality of  $\mathbf{x}^*$  is not guaranteed, and just weak Pareto optimality is assured. To obtain a Pareto optimal solution in a strict sense, we consider the augmented minimax problem

$$\left. \begin{array}{l} \text{minimize } \max_{l=1, \dots, k} \left\{ \hat{h}_l - h_l + \rho \sum_{l=1}^k (\hat{h}_l - h_l) \right\} \\ \text{subject to } \Pi_{\tilde{P}_l}(\tilde{G}_l) \geq h_l, l = 1, \dots, k \\ A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}, \end{array} \right\} \quad (13)$$

where  $\rho$  is a sufficiently small positive number.

From (8), the constraint  $\Pi_{\tilde{P}_l}(\tilde{G}_l) \geq h_l$  in (13) is equivalently replaced by the condition that there exists a  $p_l$  such that  $\mu_{\tilde{P}_l}(p_l) \geq h_l$  and  $\mu_{\tilde{G}_l}(p_l) \geq h_l$ , namely,

$$\sup_{\mathbf{s}_l} \min_{1 \leq j \leq n} \left\{ \mu_{\tilde{M}_{lj}}(s_{lj}) \mid p_l = P(\omega \mid u_l(\omega) \leq f_l), \right. \\ \left. \bar{u}_l \sim N\left(\sum_{j=1}^n s_{lj}x_j, \sum_{j=1}^n \sigma_{lj}^2 x_j^2\right) \right\} \geq h_l \quad (14)$$

and  $p_l \geq \mu_{\tilde{G}_l}^*(h_l)$ , where  $\mu_{\tilde{G}_l}^*(h_l)$  is a pseudo inverse function defined as  $\mu_{\tilde{G}_l}^*(h_l) = \sup\{p_l \mid \mu_{\tilde{G}_l}(p_l) \geq h_l\}$ . This implies that there exists a vector  $(p_l, \mathbf{s}_l, \bar{u}_l)$  such that

$$\min_{1 \leq j \leq n} \mu_{\tilde{M}_{lj}}(s_{lj}) \geq h_l, \bar{u}_l \sim N\left(\sum_{j=1}^n s_{lj}x_j, \sum_{j=1}^n \sigma_{lj}^2 x_j^2\right), \\ p_l = P(\omega \mid u_l(\omega) \leq f_l), p_l \geq \mu_{\tilde{G}_l}^*(h_l),$$

which can be equivalently transformed into the condition that there exists a vector  $(\mathbf{s}_l, \bar{u}_l)$  such that

$$\left. \begin{array}{l} \mu_{\tilde{M}_{lj}}(s_{lj}) \geq h_l, j = 1, \dots, n, \\ \bar{u}_l \sim N\left(\sum_{j=1}^n s_{lj}x_j, \sum_{j=1}^n \sigma_{lj}^2 x_j^2\right), \\ P(\omega \mid u_l(\omega) \leq f_l) \geq \mu_{\tilde{G}_l}^*(h_l). \end{array} \right\} \quad (15)$$

In view of (3), it follows that

$$\mu_{\tilde{M}_{lj}}(s_{lj}) \geq h_l \Leftrightarrow s_{lj} \in [m_{lj} - L^*(h_l)\alpha_{lj}, m_{lj} + R^*(h_l)\beta_{lj}],$$

where  $L^*(h_l)$  and  $R^*(h_l)$  are pseudo inverse functions defined as  $L^*(h_l) = \sup\{t \mid L(t) \geq h_l\}$  and  $R^*(h_l) = \sup\{t \mid R(t) \geq h_l\}$ . Hence, (15) is rewritten as the equivalent condition that there exists a  $\bar{u}$  such that

$$P(\omega \mid u_l(\omega) \leq f_l) \geq \mu_{\tilde{G}_l}^*(h_l), \\ \bar{u}_l \sim N\left(\sum_{j=1}^n \{m_{lj} - L^*(h_l)\alpha_{lj}\}x_j, \sum_{j=1}^n \sigma_{lj}^2 x_j^2\right).$$

Since  $P(\omega \mid u_l(\omega) \leq f_l)$  is transformed into

$$P \left( \omega \mid \frac{u_l(\omega) - \sum_{j=1}^n \{m_{lj} - L^*(h_l)\alpha_{lj}\}x_j}{\sqrt{\sum_{j=1}^n \sigma_{lj}^2 x_j^2}} \leq \frac{f_l - \sum_{j=1}^n \{m_{lj} - L^*(h_l)\alpha_{lj}\}x_j}{\sqrt{\sum_{j=1}^n \sigma_{lj}^2 x_j^2}} \right),$$

in consideration of

$$\frac{\bar{u}_l - \sum_{j=1}^n \{m_{lj} - L^*(h_l)\alpha_{lj}\}x_j}{\sqrt{\sum_{j=1}^n \sigma_{lj}^2 x_j^2}} \sim N(0, 1),$$

(16) is equivalently transformed as

$$\Phi \left( \frac{f_l - \sum_{j=1}^n \{m_{lj} - L^*(h_l)\alpha_{lj}\}x_j}{\sqrt{\sum_{j=1}^n \sigma_{lj}^2 x_j^2}} \right) \geq \mu_{\tilde{G}_l}^*(h_l), \quad (16)$$

where  $\Phi$  is a probability distribution function of the standard Gaussian random variable  $N(0, 1)$ .

From the monotone increasingness of  $\Phi$ , (16) is rewritten as

$$\sum_{j=1}^n \{m_{lj} - L^*(h_l)\alpha_{lj}\}x_j + \Phi^{-1}(\mu_{\tilde{G}_l}^*(h_l)) \sqrt{\sum_{j=1}^n \sigma_{lj}^2 x_j^2} \leq f, \quad (17)$$

where  $\Phi^{-1}$  is the inverse function of  $\Phi$ .

From (14)–(17), it holds that

$$\begin{aligned} \Pi_{\tilde{P}_l}(\tilde{G}_l) \geq h \Leftrightarrow & \sum_{j=1}^n \{m_{lj} - L^*(h_l)\alpha_{lj}\}x_j \\ & + \Phi^{-1}(\mu_{\tilde{G}_l}^*(h_l)) \sqrt{\sum_{j=1}^n \sigma_{lj}^2 x_j^2} \leq f. \end{aligned} \quad (18)$$

Consequently, (13) is rewritten as

$$\left. \begin{aligned} & \text{minimize} \quad \max_{l=1, \dots, k} \left\{ \hat{h}_l - h_l + \rho \sum_{l=1}^k (\hat{h}_l - h_l) \right\} \\ & \text{subject to} \quad \sum_{j=1}^n \{m_{lj} - L^*(h_l)\alpha_{lj}\}x_j \\ & \quad + \Phi^{-1}(\mu_{\tilde{G}_l}^*(h_l)) \sqrt{\sum_{j=1}^n \sigma_{lj}^2 x_j^2} \leq f_l, \quad l = 1, \dots, k \\ & \quad Ax \leq b, \quad x \geq 0. \end{aligned} \right\} \quad (19)$$

Observing that (19) is a nonconvex nonlinear programming problem, we can employ genetic algorithms such as GENOCOP III [20] or the revised GENOCOP III [21] in order to obtain an approximate solution.

Now we are ready to construct an interactive algorithm for deriving the satisficing solution for the DM from among the Pareto optimal solution set.

### Interactive satisficing method for the possibility-based probability model

- Step 1: Ask the DM to specify the membership functions  $\mu_{\tilde{G}_l}$  and the target values  $f_l$ ,  $l = 1, \dots, k$ .
- Step 2: Set the initial reference possibility levels at 1s, which can be viewed as the ideal values, i.e.,  $\hat{h}_l = 1$ ,  $l = 1, \dots, k$ .
- Step 3: For the current reference possibility levels  $\hat{h}_l$ ,  $l = 1, \dots, k$ , solve the augmented minimax problem (19).
- Step 4: The DM is supplied with the corresponding Pareto optimal solution  $x^*$ . If the DM is satisfied with the current objective function values  $h_l^*$ ,  $l = 1, \dots, k$ , then stop. Otherwise, ask the DM to update the reference possibility levels  $\hat{h}_l$ ,  $l = 1, \dots, k$  by considering the current values of objective functions  $h_l$ ,  $l = 1, \dots, k$ , and return to step 3.

It should be noted for the DM that any improvement of one objective function value can be achieved only at the expense of at least one of the other objective function values for the fixed target values  $f_l$ ,  $l = 1, \dots, k$ .

## VI. CONCLUSIONS

In this paper, we have considered multiobjective programming problems in which the coefficients of the objective functions are random fuzzy variables. Incorporating the concept of possibility measure into the stochastic programming models, we have proposed a possibility-based probability model for random fuzzy programming problems. It has been shown that the formulated problem based on the proposed model can be transformed into deterministic nonlinear programming problem. In order to obtain a satisficing solution for a decision maker from among a Pareto optimal solution set, we have constructed an interactive satisficing algorithm.

In the future, we will consider more general problems in which random fuzzy variables are involved not only in the objective function but also in the constraints. Furthermore,

extensions of the proposed models to integer problems will be discussed elsewhere.

## REFERENCES

- [1] H. Kwakernaak, "Fuzzy random variable-1," *Information Sciences*, vol. 15, pp. 1–29, 1978.
- [2] M.L. Puri and D.A. Ralescu, "Fuzzy random variables. "Journal of Mathematical Analysis and Applications" vol. 14, pp. 409–422, 1986.
- [3] H. Katagiri, E.B. Mermri, M. Sakawa, K. Kato and I. Nishizaki, "A possibilistic and stochastic programming approach to fuzzy random MST problems" *IEICE Transaction on Information and Systems*, vol. E88-D, pp. 1912–1919, 2005.
- [4] H. Katagiri and M. Sakawa, "An interactive fuzzy satisficing method for fuzzy random linear multiobjective 0-1 programming problems through the expectation optimization model," *Mathematical and Computer Modelling*, vol. 40, pp. 411–421, 2004.
- [5] H. Katagiri, M. Sakawa, K. Kato and I. Nishizaki, "Interactive multi-objective fuzzy random programming: maximization of possibility and probability," *European Journal of Operational Research*, vol. 188, pp. 530–539, 2008.
- [6] H. Katagiri and M. Sakawa, "Interactive multiobjective fuzzy random programming through the level set-based probability model," *Information Sciences*, vol. 181, pp. 1641–1650, 2011.
- [7] B. Liu, "Fuzzy random chance-constrained programming," *IEEE Transaction on Fuzzy Systems*, vol. 9, pp. 713–720, 2001.
- [8] M.K. Luhandjula, "Fuzzy stochastic linear programming: survey and future research directions," *European Journal of Operational Research*, vol. 174, pp. 1353–1367, 2006.
- [9] X. Huang, "Optimal project selection with random fuzzy parameters," *International Journal of Production Economics*, vol. 106, pp. 513–522, 2007.
- [10] X. Huang, "A new perspective for optimal portfolio selection with random fuzzy returns," *Information Sciences*, vol. 177, pp. 5404–5414, 2007.
- [11] B. Liu, "Random fuzzy dependent-chance programming and its hybrid intelligent algorithm," *Information Sciences*, vol. 141, pp. 259–271, 2002.
- [12] M. Wen and K. Iwamura, "Facility location-allocation problem in random fuzzy environment: Using  $(\alpha, \beta)$ -cost minimization model under the Hurewicz criterion," *Computers and Mathematics with Application*, vol. 55, pp. 704–713, 2008.
- [13] T. Hasuike, H. Katagiri and H. Ishii, "Portfolio selection problems with random fuzzy variable returns," *Fuzzy Sets and Systems*, vol. 18, pp. 2579–2596, 2009.
- [14] I.M. Stancu-Minasian, "Overview of different approaches for solving stochastic programming problems with multiple objective functions," R. Slowinski and J. Teghem (eds.): *Stochastic Versus Fuzzy Approaches to Multiobjective Mathematical Programming under Uncertainty*, Kluwer Academic Publishers, Dordrecht/Boston/London, pp. 71–101, 1990.
- [15] P. Kall and J. Mayer, *Stochastic Linear Programming: Models, Theory, and Computation*, 2nd Edition, Springer-Verlag, New York/Dordrecht/Heidelberg/London, 2010.
- [16] G. Infanger, *Stochastic Programming: The State of the Art in Honor of George B. Dantzig*, Springer-Verlag, New York/Dordrecht/Heidelberg/London, 2010.
- [17] S. Nahmias, "Fuzzy variables," *Fuzzy Sets and Systems*, vol. 1, pp. 97–110, 1978.
- [18] D. Dubois and H. Prade, *Fuzzy Sets and Systems*. Academic Press, New York, 1980.
- [19] L.A. Zadeh, "Fuzzy sets as a basis for a theory of possibility," *Fuzzy Sets and Systems*, vol. 1, pp. 3–28, 1978.
- [20] Z. Michalewicz and G. Nazhiyath, "GenocopIII: a co-evolutionary algorithm for numerical optimization problems with nonlinear constraints," *Proceedings of the Second IEEE International Conference on Evolutionary Computation*, pp. 647–651, 1995.
- [21] M. Sakawa, *Genetic Algorithms and Fuzzy Multiobjective Optimization*, Kluwer Academic Publishers, Boston, 2001.