

A Fuzzy Tool for the Validation of Individual Risk Premia in High-Technology Venture Valuation

The Application of a Novel Framework to Convert FCMs into Rule-Based Fuzzy Systems

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Abstract - Venture valuation is an essential technique in economics. Depending on the reason for a valuation, there are several methods applicable. It becomes even more complex when it comes to the valuation of companies that possess mainly intangible assets and whose success lies in the future. In this article a further step towards a Fuzzy Method is presented and discussed that allows the determination of a relevant parameter for a widely used valuation method. We focus on the application of a novel methodological framework to convert Fuzzy Cognitive Maps into Fuzzy rule-based Inference Systems. We demonstrate the principal feasibility and elucidate a method to optimize the functionality.

Key Words Fuzzy Cognitive Map, Fuzzy Inference Systems, Fuzzy Rules, High-Technology Ventures, Company Valuation, Risk-adequate Discount Rate

I. INTRODUCTION

A. Background

There is a substantial need with key decision makers for support instruments that are robust, reliable, and above all, applicable in business real life. Particularly in the Venture Capital world, decisions that are based on unreliable or even wrong information can lead to significant losses of the invested money. This is particularly essential when it comes to valuing a young high-technology venture before an investment transaction. The inherent peculiarities of small high-technology ventures in venture capital transactions are analyzed exemplarily in [16].

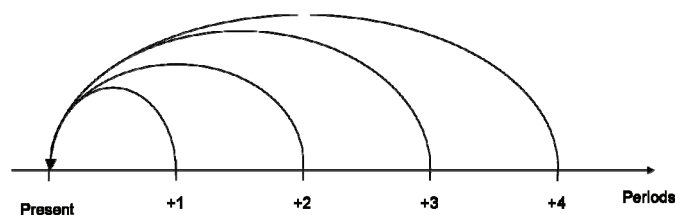


Figure 1: Principle of Future-Oriented Valuation Methods

Due to the lack of historic data, the existence of mainly tangible assets and the limited comparability to grown mature companies even in the same industry field, there is a need for specific valuation methods for young high-technology companies. A concept that has experienced a broad acceptance with practitioners in the recent years is based on the time-value-of-money concept. The main feature of Discounted Cash Flow Methods (DCF) is its future-oriented approach to valuation, i.e., prospective earnings determine the current value of the company (Fig. 1). The formal expression is shown in Equation 1. The present value PV of a company is the result of the added cash-flow CF , discounted by the discount rate r , the company is going to earn in the next i periods (Equation 1).

$$PV = \sum_{i=1}^i \frac{CF_i}{(1+r)^i}$$

Equation 1: Presented Value - Discounted Cash-Flow Method

The future development of a young high-technology company is, however, risky. Until now, no basic principle on how to consider the company or entrepreneurial risk in the context of company valuation has achieved acceptance.

One of the most popular methods with practioners is the so-called *Risk Premium Method*. This approach implies that the discount rate r consists of two components: the prime rate and an additive risk premium. The former can be relatively distinctly derived from, for example, the rate of return of risk-free fixed income assets (e.g. government bonds). The latter, however, is one the most disputed component in scientific literature on company valuation. In fact, there has not been a distinct, clear, and practicable method for exactly determining the risk premium, particularly when it comes to High-Technology Ventures. Usually, the chosen risk premia largely depends on subjective assessments of the valuer. Hence, an objective traceability is not fully given. Current research on this obvious deficit is agile, though; the aim of higher objectivity has not attained yet [3].

Obviously, the need for models and methods that can cope with uncertain prospective developments is big. Fuzzy techniques have already shown that they can be capable to

manage uncertainty and map it to formal computable structures. So, the application of Fuzzy techniques and concepts to this field is self-evident.

B. State-of-the-Art

The use of Fuzzy techniques to model relevant parameters in Finance has not been a major issue. Certain approaches have been presented, e.g., the use of fuzzy numbers to model uncertainty with investment decisions [4] or fuzzy interval methods in investment risk appraisal [5]. Takagi-Sugeno probabilistic fuzzy systems have also been used to identify relevant states of financial markets [19]. Modeling of risk has also been touched, for example when it comes to investment project risk appraisal [6]. The widely used *Capital Asset Pricing Model* (CAPM) was as well enhanced by Fuzzy concepts to model risk-return-relations under uncertainty [8], [9]. A further specific application of a Fuzzy CAPM in Insurance was presented in [7].

Besides adopting conventional Finance models with Fuzzy aspects, there have only been little advances in developing practical tools that play out the strengths of Fuzzy theory in modeling and making complex real-world systems manageable. Particularly, describing the relationship between risk and return in Venture Capital Investment has obviously been rarely touched by scientific literature. Hence, there is only a relatively small number of articles that directly deals with this issue. In [10] for example, a novel risk warning control method was presented that allows the comparison of different investment objects on the basis of risk-return-characteristics.

It becomes obvious that there is no explicit Fuzzy method that can help determine the relevant parameters for the DCF-model in the context of *High-Technology Venture* (HTV) Valuation.

C. Aim of this paper

In this article we want to discuss a novel approach to a risk-adequate company valuation. It shall allow a clear and traceable determination of the risk premium upon the basis of individual company characteristics. Essentially, our approach is based upon more realistic assumptions and does not demanding historic and comprehensive statistic data. To realize these ambitious goals, we make use of a novel method allowing the use of *Fuzzy Cognitive Maps* (FCM) as a knowledge capturing and representation tool and, in addition, as a starting point from which a rule-based *Fuzzy Inference System* (FIS) can be methodically clear derived from. As a result a crisp value for the parameter risk premium r_{ip} can be obtained.

D. Contribution

This article represents a significant further development of our original approach. The methodological fundament was presented and discussed in [1]. Therein, the initial model was established that uses verbal expert knowledge to assess the influence of specific factors on the risk of young HTVs. *Fuzzy Cognitive Maps* (FCM) were used for knowledge capturing and representation. Moreover, the FCMs can be used as the fundament for the development of rule-based *Fuzzy Inference*

Systems (FIS). A first attempt to apply this technique in the context of high-technology venture valuation was presented in [2] and [3]. The aim was to obtain a crisp parameter upon the basis of qualitative expert knowledge. Besides the starting points for further research we showed the principal feasibility of this approach.

One of the major weak points of our method was the limited accuracy and efficacy of the initial model describing the relationship between the individual company's characteristics and the inherent risk. We primarily used not fully empirically proven factors. By founding our model on the findings of statistical meta-analysis, we could present in [11] a model that realistically maps the influence factors onto the company risk. Key to this enhancement is the number and the relevance of the factors that was statistically proven by 131 single quantitative studies on this issue.

In this article a further leap ahead is presented. Besides the enhanced economic model the focus is laid on the underlying Fuzzy Inference Systems and their rule bases. A particular focus is laid on the rule definition and rule base optimization of the final FIS. The result is a far more realistic behavior of the underlying model. Thus, our model provides the principal and technical fundament for a practicable decision making tool in venture capital investments, particularly focusing on high-technology ventures in the early stage.

E. Structure

After the introduction in Section 1, the preliminaries of the used methodological framework are presented in Section 2. The novel approach is elucidated in Section 3. Besides the basics of the underlying economic model, the rule base is being optimized. Simulation outputs before and after optimization are also depicted in this Section. Subsequently, the results and further research needs are discussed in Section 4.

II. PRELEMINARIES

The detailed presentation of the methodical framework in [1], [2] is used as a conceptual fundament. Hence, in this article, only the main features of the underlying method are cursorily presented.

A. Interpretation of Nodes and Edges

The shortages of conventional FCM are overcome by transformation using the specifically developed method presented in [1]. The major assumption of the novel framework is that there is no central rule base that represents the causal relations between the nodes. In contrast, it is assumed that each causal link can be represented by a standardized rule base. A causal link of the causal strength w_{ij} between two nodes C_i and C_j depicted by an arrow in an FCM is represented by a rule-based Fuzzy Inference System FIS_{ij} (see Figure 2). Hence, each causal link in an FCM can be translated into a corresponding single-input-single-output Fuzzy Inference System. By applying this procedure, every highly-individual FCM can be translated into a network of Fuzzy Inference Systems.

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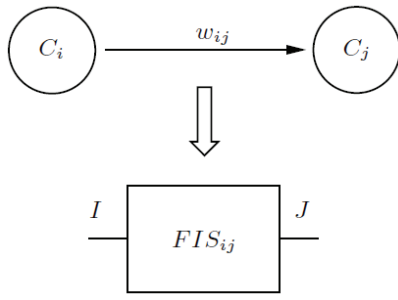


Figure 2: Representation of a single Causal Link by an FIS

B. Multiple Causal Inputs

To cope with multiple causal inputs, a method for combining multiple causal inputs is proposed being similar to the aggregation operator A proposed by Khan and Khor [17], [18]. Initially, all incoming edges are transformed to individual FIS according to the causal weight associated to them.

Thereby, the incoming signals can be weighted according to their (assumed) importance (Fig. 3). Usually, the signals are all treated equally. The outgoing signal represents the combined causal effect of the preceding cause nodes on the following effect node.

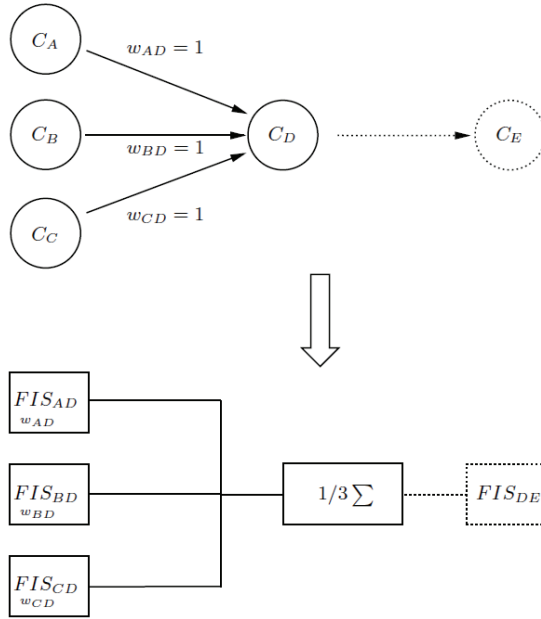


Figure 3: Interpretation of Multiple Causal Links

C. Rule-Based Causality

Each of these causalities is represented by a specific rule-based Fuzzy Inference System. The type of causality (positive or negative) as well as its strength is expressed by a specific rule base. The rule base is made for a single-input-single-output-Fuzzy Inference System. On the one hand, this decreases the calculation power needed to calculate a whole network of FIS. On the other hand, it reduces the risk of rule explosion.

The input and output parameters of the FIS correspond with the level of activation of the actual node. They are represented by I , whereas J denotes the output parameter level of activation of the effect node. Both parameters are defined on the basic set $X_{I,J} = [-1; 1] \in \mathbb{R}$. An input signal $I = 1.0$ stands for the highest activation of the actual concept. It can correspond with the highest growth of the underlying concept. In contrast to that, an input value $I = -1.0$ expresses the highest activation of the node in the negative sense; it corresponds with the largest decline of the actual concept. The parameters I and J can take 21 parameter values each. Every step at 0.1 is represented by a parameter value. The output signal J equals the value of the concept node C_j and is, thus, possibly the input signal I of a successive FIS.

Each parameter is mapped onto a Fuzzy Set. The corresponding membership function is bell-shaped or Gaussian. This form of membership function provides a more realistic modeling of the underlying concept values and their corresponding membership degrees. At the outer borders of a fuzzy set, bell-shaped membership functions allow for a less harsh end and beginning of the membership. But, the closer the value gets to center c of the fuzzy number, the stronger the membership degree grows. This non-linearity corresponds with the real-world understanding of fuzzy observations (Fig. 4).

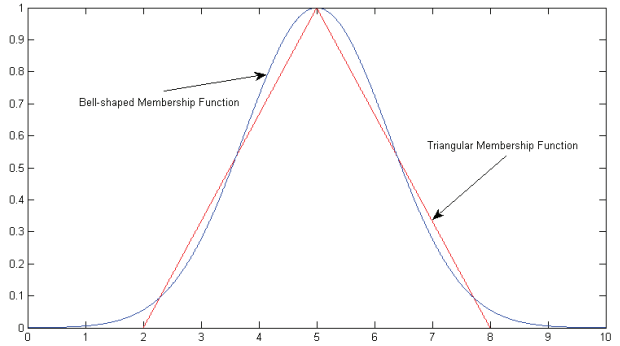


Figure 4: Triangular vs. Gaussian Membership Function

The set of membership functions representing the input and output parameter I and J is shown in Fig. 5.

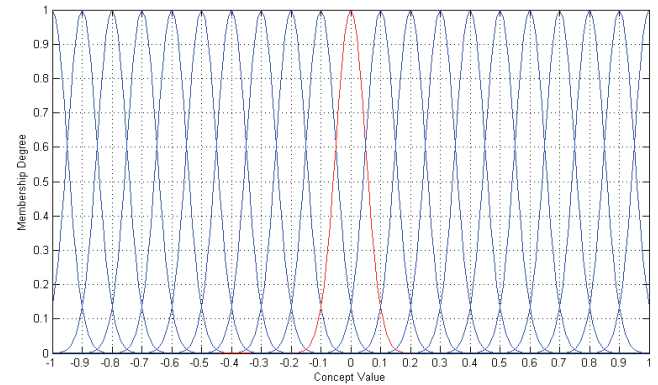


Figure 5: Membership Functions of the Parameters I and J

Next, the causal linkage between the two parameters has to be defined. To model (non-linear and non-symmetric) causality a specified rule base is designed. Each causality, and hence each causal strength is represented by a specific rule base. The IF-part of the rule consists of only one input I . This signal equals the fuzzy number of the concept node value C_i . Due to the discretization in steps of 0.1, the rule base is limited to a reasonable number of rules. Correspondingly, the THEN-part of the rule also consists of only one de-fuzzified output signal J . To determine the individual rule bases, the general understanding of the causal relationship between input and output parameter has to be highlighted. A positive causal relationship between C_i and C_j means that if C_i increases (less/much) then C_j also increases (less/much). A positive causal relation that has the causal strength $w_{ij} < 1$ leads, thus, to a weakened increase of C_j in case C_i increases. This weakening effect grows the lower the causal strength is.

For example, a concept value of $C_i = 1.0$ may lead only to a concept value $C_j = 0.2$. The same holds true for a decline when there is a positive causal relation. An exemplary rule base representing a positive causal strength $w = 1.0$ and the corresponding output profile is shown in Figure 5 and Figure 6, respectively. Analogue to this example there are specific rule bases for all possible causal weights w , $w \in [-1; 1]$. In the herein proposed methodological framework there is a total of 11 specific rule bases (increment of causality is 0.2, ranging from -1 to 1).

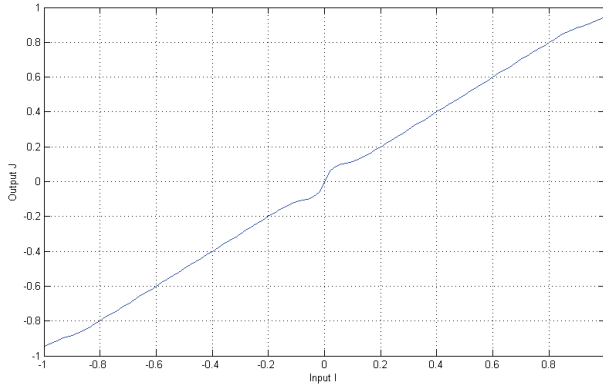


Figure 6: Output Profile of the FIS for Causality $w_{ij} = 1.0$

D. Crisp Output

To eventually obtain a crisp output, a specified component is needed: a final FIS_f. Its purpose is to take all directly preceding causal links and to 'calculate' a final output value. The input value is obtained by aggregating all incoming causal links (inputs) by using the aggregation operator (Figure 3). Its value ranges from +1 (highest increase) to -1 (highest decrease). The input value is mapped onto the corresponding output value by a specific rule base depending on the actual context. By applying this methodological framework it is possible to describe and model any complex system and its behavior by means of FCMs. Subsequently, this model of a complex system can be analyzed quantitatively, that means, a crisp value is returned.

Rule1:	IF	$I = -1.0$	THEN	$J = -1.0$
Rule2:	IF	$I = -0.9$	THEN	$J = -0.9$
Rule3:	IF	$I = -0.8$	THEN	$J = -0.8$
Rule4:	IF	$I = -0.7$	THEN	$J = -0.7$
Rule5:	IF	$I = -0.6$	THEN	$J = -0.6$
Rule6:	IF	$I = -0.5$	THEN	$J = -0.5$
Rule7:	IF	$I = -0.4$	THEN	$J = -0.4$
Rule8:	IF	$I = -0.3$	THEN	$J = -0.3$
Rule9:	IF	$I = -0.2$	THEN	$J = -0.3$
Rule10:	IF	$I = -0.1$	THEN	$J = -0.2$
Rule11:	IF	$I = 0$	THEN	$J = 0$
Rule12:	IF	$I = 0.1$	THEN	$J = 0.1$
Rule13:	IF	$I = 0.2$	THEN	$J = 0.2$
Rule14:	IF	$I = 0.3$	THEN	$J = 0.3$
Rule15:	IF	$I = 0.4$	THEN	$J = 0.4$
Rule16:	IF	$I = 0.5$	THEN	$J = 0.5$
Rule17:	IF	$I = 0.6$	THEN	$J = 0.6$
Rule18:	IF	$I = 0.7$	THEN	$J = 0.7$
Rule19:	IF	$I = 0.8$	THEN	$J = 0.8$
Rule20:	IF	$I = 0.9$	THEN	$J = 0.9$
Rule21:	IF	$I = 1.0$	THEN	$J = 1.0$

Figure 7: Rule Base for Causality $w_{ij} = 1.0$

III. APPLICATION

A. Basic Assumptions

For presenting a novel method to determine a relevant parameter for venture valuations, it is necessary to model the relationship between this parameter r_p and the influencing factors. Referring to [4] and [12], the main factors are listed in Table 1.

Table 1: Significant Meta-Factors of Success [12]

Metafactor	N	K	ρ	Expl. Var	X_s
Market Scope	1,046	10	0.21	100%	78
Industry Experience	423	4	0.11	100%	2
Marketing Experience	381	3	0.11	100%	2
Financial Resources	638	6	0.12	100%	14
Firm Age	1,890	15	0.16	87%	157
Patent Protection	453	5	0.11	100%	157
Size of Founding Team	332	5	0.13	100%	6
Supply Chain Integration	604	6	0.23	89%	41
<i>N</i> ...aggregate sample size <i>K</i> ...number of correlation in the underlying studies that build the metafactor <i>ρ</i> ...estimate of the real population correlation <i>X_s</i> ...critical number of null-studies					

Using these factors as the conceptual starting point, five experts were asked to construct an FCM upon these factors and weigh the causal interrelations between them. The result is depicted in Fig. 8.

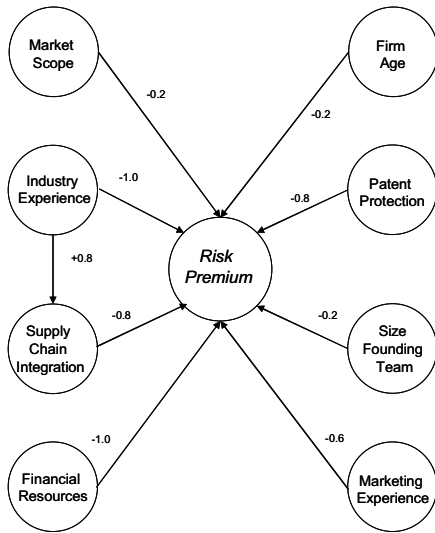


Figure 8: Expert FCM

The corresponding adjacency matrix E_{fin} is given by:

$$E_{fin} = \sum_{n=1}^k a_k E_{ij}^k = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.6 \\ 0 & 0 & 0.8 & 0 & 0 & 0 & 0 & 0 & -1.0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

B. Fuzzy Inference System and Rule Base

According to the methodological framework discussed in [1], this FCM can be converted into a network of FIS. To eventually obtain a crisp value a final FIS is designed. Its position in the network of FIS is analogue to the configuration in Fig. 3. The final FIS maps the aggregated auxiliary parameter R onto the output parameter r_{rp} that is the crisp output. The design of the final FIS is of particular relevance as it transforms the model's causal relationships into one final crisp output. To obtain a realistic behavior the underlying relationship is modeled according to empirical findings on the relationship between risk and return rates and risk premia, respectively.

We conduct the process of Fuzzy Engineering as described by Kosko [14]. For constructing the rule base, we lean on the findings on different high technology ventures and corresponding return rates and risk premia that were observed in the recent past by Achleitner [13]. According to the used methodology, the input parameter of the final FIS_f consists of the aggregated antecedent causal inflows in the parameter *Risk*

Premium. It integrates all impacting factors into one single figure.

Corresponding to the general understanding of nodes and their concept values the values R can take range in the interval -1 to 1 . Thus, $X_R = [-1; 1] \subseteq R$. The observed risk premium r_{rp} in the Early-Stage sector scatters between 10% p.a. and 75% p.a. whereas about 50% of the values lie in the interval between 20% p.a. and 50% p.a. Thus, $X_{rp} = [10; 75] \subseteq R$. Consequently, this distribution can be a guideline for determining the membership functions' form and position for representing the risk premia. The empirically observed risk premia concentrates in the interval $[20; 45]$. The borders of this interval represent the upper 75-quartile and the lower 25-quartile of the empirical distribution. This is modelled by using asymmetrically bell-shaped membership functions.

The modeling of the concentration of risk premia in the interval $[25; 50]$ is supported by centring the membership functions stepwise at every five percent (20%, 25%, 30%, as well as, 40%, 45%, 50%). Outside this interval, the membership functions are centred at every ten percent (10%, 20% and 50%, 60%, 70%).

The median $r_{rp} = 35\%$ is depicted by a special form of the bell-shaped curve. It rather covers an interval than representing a single fuzzy value. Here, the Fuzzy Number $r_{rp}^{\sim} = 35$ stands for the interval between $r_{rp} = 30$ and $r_{rp} = 40$. This should strengthen modelling the risk premia's concentration around the median. The lower and upper borders of the interval, namely the return rates $r_{rpl} = 10\%$ and $r_{rpu} = 75\%$ are represented by triangular membership functions. Thus, a clear and distinct border is modelled guaranteeing that there are no risk premia smaller than 10% p.a. or greater than 75% p.a. The resulting membership functions are shown in Fig.9. A detailed description of this process can be found in [4].

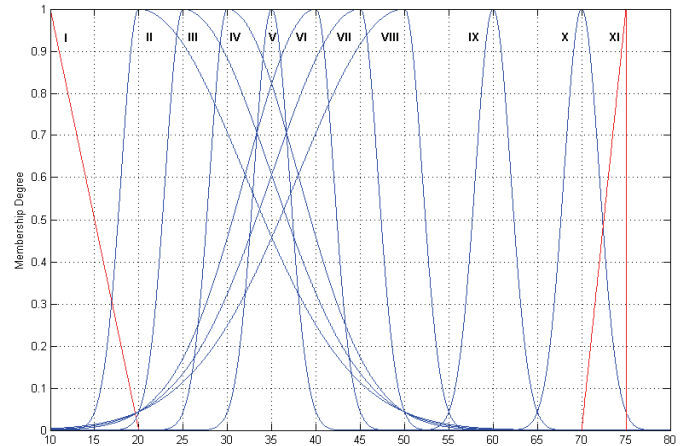


Figure 9: Membership Functions Representing the Empirically Observed Risk Premia

C. First Results

The simulation was carried out using the Fuzzy Logic Toolbox (Version V2.2.5 R2007a) for MatlabTM (Version R 2007a). The Fuzzification is realized by singleton sets. The

Implication is carried out by using the *Min*-method. The *Max*-method is used as the aggregation operator. The *Center-of-Gravity*-method is used for de-fuzzification. In Fig. 10 the obtained preliminary output profile is depicted.

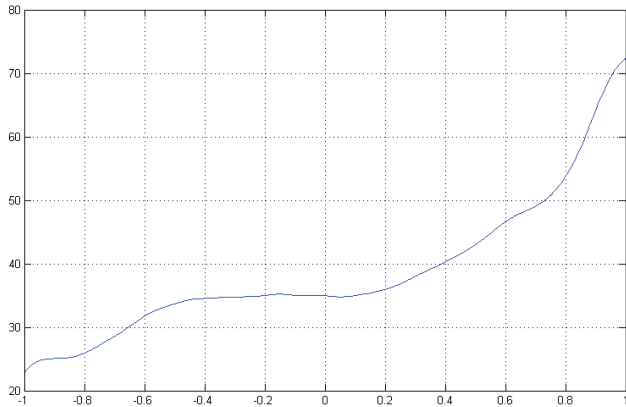


Figure 10: Output Profile of the final FIS

The depicted output profile fundamentally corresponds with the empirical findings of [12]. Below neutral point $R = 0$, the increase of the risk premium r_{tp} is rather slow. Beyond the point $R = 0$, the increase of r_{tp} is above-average denoting the tendency to increase the risk premia when the risk rises especially above a normal level. However, the mapping is still to be improved. Particularly, the peaks and unevenness of the output profile is to be bettered.

D. Rule Base Optimization

Not every rule is needed to approximate an arbitrary continuous function. The *Fuzzy Representation Theory* states that a Fuzzy Systems only needs to cover the extrema of the approximant f (Fig. 11).

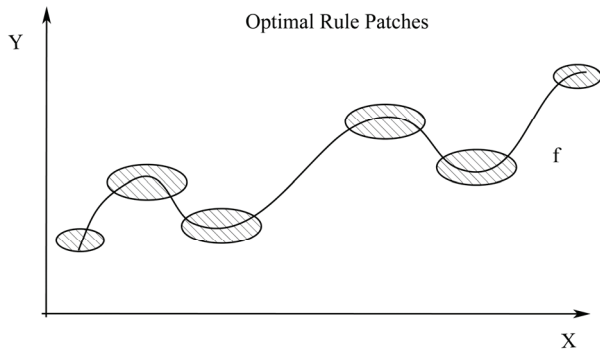


Figure 11: Rule Patches covering Extrema of the Approximant f

A proof of this theorem, made by Watkins [15], even states that two rules covering the turning points are sufficient to fully approximate the function. This, however, requires the exact knowledge about the function's shape. Despite of any theoretical limitations, this theorem gives a clear hint where to begin with rule optimization.

Table 2: Rule Base Covering the Extrema

Rule1	IF	$R = -1.0$	THEN	$r_{tp} = 10\%$
Rule2	IF	$R = 0$	THEN	$r_{tp} = 35\%$
Rule3	IF	$R = 1.0$	THEN	$r_{tp} = 75\%$

1) Rules Covering the Extrema

The rule base optimization with the FIS rule base starts with a rule coverage at the extremes $(-1.0; 10\%)$, $(1.0; 75\%)$ and at the point $(0; 35\%)$ (Table 2). The resulting output profile can be seen in Fig. 12. However, the transitions are still too abrupt and the mapping is far too symmetrical. It does not yet possess the targeted shape and mapping compared to the preliminary output profile in Fig. 10.

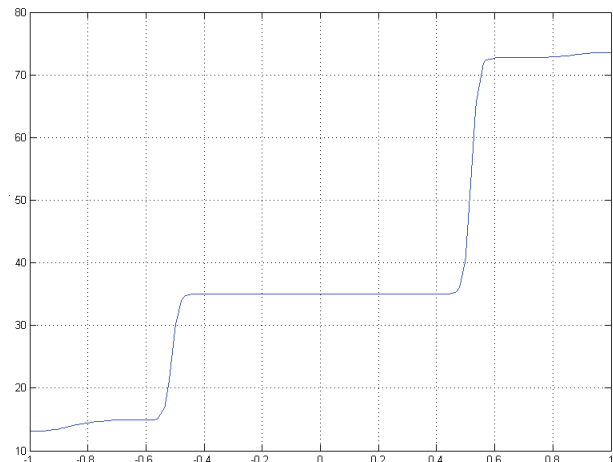


Figure 12: Output Profile after the 1st Optimization Step

2) Rules Covering the Extrema and the right-hand Side

($R > 0$). The rule base is enlarged by seven rules covering the right-hand side beyond $R = 0$ (Table 3). This results in a clearly smoother increase of r_{tp} when R rises over $R = 0$. The left-hand side $R < 0$ is still characterized by a too harsh, step-wise increase of r_{tp} (Fig. 13).

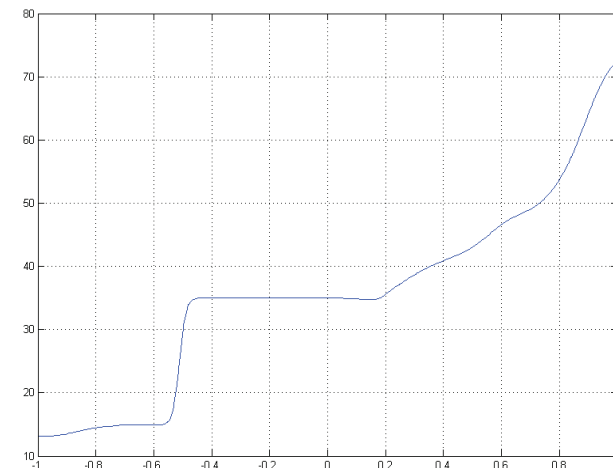


Figure 13: Output Profile after the 2nd Optimization Step

Table 3: Rule Base 2nd Optimization Step

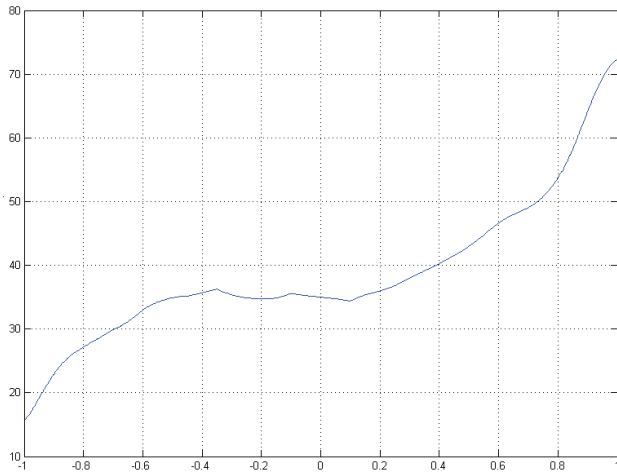
Rule1	IF	R= -1.0	THEN	$r_{rp} = 10\%$
Rule2	IF	R= 0	THEN	$r_{rp} = 35\%$
Rule3	IF	R= 0.2	THEN	$r_{rp} = 40\%$
Rule4	IF	R= 0.4	THEN	$r_{rp} = 45\%$
Rule5	IF	R= 0.5	THEN	$r_{rp} = 50\%$
Rule6	IF	R= 0.6	THEN	$r_{rp} = 50\%$
Rule7	IF	R= 0.7	THEN	$r_{rp} = 60\%$
Rule8	IF	R= 0.8	THEN	$r_{rp} = 75\%$
Rule9	IF	R= 0.9	THEN	$r_{rp} = 75\%$
Rule10	IF	R= 1.0	THEN	$r_{rp} = 75\%$

3) Rule covering the Extrema and both Sides

Consequently, this next step mainly focuses on the rule coverage of the left-hand side ($R < 0$). Particularly, the peak at $R = -0.2$ and the slight trough at $R \approx 0.2$ disrupt the continuity needed for the intended output profile. By deleting rules on the left-hand side of the mapping ($R < 0$) the peak at $R \approx -0.2$ is lessened. The same holds true for the slight trough at $R \approx 0.2$ which is mainly due to the added rule number seven (Fig. 14).

Table 4: Rule Base 3rd Optimization Step

Rule1	IF	R= -1.0	THEN	$r_{rp} = 10\%$
Rule2	IF	R= -0.9	THEN	$r_{rp} = 10\%$
Rule3	IF	R= -0.7	THEN	$r_{rp} = 25\%$
Rule4	IF	R= -0.5	THEN	$r_{rp} = 30\%$
Rule5	IF	R= -0.2	THEN	$r_{rp} = 30\%$
Rule6	IF	R= 0	THEN	$r_{rp} = 35\%$
Rule7	IF	R= 0.2	THEN	$r_{rp} = 40\%$
Rule8	IF	R= 0.3	THEN	$r_{rp} = 45\%$
Rule9	IF	R= 0.4	THEN	$r_{rp} = 50\%$
Rule10	IF	R= 0.5	THEN	$r_{rp} = 50\%$
Rule11	IF	R= 0.6	THEN	$r_{rp} = 60\%$
Rule12	IF	R= 0.7	THEN	$r_{rp} = 75\%$
Rule13	IF	R= 0.8	THEN	$r_{rp} = 75\%$
Rule14	IF	R= 0.9	THEN	$r_{rp} = 75\%$
Rule15	IF	R= 1.0	THEN	$r_{rp} = 75\%$

Figure 14: Output Profile after the 3rd Optimization Step

4) Optimal Rule Base

By reducing the rules on the left-hand side as well as on the right-hand side to finally twelve (Table 5), the behaviour of the mapping of R to r_{rp} is optimal. It is attained that there is a nearly monotonic growth of r_{rp} when R increases. But the increase of growth rate of r_{rp} depends on the actual value of R . The higher R grows, the stronger becomes the increase of r_{rp} .

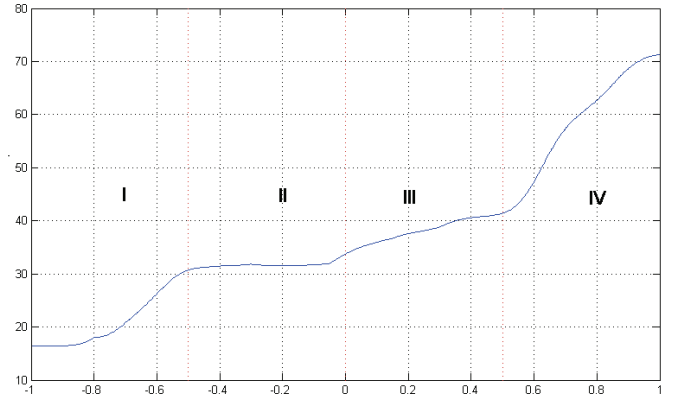


Figure 15: Simulation Results: Output Profile after Rule-Base Optimization

This optimal profile (Fig. 15) is the result of the optimized rule base (Table 5). This profile can be subdivided into four sections denoting the different behaviour of the FIS depending on the actual height of the aggregated risk R .

Table 5: Optimized Rule Base

Rule1	IF	R= -1.0	THEN	$r_{rp} = 10\%$
Rule2	IF	R= -0.7	THEN	$r_{rp} = 10\%$
Rule3	IF	R= -0.5	THEN	$r_{rp} = 20\%$
Rule4	IF	R= -0.4	THEN	$r_{rp} = 25\%$
Rule5	IF	R= -0.2	THEN	$r_{rp} = 25\%$
Rule6	IF	R= -0.1	THEN	$r_{rp} = 30\%$
Rule7	IF	R= 0	THEN	$r_{rp} = 30\%$
Rule8	IF	R= 0.1	THEN	$r_{rp} = 40\%$
Rule9	IF	R= 0.2	THEN	$r_{rp} = 40\%$
Rule10	IF	R= 0.4	THEN	$r_{rp} = 45\%$
Rule11	IF	R= 0.7	THEN	$r_{rp} = 60\%$
Rule12	IF	R= 1.0	THEN	$r_{rp} = 75\%$

Section I. In Section I, the risk parameter is below -0.5. Obviously, the risk is significantly smaller than usual. This is also expressed by a rather slow growth of the risk premium r_{rp} in the interval $[-1; 0.8)$. In the range $-0.8 < R \leq -0.5$, however, the risk premium r_{rp} grows stronger than in the preceding sector. This denotes the positive relationship between risk and risk premium even in cases that appear to be lesser risky than normal.

Section II. The second section is characterized by an almost constant mapping of the risk parameter R to the risk premium r_{rp} . This signifies the vague understanding of risk being only somewhat smaller than usually observed. The interval $-0.5 < R < 0$ stands for the wider range of high-

technology ventures whose risk is, at most, normal. But their risk profile not justifies yet a significant abatement of the risk premium r_{rp} .

Section III. Closely before R becomes zero or positive, the return rate r begins to grow steadily. This indicates that in case the high-technology venture appears to be more risky than average the required return rate r is used as a compensation for every unit risk that is taken more. This nearly linear relationship spans over the whole section III ($0 < R < 0.5$).

Section IV. After the risk parameter R has grown above 0.5, the growth of r_{rp} becomes above-average. This signifies that when a high-technology venture appears to be significantly riskier than the average ($R > 0.5$), the growth of the return rate r is no longer linear. It nearly becomes exponential. This is due to the fact that an investor only engages in a more-than-usual risky venture if there is an exceptional risk premium. Summing up, by optimizing the rule base, a more realistic behaviour of our model could be attained.

IV. DISCUSSION

In prior articles [1], [2], [3], [4], the principal feasibility of our approach was presented and discussed. In this article, we presented a further development of the approach. In detail, it comprises an enhanced economic model based upon statistically proven factors. Moreover, an optimized rule base was proposed. By these features, a way was demonstrated how to validate the risk premium, the most disputed component in DCF-valuation methods. Besides these promising results, there is still a lot of work to be done.

The herein proposed rule base is built and optimized manually. The conceptual basis for this is the theorem elaborated by Watkins [15]. Hence, future research will be focused on integrating learning algorithms to build the final rule base. This can lead to an even more objective rule base and even more realistically modeled system behavior. One suitable approach has already been identified: in [11] genetic algorithms are used to build and to optimize the rule base. The combination of our methodological framework with genetic algorithms for rule base building seems promising since genetic algorithm, as bio-inspired optimization methods in general, does not demand for an exact mathematical model. This feature significantly alleviates the application in the here presented context in which exact are intentionally avoided.

An improvement of the modeling of the economic relationships and the determination of the relevant factors is out of the scope of this article. The authors will look out for advancements on this topic. The herein used model provides the flexibility to adapt changing parameters and concepts.

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REFERENCES

- [1] L. Krueger, K2F – A Novel Framework for Converting Fuzzy Cognitive Maps into Rule-Based Fuzzy Inference Systems”, in Lecture Notes on Artificial Intelligence and Soft Computing, L. Rutkowski et al. (Eds), Vol.6113, Pt. 1, 2010, pp. 128-135
- [2] L. Krueger, M. Klofsten, P. Heydebreck et al. “A Fuzzy Cognitive Map-based Approach to the Determination of appropriate Discount Rates for the Valuation of High-Technology Ventures in the Early Stages of Venture Capital Investments”, in: Proceedings of the 14th International Business Information Management Association Conference IBIMA, June 23-23, 2010, Istanbul, Turkey
- [3] L. Krueger, M. Klofsten, P. Heydebreck “ K2F – A Methodical Framework to use Fuzzy Cognitive Maps for the Determination of adequate Discount Rate in Venture Valuations”, in Communications of the IBIMA, in press Jan 2011
- [4] L. Krueger, P. Heydebreck, R. Salomon et al. “Validation of Subjectively Determined Risk Premia in High-Technology Ventures Investments using a Novel Framework for Converting Fuzzy Cognitive Maps into Rule-Based Fuzzy Inference Systems”, 2011, unpublished
- [5] R. Simonelli “Fuzziness in Valuing Financial Instruments by Certainty Equivalents”, in European Journal of Operational research, 2001, Vol. 135, Is. 2, 2011, pp. 296-302
- [6] A. Serguieva “Fuzzy Interval Methods in Investment Risk Appraisal”, in: Fuzzy Sets and Systems, Vol. 142, Is. 3, 2004, pp. 443-466
- [7] B. Rebasz “Fuzziness and Randomness in Investment Project Risk Appraisal”, in: Computer Operations Research, Vol. 34, Is.1, 2007, pp. 199-210
- [8] L.-H. Lai “Underwriting Systematic Risk and Profit Margin in Fuzzy CAPM and ICAPM Models: The Case of Aviation Coverage”, in: Contemporary Management Research, Vol. 5, No. 4, 2009, pp. 387-396
- [9] S.K. Blatt et al. “Capital Asset Pricing Modell under Fuzzy Information”, in: Proceedings of the Annual Conference of the Administrative Sciences Association of Canada, ASAC Conference June 14-17, 2003, Halifax, Nova Scotia, Canada
- [10] Hasuikie, T. et al. “Multiobjective Random Fuzzy Portfolio Selection Problems based on CAPM”, in: Systems, Man and Cybernetics 2009, Oct. 11-14, 2009, San Antonio, TX, USA
- [11] O. Cordon, F. Herra, P. Villar, “Generating the Knowledge Base of a Fuzzy Rule-Based System by the Genetic Learning of the Data Base”, in: IEEE Transactions on Fuzzy Systems, Vol. 9 Is. 4, 2001, pp. 667-674
- [12] Song, M. Podoyntsyna K., van der Bij H., J. Halman, „Success Factors in New Ventures. A Meta Analysis”, in: Journal of Product Innovation Management, 25, 2008, pp. 7-27
- [13] A. Achleitner, E. Nathusius “Venture Valuation and Venture Capital Financing “ (German), in: Business Studies (Wirtschaftswissenschaftliches Studium), No.3, 2004, pp. 134-139
- [14] B. Kosko, “Fuzzy Engineering”, Prentice Hall, 1997, New Jersey
- [15] F.A. Watkins “The Representation Problem for Additive Fuzzy Systems”, in Proceedings of the IEEE FUZZ-95, Vol. 1, pp.117-122, 1995
- [16] M. Klofsten, M. Jonsson, J. Simon, „Supporting the Pre-Commercialisation Stages of Technology-based Firms: the Effects of Small-Scale Venture Capital“, in: International Journal of Venture Capital, Vol. 1, Is. 1, 1999, pp. 43-52
- [17] M.S. Khan and S.W. Khor “Framework for Fuzzy Rule-Based Cognitive Maps”, Lecture Notes in Computer Science, (2004), 454-463
- [18] M.S. Khan A. Chong, M. Quaddus “Fuzzy Cognitive Maps and Intelligent Decision Support - A Review”, Journal of Systems Research and Information Science, 26, (2004), 257-268
- [19] J. van den Berg, U. Kaymak, W.M. van den Bergh „Financial Markets Analysis by using a probabilistic Fuzzy Modelling Approach”, in: International Journal of Approximate Reasoning, Vol. 35, Is.3, (2004), 291-305