

Short Term Load Forecasting Using Interval Type-2 Fuzzy Logic Systems

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Abstract—Accurate Short Term Load Forecasting (STLF) is essential for a variety of decision making processes. However, forecasting accuracy may drop due to presence of uncertainty in the operation of energy systems or unexpected behavior of exogenous variables. This paper proposes the application of Interval Type-2 Fuzzy Logic Systems (IT2 FLSs) for the problem of STLF. IT2 FLSs, with extra degrees of freedom, are an excellent tool for handling prevailing uncertainties and improving the prediction accuracy. Experiments conducted with real datasets show that IT2 FLS models appropriately approximate future load demands with an acceptable accuracy. Furthermore, they demonstrate an encouraging degree of accuracy superior to feedforward neural networks used in this study.

Index Terms—Load forecasting, type-2 fuzzy logic.

I. INTRODUCTION

By definition, forecasting is a phenomenon of calculating or estimating a measure in the next coming time periods. Tools and models for accurate forecasting of electric power load are essential to the operation and planning of utility companies. Load forecasts are used by participants in electric energy generation, transmission, distribution, and markets for a variety of decision-making processes, such as economy dispatch, unit commitment, hydro-thermal coordination, transaction evaluation, and expansion planning.

There has always been a need for accurate forecasting of future load demands. However, the need has intensified in the last decade due to deregulation of the energy industries in developed countries. As the energy prices may be boosted by a factor of ten or more during periods of peak demands, precise short and medium term load forecasting becomes vitally important to the utilities. Taking into account this as well as rapid fluctuations of demands and abrupt changes in weather condition, access to reliable models for accurate prediction of load demand is essential.

Studies conducted by scholars and industry practitioners for load forecasting can be divided into two broad categories: (i) traditional methods, such as Box-Jenkins models, regression models, Autoregressive Integrated Moving Average (ARIMA) (in general time series), Kalman filtering models, and splines [1] [2], [3] [4]; and (ii) Artificial Intelligence (AI) methods, such as Neural Networks (NNs), Fuzzy Logic Systems (FLSs), and expert systems. Models in the former category are typically linear and cannot cope with the level of nonlinearity and seasonality present in the behavior of electric loads. There exists no best method that always outperforms others in the

field of load forecasting. However, recent reviewing studies in the field of prediction and forecasting show that AI-based models, and in particular NNs, often outperform the traditional modeling techniques in terms of prediction accuracy and generalization power [5].

Both NN and FLSs are universal approximators with the capability of identification and approximation of nonlinear relationships between independent (inputs) and dependent (targets) variables to any arbitrary degree of accuracy [6] [7]. While black-box NN models have their excellent learning capabilities, transparent FLS models can simultaneously handle quantitative and qualitative information, and as so uncertainties. Both model types are of the best alternatives for modeling, prediction, and forecasting purposes. A good review of NN-based Short Term Load Forecasting (STLF) can be found in [6] and [7] (and references within)¹. Recently Khosravi et al. [9] [10] investigated the application of NN-based prediction intervals for quantifying uncertainties associated with forecasted loads. They showed that prediction intervals provide more information about uncertainties present in the process of load forecasting. However, construction of prediction intervals using the proposed methods is computationally expensive.

Studies on application of FLSs and neuro-fuzzy systems for STLF can be found in [11] [12] [13]. The Achilles heel of traditional FLSs is their precise Type-1 Fuzzy Sets (T1 FSs). Once the type-1 Membership Functions (MFs) are selected and adjusted, all uncertainties disappear. Type-2 FLS (T2 FLS) is a new emerging paradigm that uses Type-2 Fuzzy Sets (T2 FS). T2 FSs are characterized by a three dimensional fuzzy MF including a footprint of uncertainty. The footprint of uncertainty and the third dimension of MFs provide additional degrees of freedom that make it possible to directly model and handle uncertainties. Therefore, a T2 FLS has the potential to outperform a Type-1 FLS (T1 FLS) [14].

Zadeh [15] originally introduced the concept of type-2 fuzzy sets as an extension of the concept of an ordinary type-1 fuzzy set in 1975. Mendel [16] and Karnik et al. [17] [18] further developed the theory of T2 FLSs. The structure of rules in the T2 FLS and its inference engine are similar to those in T1 FLS. A T2 FLS includes a fuzzifier, a rule base, fuzzy inference engine, and an output processing unit composed of a type reducer and a defuzzifier (Fig. 1).

¹ Similar studies for the case of electricity price forecasting can be found in [8].

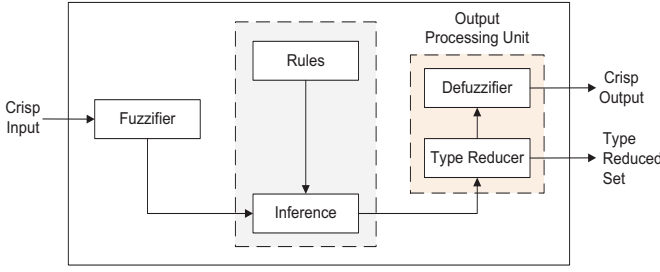


Fig. 1. Structure of a T2 FLS.

The computational complexity of general T2 FLSs has until recently hindered their widespread use in practical applications. As a restricted class of T2 FLSs, Interval T2 FLSs (IT2 FLSs) [19] were introduced to avoid massive computational requirement (mainly due to type reduction stage). IT2 FLSs are characterized by secondary MFs that only take the value of 1 over their domain. Such a restriction greatly reduces the computational burden of performing inference compared with general T2 FLSs. In the last few years, T2 FLSs, and in particular, to IT2 FLSs have drawn a great deal of attention from both academia and industry. IEEE Computational Intelligence Magazine, February 2007 contains several papers reviewing theory and application of T2 FLSs and IT2 FLSs [20] [21] [22]. Also more recent work on application of IT2 FLSs and their combination with neural networks can be found in [23] [24].

Motivated by these recent trends in academia, this paper investigates the use of IT2 FLSs for STLF problem. Takagi-Sugeno-Kang (TSK) fuzzy inference systems are used for developing IT2 TSK FLSs for load forecasting. It is expected that load forecasting accuracy will be improved due to excellent capabilities of IT2 FLSs in handling uncertainties in data. Feedforward NN models are also used for STLF problem and their forecasting performance and accuracy is compared with performance of IT2 TSK FLS models.

The rest of this article is organized as follows: Section II introduces IT2 TSK FLSs. Section III describes how IT2 TSK FLSs are developed and trained in this study. Experimental procedure and used datasets are discussed in Section IV. Simulation results are demonstrated in Section V. Finally, we draw conclusions and discuss future work in section VI.

II. INTERVAL TYPE-2 TSK FUZZY LOGIC SYSTEMS

IT2 TSK FLSs are TSK fuzzy models, where (at least) antecedent MFs are IT2 FSs. As per this definition, three classes of models can be formed:

- A2-C1: Antecedents are IT2 FSs, and consequents are interval T1 FSs.
- A2-C0: Antecedents are similar to the case of A2-C1 model, however, consequent parameters are crisp (similar to the case of traditional type-1 TSK fuzzy models).
- A1-C1: Antecedents and consequents are both type-1 fuzzy sets.

We use class A2-C1 in this study, as it is the most general form of IT2 TSK FLS models with the largest degrees of freedom. The extra flexibility of these models allows for efficient handling of uncertainties and minimizing their effects on the quality of forecasts.

In an IT2 TSK FLS (A2-C1) with a rule base of M rules in which each rule has p antecedents, let the l th rule be denoted by R^l ,

R^l : If x_1 is $\tilde{F}_1^l$, x_2 is $\tilde{F}_2^l, \dots$, and x_p is $\tilde{F}_p^l$, then

$$y^l = C_0^l + \sum_{i=1}^p C_i^l x_i \quad (1)$$

where $l = 1, \dots, M$. $\tilde{F}_i^l$ is the i th IT2 FS ($i = 1, \dots, p$) composed of a lower and upper bound MFs,

$$\mu_{\tilde{F}_i^l}(x_i) = [\underline{\mu}_{\tilde{F}_i^l}(x_i), \bar{\mu}_{\tilde{F}_i^l}(x_i)] \quad (2)$$

C_i^l is also an interval T1 FS, where its center and spread are c_i^l and s_i^l respectively,

$$C_i^l = [c_i^l - s_i^l, c_i^l + s_i^l] \quad (3)$$

Given an input $x = (x_1, x_2, \dots, x_p)$, the result of the input and antecedent operations (firing strength) is an interval type-1 set, $F^l = [\underline{f}^l, \bar{f}^l]$, where,

$$\underline{f}^l(x) = \underline{\mu}_{\tilde{F}_1^l}(x_1) * \underline{\mu}_{\tilde{F}_2^l}(x_2) * \dots * \underline{\mu}_{\tilde{F}_p^l}(x_p) \quad (4)$$

$$\bar{f}^l(x) = \bar{\mu}_{\tilde{F}_1^l}(x_1) * \bar{\mu}_{\tilde{F}_2^l}(x_2) * \dots * \bar{\mu}_{\tilde{F}_p^l}(x_p) \quad (5)$$

where $*$ represents a t-norm. It is assumed that singleton fuzzyfier is used in obtaining (4) and (5).

y^l in (1) is the output from the l th If-Then rule, which is a T1 FS, $Y^l = [y_L^l, y_R^l]$. y_L^l and y_R^l are evaluated as,

$$y_L^l = \sum_{i=1}^p c_i^l x_i + c_0^l - \sum_{i=1}^p s_i^l |x_i| - s_0^l \quad (6)$$

$$y_R^l = \sum_{i=1}^p c_i^l x_i + c_0^l + \sum_{i=1}^p s_i^l |x_i| - s_0^l \quad (7)$$

The final output of the IT2 TSK FLS model is obtained through combining the outcomes of M rules, as shown in (8).

$$Y = [y_L, y_R] = \frac{\int_{y^1 \in [y_L^1, y_R^1]} \dots \int_{y^M \in [y_L^M, y_R^M]} \frac{1}{\sum_{l=1}^M \bar{f}^l y^l}}{\sum_{l=1}^M \bar{f}^l} \quad (8)$$

Before generating a crisp output, the outputs of the inference engine should be type-reduced and then defuzzified.

Unfortunately, there is no direct theoretical solution (closed-form formula) for calculation of y_L and y_R in (8). However, they can be calculated using the iterative Karnik-Mendel (KM) procedure [18] for type reduction (transferring a T2 FS into a T1 FS using the concept of center of sets). In the KM algorithm, y_L^l are reordered in ascending order. A switch point, L , is iteratively found that minimizes the value of y_L . The same procedure can be applied for calculation of y_R , where a switch point, R , is determined for maximizing y_R . y_L and y_R are given below,

$$y_L = \frac{\sum_{l=1}^L \bar{f}_i^l y_L^l + \sum_{l=L+1}^M \underline{f}_i^l y_L^l}{\sum_{l=1}^L \bar{f}_i^l + \sum_{l=L+1}^M \underline{f}_i^l} \quad (9)$$

$$y_R = \frac{\sum_{l=1}^R \underline{f}_i^l y_R^l + \sum_{l=R+1}^M \bar{f}_i^l y_R^l}{\sum_{l=1}^R \underline{f}_i^l + \sum_{l=R+1}^M \bar{f}_i^l} \quad (10)$$

Details of the KM type reduction algorithm can be found in [18] [25]. Finally, the defuzzified crisp output from the IT2 TSK FLS is the mean of y_L and y_R ,

$$y = \frac{y_L + y_R}{2} \quad (11)$$

III. STRUCTURE AND TRAINING OF IT2 TSK FLS MODELS

A. Membership Functions of IT2 TSK FLS

The performance of an IT2 TSK FLS depends on several factors, such as type and quantity of MFs, training algorithm, number of inputs, and the amount of available data for training. The antecedent MFs used in this study are assumed to be Gaussian with a fixed mean and uncertain standard deviations,

$$\mu_{\bar{F}_i^l}(x_{i,k}) = \exp \left[-\frac{1}{2} \left(\frac{x_{i,k} - m_i^l}{\sigma_{i,1}^l} \right)^2 \right] = \mathcal{N}(m_i^l, [\sigma_{i,1}^l, \sigma_{i,2}^l]) \quad (12)$$

where $i = 1, \dots, p$, $l = 1, \dots, M$, and k indicates the sample index. The upper and lower MFs are,

$$\mu_{\bar{F}_i^l}(x_{i,k}) = \mathcal{N}(m_i^l, \sigma_{i,1}^l) \quad (13)$$

$$\mu_{\underline{F}_i^l}(x_{i,k}) = \mathcal{N}(m_i^l, \sigma_{i,2}^l) \quad (14)$$

Fig. 2 shows the graphical representation of these two MFs for arbitrary chosen values for m_i^l , $\sigma_{i,1}^l$, and $\sigma_{i,2}^l$. The uniformly shaded region is the footprint of uncertainty for the IT2 Gaussian MF.

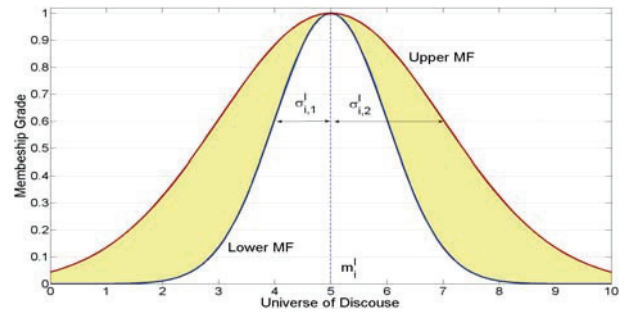


Fig. 2. The IT2 Gaussian MFs with fixed mean and uncertain standard deviation.

B. Genetic Algorithm for Training

The Genetic Algorithm (GA) is used for adjusting parameters of the IT2 TSK FLS models. By simultaneously exploring different parts of the parameter space, GAs increase the probability of finding the global solution to the minimization problem. The GA includes three main operators, namely reproduction (elitism), crossover, and mutation. Offsprings (new populations) are generated through crossover and mutation. Reproduction is a process in which individual chromosomes are copied according to their scaled fitness function values. Mutation introduces random changes to the chromosomes by altering the value to a gene with a probability called the mutation rate. Crossover operator determines how the GA combines two parents to form an offspring (crossover child) for the next generation.

Once a decision on the number of inputs and MFs per input is made, we can code the IT2 TSK FLS parameters into a chromosome. In each rule, premise parameters (mean and two standard deviations of each input membership function) and consequent parameters, as shown in (3), are coded as real variables and allowed to take real values. The Root Mean Squared Error (RMSE) is considered as the fitness function,

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (Act_i - Pre_i)^2} \quad (15)$$

where Act_i and Pre_i are the i th actual load and the i th forecasted load. n is also the number of load samples.

Parameters of premise and consequent parts of IT2 TSK FLS (A2-C1) are randomly initialized. Exploration of solution space continues until termination conditions are met. The maximum number of iterations, a minimum convergence speed, and a satisfactory small value of RMSE are the stopping criteria used in our experiments. After termination of the GA optimization, test samples are used for examining performance of the trained model.

Table I lists parameters used in the optimization algorithm. The cross over method is single point. This randomly selects a crossover point within a chromosome, and then interchanges the two parent chromosomes at this point to produce two new offspring. Parameters summarized in Table I are obtained through trial and error. The preliminary experiments were

TABLE I

PARAMETERS USED IN EXPERIMENTS AND GA-BASED TRAINING METHOD

Parameter	Numerical value
Population size	100
Parent selection	Stochastic uniform
Reproduction (elite count)	2
Cross over fraction (single point)	0.8
Number of MFs	3
Type of MFs	Gaussian (Fig. 2)
D_{train}	80% of all samples
D_{test}	20% of all samples

carried out with the aim to ensure optimization convergence within a short time (minimum number of generations), but to avoid premature convergence.

IV. DATA AND EXPERIMENT PROCEDURE

A. Data

The dataset in this study is the same that used previously in [9] and [10]. The purpose is to forecast the two-day-ahead loads based on past observations. The inputs to the IT2 TSK FLS models have been listed in Table II. These inputs are the mean of loads in the last three hours, the current temperature, and the day of week (coded as a number between 1 and 7). Other combinations of inputs and inclusion of lagged loads (up to even one year ago) can improve the quality of load forecasting results, however, determination of such a subset is beyond the scope of this paper. Furthermore, inclusion of more inputs will substantially increase the complexity of developed models.

The profile of loads for three years is shown in Fig 3. There is seasonal behavior with rapid fluctuations in the amount of consumed loads. Load peaks occur in summer, where load demand is minimum in winter. Additionally, there is an upward trend in the curve, which indicates an annual growth load demand.

IT2 TSK FLSs are data-driven techniques, and their forecasting performance intimately hinges on data used for their training. With the purpose of an objective assessment, the available dataset is split into training samples (80%, D_{train}) and test samples (20%, D_{test}). The IT2 TSK FLS is first trained using training samples, where the fitness function (RMSE) is minimized using the GA.

All available data are pre-processed to have zero mean and unit variance. This is done in particular for inputs (those listed in Table II) to give them an equal chance for contribution to models and load forecasting.

For the purpose of comparison, we benchmark performance of IT2 TSK FLS models by feedforward Neural Network (NNs). A two layer NN with 10 neurons in each hidden layer is trained using the Levenberg-Marquardt training algorithm. Simulations and forecasting procedure are repeated twenty times and averaged results are reported. This is done to avoid misleading results and subjective judgment due to random initialization of NN parameters.

TABLE II

INPUTS OF IT2 TSK FLS FOR STLFL

Input	Quantity
Load demand in the last three hours	3
Current temperature	1
Day of the week	1

B. Performance Evaluation

Prediction performance of the developed models for STLFL can be measured using three statistical measures. Coefficient of determination, designated by R^2 , is one of the best choices, as it provides a measure of target variation captured by the model. R^2 is calculated as follows,

$$R^2 = 1 - \frac{\sum_{i=1}^n (Act_i - Pre_i)^2}{\sum_{i=1}^n (Act_i - \overline{Act})^2} \quad (16)$$

where Act_i , Pre_i , and $\overline{Act}$ are the i th actual load, the i th forecasted load, and the mean of loads, respectively. A coefficient of determination close to unity (or 100%) is an indication that forecasted values are close to actual data, and performance of the developed model is acceptable.

The two other performance measures used are RMSE (which also used as the fitness function) and correlation coefficient.

V. RESULTS AND ANALYSIS

The convergence behavior of the optimization algorithm and minimization of the fitness function (RMSE) is illustrated in Fig. 4. The fitness function is reduced several orders of magnitude from its initial value. The fitness function continuously decreases at a high rate up to generation 60. The optimization process then slows and converges to the optimal solution at iteration 109. During optimization, the fitness function drops from 0.5019 to 0.1283. This indicates that randomly selected initial values for IT2 TSK FLS (A2-C1) models are not optimal, leading to a large forecasting error.

Fig. 5 shows the plot of forecasted load demands versus their actual values. It is evident that there is a good agreement between the observed and forecasted load demands. The correlation coefficient between these values is 89.92%, which is sufficiently large. Also, forecasting errors are shown in Fig. 6. In some cases, the gap between actual and forecasted values is large. The maximum forecasting error (underestimation) is 3185 MWs. Such a large error can be due to unknown patterns in data or lack of informative variables.

Table III summarizes quantitative values of three performance measures for results obtained from IT2 TSK FLS (A2-C1) and NN models. These results are for test samples, which were not used for model training. The performance measures for NNs are averaged values obtained from 20 replicates of experiments. The main conclusions from the results listed in Table III are as follows:

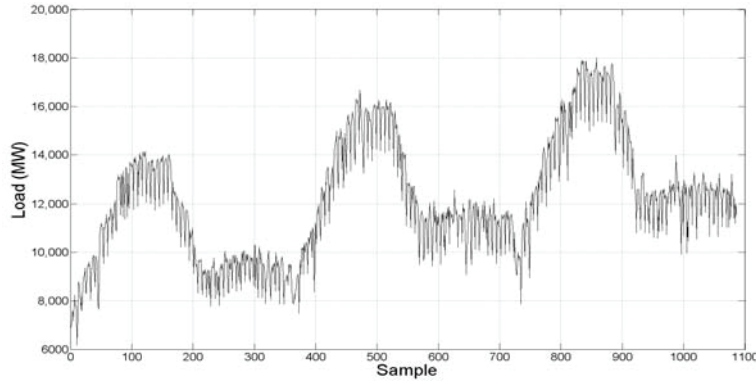


Fig. 3. The profile of loads used in this study.

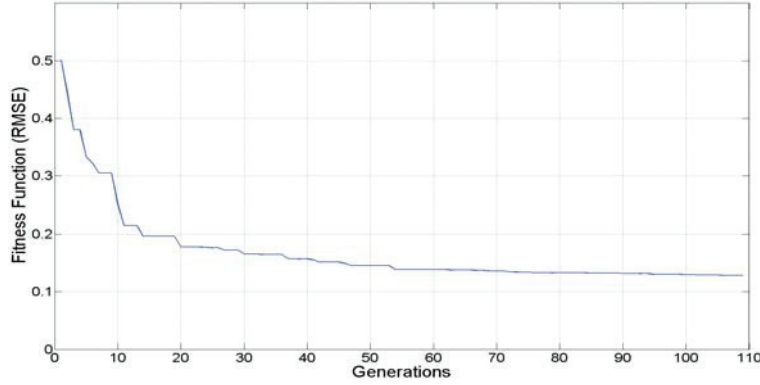


Fig. 4. Illustration of the convergence history of optimization algorithm for minimization of fitness function (RMSE) and adjusting parameters of IT2 TSK FLS (A2-C1).

- IT2 TSK FLS (A2-C1) models accurately forecast two day ahead load demands for test samples. A large R^2 and a small RMSE are indications of satisfactory results.
- The performance of IT2 TSK FLS (A2-C1) model is superior to multilayer NN models for STLF. This superiority is due to use of IT2 MFs in our forecasting models. This supports the claim that T2 FLSs have an excellent capability in handling uncertainties and minimizing their effects.
- NN models cannot appropriately cope with uncertainties in data. These uncertainties significantly degrade performance of NN models and cause a large forecasting error. This is due to fact that NNs are deterministic models and show a poor performance in case of multivalued targets [9] [26] [10] [27].

TABLE III
PERFORMANCE MEASURES FOR IT2 TSK FLS AND NN MODELS

Performance Measure	Neural Network	IT2 TSK FLS (A2-C1)
Correlation coefficient	86.48%	89.92%
RMSE	0.1685	0.1394
R^2	62.52%	77.02%

STLF performance of IT2 TSK FLS (A2-C1) models can be further improved through the use of other types of IT2 MFs

with more degrees of freedom. An example is the Gaussian MF with uncertain mean and standard deviation. The number of MFs per inputs is 3 in experiments conducted (totally 81 rules). Increasing resolution of models by considering more MFs will improve the forecasting performance of models. The forecasting error for more complicate models (with more MFs) will also be further decreased in case of applying more generations in GAs for training of models. Inclusion of more inputs and using a validation set will also improve the accuracy of forecasting results obtained from both IT2 TSK FLS (A2-C1) and NN models.

VI. CONCLUSION

In this paper, interval type-2 TSK fuzzy logic systems are used for the problem of short term load forecasting. Lagged load demands and weather and calendar information are used as inputs to the forecasting model. Created models are trained using the genetic algorithm, where mean squared error is the cost function. The conclusion of this research is that short term power demand can be accurately forecasted using interval type-2 TSK fuzzy logic systems. The extra degrees of freedom of these models provide the analyzers with sufficient capacity for modeling nonlinear relationships and handling of uncertainties. The coefficient of determination between actual and forecasted loads is 77%, which is approximately 15%

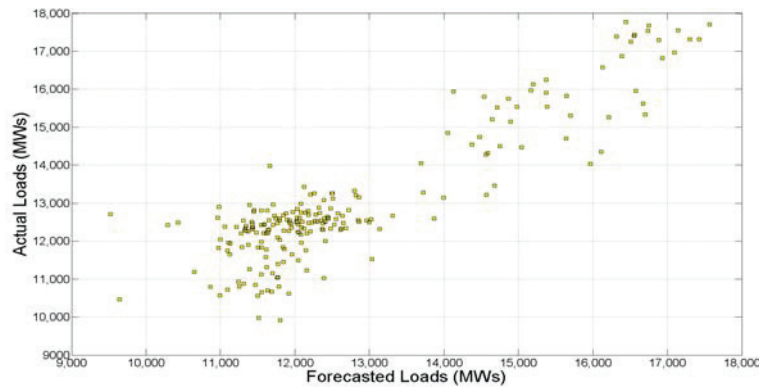


Fig. 5. Forecast vs. actual power demand with a correlation coefficient equal to 89.92%.

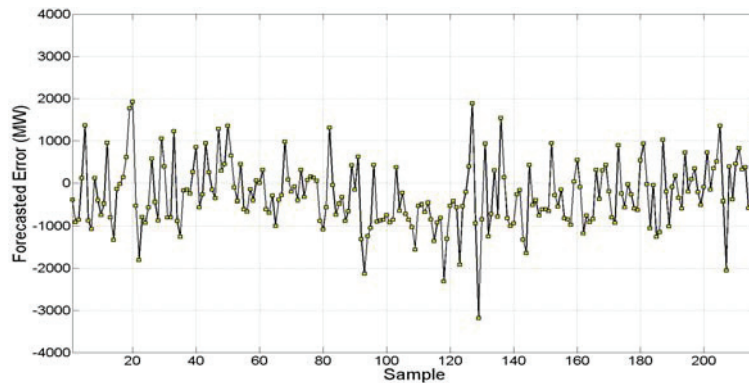


Fig. 6. Forecasting error for test samples (D_{test}).

greater than coefficient of determination for results obtained using neural networks.

IT2 TSK FLS (A2-C1) models used in this study can also be applied to the problem of medium and long term load forecasting. It is expected that these models will efficiently handle uncertainties and minimize their effects on forecasted long and short term load demands. Electricity price estimation is another research field, where these models could be used to handle prevailing uncertainties. Further research in these fields is continuing.

The work presented in this paper is a pioneer study on application of IT2 TSK FLS (A2-C1) for STLF problem. It will have a significant impact on the way artificial intelligence methods are used to forecast power load demands. It is expected that results of this paper will encourage other research groups to devote further effort into the application of IT2 FLS models for addressing problems in the power engineering field.

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