

An Adaptive Type-2 Fuzzy Logic Controller for Dynamic Positioning

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Abstract—This paper presents an indirect adaptive type-2 fuzzy logic controller (FLC) for dynamic positioning (DP) vessels with attached thrusters under various time-varying hydrodynamic disturbances. Approximation-based adaptive control technique in combination with type-2 fuzzy logic system (FLS) is employed in the design of the control. The stability of the design is demonstrated through Lyapunov analysis where globally asymptotical convergence of the regulation error is guaranteed. Rigorous analysis shows the resultant closed-loop system is passive. The proposed adaptive type-2 FLC is able to reject the hydrodynamic disturbances without exact information on them. Comparative simulations with linear proportional derivative (PD) controller and adaptive type-1 FLC are carried out. The proposed technique is found to be effective, robust, and has better performance.

Index Terms—Adaptive control, type-2 fuzzy logic system, function approximation, dynamic positioning.

I. INTRODUCTION

The offshore exploration and exploitation of hydrocarbons have opened up an era of dynamic positioning (DP) vessels. DP systems are widely used in situations where mooring or anchoring is not feasible due to deep water, congested structures such as pipelines on the sea bottom or other economical issues. DP systems maintain floating structures in fixed position and heading for marine operation purpose exclusively by means of active propellers and thrusters. In the 1960s the first DP system was introduced for horizontal modes of motion (surge, sway, and yaw) using PID controllers in cascaded with low-pass or notch filter. In the 1970s more advanced control techniques based on optimal control and linear Kalman filter theory was proposed in [1]. This work was later improved and extended [2], [3]. In the 1990s nonlinear control techniques were employed in several works [4]. More recently, supervisory switched hybrid controllers for DP from calm to extreme sea conditions and from transit to station keeping operation were presented [5].

In parallel with the developments in control techniques, vessel modeling techniques have been improving as well [3]. One of the newest vessel models was presented in [6], [7], and there is still a lack of effective controller for this model due to the complex time-varying hydrodynamic disturbances presented in this model. One intuitive solution is adaptive control technique. For this model, traditional model-based

adaptive control technique is not applicable since it is generally useful only when dealing with systems in which the dynamics are linear in the parameters, the regressors are exactly known, and the uncertainties are parametric and time-invariant [8]. As these disturbances are time-varying and changing for different sea conditions, approximation-based adaptive control technique [9], [10] is adopted to compensate for the disturbances and functional uncertainties. Such approximators can utilize a standard regressor function whose structure is independent of the disturbances' dynamic characteristics.

Type-1 fuzzy logic system (FLS) has been found to be able to approximate continuous nonlinear functions to any desired accuracy over a compact set [11]–[14], thus could be such an approximator. In recent years, studies on type-2 fuzzy sets and FLSs have drawn much attention [15]–[18]. A type-2 fuzzy set is characterized by a fuzzy membership function whose membership value for each element is a fuzzy set in $[0,1]$, unlike a type-1 fuzzy set whose membership value is a crisp number in $[0,1]$. The membership functions of type-2 fuzzy sets are three dimensional, and it is the new third dimension that provides additional degrees of freedom that equip type-2 FLS with the potential to outperform its type-1 counterpart. Type-2 FLS is a fuzzy system where at least one of the antecedent or consequent fuzzy sets is a type-2 fuzzy set. The type-2 FLS has already been successfully applied to several engineering problems [19]–[22] and studied from lots of points of view. One interesting research point of view is whether the type-2 FLS could be a universal approximator for different application, and this paper is an effort along this direction.

By combining approximation-based adaptive control technique and type-2 FLS, an indirect adaptive type-2 fuzzy logic controller (FLC) is proposed. It overcomes the limitations of model-based adaptive control to provide an effective control algorithm for the newly developed vessel model [6], [7]. And the DP problem is treated as an application to investigate the approximation property of type-2 FLS.

The rest of the paper is organized as follows. Section II describes the singleton interval type-2 (IT2) FLS and expresses it in the linear in the parameters form. Next, the modeling of marine vessels in two complexity levels is introduced in

Section III. Then the design and stability analysis of the adaptive type-2 FLC is delineated in Section IV, the passivity property of the resultant closed-loop system is illustrated in this section as well. After that, Section V describes the simulation results of the closed-loop system. Comparisons with other controllers are also shown in this section. Finally, conclusions are drawn in Section VI.

II. DESCRIPTION OF SINGLETON IT2 FLS

In this paper, we consider a singleton IT2 FLS whose basic configuration is depicted in Fig. 1. It consists of fuzzifier, rule base, fuzzy inference engine, type-reducer, and defuzzifier. Here, a multi-input, single-out fuzzy system is described. And a multi-output system can always be separated into a group of single-output systems.

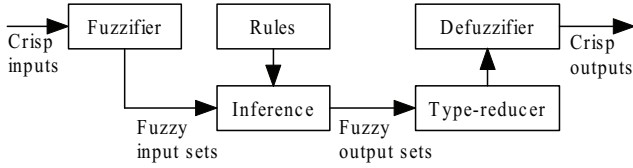


Fig. 1. A singleton interval type-2 FLS.

1) *Fuzzification*: The fuzzifier maps a crisp point $\mathbf{x} = (x_1, \dots, x_n) \in \mathbf{U} \subset \mathbb{R}^n$ into a fuzzy set $\tilde{A}_{\mathbf{x}}$ in $\mathbf{U}$. Since singleton fuzzification is employed, the membership grade of an input $\mathbf{x}$ in the fuzzy set $\tilde{A}_{\mathbf{x}}$ is as follows.

$$\mu_{\tilde{A}_{\mathbf{x}}}(\mathbf{x}) = \begin{cases} 1; & \mathbf{x} = \mathbf{x}' \\ 0; & \mathbf{x} \neq \mathbf{x}' \end{cases} \quad (1)$$

2) *Rules*: Rules are the heart of a FLS, and can be extracted from numerical data or provided by experts. Consider a FLS having n inputs and one output, then the l th rule in the rule base of the IT2 FLS is

$$R^l: \text{IF } x_1 \text{ is } \tilde{F}_1^l \text{ and } \dots \text{ and } x_n \text{ is } \tilde{F}_n^l \text{ THEN } y \text{ is } \tilde{G}^l \quad (2)$$

where $l = 1, 2, \dots, M$, x_i ($i = 1, 2, \dots, n$) and y are the input and output variables to the fuzzy system respectively, $\tilde{F}_i^l$ and $\tilde{G}^l$ are labels of fuzzy sets in $\mathbf{U}_i$ and $\mathbf{R}$, respectively.

3) *Fuzzy Inference Engine*: The fuzzy inference engine combines fuzzy rules with fuzzy logic principles to provide a mapping from fuzzy input sets in $\mathbf{U}$ to fuzzy output sets in $\mathbf{R}$. For the singleton IT2 FLS, based on the sup-star compositional rule of inference, the output consequent set corresponding to rule R^l can be represented as

$$\mu_{\tilde{B}^l}(y) = \mu_{\tilde{G}^l}(y) \cap \left[\prod_{i=1}^n \mu_{\tilde{F}_i^l}(x'_i) \right]. \quad (3)$$

where the firing set $\prod_{i=1}^n \mu_{\tilde{F}_i^l}(x'_i) \equiv F^l(x')$ is an interval type-1 set, i.e.,

$$F^l(x') = \left[\underline{f}^l(x'), \bar{f}^l(x') \right] \quad (4)$$

$$\underline{f}^l(x') = \underline{\mu}_{\tilde{F}_1^l}(x'_1) \star \dots \star \underline{\mu}_{\tilde{F}_n^l}(x'_n) \quad (5)$$

$$\bar{f}^l(x') = \bar{\mu}_{\tilde{F}_1^l}(x'_1) \star \dots \star \bar{\mu}_{\tilde{F}_n^l}(x'_n). \quad (6)$$

Here $\underline{\mu}_{\tilde{F}_i^l}(x'_i)$ is the lower membership grade of $\tilde{F}_i^l$, whereas $\bar{\mu}_{\tilde{F}_i^l}(x'_i)$ is the upper membership grade of $\tilde{F}_i^l$, symbol $\star$ denotes the t-norm, which corresponds to the conjunction "and" in (2). The fired output consequent set, $\mu_{\tilde{B}^l}(y)$, for the l th rule is the IT2 fuzzy set

$$\mu_{\tilde{B}^l}(y) = \int_{b^l \in [\underline{f}^l \star \underline{\mu}_{\tilde{G}^l}(y), \bar{f}^l \star \bar{\mu}_{\tilde{G}^l}(y)]} 1/b^l \quad (7)$$

where $\underline{\mu}_{\tilde{G}^l}$ and $\bar{\mu}_{\tilde{G}^l}$ are the lower and upper membership grades of consequent IT2 fuzzy set, $\tilde{G}^l$.

4) *Type-Reduction and Defuzzification*: The outputs of the inference engine are IT2 fuzzy sets which need to be type-reduced and then defuzzified to produce a crisp output. Assume maximum t-conorm, product t-norm, product implication, and center of sets type reduction are used. Then in the type-reduction, the type-reduced set is an interval type-1 fuzzy set $[y'_l, y'_r]$. The outputs y'_l and y'_r can be computed using the Karnik-Mendel iterative procedure. In this procedure, assume singleton consequent sets are used, the consequent parameters are reordered in ascending order. Let $\mathbf{y} = (y^1, \dots, y^M)^T$ denote the original rule-ordered consequent values, and let $\hat{\mathbf{y}} = (\hat{y}^1, \dots, \hat{y}^M)^T$ denote the reordered sequences, where $\hat{y}^1 \leq \hat{y}^2 \leq \dots \leq \hat{y}^M$. The relationship between $\mathbf{y}$ and $\hat{\mathbf{y}}$ is

$$\hat{\mathbf{y}} = \mathbf{Q}\mathbf{y} \quad (8)$$

where $\mathbf{Q}$ is $M \times M$ permutation matrix with elementary vectors as columns, and these vectors are arranged to move elements in $\mathbf{y}$ to new locations in ascending order in the transformed vector $\hat{\mathbf{y}}$. The original rule firing strength orders $\underline{\mathbf{f}} = (\underline{f}^1, \underline{f}^2, \dots, \underline{f}^M)^T$ and $\bar{\mathbf{f}} = (\bar{f}^1, \bar{f}^2, \dots, \bar{f}^M)^T$ are reordered accordingly. And the new orders for $\underline{\mathbf{f}}$ and $\bar{\mathbf{f}}$ are $\mathbf{Q}\underline{\mathbf{f}}$ and $\mathbf{Q}\bar{\mathbf{f}}$ respectively. According to [23], [24], the output y'_l can be computed as

$$\begin{aligned} y'_l &= \frac{\sum_{i=1}^L (\mathbf{Q}\bar{\mathbf{f}})_i \hat{y}^i + \sum_{i=L+1}^M (\mathbf{Q}\underline{\mathbf{f}})_i \hat{y}^i}{\sum_{i=1}^L (\mathbf{Q}\bar{\mathbf{f}})_i + \sum_{i=L+1}^M (\mathbf{Q}\underline{\mathbf{f}})_i} \\ &= \frac{\bar{\mathbf{f}}^T \mathbf{Q}^T \mathbf{E}_1^T \mathbf{E}_1 \mathbf{Q} \mathbf{y} + \underline{\mathbf{f}}^T \mathbf{Q}^T \mathbf{E}_2^T \mathbf{E}_2 \mathbf{Q} \mathbf{y}}{\mathbf{p}_l^T \mathbf{Q} \bar{\mathbf{f}} + \mathbf{g}_l^T \mathbf{Q} \underline{\mathbf{f}}} \end{aligned} \quad (9)$$

where L and R denote the left and right crossover points respectively,

$$\mathbf{p}_l = \underbrace{(1, 1, \dots, 1, 0, \dots, 0)^T}_{L} \in \mathbb{R}^{M \times 1}$$

$$\mathbf{g}_l = (0, \dots, 0, \underbrace{1, 1, \dots, 1}_{M-L})^T \in \mathbb{R}^{M \times 1}$$

$$\mathbf{E}_1 = (\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_L, \mathbf{0}, \dots, \mathbf{0})^T \in \mathbb{R}^{L \times M}$$

$$\mathbf{E}_2 = (\mathbf{0}, \dots, \mathbf{0}, \mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_{M-L})^T \in \mathbb{R}^{(M-L) \times M}$$

and where $\mathbf{e}_i \in R^L$ and $\varepsilon_i \in R^{M-L}$ are elementary vectors. Similarly, the output y'_r can be expressed as

$$y'_r = \frac{\sum_{i=1}^R (\mathbf{Q}\mathbf{f})_i \hat{y}^i + \sum_{i=R+1}^M (\mathbf{Q}\bar{\mathbf{f}})_i \hat{y}^i}{\sum_{i=1}^R (\mathbf{Q}\mathbf{f})_i + \sum_{i=R+1}^M (\mathbf{Q}\bar{\mathbf{f}})_i} \quad (10)$$

$$= \frac{\mathbf{f}^T \mathbf{Q}^T \mathbf{E}_3^T \mathbf{E}_3 \mathbf{Q} \mathbf{y} + \bar{\mathbf{f}}^T \mathbf{Q}^T \mathbf{E}_4^T \mathbf{E}_4 \mathbf{Q} \mathbf{y}}{\mathbf{p}_r^T \mathbf{Q}\mathbf{f} + \mathbf{g}_r^T \mathbf{Q}\bar{\mathbf{f}}}$$

where

$$\mathbf{p}_r = \underbrace{(1, 1, \dots, 1, 0, \dots, 0)}_R^T \in R^{M \times 1}$$

$$\mathbf{g}_r = \underbrace{(0, \dots, 0, 1, 1, \dots, 1)}_{M-R}^T \in R^{M \times 1}$$

$$\mathbf{E}_3 = (\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_R, \mathbf{0}, \dots, \mathbf{0})^T \in R^{R \times M}$$

$$\mathbf{E}_4 = (\mathbf{0}, \dots, \mathbf{0}, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_{M-R})^T \in R^{(M-R) \times M}$$

and where $\mathbf{e}_i \in R^R$ and $\varepsilon_i \in R^{M-R}$ are elementary vectors. Finally, by computing the average of y'_l and y'_r , the interval set is defuzzified to generate the system output as

$$y' = \frac{y'_l + y'_r}{2}. \quad (11)$$

View $\boldsymbol{\theta} = \mathbf{y} = (y^1, \dots, y^M)^T$ as adjustable parameters, then (11) can be written as

$$y' = \boldsymbol{\phi}^T \boldsymbol{\theta} \quad (12)$$

where

$$\boldsymbol{\phi}^T = \frac{\bar{\mathbf{f}}^T \mathbf{Q}^T \mathbf{E}_1^T \mathbf{E}_1 \mathbf{Q} + \mathbf{f}^T \mathbf{Q}^T \mathbf{E}_2^T \mathbf{E}_2 \mathbf{Q}}{2(\mathbf{p}_l^T \mathbf{Q}\bar{\mathbf{f}} + \mathbf{g}_l^T \mathbf{Q}\mathbf{f})} \quad (13)$$

$$+ \frac{\mathbf{f}^T \mathbf{Q}^T \mathbf{E}_3^T \mathbf{E}_3 \mathbf{Q} + \bar{\mathbf{f}}^T \mathbf{Q}^T \mathbf{E}_4^T \mathbf{E}_4 \mathbf{Q}}{2(\mathbf{p}_r^T \mathbf{Q}\mathbf{f} + \mathbf{g}_r^T \mathbf{Q}\bar{\mathbf{f}})}$$

is a regressive vector. Now as shown in (12), the singleton IT2 FLS is expressed as linear in the parameters.

III. MODELING OF MARINE VESSELS

The model of positioned vessels may be divided into two groups [25], namely process plant model and control plant model. The process plant model is used for numerical analysis of the performance and stability of the closed-loop system by simulating the real plant dynamics as closely as possible. It includes process disturbance, sensor outputs and control inputs. The control plant model, which is used for controller design and theoretical stability analysis (e.g., in the sense of Lyapunov), is a simplified form of the process plant model.

A. Process Plant Model

1) *Kinematics*: In DP, the motions and state variables of the system are conventionally defined and measured with respect to two reference frames as shown in Fig. 2, namely earth-fixed frame and body-fixed frame. The earth-fixed frame, denoted as $X_E Y_E Z_E$, is used for global guidance and navigation. The body-fixed frame, denoted as XYZ , is fixed to the vessel with its origin located at the vessel's center of gravity. Let

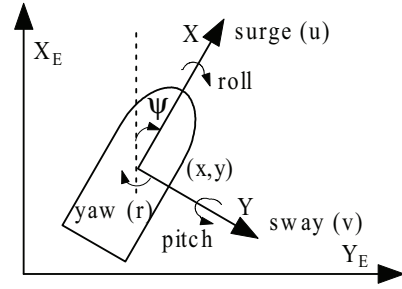


Fig. 2. 2D Earth-fixed and body-fixed reference frames.

$\boldsymbol{\eta}' = [x, y, z, \phi, \theta, \psi]^T$ be the vector denoting the position and orientation of the vessel relative to an earth-fixed frame, and let $\boldsymbol{\nu}' = [u, v, w, p, q, r]^T$ represent the translation and rotation velocities of the vessel decomposed in a body-fixed frame. These six modes are referred to as the surge, sway, heave, roll, pitch, yaw modes of a vessel. Then, the transformation between the earth-fixed and body-fixed velocity vectors is

$$\dot{\boldsymbol{\eta}}' = \mathbf{J}'(\boldsymbol{\eta}') \boldsymbol{\nu}'. \quad (14)$$

where $\mathbf{J}'(\boldsymbol{\eta}')$ is a state dependent transformation matrix, whose definition can be found in [3].

2) *Dynamics*: The six degrees of freedom (DOF) model formulation is based on [6], [7]. The equation of motion, which is in body-fixed frame, for the nonlinear low frequency model of a floating vessel traveling in low speed is given by

$$\mathbf{M}' \dot{\boldsymbol{\nu}}' + \bar{\mathbf{B}}' \boldsymbol{\nu}'_r + \mathbf{G}' \boldsymbol{\eta}' + \boldsymbol{\mu}' = \boldsymbol{\tau}' + \boldsymbol{\tau}'_{\text{CFD}} + \boldsymbol{\tau}'_{\text{wave}} \quad (15)$$

where $\mathbf{M}' = \mathbf{M}'_{RB} + \bar{\mathbf{A}}' \in R^{6 \times 6}$ is the sum of the system inertia matrix and added mass matrix, $\bar{\mathbf{B}}' \in R^{6 \times 6}$ is the potential damping matrix, $\mathbf{G}' \in R^{6 \times 6}$ is the restoring matrix, $\boldsymbol{\mu}' \in R^6$ is computed using a state space model to represent the fluid memory effect,

$$\dot{\boldsymbol{\chi}} = \mathbf{A}_r \boldsymbol{\chi} + \mathbf{B}_r \boldsymbol{\nu}'_r \quad (16)$$

$$\boldsymbol{\mu}' = \mathbf{C}_r \boldsymbol{\chi} + \mathbf{D}_r \boldsymbol{\nu}'_r \quad (17)$$

and where $(\mathbf{A}_r, \mathbf{B}_r, \mathbf{C}_r, \mathbf{D}_r)$ are constant matrices of appropriate dimensions. $\boldsymbol{\tau}' \in R^6$ is control force vector produced by thruster system, $\boldsymbol{\tau}'_{\text{CFD}} \in R^6$ is cross-flow drag and surge resistance vector, $\boldsymbol{\tau}'_{\text{wave}} \in R^6$ is wave load vector, which is computed according to Response Amplitude Operator tables, and $\boldsymbol{\nu}'_r = \boldsymbol{\nu}' - \boldsymbol{\nu}'_c \in R^6$ is the relative velocity vector between vessel velocity vector $\boldsymbol{\nu}'$ and sea current velocity vector $\boldsymbol{\nu}'_c$. All the detailed definition and computation of the above matrices and vectors can be found in [3], [6] and references therein.

B. Control Plant Model

To facilitate the controller design and analytical stability analysis, the process plant model presented in previous subsection is simplified based on the following assumptions.

Assumption 1: The motions along heave, roll, and pitch directions are ignored. In fact, this is an appropriate assumption since a conventional ship is metacentric stable which implies that there exist restoring force and moments along those

directions. As a result of this assumption, the dimension of the restoring matrix $\mathbf{G}' \in R^{6 \times 6}$ in (15) is reduced to yield $\mathbf{G} \in R^{3 \times 3} = \mathbf{0}$.

Assumption 2: The state variables, i.e. displacements and velocities along surge, sway, and yaw directions, are measured by on-board devices such as GPS, gyro system, and accelerometers. This is a acceptable assumption due to the development of integration system of inertial sensors and satellite navigation system.

Assumption 3: The effect of sea current is ignored. This assumption is for the convenience of theoretical analysis.

Assumption 4: The various kinds of time-varying hydrodynamic disturbances, i.e. μ' , τ'_{CFD} , and τ'_{wave} in (15), could be estimated by singleton IT2 FLSs in the form of

$$\begin{aligned}\tau'_d &= -\Phi'(\nu')\Theta' \\ &= \tau'_{\text{CFD}} + \tau'_{\text{wave}} - \mu'\end{aligned}\quad (18)$$

where

$$\Theta' = (\theta_1'^T, \theta_2'^T, \dots, \theta_6'^T)^T \in R^{6M \times 1} \quad (19)$$

is a weighting vector and

$$\Phi'(\nu') = \text{diag}[\phi_1'^T, \phi_2'^T, \dots, \phi_6'^T] \in R^{6 \times 6M} \quad (20)$$

is a regressive matrix, M is the number of rules for each sub-FLS. So far, this assumption is not well established, since there is still a lack of rigorous proof that singleton IT2 FLS is universal approximator, but the assumption will be checked by subsequent simulations. The situation is different for type-1 case, as there has been a lot of works showing that type-1 FLS is universal approximator [11]–[14].

Application of Assumptions 1-4 to (14) and (15) yields the following control plant model for DP.

$$\dot{\eta} = \mathbf{J}(\eta)\nu \quad (21)$$

$$\mathbf{M}\dot{\nu} + \bar{\mathbf{B}}\nu = \tau - \Phi(\nu)\Theta \quad (22)$$

where $\eta = [x, y, \psi]^T$ and $\nu = [u, v, r]^T$ are the vectors representing displacement and velocity along surge, sway, and yaw, $\mathbf{J}(\eta) \in R^{3 \times 3}$ is the rotation matrix, note that $\mathbf{J}\mathbf{J}^T = \mathbf{I}$ ($\mathbf{I}$ is the identity matrix), $\mathbf{M} \in R^{3 \times 3}$ is the system inertia matrix including added mass, $\bar{\mathbf{B}} \in R^{3 \times 3}$ is the potential damping matrix,

$$\Theta = (\theta_1^T, \theta_2^T, \theta_3^T)^T \in R^{3M \times 1} \quad (23)$$

is the weighting vector and

$$\Phi(\nu) = \text{diag}[\phi_1^T(\nu), \phi_2^T(\nu), \phi_3^T(\nu)] \in R^{3 \times 3M} \quad (24)$$

is the regressive matrix. Thus, here three sub-FLSs are used to estimate the hydrodynamic disturbances.

IV. ADAPTIVE FUZZY LOGIC CONTROLLER DESIGN

The control objective for DP is to position and orientate the vessel to a fixed placement via attached thruster system, which actually is a regulation problem. In this section, an indirect adaptive fuzzy logic controller for DP is proposed, the regulation error is proved to globally asymptotically converge to zero. Rigorous analysis shows the resultant closed-loop system is passive.

A. Control and Adaptive Law

The proposed control and adaptive law for plant (21), (22) are as follows.

$$\tau = -\lambda_1\nu - \lambda_2\mathbf{J}^T(\eta)e + \Phi(\nu)\hat{\Theta} \quad (25)$$

$$\dot{\hat{\Theta}} = \lambda_3\Phi^T(\nu)(\lambda_4\nu + \frac{2\mathbf{J}^T(\eta)e}{1+2e^Te}) \quad (26)$$

where $e = \eta - \eta_d \in R^3$ is the control error vector between vessel displacement vector η and the desired vessel placement vector η_d , $\tilde{\Theta} = \Theta - \hat{\Theta} \in R^{3M}$ is the error vector between true Θ in (22) and adapted $\hat{\Theta}$, and λ_i ($i = 1, \dots, 4$) are positive constant.

Application of control and adaptive law (25), (26) to the plant (21), (22) yields the resultant closed-loop system

$$\mathbf{M}\dot{\nu} + \bar{\mathbf{B}}\nu = -\lambda_1\nu - \lambda_2\mathbf{J}^T(\eta)e - \Phi(\nu)\tilde{\Theta} \quad (27)$$

$$\dot{\tilde{\Theta}} = \lambda_3\Phi^T(\nu)(\lambda_4\nu + \frac{2\mathbf{J}^T(\eta)e}{1+2e^Te}) \quad (28)$$

$$\dot{e} = \dot{\eta} = \mathbf{J}(\eta)\nu. \quad (29)$$

B. Stability Analysis

Theorem 1: The states (e, ν) of the closed-loop system (27)-(29) globally asymptotically converge to $(\mathbf{0}, \mathbf{0})$ if

$$\lambda_4 > \max\left\{\frac{2\lambda_M}{\sqrt{\lambda_2\lambda_m}}, \frac{(\lambda_d + 2\lambda_1)^2 + 16\lambda_2\lambda_M}{4\lambda_2\lambda_d + 8\lambda_1\lambda_2}\right\}, \quad (30)$$

where λ_m and λ_M are the minimum and maximum eigenvalue of matrix $\mathbf{M}$ respectively, and λ_d is the minimum eigenvalue of matrix $\bar{\mathbf{B}} + \bar{\mathbf{B}}^T$.

Proof: Consider the following Lyapunov function candidate.

$$V = \frac{\lambda_2\lambda_4}{2}e^Te + \frac{\lambda_4}{2}\nu^T\mathbf{M}\nu + \frac{2\nu^T\mathbf{M}\mathbf{J}^Te}{1+2e^Te} + \frac{1}{2\lambda_3}\tilde{\Theta}^T\tilde{\Theta} \quad (31)$$

To make sure the Lyapunov function candidate V positive definite, due to the fact $\|\mathbf{J}\| = 1$ and $\lambda_m\mathbf{I} \leq \mathbf{M} \leq \lambda_M\mathbf{I}$, we have

$$\begin{aligned}V_1 &= \frac{\lambda_2\lambda_4}{2}e^Te + \frac{\lambda_4}{2}\nu^T\mathbf{M}\nu + \frac{2\nu^T\mathbf{M}\mathbf{J}^Te}{1+2e^Te} \\ &\geq \frac{\lambda_2\lambda_4}{2}e^Te + \frac{\lambda_4\lambda_m}{2}\nu^T\nu - \frac{2\lambda_M}{1+2e^Te}\nu^Te,\end{aligned}$$

If $4\lambda_M^2 - \lambda_4^2\lambda_2\lambda_m < 0$ i.e., $\lambda_4 > 2\lambda_M/\sqrt{\lambda_2\lambda_m}$, then V_1 is positive definite, additionally,

$$V_2 = \frac{1}{2\lambda_3}\tilde{\Theta}^T\tilde{\Theta}$$

is positive definite, thus $V = V_1 + V_2$ is positive definite.

Differentiation of V along the trajectory of the closed-loop system (27)-(29) yields

$$\begin{aligned}\dot{V} &= -\lambda_4\nu^T\bar{\mathbf{B}}\nu - \lambda_1\lambda_4\nu^T\nu + \frac{2\nu^T\mathbf{M}\nu}{1+2e^Te} - \frac{2e^T\mathbf{J}\bar{\mathbf{B}}\nu}{1+2e^Te} \\ &\quad - \frac{2\lambda_1e^T\mathbf{J}\nu}{1+2e^Te} - \frac{2\lambda_2e^Te}{1+2e^Te} - \frac{8e^T\nu\nu^T\mathbf{M}e}{(1+2e^Te)^2}.\end{aligned}$$

Applying $\lambda_d\mathbf{I} \leq \bar{\mathbf{B}} + \bar{\mathbf{B}}^T$, we have

$$\dot{V} \leq -\left(\frac{\lambda_4\lambda_d}{2} + \lambda_1\lambda_4 - 2\lambda_M\right)\nu^T\nu - (\lambda_d + 2\lambda_1)e^T\nu - 2\lambda_2e^Te,$$

If $(\lambda_d + 2\lambda_1)^2 - 8\lambda_2((\lambda_4\lambda_d)/2 + \lambda_1\lambda_4 - 2\lambda_M) < 0$ i.e.,

$$\lambda_4 > \frac{(\lambda_d + 2\lambda_1)^2 + 16\lambda_2\lambda_M}{4\lambda_2\lambda_d + 8\lambda_1\lambda_2}$$

then $\dot{V} \leq 0$, which means the derivative of the Lyapunov function candidate is semi-negative definite. Moreover, the largest invariant set contained in the set $\dot{V} \equiv 0$ is contained in the set $(e, \nu) = (0, 0)$.

Now, according to Krasovskii-La Salle Theorem, we may conclude the point (e, ν) globally asymptotically converges to $(0, 0)$, and $\dot{\tilde{\Theta}} = 0$, which means $\tilde{\Theta}$ is bounded, if

$$\lambda_4 > \max\left\{\frac{2\lambda_M}{\sqrt{\lambda_2\lambda_m}}, \frac{(\lambda_d + 2\lambda_1)^2 + 16\lambda_2\lambda_M}{4\lambda_2\lambda_d + 8\lambda_1\lambda_2}\right\}.$$

■

Remark 1: For plant (21), (22), an intuitive adaptive law may be

$$\dot{\tilde{\Theta}} = \lambda_5 \Phi^T(\nu)\nu, \quad (32)$$

but rigorous analysis indicates that the regulation error using such adaptive law cannot be asymptotically convergence.

Remark 2: Here, the theorem is developed for the case where the gains λ_i ($i = 1, \dots, 4$) are scalar, the case where the gains are matrices can be easily extended to in a similar approach.

C. Passivity Interpretation

Passivity theory gives a framework for the design and analysis of control systems using an input-output description according to energy-related considerations. And the input-output description further allows for a modular approach to control systems design and analysis [26]. As illustrated in Fig. 3, the closed-loop system (27)-(29) can be interpreted as the negative feedback interconnection of two subsystems with respective inputs u_1, u_2 and outputs y_1 and y_2 , with $y_1 = u_2$ and $y_2 = -u_1$, and the two subsystems as follows.

Subsystem 1:

$$\begin{cases} \mathbf{M}\dot{\nu} + \bar{\mathbf{B}}\nu = -\lambda_1\nu - \lambda_2\mathbf{J}^T(\eta)e + u_1 \\ y_1 = \lambda_4\nu + \frac{2\mathbf{J}^T(\eta)e}{1+2e^Te} \end{cases} \quad (33)$$

Subsystem 2:

$$\begin{cases} \dot{\tilde{\Theta}} = \lambda_3\Phi^T(\nu)u_2 \\ y_2 = \Phi(\nu)\tilde{\Theta} \end{cases} \quad (34)$$

Theorem 2: The closed-loop system (27)-(29) is passive if (30) is satisfied.

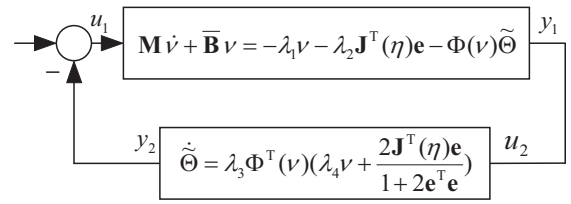


Fig. 3. Closed-loop equivalent representation.

Proof: For subsystem 1, its supply rate is

$$\begin{aligned} & \int_0^t u_1^T y_1 ds \\ &= \int_0^t (\lambda_4\nu + \frac{2\mathbf{J}^T e}{1+2e^T e})^T (\mathbf{M}\dot{\nu} + \bar{\mathbf{B}}\nu + \lambda_1\nu + \lambda_2\mathbf{J}^T e) ds \\ &= \int_0^t [\lambda_2\lambda_4\nu^T \mathbf{J}^T e + \frac{d}{ds}(\frac{\lambda_4}{2}\nu^T \mathbf{M}\nu + \frac{2\nu^T \mathbf{M}\mathbf{J}^T e}{1+2e^T e})] ds \\ &+ \int_0^t [\lambda_4\nu^T \bar{\mathbf{B}}\nu + \lambda_1\lambda_4\nu^T \nu - \frac{2\nu^T \mathbf{M}\nu}{1+2e^T e} + \frac{2e^T \mathbf{J}\bar{\mathbf{B}}\nu}{1+2e^T e} \\ &+ \frac{2\lambda_1 e^T \mathbf{J}\nu}{1+2e^T e} + \frac{2\lambda_2 e^T e}{1+2e^T e} + \frac{8e^T \nu \nu^T \mathbf{M}e}{(1+2e^T e)^2}] ds, \end{aligned}$$

if (30) is satisfied,

$$\int_0^t u_1^T y_1 ds \geq V_1(t) - V_1(0),$$

which means the subsystem 1 is passive with supply rate $\int_0^t u_1^T y_1 ds$ and storage function V_1 , which has been defined during the proof for Theorem 1.

For subsystem 2 its supply rate is

$$\begin{aligned} \int_0^t u_2^T y_2 ds &= \int_0^t \tilde{\Theta}^T \Phi^T(\nu) u_2 ds \\ &= \int_0^t \tilde{\Theta}^T \frac{1}{\lambda_3} \dot{\tilde{\Theta}} ds \\ &\geq V_2(t) - V_2(0) \end{aligned}$$

which means the subsystem 2 is passive with supply rate $\int_0^t u_2^T y_2 ds$ and storage function V_2 .

According to the fact that the negative feedback interconnection of two passive systems is passive, the closed-loop system (27)-(29) is passive. ■

V. SIMULATION STUDIES

The simulation studies are performed on the platform of Marine System Simulator developed in Norwegian University of Science and Technology [27]. The S-175 container ship is used as case study. The subsequent simulation results are obtained for following seas with the International Towing Tank Conference (ITTC) wave spectrum using significant wave height $H_s = 5$ m and peak frequency $\omega_0 = 0.56$ rad/s. The significant wave height H_s is used to classify the type of sea, $H_s = 5$ m corresponds to very rough sea with large waves. Even though ignored during Lyapunov analysis, the sea current is set with speed $V_c = 2$ m/s and direction $\beta_c = 30^\circ$

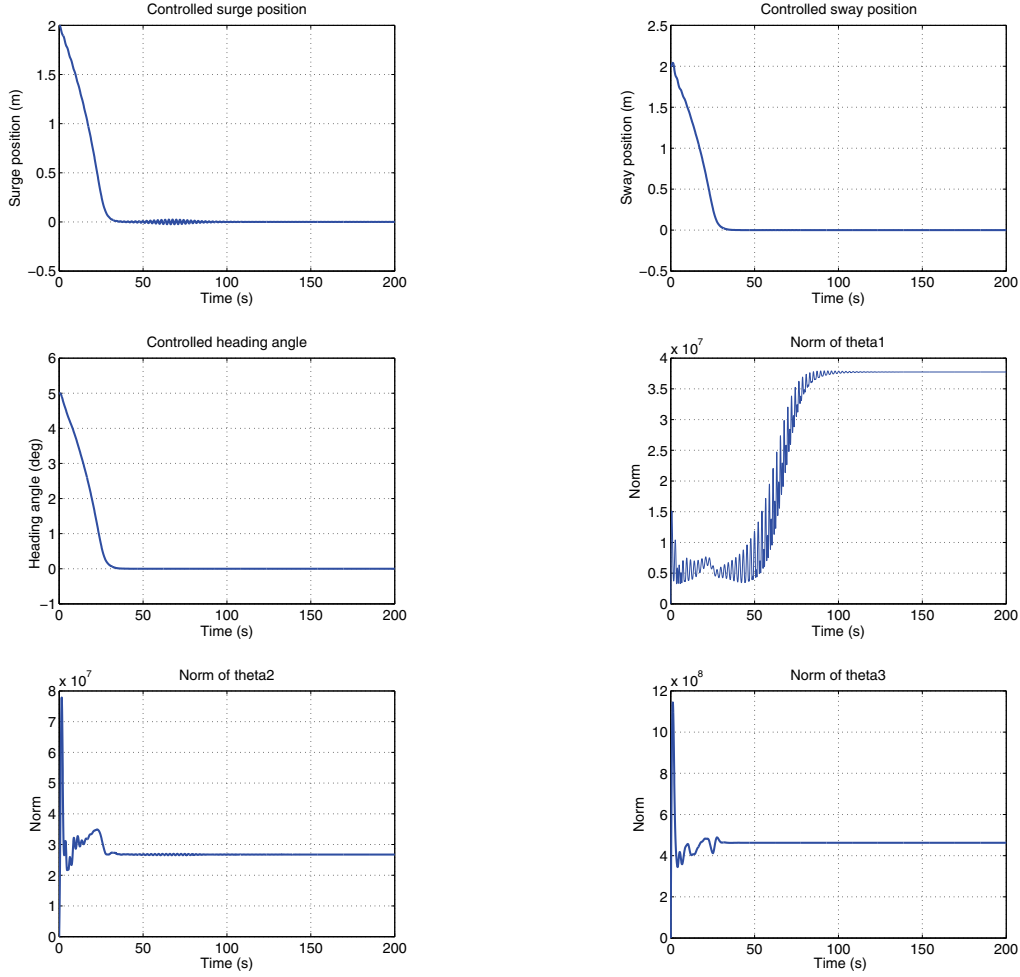


Fig. 4. Plots 1-3 show the controlled position and heading angle of the vessel. Plots 4-6 show the norms of the adapted weighting vectors.

in the following simulations. The control objective is set to regulate the vessel to state $[\eta_d^T, \nu_d^T] = [0, 0]$ from initial state $[\eta_0^T, \nu_0^T] = [2, 2, 5^\circ, 0, 0, 0]$.

A. Closed-loop Performance

Although the controller (25), (26) is designed based on the control plant model (21), (22), it is tested with the process plant model (14), (15). As indicated in (12), (23), and (24), three singleton IT2 sub-FLSs are used to approximate the complex hydrodynamic disturbances. In each sub-system, each of the input domain is partitioned by three fuzzy membership functions labeled as **N**, **Z**, and **P**, thus there are 27 rules and 27 adjustable consequent parameters accordingly. The primary membership functions of the antecedent IT2 fuzzy sets are chosen to be Gaussian function with uncertain standard deviation, i.e.,

$$\mu_{\tilde{F}_i^j}(x_j) = \exp\left[-\frac{1}{2}\left(\frac{x_j - m_i}{\sigma}\right)^2\right] \quad \sigma \in [\underline{\sigma}, \bar{\sigma}] \quad (35)$$

where $i = 1, 2, 3$ is the index of three fuzzy membership functions for j th input and $j = 1, 2, 3$ is the index of three inputs. The inputs to each sub-system is scaled properly so that

the means for the Gaussian functions are fixed at $m_1 = -1$, $m_2 = 0$, and $m_3 = 1$ for each input, and the standard deviation for all of the lower membership function is fixed at $\underline{\sigma} = 0.6$, whereas the one for all of the upper membership function is $\bar{\sigma} = 0.8$. Set $\lambda_1 = 4.3 \times 10^6$, $\lambda_2 = 4.3 \times 10^5$, $\lambda_3 = 5 \times 10^8$, and $\lambda_4 = 5$ in (25) and (26). The controlled position and heading angle of the vessel are shown in plot 1-3 of Fig. 4, from which it can be observed that the closed-loop system is stable, the control performance is satisfactory despite the time-varying hydrodynamic disturbances, and the sub-FLSs efficiently approximate the complex disturbances although universal approximation property for IT2 FLS has not been established yet. Plot 4-6 of Fig. 4 show the norms of the adapted weighting vectors $\|\hat{\theta}_i\|$ ($i = 1, 2, 3$), they verify the boundedness of $\hat{\Theta}$. Note that the norm of weighting vector $\hat{\theta}_1$ shows fluctuation during the simulation, it is because the disturbances along the surge direction fluctuate heavily at the start of the simulation.

B. Comparison with a PD Controller

Comparative study between the designed controller and a linear proportional derivative (PD) controller is presented. The

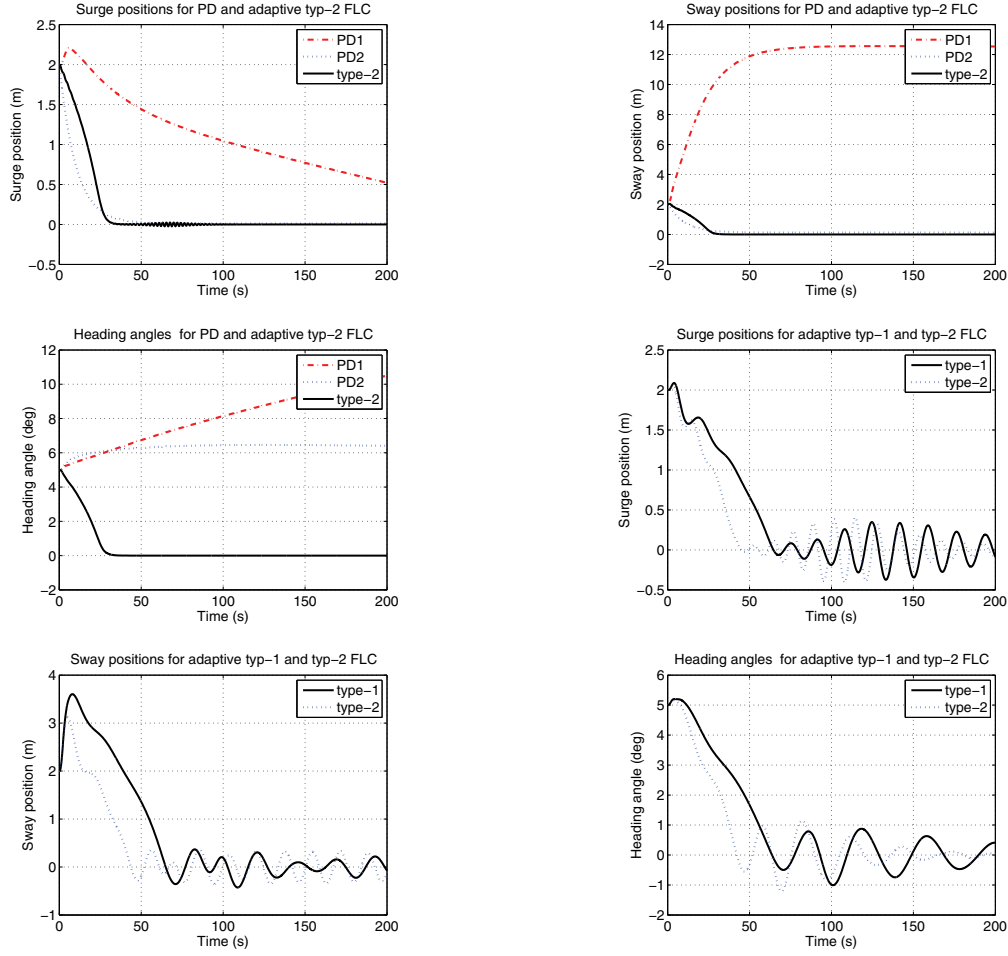


Fig. 5. Plots 1-3 show the positions and heading angles of the vessel for two PD controllers and the adaptive type-2 FLC. Plots 4-6 show the positions and heading angles of the vessel for the adaptive type-1 and type-2 FLC.

PD controller is in the form of

$$\tau = -\lambda_5 \nu - \lambda_6 \mathbf{J}^T(\eta) e. \quad (36)$$

Two sets of control gains are implemented for the PD controller, $\lambda_5^1 = 4.3 \times 10^6$, $\lambda_6^1 = 4.3 \times 10^5$ and $\lambda_5^2 = 4.3 \times 10^8$, $\lambda_6^2 = 4.3 \times 10^7$, meanwhile the parameter settings for the adaptive type-2 FLC are kept same as the one in previous subsection. The results are shown in plot 1-3 of Fig. 5, from which it can be seen that with same proportion and derivative gains, the designed controller achieves satisfactory performance, whereas the PD controller is unstable. With its proportion and derivative gains increasing, the PD controller can be stable, but still with large steady state error due to disturbances. Besides, large control gains are not recommended in practice as they reduce robustness and cause large overshoots. Hence, the designed controller performs much better than the PD controller and has better disturbance rejection property.

C. Comparison with an Adaptive Type-1 FLC

Replace the components in (24) and correspondingly in (25) and (26) with fuzzy basis function vector [10], [11],

$$\phi^T = (\phi^1(\mathbf{x}), \dots, \phi^M(\mathbf{x})) \quad (37)$$

where fuzzy basis function is defined as

$$\phi^l(\mathbf{x}) = \frac{\prod_{i=1}^n \mu_{F_i^l}(x_i)}{\sum_{l=1}^M (\prod_{i=1}^n \mu_{F_i^l}(x_i))}, \quad (38)$$

the controller (25), (26) becomes an adaptive type-1 FLC. In this subsection, the adaptive type-2 FLC is compared with such an adaptive type-1 FLC. For each sub-system in the adaptive type-2 FLC, the parameter settings are kept identical as the one in previous subsection, whereas for each sub-system in type-1 case, each of the input domain is partitioned by five fuzzy membership functions labeled as **NL**, **NS**, **Z**, **PS**, and **PL**, thus there are 125 rules and 125 adjustable consequent parameters accordingly in contrast to 27 rules and parameters for type-2 case. The membership functions of the antecedent sets for each input are chosen as Gaussian functions, i.e.,

$$\mu_{F_i^j}(x_j) = \exp\left[-\frac{1}{2}\left(\frac{x_j - m_i}{\sigma}\right)^2\right]. \quad (39)$$

where $i = 1, \dots, 5$ is the index of five fuzzy membership functions for j th input and $j = 1, 2, 3$ is the index of three inputs. While the means are set as $(m_1, \dots, m_5) =$

$(-1, -0.5, 0, 0.5, 1)$, the standard deviations for all the Gaussian functions are fixed at $\sigma = 0.4$. Set $\lambda_1 = 4.3 \times 10^6$, $\lambda_2 = 4.3 \times 10^5$, $\lambda_3 = 1.5 \times 10^7$, and $\lambda_4 = 5$ in (25) and (26) for both adaptive type-1 and type-2 FLCs, the resultant positions and heading angles of the vessel are shown in plot 4-6 of Fig. 5, from which it can be observed that the rising and settling time for adaptive type-2 FLC is shorter than that for type-1 case, even though type-1 case uses more rules. To quantify the performance differences between adaptive type-1 and type-2 FLCs, the integral of time-weighted absolute errors (ITAE) for both cases in the following form are calculated.

$$P_x = \int_0^T t|x|dt \quad (40)$$

$$P_y = \int_0^T t|y|dt \quad (41)$$

$$P_\psi = \int_0^T t|\psi|dt \quad (42)$$

where T is the total simulation time. The results are shown in Table I, and the ITAE for the type-2 case are smaller than those for type-1 case along all the three degrees of freedom. Regarding the computational burden, type-reduction for type-2 case does take some time, but the time is very little as only a few rules are fired simultaneously and it has been proved Karnik-Mendel algorithm is super-exponentially convergent [28]. In addition, the real-time application of the type-2 FLS has been reported [29]. Thus, type-reduction should not be an issue that causes any stability problem. In a word, it seems that the type-2 FLS has better approximation property with less rules than its type-1 counterpart, and rigorous theoretical examination is needed.

TABLE I
ITAE FOR ADAPTIVE TYPE-1 AND TYPE-2 FLCs

ITAE	P_x	P_y	P_ψ
Type-1	4604	5960	210
Type-2	3252	4951	121

VI. CONCLUSIONS

In this paper, an indirect adaptive type-2 FLC has been designed for DP vessels with attached thrusters in the presence of complex hydrodynamic disturbances. It has been proved that the regulation error under the proposed control globally asymptotically converges to zero and the closed-loop system under the proposed control is passive. Simulation results have demonstrated that the adaptive type-2 FLC is effective and robust. When compared against a PD controller, the proposed controller has better control performance and disturbance rejection property. Comparison with its type-1 counterpart suggests type-2 FLS has better approximation property, but rigorous theoretical examination is necessary.

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