

Passive Fuzzy Control for Uncertain Nonlinear Stochastic Inverted Pendulum Robot System

Wen-Jer Chang

Department of Marine Engineering
National Taiwan Ocean University
Keelung 20224, Taiwan, R.O.C.
e-mail: wjchnag@mail.ntou.edu.tw

Sin-Sian Jheng

Department of Marine Engineering
National Taiwan Ocean University
Keelung 20224, Taiwan, R.O.C.
e-mail: M98660024@mail.ntou.edu.tw

Cheung-Chieh Ku

Department of Electrical Engineering
National Taiwan Ocean University
Keelung 20224, Taiwan, R.O.C.
e-mail: D95530001@mail.ntou.edu.tw

Abstract—This paper presents a passive fuzzy controller design method for stabilizing the uncertain nonlinear stochastic inverted pendulum robot system. Through Takagi-Sugeno (T-S) fuzzy modeling approach, the complex nonlinear properties can be described by combining several linear sub-systems and determined membership functions. The Itô stochastic differential equation is employed to represent dynamics of systems in this paper. Applying the modeling approach, the T-S fuzzy model with multiplicative noise can be built for describing the nonlinear stochastic system. Besides, the uncertainties of the controlled system are considered in this paper for dealing with molding errors and varying parameters. Hence, the robust control problem is also discussed and investigated in this paper. Finally, the numerical simulations are proposed to show the effectiveness of the present design approach.

Keywords—passive; uncertain; T-S fuzzy model; multiplicative noise

I. INTRODUCTION

There have been many researches on control of a two wheeled inverted pendulum system. Over the past few years, the inverted pendulum robot control problem has been investigated in many literatures [1-2]. And, the researchers have applied the pendulum system to simulate human stance and walk behaviors. However, the human stance and walk behaviors control problem is a complicated and difficult problem due to the nonlinearities of system.

In some circumstances, we may encounter real systems with random noise which makes the systems stochastic. When the stochastic behavior exists in the system, the related model can be called as systems with multiplicative noise [3]. Since stochastic modeling approach plays important role, many efforts have been proposed for stochastic system. Furthermore, parameter uncertainty often also appears in the real system due to the model inaccuracies and/or changes in the environment. With the parameter uncertainties, the control problem of the system considered in this paper are more difficult and challenger to be solved and investigated, than the same problems of deterministic systems. Based on the above motivation, the control problem of the inverted pendulum robot model with multiplicative noise is studied in this paper.

As well known that the stability of complex nonlinear systems is hard to be analyzed and synthesized. Recently, the fuzzy logic has been widely applied to approximate the nonlinearities of the system. And then, since Takagi-Sugeno [4], proved a model-based fuzzy approximating matched called T-S model, there have been dramatic progress in stability analysis and control design of nonlinear systems this model-based fuzzy systems. Referring to [4], many practical nonlinear systems have been successful represented via T-S fuzzy model. Thus, the inverted pendulum robot system with multiplicative noise is represented by T-S fuzzy model in this paper. Besides, the Parallel Distributed Compensation (PDC) concept [5] is applied to design the fuzzy controller for T-S fuzzy models. Through the T-S fuzzy model and PDC design concept, the linear control criteria can be applied to analyze the complex nonlinear systems.

Furthermore, consider the attenuation performance, the passivity theory [6] provides a useful tool to discuss the effect of disturbance on the systems. With the passivity theory, many efforts [7-8] have been proposed to design fuzzy controller for T-S fuzzy model with external disturbance. And, based on Lyapunov function, the stability conditions were derived into Linear Matrix Inequality (LMI) problems. According to the literatures [9], we can find that passivity theory provides several kinds of the performance: for example, H_∞ control and L_2 control. Based on the definition of supply rate, the different performance can be determined with passivity theory.

The purpose of this paper is to concentrate on the passive fuzzy controller design problem for uncertain nonlinear stochastic inverted pendulum robot system with multiplicative noises. The proposed fuzzy controller design methodology is developed by using the concept of PDC and the Lyapunov stability criterion. And the passivity theory is employed to guarantee disturbance attenuation performance of the system. Based on the LMI technique [10], one can find a suitable common positive definite matrix and feedback gains for satisfying the derived stability conditions. Then, the PDC-based passive fuzzy controller can be designed by obtained feedback gains. Thus, according to the proposed stability criterion, the nonlinear stochastic inverted pendulum robot system driven by designed fuzzy controller is strictly input passive and robustly asymptotically mean square stable.

Finally, a simulations result is proposed to show the effectiveness and application of the proposed design method of this paper.

The organization of this paper is structured as follows. The dynamic model of simple human stance system with multiplicative noise is described in Section II. The discrete-time stochastic T-S fuzzy system of the nonlinear system is described in Section III. The fuzzy controller design method is proposed in Section IV. Through applying the proposed design approach, simulation results are provided in Section V. Finally, concluding remarks are made in Section VI.

II. THE DYNAMIC MODEL OF SIMPLE HUMAN STANCE SYSTEM

In this section, the inverted pendulum robot system is introduced. Referring to Fig. 1, a simplified mathematical dynamic model for describing inverted pendulum robot system is proposed to simulate the stance of human [11] as follows.

$$\begin{aligned}
 x_1(k+1) &= x_1(k) + T x_2(k) + 0.1 v(k) \\
 x_2(k+1) &= x_2(k) + \frac{T}{M+m(\sin x_3(k))^2} (u(k) + m l x_4^2(k) \\
 &\quad \times \sin x_3(k) - b x_2(k) - m g \cos x_3(k) \sin x_3(k)) \\
 x_3(k+1) &= x_3(k) + T x_4(k) \\
 x_4(k+1) &= x_4(k) + \frac{T}{l(M+m(\sin x_3(k))^2)} ((M+m)g \\
 &\quad \times \sin x_3(k) - u(k) \cos x_3(k) + b x_2(k) \cos x_3(k) \\
 &\quad - m l x_4^2(k) \sin x_3(k) \cos x_3(k)) \\
 y(k) &= x_1(k) + v(k)
 \end{aligned} \quad (1)$$

where

- m is the mass of black on the pendulum.
- l is the length of pendulum.
- g is the acceleration due to gravity.
- b is coefficient of viscous friction for motion of cart.
- u is the applied force.
- v is the external disturbance input.
- M is the mass of the cart.
- T is the sampling period.

The four state variables stand for $x_1(k) = x$, $x_2(k) = \dot{x}$, $x_3(k) = \theta$, $x_4(k) = \dot{\theta}$ with x the position of the cart, $y(k)$ is the system output, $v(k)$ is the external disturbance, and θ the angle of pendulum makes with vertical. This model is obtained by discretizing the continuous time model via Euler's method with $T = 0.1$ s, $b = 12.98$ Kg/s, $M = 1.378$ Kg,

$$l = 0.325 \text{ m}, g = 9.8 \text{ m/s}^2, m = 0.051 \text{ Kg}.$$

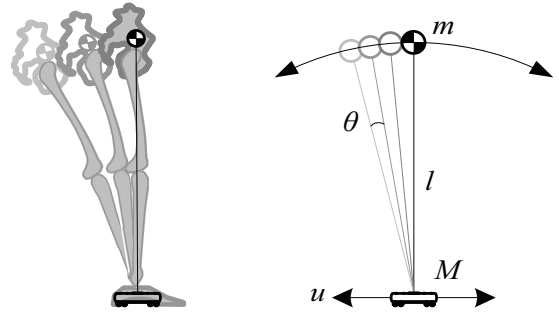


Fig. 1 Inverted pendulum robot system

Considering parameter uncertainties and stochastic behaviors, the modified dynamic model for the inverted pendulum robot system can be described as follows:

$$\begin{aligned}
 x_1(k+1) &= x_1(k) + 0.1 x_2(k) + 0.01 x_1(k) w(k) + \delta_1 x_1(k) \\
 &\quad + 0.1 v(k) + 0.001 v(k) w(k) + \delta_2 u(k) + \delta_3 v(k) \\
 x_2(k+1) &= x_2(k) + \frac{0.1}{1.378 + 0.051(\sin x_3(k))^2} (u(k) - 12.98 x_2 \\
 &\quad + 0.0166 x_4^2(k) \sin x_3(k) - 0.4998 \cos x_3(k) \\
 &\quad \times \sin x_3(k) + 0.1 x_2(k) w(k) + 0.000726 u(k) w(k)) \\
 x_3(k+1) &= x_3(k) + 0.1 x_4(k) + 0.1 x_3(k) w(k) \\
 x_4(k+1) &= x_4(k) + \frac{0.1}{0.4478(0.0166(\sin x_3(k))^2)} \\
 &\quad \times (14.0042 \sin x_3(k) - u(k) \cos x_3(k) \\
 &\quad + 12.98 x_2(k) \cos x_3(k) \\
 &\quad - 0.0166 x_4^2(k) \sin x_3(k) \cos x_3(k) \\
 &\quad + 0.1 x_4(k) w(k) - 0.002233 u(k) w(k)) \\
 y(k) &= x_1(k) + v(k)
 \end{aligned} \quad (2)$$

where δ_1 , δ_2 and δ_3 are the perturbation, which denoted by $\delta_1 \in [-0.001, 0.001]$, $\delta_2 = \delta_3 \in [-0.0002, 0.0002]$, and $w(k)$ is discrete Brownian Motion [12]. Using fuzzy modeling technique one can obtain T-S fuzzy model of (2) in the following section.

III. THE DISCRETE STOCHASTIC T-S FUZZY MODELS

In order to build the T-S fuzzy model of the considered inverted pendulum robot system (2), two operating points are chosen to obtain the linearized models with the sector linearization technique [10]. The operating points are chosen as follows.

$$\begin{aligned} (x_1(k), x_2(k), x_3(k), x_4(k), u(k))_{\text{oper1}} &= (0 \ 0 \ \pm 88^\circ \ 0 \ 0) \\ \text{and} \\ (x_1(k), x_2(k), x_3(k), x_4(k), u(k))_{\text{oper2}} &= (0 \ 0 \ 0 \ 0 \ 0). \end{aligned}$$

In the points $(x_1(k), x_2(k), x_3(k), x_4(k), u(k))_{\text{oper2}}$ is the equilibrium point and the others is the off-equilibrium point. According to the points, the membership function can be determined as Fig. 2. Through the above two linear subsystems and defining membership functions as Fig. 2, one can obtain the following T-S fuzzy model for the inverted pendulum robot system (2) such as.

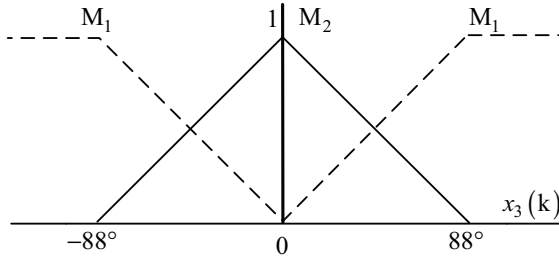


Fig. 2 Membership functions of $x_3(k)$

Rule 1: IF $x_3(k)$ is about M_1 THEN

$$\begin{aligned} x(k+1) &= \mathbf{A}_1(k)x(k) + \mathbf{B}_1(k)u(k) + \mathbf{D}_1(k)v(k) \\ &\quad + (\bar{\mathbf{A}}_1x(k) + \bar{\mathbf{B}}_1u(k) + \bar{\mathbf{D}}_1v(k))w(k) \\ y(k) &= \mathbf{C}_1x(k) + \mathbf{E}_1v(k) \end{aligned} \quad (3)$$

Rule 2: IF $x_3(k)$ is about M_2 THEN

$$\begin{aligned} x(k+1) &= \mathbf{A}_2(k)x(k) + \mathbf{B}_2(k)u(k) + \mathbf{D}_2(k)v(k) \\ &\quad + (\bar{\mathbf{A}}_2x(k) + \bar{\mathbf{B}}_2u(k) + \bar{\mathbf{D}}_2v(k))w(k) \\ y(k) &= \mathbf{C}_2x(k) + \mathbf{E}_2v(k) \end{aligned} \quad (4)$$

where $x(k) = [x_1(k) \ x_2(k) \ x_3(k) \ x_4(k)]^T$,

$$\begin{aligned} \mathbf{A}_i(k) &= \mathbf{A}_i + \Delta\mathbf{A}_i(k), \quad \mathbf{B}_i(k) = \mathbf{B}_i + \Delta\mathbf{B}_i(k), \\ \mathbf{D}_i(k) &= \mathbf{D}_i + \Delta\mathbf{D}_i(k), \\ [\Delta\mathbf{A}_i(k) \ \Delta\mathbf{B}_i(k) \ \Delta\mathbf{D}_i(k)] &= \mathbf{H}_i\Delta(k)[\mathbf{R}_i \ \mathbf{R}_{Bi} \ \mathbf{R}_{Di}], \quad (5) \\ \Delta(k)^T\Delta(k) &\leq \mathbf{I} \quad (6) \end{aligned}$$

and

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0.1 & 0 & 0 \\ 0 & 0.0916 & -0.0008 & 0 \\ 0 & 0 & 1 & 0.1 \\ 0 & 0.0975 & 1.9622 & 1 \end{bmatrix},$$

$$\mathbf{A}_2 = \begin{bmatrix} 1 & 0.1 & 0 & 0 \\ 0 & 0.0581 & -0.0363 & 0 \\ 0 & 0 & 1 & 0.1 \\ 0 & 2.8983 & 3.127 & 1 \end{bmatrix}, \quad \mathbf{B}_1 = \mathbf{B}_2 = \begin{bmatrix} 0 \\ 0.0726 \\ 0 \\ -0.2233 \end{bmatrix},$$

$$\mathbf{D}_1 = \mathbf{D}_2 = \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \bar{\mathbf{A}}_1 = \bar{\mathbf{A}}_2 = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix},$$

$$\bar{\mathbf{B}}_1 = \bar{\mathbf{B}}_2 = 0.01 \times \begin{bmatrix} 0 \\ 0.0726 \\ 0 \\ -0.2233 \end{bmatrix}, \quad \bar{\mathbf{D}}_1 = \bar{\mathbf{D}}_2 = 0.01 \times \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\Delta\mathbf{A}_1 = \Delta\mathbf{A}_2 = \begin{bmatrix} \delta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \mathbf{H}\Delta(k)\mathbf{R}, \quad \mathbf{E}_1 = \mathbf{E}_2 = \mathbf{I}$$

$$\mathbf{H} = [0.02 \ 0 \ 0 \ 0]^T, \quad \mathbf{R} = [0.05 \ 0 \ 0 \ 0],$$

$$\mathbf{R}_{B1} = \mathbf{R}_{B2} = \mathbf{R}_{D1} = \mathbf{R}_{D2} = 0.01, \quad \mathbf{C}_1 = \mathbf{C}_2 = [1 \ 0 \ 0 \ 0]. \quad (7)$$

where $\mathbf{I}$ is identity matrix and the notation $\Delta(k)$ is time-varying matrix function. In this simulation, we assume that $\Delta(k) = \sin(k)$. $E\{w(k)\} = 0$, $E\{w(k)^2\} = 1$, the noise $w(k)$ is assumed independent with $x(k)$. Furthermore, $E\{\bullet\}$ denotes standard expected operator. The uncertain matrices $\Delta\mathbf{A}_i(k)$, $\Delta\mathbf{B}_i(k)$, $\Delta\mathbf{D}_i(k)$ are said to be admissible if both (5) and (6) hold. Given a pair of $(x(k), u(k))$, the final outputs of the fuzzy system can be inferred as follows.

$$\begin{aligned} x(k+1) &= \sum_{i=1}^2 h_i(x_3(k)) (\mathbf{A}_i(k)x(k) + \mathbf{B}_i(k)u(k) \\ &\quad + \mathbf{D}_i(k)v(k) + (\bar{\mathbf{A}}_i x(k) + \bar{\mathbf{B}}_i u(k) + \bar{\mathbf{D}}_i v(k))w(k)) \\ y(k) &= \sum_{i=1}^2 h_i(x_3(k)) (\mathbf{C}_i x(k) + \mathbf{E}_i v(k)) \end{aligned} \quad (8)$$

$$\text{where } h_i(x_3(k)) = \frac{M_i(x_3(k))}{\sum_{i=1}^2 M_i(x_3(k))}, \quad h_i(x_3(k)) \geq 0,$$

$M_i(x_3(k)) = \prod_{j=1}^r M_j(x_3(k))$ and $\sum_{i=1}^2 h_i(x_3(k)) = 1$. And the classical PDC law is given by

$$u(k) = \sum_{i=1}^2 h_i(x_3(k)) \mathbf{F}_i x(k) \quad (9)$$

Substituting (9) into (8), the closed-loop model can be written as

$$\begin{aligned} x(k+1) &= \sum_{i=1}^2 \sum_{j=1}^2 h_i(x_3(k)) h_j(x_3(k)) (\mathbf{G}_{ij}(k)x(k) + \mathbf{D}_i(k) \\ &\quad \times v(k) + (\bar{\mathbf{G}}_{ij}x(k) + \bar{\mathbf{D}}_i v(k))w(k)) \\ y(k) &= \sum_{i=1}^2 h_i(x_3(k)) (\mathbf{C}_i x(k) + \mathbf{E}_i v(k)) \end{aligned} \quad (10)$$

$$\text{where } \mathbf{G}_{ij}(k) = \mathbf{A}_i(k) + \mathbf{B}_i(k)\mathbf{F}_j, \quad \bar{\mathbf{G}}_{ij} = \bar{\mathbf{A}}_i + \bar{\mathbf{B}}_i\mathbf{F}_j. \quad (11)$$

In the following, we introduce the following lemma and definition, which will be used in the derivation of our main results in the next sections.

Definition 1 [8]

Considering the system (1), then the disturbance $v(k)$ and output $y(k)$ are said to be strictly input passive if

$$E \left\{ 2 \sum_{k=0}^{k_q} y^T(k) v(k) \right\} > E \left\{ \sum_{k=0}^{k_q} \gamma v^T(k) v(k) \right\} \quad (12)$$

where $\gamma \geq 0$ and terminal time $k_q > 0$. #

Lemma 1 [13]

Let $\mathbf{Q}$, $\mathbf{H}$, $\mathbf{S}$, $\boldsymbol{\beta}$ and $\boldsymbol{\Psi}$ be real matrices with appropriate dimensions such that $\boldsymbol{\beta} > 0$ and $\boldsymbol{\Psi}^T \boldsymbol{\Psi} \leq \mathbf{I}$. Then, for any scalar $\varepsilon > 0$ such that $\boldsymbol{\beta} - \varepsilon \mathbf{H} \mathbf{H}^T > 0$, one has

$$(\mathbf{Q} + \mathbf{H} \boldsymbol{\Psi} \mathbf{S})^T \boldsymbol{\beta}^{-1} (\mathbf{Q} + \mathbf{H} \boldsymbol{\Psi} \mathbf{S}) \leq \mathbf{Q}^T (\boldsymbol{\beta} - \varepsilon \mathbf{H} \mathbf{H}^T)^{-1} \mathbf{Q} + \varepsilon^{-1} \mathbf{S}^T \mathbf{S}. \quad (13)$$

IV. ROBUST PASSIVE CONTROL DESIGN

In this section, for analyzing and synthesizing the stability of closed-loop system (10), the sufficient stability condition is developed via passivity theory and Lyapunov function.

Theorem 1

Given a dissipative rate $\gamma \geq 0$, if there exists a positive definite matrix $\mathbf{P} > 0$, and scalar $\varepsilon_i > 0$ for satisfying the following conditions, then the closed-loop system (10) is strictly input passive and robustly asymptotically mean square stable.

$$\begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{21}^T \\ \bar{\Phi}_{21} & \bar{\Phi}_{22} \end{bmatrix} < 0 \text{ for } i, j = 1, 2 \quad (14)$$

where

$$\begin{aligned} \bar{\Phi}_{11} &= \mathbf{G}_{ij}^T (\mathbf{P}^{-1} - \varepsilon_i \mathbf{H}_i \mathbf{H}_i^T)^{-1} \mathbf{G}_{ij} + \varepsilon_i^{-1} \mathbf{R}_{ij}^T \mathbf{R}_{ij} + \bar{\mathbf{G}}_{ij}^T \mathbf{P} \bar{\mathbf{G}}_{ij} - \mathbf{P} \\ \bar{\Phi}_{21} &= \Phi_{21} - \mathbf{C}_i, \quad \bar{\Phi}_{22} = \Phi_{22} - \mathbf{E}_i^T - \mathbf{E}_i + \gamma \mathbf{I} \\ \Phi_{21} &= \mathbf{D}_i^T (\mathbf{P}^{-1} - \varepsilon_i \mathbf{H}_i \mathbf{H}_i^T)^{-1} \mathbf{G}_{ij} + \varepsilon_i^{-1} \mathbf{R}_{Di}^T \mathbf{R}_{ij} + \bar{\mathbf{D}}_i^T \mathbf{P} \bar{\mathbf{G}}_{ij} \\ \Phi_{22} &= \mathbf{D}_i^T (\mathbf{P}^{-1} - \varepsilon_i \mathbf{H}_i \mathbf{H}_i^T)^{-1} \mathbf{D}_i + \varepsilon_i^{-1} \mathbf{R}_{Di}^T \mathbf{R}_{Di} + \bar{\mathbf{D}}_i^T \mathbf{P} \bar{\mathbf{D}}_i \end{aligned} \quad (15)$$

Proof:

Let us choose the Lyapunov function $V(x(k)) = x^T(k) \mathbf{P} x(k)$. Firstly, by calculating difference and taking mathematical expectation of $V(x(k))$ one has.

$$\begin{aligned} E \{ \Delta V(x(k)) \} &= E \{ x^T(k+1) \mathbf{P} x(k+1) - x^T(k) \mathbf{P} x(k) \} \\ &= E \left\{ \sum_{i=1}^2 \sum_{j=1}^2 h_i(x_3(k)) h_j(x_3(k)) \left((\mathbf{G}_{ij}(k)x(k) + \mathbf{D}_i(k)v(k)) \right)^T \right. \\ &\quad \times \mathbf{P} (\mathbf{G}_{ij}(k)x(k) + \mathbf{D}_i(k)v(k)) + (\bar{\mathbf{G}}_{ij}x(k) + \bar{\mathbf{D}}_i v(k))^T \\ &\quad \times \mathbf{P} (\bar{\mathbf{G}}_{ij}x(k) + \bar{\mathbf{D}}_i v(k)) - x^T(k) \mathbf{P} x(k) \left. \right\} \end{aligned} \quad (16)$$

Using the Lemma 1, the following relations can be obtained.

$$\begin{aligned} &(\mathbf{G}_{ij}(k)x(k) + \mathbf{D}_i(k)v(k))^T \mathbf{P} (\mathbf{G}_{ij}(k)x(k) + \mathbf{D}_i(k)v(k)) \\ &\leq \xi^T(k) \left[\begin{bmatrix} \mathbf{G}_{ij}^T \\ \mathbf{D}_i^T \end{bmatrix} (\mathbf{P}^{-1} - \varepsilon_i \mathbf{H}_i \mathbf{H}_i^T)^{-1} \begin{bmatrix} \mathbf{G}_{ij} & \mathbf{D}_i \end{bmatrix} \right. \\ &\quad \left. + \varepsilon_i^{-1} \begin{bmatrix} \mathbf{R}_{ij}^T \\ \mathbf{R}_{Di}^T \end{bmatrix} \begin{bmatrix} \mathbf{R}_{ij}^T \\ \mathbf{R}_{Di}^T \end{bmatrix}^T \right] \xi(k), \end{aligned} \quad (17)$$

where $\mathbf{G}_{ij}(k) = \mathbf{G}_{ij} + \mathbf{H}_i \Delta \mathbf{R}_{ij}$, $\mathbf{R}_{ij} = \mathbf{R}_i + \mathbf{R}_{Bi} \mathbf{F}_j$ and $\xi(k) = \begin{bmatrix} x(k) \\ v(k) \end{bmatrix}$.

From (17), one has following inequality from (16)

$$\begin{aligned} &E \{ \Delta V(x(k)) \} \\ &\leq E \left\{ \sum_{i=1}^2 \sum_{j=1}^2 h_i(x_3(k)) h_j(x_3(k)) \left\{ \xi^T(k) \begin{bmatrix} \bar{\Phi}_{11} & \Phi_{21}^T \\ \Phi_{21} & \bar{\Phi}_{22} \end{bmatrix} \xi(k) \right\} \right\} \end{aligned} \quad (18)$$

Let us introduce the following performance function

$$J = E \left\{ \sum_{k=0}^{k_q} (\gamma v^T(k) v(k) - 2y^T(k) v(k)) \right\} \quad (19)$$

for all $v(k) \neq 0$ with zero initial condition. From (19), one has

$$J \leq E \left\{ \sum_{k=0}^{k_q} (\gamma v^T(k) v(k) - 2y^T(k) v(k) + \Delta V(x(k))) \right\} \\ \equiv E \left\{ \sum_{k=0}^{k_q} \Theta(k) \right\} \quad (20)$$

where

$$\Theta(k) = \Delta V(x(k)) + \gamma v^T(k) v(k) - 2y^T(k) v(k) \quad (21)$$

From (18) and (10), one has

$$\Theta(k) \leq \sum_{i=1}^r \sum_{j=1}^r h_i(x_3(k)) h_j(x_3(k)) \{ \xi^T(k) \Psi \xi(k) \} \quad (22)$$

where $\Psi = \begin{bmatrix} \bar{\Phi}_{11} & \bar{\Phi}_{12} \\ \bar{\Phi}_{21} & \bar{\Phi}_{22} \end{bmatrix}$. Obviously, if the stability condition (14) holds, then one has $\Psi < 0$ and $\Theta(k) < 0$. According to $\Theta(k) < 0$, one has the following relations from (20).

$$J < 0 \text{ and } E \left\{ 2 \sum_{k=0}^{k_q} y^T(k) v(k) \right\} > E \left\{ \sum_{k=0}^{k_q} \gamma v^T(k) v(k) \right\} \quad (23)$$

Since the (23) is equivalent to (12) of Definition 1, the closed-loop system (10) is strictly input passive. Besides, the stability of system is necessary performance for controlled system. Hence, in the following, the stability of system is discussed. If the condition (14) of Theorem 1 is held i.e., $\Psi < 0$, thus $\Theta(k) < 0$ from (22). By assuming the $v(k) = 0$, the $\Delta V(x(k)) < 0$ can be found from (21) due to $\Theta(k) < 0$. Referring to [14], one can find that the system is robustly asymptotically mean square stable due to $E\{\Delta V(x(k))\} < 0$.

So, if the stability condition (14) holds then the closed-loop system (10) is strictly input passive and robustly asymptotically mean square stable. The proof is completed.

#

In Theorem 1, the stability condition (14) is not strict LMI problem for applying convex optimization algorithm. For the reason, one can directly convert (14) into LMI form with Schur complement and variables setting technique. Since the limitation of paper, the convention process is omitted here. In the following section, the proposed method of this paper is employed to design PDC-based fuzzy controller for stabilizing system (2).

V. SIMULATION RESULTS

Consider the closed-loop system (10), and choose the dissipative rate $\gamma = 1$. Applying Theorem 1 and using the MATLAB LMI toolbox, one can obtain

$$P = \begin{bmatrix} 16.8513 & 6.4704 & 22.3447 & 4.3105 \\ 6.4704 & 3.6448 & 12.0250 & 2.2749 \\ 22.3447 & 12.0250 & 45.1505 & 8.6407 \\ 4.3105 & 2.2749 & 8.6407 & 1.6566 \end{bmatrix},$$

$\varepsilon_1 = 3.6974$ and $\varepsilon_2 = 3.4109$. Subsequently, one can obtain the feedback gain

$$F_1 = [17.6175 \quad 3.5721 \quad 53.3833 \quad 11.3578] \text{ and } F_2 = [17.5728 \quad 24.1477 \quad 61.6890 \quad 11.3458].$$

According to the PDC concept, the fuzzy controller can be structured via state feedback gains with control law (9) designed controller, the responses of considered system are stated in Figs. 3-6 with initial condition $x(k) = [0 \quad 0 \quad 0.8 \quad 0]^T$ and $v(k)$ is a zero-mean white noise with variance 0.01. From the simulation results of Figs. 3-6, one can find that the closed-loop system is robustly asymptotically mean square stable. Besides, the following equation is introduced to check the strictly input passive of system.

$$\frac{E \left\{ \sum_{k=0}^{k_q} v^T(k) v(k) \right\}}{E \left\{ 2 \sum_{k=0}^{k_q} y^T(k) v(k) \right\}} = 0.4958 \quad (24)$$

Based on Definition 1, the strictly input passivity of system (2) can be achieved by fuzzy controller (9) due to the value in (24) is less than one. From the Figs. 3-6 and (24), the strictly input passivity and robustly asymptotically mean square stability of system (2) is achieved by employing the designed PDC-based fuzzy controller (9).

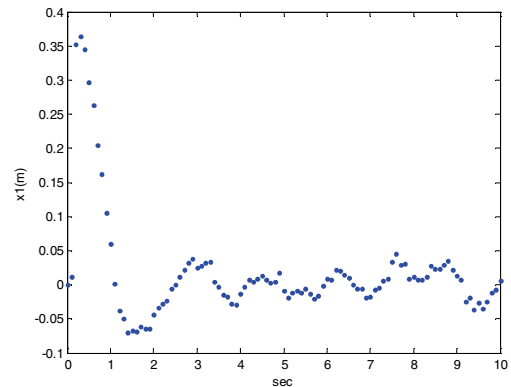


Fig. 3 Responses of $x_1(k)$

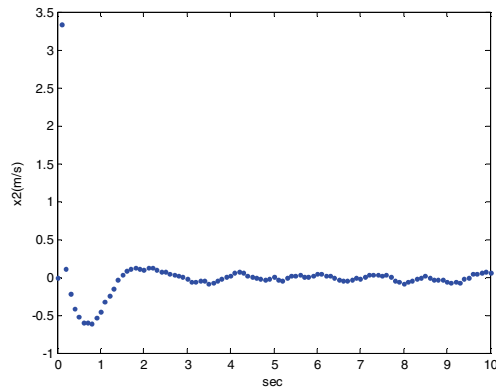


Fig. 4 Responses of $x_2(k)$

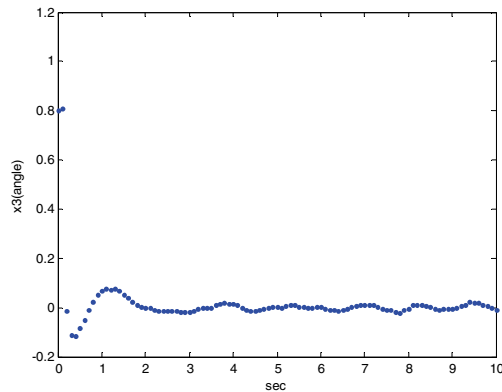


Fig. 5 Responses of $x_3(k)$

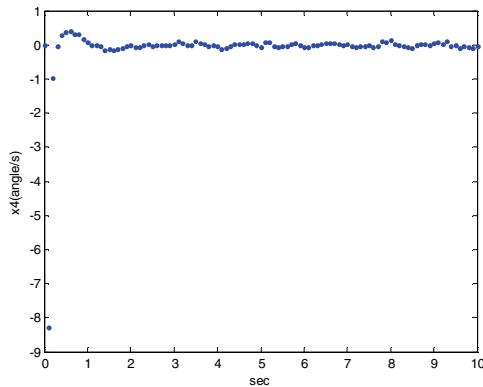


Fig. 6 Responses of $x_4(k)$

VI. CONCLUSIONS

In this paper, the control problem of nonlinear inverted pendulum system with multiplicative noises has been studied.

The T-S fuzzy model was build to represent the original complex nonlinear system. In order to simulate the practical operation conditions, the stochastic behavior and uncertainties of system have been considered during investigation and discussion in this paper. Expect for stability of system, the attenuation performance was also concerned in this paper. Referring to the simulation results, the strictly input passivity and robustly asymptotically mean square stability of nonlinear inverted pendulum robot system with multiplicative noise can be achieved designed PDC-based fuzzy controller.

REFERENCES

- [1] H. J. Lee and S. Jung, "Control of a Mobile Inverted Pendulum Robot System," *Proc. IEEE International Conf. on Control, Automation and Syst.*, pp. 217–222, Oct., 2008.
- [2] L. F. Lupian and R. Avila, "Stabilization of a Wheeled Inverted Pendulum by a Continuous-Time Infinite-Horizon LQG Optimal Controller," *Proc. IEEE Latin American Robotic Symposium*, pp. 65–70, Oct., 2008.
- [3] L. E. Ghaoui, "State-Feedback Control of Systems with Multiplicative Noise via Linear Matrix Inequalities," *Syst. and Control Letters*, vol. 24, pp. 223–228, Feb. 1995.
- [4] H. O. Wang, K. Tanaka and M. Griffin, "An Approach to Fuzzy Control of Nonlinear System: Stability and Design Issues," *IEEE Trans. on Fuzzy syst.*, vol. 4, pp. 14–23, Feb., 1996.
- [5] K. Tanaka and M. Sano, "A Robust Stabilization Problem of Fuzzy Control Systems and Its Application to Backing Up Control of a Truck-Trailer," *IEEE Trans. Fuzzy syst.*, vol. 2, pp. 119–134, May, 1994.
- [6] C. I. Byrnes and W. Lin, "Discrete-Time Lossless Systems, Feedback Equivalence and Passivity," *Proc. IEEE Conf., Decision and Control*, vol. 2, pp. 1775–1781, Dec., 1993.
- [7] H. J. Uang, "On the Dissipativity of Nonlinear Systems: Fuzzy Control Approach," *Fuzzy Sets and Syst.*, vol. 156, pp. 185–207, Dec., 2005.
- [8] W. J. Chang, M. W. Chen and C. C. Ku, "Passive Fuzzy Controller Design for a Model Car via Discrete T-S Fuzzy Model with Multiplicative Noise," *Proc. IEEE International Conf. on Information and Automation*, pp. 1193–1198, June, 2009.
- [9] R. Lozano, B. Brogliato, O. Egeland and B. Maschke, *Dissipative Systems Analysis and Control Theory and Application*, Springer, London, 2000.
- [10] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, John Wiley and Sons, Inc., New York, 2001.
- [11] D. Wang and J. Huang, "A Neural-Network-Based Approximation Method for Discrete-Time Nonlinear Servomechanism Problem," *IEEE Trans. on Neural Networks*, vol. 12, pp. 591–597, May, 2001.
- [12] J. Liang, Z. Wang, Y. Liu, and X. Liu, "Global Synchronization Control of General Delayed Discrete-Time Networks with Stochastic Coupling and Disturbances," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 38, pp. 1073–1083, Aug., 2008.
- [13] Y. Wang, L. Xie, and C. E. de Souza, "Robust Control of a Class of Uncertain Nonlinear Systems," *Syst. and Control Letters*, vol. 19, pp. 139–149, Aug., 1992.
- [14] G. Eli, S. Uri and Y. Isaac, *H_∞ Control and Estimation of State–Multiplicative Linear Systems*, Springer, London, 2005.