

LMI-Based Stability Conditions for Interval Type-2 Fuzzy-Model-Based Control Systems

H.K. Lam¹, Mohammad Narimani² and L.D. Seneviratne³

Division of Engineering,
King's College London,
Strand, London,

WC2R 2LS, United Kingdom

E-mails: ¹hak-keung.lam@kcl.ac.uk, ²mohammad.narimani@kcl.ac.uk, ³lakmal.seneviratne@kcl.ac.uk

Abstract—This paper investigates the stability of the interval type-2 (IT2) fuzzy-model-based (FMB) control systems. An IT2 T-S fuzzy model is developed to represent the nonlinear plant subject to parameter uncertainties, which are captured by the lower and upper membership functions. An IT2 fuzzy model is then proposed to close the feedback loop. It is not required that the IT2 fuzzy controller shares the same premise membership functions or the same number of fuzzy rules as those of the IT2 T-S fuzzy model. Consequently, it offers a greater design flexibility to the IT2 fuzzy controllers. However, the mismatched premise membership functions are not favourable to the development of stability conditions and thus leads to a conservative stability analysis result. In this paper, with the consideration of the lower and upper membership functions, which carry the information of the nonlinearities and parameter uncertainties of the nonlinear plant, the stability of the IT2 FMB control systems is investigated based on the Lyapunov stability theory. Stability conditions in terms of linear matrix inequalities are developed to guarantee the stability of the IT2 FMB control systems and synthesize the IT2 fuzzy controller. A simulation example is given to demonstrate the effectiveness of the proposed approach.

Index Terms—Fuzzy Controller, Fuzzy-Model-Based Control, Interval Type-2, Linear Matrix Inequalities (LMIs), Stability Analysis.

I. INTRODUCTION

Type-1 fuzzy-model-based (FMB) control [1] is a promising control approach for complex and ill-defined nonlinear plants, represented by the type-1 Takagi-Sugeno (T-S) fuzzy model [2], [3]. Based on the Lyapunov stability theory, a set of stability conditions in the form of linear matrix inequalities (LMIs) [4], [5] was obtained to guarantee the system stability. The work in [4], [5] provides the basic stability analysis procedure for the type-1 FMB control systems. More relaxed LMI-based stability conditions were obtained in [7]–[12]. Further relaxed stability conditions were obtained in [13]–[20] by considering the information of the type-1 membership functions.

In general, for the type-1 T-S FMB control system under the parallel distributed compensation (PDC) design [4], [5], the fuzzy controller needs to share the same premise membership function as those of the T-S fuzzy model. When the nonlinear plant is subject to parameter uncertainties, the membership functions of the T-S fuzzy model will contain the uncertain parameters and lead to the grades of membership uncertain in value. Consequently, the advantage of the PDC design given to the stability analysis will be vanished.

Type-2 fuzzy sets [21]–[24] have been shown that it is superior to the type-1 ones on dealing with uncertainties. With the lower and upper membership functions, the uncertainties can be captured and the information of the uncertainties is embedded in the footprint of uncertainty (FOU) [21]. In recent years, some properties and theories of type-2 fuzzy sets have been developed to theoretically support the design and implementation of type-2 fuzzy logic systems. However, it is mainly for general type-2 fuzzy logic systems but not mainly for the purposes of fuzzy control. In type-1 FMB control approach, the type-1 T-S fuzzy model [2], [3] plays an important role representing the dynamics of the nonlinear plant to facilitate the stability analysis and control synthesis. A basic interval type-2 (IT2) T-S fuzzy model was proposed in [25] for a limited class of nonlinear plants. We extended the idea and proposed a more general IT2 T-S fuzzy model [26] for a wider class of nonlinear plants to support the analysis and development of IT2 FMB control systems. A preliminary stability analysis result was reported in [26] for an IT2 FMB control systems under perfect premise matching that the IT2 T-S fuzzy model and IT2 fuzzy controller share the same lower and upper membership functions (which implies the same number of fuzzy rules). A set of LMI-based stability conditions was obtained to guarantee the system stability.

In this paper, we investigate the stability of IT2 FMB control systems under imperfect premises matching. As it is not required that the IT2 T-S fuzzy model and IT2 fuzzy controller share the same lower and upper membership functions, and the same number of fuzzy rules, it enhances the design flexibility of the IT2 fuzzy controller and is able to lower the controller complexity by using a smaller number of fuzzy rules and simple membership functions. However, it makes the stability analysis more difficulty and leads to more conservative stability conditions. To facilitate the stability analysis of IT2 FMB control system, an IT2 T-S fuzzy model [26] is employed to represent the nonlinear plant subject to parameter uncertainties, captured by the lower and upper membership functions. An IT2 fuzzy controller with different lower and upper membership functions is employed to close the feedback loop. We propose a favourable form to represent the lower and upper membership functions for the stability analysis and LMI-based stability conditions are obtained to guarantee the system stability and

synthesize the IT2 fuzzy controller.

The organization of this paper is as follows. In Section II, the IT2 fuzzy model representing the nonlinear plant subject to parameter uncertainties, IT2 fuzzy controller and IT2 FMB control systems are presented. In Section III, LMI-based stability conditions are derived based on the Lyapunov stability theory for the IT2 FMB control systems. In Section IV, a simulation example is given to illustrate the merits of the proposed IT2 FMB control scheme. In Section V, a conclusion is drawn.

II. IT2 TS FUZZY MODEL, IT2 FUZZY CONTROLLER AND IT2 FMB CONTROL SYSTEM

An IT2 FMB control system is referred to a closed-loop system consisting of the IT2 T-S fuzzy model [25], [26] and the IT2 fuzzy controller connected in a closed loop.

A. IT2 T-S Fuzzy Model

Consider an IT2 T-S fuzzy model [25], [26] with p rules of the following format of which the antecedent of each rule contains some IT2 fuzzy sets and the consequent is a linear dynamical system.

Rule i : IF $f_1(\mathbf{x}(t))$ is $\tilde{M}_1^i$ AND $\dots$ AND $f_\Psi(\mathbf{x}(t))$ is $\tilde{M}_\Psi^i$
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)$ (1)

where $\tilde{M}_\alpha^i$ is an IT2 fuzzy set of rule i corresponding to the function $f_\alpha(\mathbf{x}(t))$, $\alpha = 1, 2, \dots, \Psi$; $i = 1, 2, \dots, p$; Ψ is a positive integer; $\mathbf{x}(t) \in \mathbb{R}^n$ is the system state vector; $\mathbf{A}_i \in \mathbb{R}^{n \times n}$ and $\mathbf{B}_i \in \mathbb{R}^{n \times m}$ are the known system and input matrices, respectively; $\mathbf{u}(t) \in \mathbb{R}^m$ is the input vector. The firing strength of the i -th rule is the following interval sets.

$$W_i(\mathbf{x}(t)) = [\underline{w}_i(\mathbf{x}(t)), \overline{w}_i(\mathbf{x}(t))] , i = 1, 2, \dots, p \quad (2)$$

where

$$\underline{w}_i(\mathbf{x}(t)) = \prod_{\alpha=1}^{\Psi} \underline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t))) \geq 0, \quad (3)$$

$$\overline{w}_i(\mathbf{x}(t)) = \prod_{\alpha=1}^{\Psi} \overline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t))) \geq 0, \quad (4)$$

in which $\underline{w}_i(\mathbf{x}(t))$ and $\overline{w}_i(\mathbf{x}(t))$ denote the lower and upper grades of membership, respectively; $\underline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t))) \geq 0$ and $\overline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t))) \geq 0$ denote the lower and upper membership functions, respectively. It exhibits the property that $\overline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t))) \geq \underline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(\mathbf{x}(t)))$ leading to $\overline{w}_i(\mathbf{x}(t)) \geq \underline{w}_i(\mathbf{x}(t))$ for all i . The inferred IT2 T-S fuzzy model [26] is defined as follows.

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t))(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)) \quad (5)$$

where

$$\tilde{w}_i(\mathbf{x}(t)) = \underline{\alpha}_i(\mathbf{x}(t))\underline{w}_i(\mathbf{x}(t)) + \overline{\alpha}_i(\mathbf{x}(t))\overline{w}_i(\mathbf{x}(t)) \geq 0 \quad \forall i, \quad (6)$$

$$\sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t)) = 1, \quad (7)$$

in which $0 \leq \underline{\alpha}_i(\mathbf{x}(t)) \leq 1$ and $0 \leq \overline{\alpha}_i(\mathbf{x}(t)) \leq 1$ are nonlinear functions and exhibit the property of $\underline{\alpha}_i(\mathbf{x}(t)) + \overline{\alpha}_i(\mathbf{x}(t)) = 1$ for all i .

B. IT2 Fuzzy Controller

An IT2 fuzzy controller with c rules of the following format is proposed to stabilize the nonlinear plant represented by the IT2 T-S fuzzy model (5).

Rule j : IF $g_1(\mathbf{x}(t))$ is $\tilde{N}_1^j$ AND $\dots$ AND $g_\Omega(\mathbf{x}(t))$ is $\tilde{N}_\Omega^j$
THEN $\mathbf{u}(t) = \mathbf{G}_j\mathbf{x}(t)$ (8)

where $\tilde{N}_\beta^j$ is an IT2 fuzzy set of rule j corresponding to the function $g_\beta(\mathbf{x}(t))$, $\beta = 1, 2, \dots, \Omega$; $j = 1, 2, \dots, c$; Ω is a positive integer; $\mathbf{G}_j \in \mathbb{R}^{m \times n}$, $j = 1, 2, \dots, c$, are the constant feedback gains to be determined. The firing strength of the j -th rule is the following interval sets.

$$M_j(\mathbf{x}(t)) = [\underline{m}_j(\mathbf{x}(t)), \overline{m}_j(\mathbf{x}(t))] , j = 1, 2, \dots, c \quad (9)$$

where

$$\underline{m}_j(\mathbf{x}(t)) = \prod_{\beta=1}^{\Omega} \underline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\mathbf{x}(t))) \geq 0, \quad (10)$$

$$\overline{m}_j(\mathbf{x}(t)) = \prod_{\beta=1}^{\Omega} \overline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\mathbf{x}(t))) \geq 0, \quad (11)$$

in which $\underline{m}_j(\mathbf{x}(t))$ and $\overline{m}_j(\mathbf{x}(t))$ denote the lower and upper grades of membership, respectively; $\underline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\mathbf{x}(t))) \geq 0$ and $\overline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\mathbf{x}(t))) \geq 0$ denote the lower and upper membership functions, respectively. It exhibits the property that $\underline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\mathbf{x}(t))) \geq \overline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\mathbf{x}(t)))$ leading to $\overline{m}_j(\mathbf{x}(t)) \geq \underline{m}_j(\mathbf{x}(t))$ for all j . The inferred IT2 fuzzy controller is defined as follows.

$$\mathbf{u}(t) = \sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t))\mathbf{G}_j\mathbf{x}(t) \quad (12)$$

where

$$\begin{aligned} \tilde{m}_j(\mathbf{x}(t)) &= \frac{\underline{\beta}_j(\mathbf{x}(t))\underline{m}_j(\mathbf{x}(t)) + \overline{\beta}_j(\mathbf{x}(t))\overline{m}_j(\mathbf{x}(t))}{\sum_{k=1}^c (\underline{\beta}_k(\mathbf{x}(t))\underline{m}_k(\mathbf{x}(t)) + \overline{\beta}_k(\mathbf{x}(t))\overline{m}_k(\mathbf{x}(t)))} \geq 0 \quad \forall j, \end{aligned} \quad (13)$$

$$\sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t)) = 1, \quad (14)$$

in which $0 \leq \underline{\beta}_j(\mathbf{x}(t)) \leq 1$ and $0 \leq \overline{\beta}_j(\mathbf{x}(t)) \leq 1$ are predefined functions and exhibit the property of $\underline{\beta}_j(\mathbf{x}(t)) + \overline{\beta}_j(\mathbf{x}(t)) = 1$ for all j .

C. IT2 FMB Control Systems

An IT2 FMB control system is formed by connecting the IT2 T-S fuzzy model (5) and the IT2 fuzzy controller (12) in a closed loop. From (5) and (12), considering the property of $\sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t)) = \sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t)) = \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i(\mathbf{x}(t)) \tilde{m}_j(\mathbf{x}(t)) = 1$, we have,

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \sum_{i=1}^p \tilde{w}_i(\mathbf{x}(t)) (\mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \sum_{j=1}^c \tilde{m}_j(\mathbf{x}(t)) \mathbf{G}_j \mathbf{x}(t)) \\ &= \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i(\mathbf{x}(t)) \tilde{m}_j(\mathbf{x}(t)) (\mathbf{A}_i + \mathbf{B}_i \mathbf{G}_j) \mathbf{x}(t). \end{aligned} \quad (15)$$

The control objective is to determine the feedback gains, $\mathbf{G}_j$, such that the IT2 fuzzy controller (12) is able to drive the system states towards the origin, i.e., $\mathbf{x}(t) \rightarrow \mathbf{0}$ as time $t \rightarrow \infty$.

The stability of the FMB based control system in the form of (15) was investigated in [4] and guaranteed to be asymptotically stable if the LMI-based stability conditions in the following theorem are satisfied.

Theorem 1: [4] The FMB control system in the form of (15) is guaranteed to be asymptotically stable if there exist matrices $\mathbf{N}_j \in \mathbb{R}^{m \times n}$, $j = 1, 2, \dots, c$, $\mathbf{X} = \mathbf{X}^T \in \mathbb{R}^{n \times n}$ such that the following LMIs are satisfied.

$$\mathbf{X} > 0;$$

$$\mathbf{Q}_{ij} = \mathbf{A}_i \mathbf{X} + \mathbf{X} \mathbf{A}_i^T + \mathbf{B}_i \mathbf{N}_j + \mathbf{N}_j^T \mathbf{B}_i^T < 0 \quad \forall i, j,$$

where the feedback gains are defined as $\mathbf{G}_j = \mathbf{N}_j \mathbf{X}^{-1}$ for all j .

However, without considering the information of the membership functions in the development of the stability conditions in Theorem 1 are very conservative. It can be seen from Theorem 1 that employing different feedback gains does not relax the stability conditions (or make the stability conditions to be satisfied easier). The stability conditions can be reduced to $\mathbf{Q}_{ij} = \mathbf{A}_i \mathbf{X}_j + \mathbf{X}_j \mathbf{A}_i^T + \mathbf{B}_i \mathbf{N} + \mathbf{N}^T \mathbf{B}_i^T < 0$ for all i by choosing a common feedback gain, i.e., $\mathbf{N} = \mathbf{N}_j$ for all j . Consequently, the fuzzy controller becomes a linear controller.

In this paper, the IT2 fuzzy control concept is employed to investigate the stability of the IT2 FMB control system (15). In order to facilitate the stability analysis, the state space of interest denoted as Φ is divided into q connected sub-state spaces denoted as Φ_k , $k = 1, 2, \dots, q$ such that $\Phi = \bigcup_{k=1}^q \Phi_k$. We express the IT2 FMB control system (15) in the following favourable form.

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij}(\mathbf{x}(t)) (\mathbf{A}_i + \mathbf{B}_i \mathbf{G}_j) \mathbf{x}(t) \quad (16)$$

where

$$\begin{aligned} \tilde{h}_{ij}(\mathbf{x}(t)) &\equiv \tilde{w}_i(\mathbf{x}(t)) \tilde{m}_j(\mathbf{x}(t)) \\ &= \underline{\gamma}_{ij}(\mathbf{x}(t)) \underline{h}_{ij}(\mathbf{x}(t)) + \overline{\gamma}_{ij}(\mathbf{x}(t)) \overline{h}_{ij}(\mathbf{x}(t)) \geq 0 \quad \forall i, j; \end{aligned} \quad (17)$$

$$\sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij}(\mathbf{x}(t)) = 1, \quad (18)$$

where $0 \leq \underline{\gamma}_{ij}(\mathbf{x}(t)) \leq 1$ and $0 \leq \overline{\gamma}_{ij}(\mathbf{x}(t)) \leq 1$ are two functions, which are not necessary to be known, exhibiting the property that $\underline{\gamma}_{ij}(\mathbf{x}(t)) + \overline{\gamma}_{ij}(\mathbf{x}(t)) = 1$ for all i and j . The main purpose of these two nonlinear functions is to reconstruct $\tilde{h}_{ij}(\mathbf{x}(t)) \equiv \tilde{w}_i(\mathbf{x}(t)) \tilde{m}_j(\mathbf{x}(t))$ by representing it as a linear combination of $\underline{h}_{ij}(\mathbf{x}(t))$ and $\overline{h}_{ij}(\mathbf{x}(t))$ as shown in (17). The nonlinear functions $\underline{h}_{ij}(\mathbf{x}(t))$ and $\overline{h}_{ij}(\mathbf{x}(t))$ exhibiting the property of $\underline{h}_{ij}(\mathbf{x}(t)) \leq \tilde{w}_i(\mathbf{x}(t)) \tilde{m}_j(\mathbf{x}(t)) \leq \overline{h}_{ij}(\mathbf{x}(t))$ for all i, j and $\mathbf{x}(t)$ (or a predefined state space of interest) denote the lower and upper bounds of $\tilde{w}_i(\mathbf{x}(t)) \tilde{m}_j(\mathbf{x}(t))$ (which are characterized by the lower and upper membership functions of the IT2 fuzzy model and the IT2 fuzzy controller, i.e., $\underline{w}_i(\mathbf{x}(t))$, $\overline{w}_i(\mathbf{x}(t))$, $\underline{m}_j(\mathbf{x}(t))$ and $\overline{m}_j(\mathbf{x}(t))$), and are defined as follows.

$$\begin{aligned} \underline{h}_{ij}(\mathbf{x}(t)) &= \sum_{k=1}^q \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_n=1}^2 \\ &\quad \times \prod_{r=1}^n v_{ri_r k}(x_r(t)) \underline{\delta}_{ij i_1 i_2 \dots i_n k} \quad \forall i, j, k, \end{aligned} \quad (19)$$

$$\begin{aligned} \overline{h}_{ij}(\mathbf{x}(t)) &= \sum_{k=1}^q \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_n=1}^2 \\ &\quad \times \prod_{r=1}^n v_{ri_r k}(x_r(t)) \overline{\delta}_{ij i_1 i_2 \dots i_n k} \quad \forall i, j, k, \end{aligned} \quad (20)$$

where $0 \leq \underline{\delta}_{ij i_1 i_2 \dots i_n k} \leq 1$ and $0 \leq \overline{\delta}_{ij i_1 i_2 \dots i_n k} \leq 1$ are constant scalars to be determined. The functions $v_{ri_s k}(x_r(t))$ are defined such that $0 \leq v_{ri_s k}(x_r(t)) \leq 1$ and $v_{r1k}(x_r(t)) + v_{r2k}(x_r(t)) = 1$ for $r, s = 1, 2, \dots, n$; $i_r = 1, 2$; $\mathbf{x} \in \Phi_k$; otherwise, $v_{ri_s k}(x_r(t)) = 0$. As a result, we have the property that $\sum_{k=1}^q \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{ri_r k}(x_r(t)) = 1$, which is used in the stability analysis.

III. STABILITY ANALYSIS

In this section, we will investigate the stability of the IT2 FMB control system (15) through the Lyapunov stability theory. A set of LMI-based stability conditions will be derived to guarantee the system stability and facilitate the control synthesis. For brevity, in the following analysis, the time t associated with the variables is dropped for situation without ambiguity, e.g., $\mathbf{x}(t)$ is denoted as $\mathbf{x}$. The variables $\underline{w}_i(\mathbf{x}(t))$, $\overline{w}_i(\mathbf{x}(t))$, $\underline{w}_i(\mathbf{x}(t))$, $\underline{m}_j(\mathbf{x}(t))$, $\overline{m}_j(\mathbf{x}(t))$, $\tilde{m}_j(\mathbf{x}(t))$, $\tilde{h}_{ij}(\mathbf{x}(t))$, $v_{1i_1 k}(x_1(t))$, $v_{2i_2 k}(x_2(t))$, $\dots$, $v_{ni_n k}(x_n(t))$ are denoted by $\underline{w}_i$, $\overline{w}_i$, $\tilde{w}_i$, $\underline{m}_j$, $\overline{m}_j$, $\tilde{m}_j$, $\tilde{h}_{ij}$, $v_{1i_1 k}$, $v_{2i_2 k}$, $\dots$, $v_{ni_n k}$, respectively. Furthermore, the property of $\sum_{i=1}^p \tilde{w}_i = \sum_{j=1}^c \tilde{m}_j = \sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i \tilde{m}_j = \sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij} = 1$ is utilized.

We consider the following quadratic Lyapunov function candidate to investigate the stability of the IT2 FMB control systems (15) expressed in the form of (16).

$$V = \mathbf{x}^T \mathbf{P} \mathbf{x}, \quad (21)$$

where $0 < \mathbf{P} = \mathbf{P}^T \in \mathbb{R}^{n \times n}$.

Denote $\mathbf{z} = \mathbf{P}^{-1}\mathbf{x}$ and $\mathbf{X} = \mathbf{P}^{-1}$. Define the feedback gains $\mathbf{G}_j = \mathbf{N}_j\mathbf{X}^{-1}$ where $\mathbf{N}_j \in \mathbb{R}^{m \times n}$, $j = 1, 2, \dots, c$, are matrices to be determined. From (16) and (21), we have,

$$\begin{aligned}\dot{V} &= \dot{\mathbf{x}}^T \mathbf{P} \mathbf{x} + \mathbf{x}^T \mathbf{P} \dot{\mathbf{x}} \\ &= \sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij} \mathbf{x}^T ((\mathbf{A}_i + \mathbf{B}_i \mathbf{G}_j)^T \mathbf{P} + \mathbf{P}(\mathbf{A}_i + \mathbf{B}_i \mathbf{G}_j)) \mathbf{x} \\ &= \sum_{i=1}^p \sum_{j=1}^c (\underline{\gamma}_{ij} \underline{h}_{ij} + \bar{\gamma}_{ij} \bar{h}_{ij}) \mathbf{z}^T \mathbf{Q}_{ij} \mathbf{z},\end{aligned}\quad (22)$$

where $\mathbf{Q}_{ij} = \mathbf{A}_i \mathbf{X} + \mathbf{X} \mathbf{A}_i^T + \mathbf{B}_i \mathbf{N}_j + \mathbf{N}_j^T \mathbf{B}_i^T$.

Recalling the property that $\bar{h}_{ij} \geq \underline{h}_{ij} \geq 0$, $0 \leq \underline{\gamma}_{ij} \leq 1$, $0 \leq \bar{\gamma}_{ij} \leq 1$ and $\underline{\gamma}_{ij} + \bar{\gamma}_{ij} = 1$ for all i and j , we introduce some slack matrices to the stability analysis through the following inequalities using the S -procedure [6].

$$\left(\sum_{i=1}^p \sum_{j=1}^c (\underline{\gamma}_{ij} \underline{h}_{ij} + \bar{\gamma}_{ij} \bar{h}_{ij}) - 1 \right) \mathbf{M} = \mathbf{0}, \quad (23)$$

$$- \sum_{i=1}^p \sum_{j=1}^c (1 - \underline{\gamma}_{ij}) (\underline{h}_{ij} - \bar{h}_{ij}) \mathbf{W}_{ij} \geq \mathbf{0}, \quad (24)$$

where $\mathbf{M} = \mathbf{M}_{ij}^T \in \mathbb{R}^{n \times n}$ are arbitrary matrices and $0 \leq \mathbf{W}_{ij} = \mathbf{W}_{ij}^T \in \mathbb{R}^{n \times n}$.

From (22), (23) and (24), we have

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^p \sum_{j=1}^c (\underline{\gamma}_{ij} \underline{h}_{ij} + (1 - \underline{\gamma}_{ij}) \bar{h}_{ij}) \mathbf{z}^T \mathbf{Q}_{ij} \mathbf{z} \\ &\quad - \sum_{i=1}^p \sum_{j=1}^c (1 - \underline{\gamma}_{ij}) (\underline{h}_{ij} - \bar{h}_{ij}) \mathbf{z}^T \mathbf{W}_{ij} \mathbf{z} \\ &\quad + \left(\sum_{i=1}^p \sum_{j=1}^c (\underline{\gamma}_{ij} \underline{h}_{ij} + (1 - \underline{\gamma}_{ij}) \bar{h}_{ij}) - 1 \right) \mathbf{z}^T \mathbf{M} \mathbf{z} \\ &= \mathbf{z}^T \left(\sum_{i=1}^p \sum_{j=1}^c (\bar{h}_{ij} \mathbf{Q}_{ij} - (\underline{h}_{ij} - \bar{h}_{ij}) \mathbf{W}_{ij} + \bar{h}_{ij} \mathbf{M}) - \mathbf{M} \right) \mathbf{z} \\ &\quad + \sum_{i=1}^p \sum_{j=1}^c \underline{\gamma}_{ij} (\underline{h}_{ij} - \bar{h}_{ij}) \mathbf{z}^T (\mathbf{Q}_{ij} + \mathbf{W}_{ij} + \mathbf{M}) \mathbf{z}.\end{aligned}\quad (25)$$

It can be seen from (21) that $V > 0$ for all $\mathbf{x} \neq \mathbf{0}$. From (25), if the inequalities, $\sum_{i=1}^p \sum_{j=1}^c (\bar{h}_{ij} \mathbf{Q}_{ij} - (\underline{h}_{ij} - \bar{h}_{ij}) \mathbf{W}_{ij} + \bar{h}_{ij} \mathbf{M}) - \mathbf{M} < 0$ and $\mathbf{Q}_{ij} + \mathbf{W}_{ij} + \mathbf{M} > 0$ (as $\underline{\gamma}_{ij} (\underline{h}_{ij} - \bar{h}_{ij}) \leq 0$) for all i and j are satisfied, we have $\dot{V} < 0$ for all $\mathbf{x} \neq \mathbf{0}$. As a result, the IT2 FMB control system is guaranteed to be asymptotically stable in the sense of Lyapunov. The second set of inequalities is in the form of LMI, however, the first one is not due to the existence of the lower and upper membership functions $\underline{h}_{ij}$ and $\bar{h}_{ij}$. With the expression of (19) and (20), recalling that $\sum_{k=1}^q \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{ri_r k} = 1$, the second

first inequality can be written as follows.

$$\begin{aligned}&\sum_{k=1}^q \sum_{i_1=1}^2 \sum_{i_2=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{ri_r k} \left(\sum_{i=1}^p \sum_{j=1}^c (\bar{\delta}_{ij i_1 i_2 \dots i_n k} \mathbf{Q}_{ij} \right. \\ &\quad \left. - (\underline{\delta}_{ij i_1 i_2 \dots i_n k} - \bar{\delta}_{ij i_1 i_2 \dots i_n k}) \mathbf{W}_{ij} + \bar{\delta}_{ij i_1 i_2 \dots i_n k} \mathbf{M} \right) \\ &\quad - \mathbf{M} < 0\end{aligned}\quad (26)$$

It can be seen that the inequality $\sum_{i=1}^p \sum_{j=1}^c (\bar{h}_{ij} \mathbf{Q}_{ij} - (\underline{h}_{ij} - \bar{h}_{ij}) \mathbf{W}_{ij} + \bar{h}_{ij} \mathbf{M}) - \mathbf{M} < 0$ can be achieved if $\sum_{i=1}^p \sum_{j=1}^c (\bar{\delta}_{ij i_1 i_2 \dots i_n k} \mathbf{Q}_{ij} - (\underline{\delta}_{ij i_1 i_2 \dots i_n k} - \bar{\delta}_{ij i_1 i_2 \dots i_n k}) \mathbf{W}_{ij} + \bar{\delta}_{ij i_1 i_2 \dots i_n k} \mathbf{M}) - \mathbf{M} < 0$ for all $i_1, i_2, \dots, i_n$ and k .

Referring to (26), the advantages of representing the IT2 FMB control system (15) in the form of (16) can be seen. The membership functions $\tilde{h}_{ij}$ are reconstructed by a linear combination of the lower and upper membership functions $\underline{h}_{ij}$ and $\bar{h}_{ij}$. Consequently, as seen from (25), the stability of the IT2 FMB control system is determined by the lower and upper membership functions $\underline{h}_{ij}$ and $\bar{h}_{ij}$. By expressing $\underline{h}_{ij}$ and $\bar{h}_{ij}$ in the form of (19) and (20), respectively, they are characterized by the constant scalars $\underline{\delta}_{ij i_1 i_2 \dots i_n k}$ and $\bar{\delta}_{ij i_1 i_2 \dots i_n k}$. Furthermore, as the cross terms $\prod_{r=1}^n v_{ri_r k}(x_r(t))$ are independent of i and j , they can be extracted as shown in (26) to facilitate the stability analysis. With these favourable properties, we only need to check $\sum_{i=1}^p \sum_{j=1}^c (\bar{\delta}_{ij i_1 i_2 \dots i_n k} \mathbf{Q}_{ij} - (\underline{\delta}_{ij i_1 i_2 \dots i_n k} - \bar{\delta}_{ij i_1 i_2 \dots i_n k}) \mathbf{W}_{ij} + \bar{\delta}_{ij i_1 i_2 \dots i_n k} \mathbf{M}) - \mathbf{M} < 0$ at some discrete points ($\underline{\delta}_{ij i_1 i_2 \dots i_n k}$ and $\bar{\delta}_{ij i_1 i_2 \dots i_n k}$) instead of every single point of the lower and upper membership functions $\underline{h}_{ij}$ and $\bar{h}_{ij}$ to guarantee the holding of the inequality (26).

Based on the Lyapunov stability theory, the IT2 FMB control system (15) is guaranteed to be asymptotically stable if the LMI-based stability conditions summarized in the following theorem are satisfied.

Theorem 2: The IT2 FMB control system (15) under imperfect premise matching, formed by a nonlinear plant (represented by the IT2 T-S fuzzy model (5)) and an IT2 fuzzy controller (12) connected in a closed loop, is guaranteed to be asymptotically stable if there exist matrices $\mathbf{M} = \mathbf{M} \in \mathbb{R}^{n \times n}$, $\mathbf{N}_j \in \mathbb{R}^{m \times n}$, $\mathbf{X} = \mathbf{X}^T \in \mathbb{R}^{n \times n}$, $\mathbf{W}_{ij} = \mathbf{W}_{ij}^T \in \mathbb{R}^{n \times n}$, $i = 1, 2, \dots, p$; $j = 1, 2, \dots, c$, such that the following LMIs are satisfied.

$$\mathbf{X} > \mathbf{0}; \quad (27)$$

$$\mathbf{W}_{ij} \geq \mathbf{0}, \forall i, j; \quad (28)$$

$$\mathbf{Q}_{ij} + \mathbf{W}_{ij} + \mathbf{M} > \mathbf{0}, \forall i, j; \quad (29)$$

$$\begin{aligned}&\sum_{i=1}^p \sum_{j=1}^c (\bar{\delta}_{ij i_1 i_2 \dots i_n k} \mathbf{Q}_{ij} - (\underline{\delta}_{ij i_1 i_2 \dots i_n k} - \bar{\delta}_{ij i_1 i_2 \dots i_n k}) \mathbf{W}_{ij} \\ &\quad + \bar{\delta}_{ij i_1 i_2 \dots i_n k} \mathbf{M}) - \mathbf{M} < \mathbf{0}, \forall i_1, i_2, \dots, i_n, k;\end{aligned}\quad (30)$$

where $\underline{\delta}_{ij i_1 i_2 \dots i_n k}$ and $\bar{\delta}_{ij i_1 i_2 \dots i_n k}$, $i = 1, 2, \dots, p$; $j = 1, 2, \dots, c$; $i_1, i_2, \dots, i_n = 1, 2$; $k = 1, 2, \dots, q$ are pre-defined constant scalars satisfying (19) and (20);

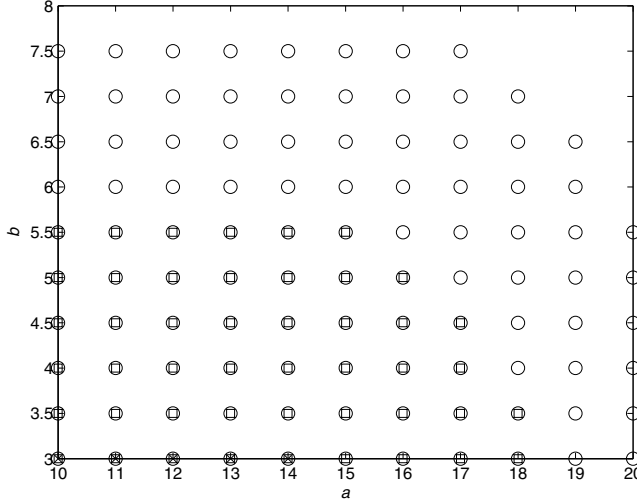


Fig. 1. Stability regions given by the stability conditions in Theorem 2 for Case 1 ('x'); Case 2 ('□'); and Case 3 ('o').

$\mathbf{Q}_{ij} = \mathbf{A}_i \mathbf{X} + \mathbf{X} \mathbf{A}_i^T + \mathbf{B}_i \mathbf{N}_j + \mathbf{N}_j^T \mathbf{B}_i^T$ for all i and j ; and the feedback gains are defined as $\mathbf{G}_j = \mathbf{N}_j \mathbf{X}^{-1}$ for all j .

Remark 1: It should be noted that the LMI-based stability conditions in Theorem 2 are for the IT2 FMB control systems under imperfect premise matching that the IT2 fuzzy controller does not require sharing the same lower and upper premise membership functions and the same number of fuzzy rules as those of the IT2 T-S fuzzy model. As a result, compared with the stability conditions for the IT2 FMB control systems under perfect premise matching [26], it offers a higher design flexibility to the IT2 fuzzy controller.

Remark 2: It can be shown that the stability conditions in Theorem 1 is a particular case of Theorem 2 (the solution of the stability conditions in Theorem 1 is that of Theorem 2). Referring to Theorem 1, if there exists a solution, we have $\mathbf{X} > 0$ and $\mathbf{Q}_{ij} < 0$ for all i and j . Referring to Theorem 2, choosing $\mathbf{M} = \varepsilon_1 \mathbf{I} > 0$ where $\varepsilon_1 > 0$. The LMIs (28) and (29) must be satisfied by choosing $\mathbf{W}_{ij} = -\mathbf{Q}_{ij} + (-\varepsilon_1 + \varepsilon_2) \mathbf{I} > 0$ for all i and j with sufficiently small non-zero positive value of ε_1 and ε_2 . As a result, recalling that $\bar{\delta}_{ij_1 i_2 \dots i_n k} \geq \underline{\delta}_{ij_1 i_2 \dots i_n k} \geq 0$, the LMIs in (30) become $\sum_{i=1}^p \sum_{j=1}^c (\bar{\delta}_{ij_1 i_2 \dots i_n k} \varepsilon_2 \mathbf{I} - \underline{\delta}_{ij_1 i_2 \dots i_n k} \mathbf{W}_{ij}) - \varepsilon_1 \mathbf{I} < 0$ for all $i_1, i_2, \dots, i_n$ and k , which will be satisfied by a sufficiently small value of ε_2 .

IV. SIMULATION EXAMPLE

We consider a nonlinear plant represented by a 3-rule IT2 T-S fuzzy model in the form of (5), with $\mathbf{A}_1 = \begin{bmatrix} 1.59 & -7.29 \\ 0.01 & 0 \end{bmatrix}$, $\mathbf{A}_2 = \begin{bmatrix} 0.02 & -4.64 \\ 0.35 & 0.21 \end{bmatrix}$, $\mathbf{A}_3 = \begin{bmatrix} -a & -4.33 \\ 0 & 0.05 \end{bmatrix}$, $\mathbf{B}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\mathbf{B}_2 = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$, $\mathbf{B}_3 =$

TABLE I
PARAMETER VALUES FOR $\underline{\beta}_j, \bar{\beta}_j, d_1$ AND d_2 .

	Case 1	Case 2	Case 3
$\underline{\beta}_j$	1	0.5	0
$\bar{\beta}_j$	0	0.5	1
d_1	0.3	0.3	0.25
d_2	0.25	0.15	0.15

$\begin{bmatrix} -b+6 \\ -1 \end{bmatrix}$, $\mathbf{x} = [x_1 \ x_2]^T$, a and b are constant system parameters. For demonstration purposes, the actual membership functions are chosen as $\tilde{w}_1(x_1) = \mu_{M_1^1}(x_1) = 1 - \frac{1}{1+e^{-(x_1+4+\sigma(t))}}$, $\tilde{w}_2(x_1) = \mu_{M_1^2}(x_1) = 1 - \tilde{w}_1(x_1) - \tilde{w}_3(x_1)$ and $\tilde{w}_3(x_1) = \mu_{M_1^3}(x_1) = \frac{1}{1+e^{-(x_1-4+\sigma(t))}}$ where $\sigma(t) \in [-0.1, 0.1]$ represents the parameter uncertainty. The lower and upper membership functions for the IT2 T-S fuzzy model are chosen to be $\underline{w}_1(x_1) = \underline{\mu}_{\tilde{M}_1^1}(x_1) = 1 - \frac{1}{1+e^{-(x_1+4+d_1)}}$, $\underline{w}_3(x_1) = \underline{\mu}_{\tilde{M}_1^3}(x_1) = \frac{1}{1+e^{-(x_1-4-d_1)}}$, $\bar{w}_1(x_1) = \bar{\mu}_{\tilde{M}_1^1}(x_1) = 1 - \frac{1}{1+e^{-(x_1+4-d_1)}}$, $\bar{w}_3(x_1) = \bar{\mu}_{\tilde{M}_1^3}(x_1) = \frac{1}{1+e^{-(x_1-4+d_1)}}$, $\underline{w}_2(x_1) = \underline{\mu}_{\tilde{M}_1^2}(x_1) = 1 - \bar{\mu}_{\tilde{M}_1^1}(x_1) - \bar{\mu}_{\tilde{M}_1^3}(x_1)$ and $\bar{w}_2(x_1) = \bar{\mu}_{\tilde{M}_1^2}(x_1) = 1 - \underline{\mu}_{\tilde{M}_1^1}(x_1) - \underline{\mu}_{\tilde{M}_1^3}(x_1)$ where d_1 is a constant to be determined.

A 2-rule fuzzy controller in the form of (12) is employed to stabilize the nonlinear plant. The lower and upper membership functions are chosen as $\underline{m}_1(x_1) = \underline{\mu}_{\tilde{N}_1^1}(x_1) = 1 - \frac{1}{e^{\frac{-x_1+d_2}{2}}}$, $\bar{m}_1(x_1) = \bar{\mu}_{\tilde{N}_1^1}(x_1) = 1 - \frac{1}{e^{\frac{-x_1-d_2}{2}}}$, $\underline{m}_2(x_1) = \underline{\mu}_{\tilde{N}_1^2}(x_1) = 1 - \bar{\mu}_{\tilde{N}_1^1}(x_1)$ and $\bar{m}_2(x_1) = \bar{\mu}_{\tilde{N}_1^2}(x_1) = 1 - \underline{\mu}_{\tilde{N}_1^1}(x_1)$. From (13), we have $\tilde{m}_j(x_1) = \frac{\underline{\beta}_j \underline{m}_j(x_1) + \bar{\beta}_j \bar{m}_j(x_1)}{\sum_{k=1}^c (\underline{\beta}_k \underline{m}_k(x_1) + \bar{\beta}_k \bar{m}_k(x_1))}$ for $j = 1, 2$, where $\underline{\beta}_j$ and $\bar{\beta}_j$ are chosen to be a constant; d_2 is a constant to be determined, in this example.

To determine the lower and upper membership functions $\underline{h}_{ij}(x_1)$ and $\bar{h}_{ij}(x_1)$, we consider $x_1 \in [-10, 10]$ and divide the state space of x_1 into 20 regions, i.e., $\phi_k : \underline{x}_{1,k} \leq x_1 < \bar{x}_{1,k}$, $k = 1, 2, \dots, 19$ and $\phi_{20} : \underline{x}_{1,20} \leq x_1 \leq \bar{x}_{1,20}$, where $\underline{x}_{1,k} = (k-11)$ and $\bar{x}_{1,k} = (k-10)$. The lower and upper membership functions $\underline{h}_{ij}(x_1)$ and $\bar{h}_{ij}(x_1)$ can be defined by choosing $v_{11k}(x_1) = 1 - \frac{x_1 - \underline{x}_{1,k}}{\bar{x}_{1,k} - \underline{x}_{1,k}}$ and $v_{12k}(x_1) = 1 - v_{11k}(x_1)$; and the constant scalars as $\underline{\delta}_{ij1k} = \underline{w}_i(\underline{x}_{1,k}) \underline{m}_j(\underline{x}_{1,k})$, $\underline{\delta}_{ij2k} = \underline{w}_i(\bar{x}_{1,k}) \underline{m}_j(\bar{x}_{1,k})$, $\bar{\delta}_{ij1k} = \bar{w}_i(\underline{x}_{1,k}) \bar{m}_j(\underline{x}_{1,k})$, $\bar{\delta}_{ij2k} = \bar{w}_i(\bar{x}_{1,k}) \bar{m}_j(\bar{x}_{1,k})$ for all k .

The stability conditions in Theorem 2 are employed to check the stability of the IT2 FMB control system with the Matlab LMI toolbox. We consider three cases with different values of $\underline{\beta}_j, \bar{\beta}_j, d_1$ and d_2 , as shown in Table I, to test the stability conditions in Theorem 2. The values of d_1 and d_2 are chosen such that $\bar{h}_{ij}(x_1)$ in the form of (17) are within the lower and upper membership functions defined in (19) and (20), respectively. Considering $10 \leq a \leq 20$ at the interval of 1 and $3 \leq b \leq 8$ at the interval of 0.5, with the $\underline{\beta}_j = \bar{\beta}_j = 0.5$, the stability regions are shown in Fig. 1 indicated by 'x',

‘□’ and ‘○’ corresponding to Case 1 to Case 3, respectively. It can be seen that different values of $\underline{\beta}_j$ and $\overline{\beta}_j$, leading to different values of d_1 and d_2 , will produce different size of stability regions. It demonstrates a trend that Case 3 produces a larger stability region than Case 2 while Case 3 produces a larger stability region than Case 1 due to different values of d_1 and d_2 . For comparison purposes, Theorem 1 is employed to check the stability of the IT2 FMB control system. However, no feasible solution can be obtained. It should be noted that the IT2 FMB control system is under imperfect premise matching, the stability conditions in [26] for perfect premise matching cannot be applied in this example.

V. CONCLUSION

The stability of IT2 FMB control systems subject to parameter uncertainties under imperfect matching has been investigated. It is not required that both the IT2 T-S fuzzy model and the IT2 fuzzy controller share the same number of rules and/or the same set of premise membership functions. Thus, it offers a greater design flexibility for the IT2 fuzzy controller and is possible to lower the controller complexity by employing a smaller number of rules and simple membership functions. Through the lower and upper membership functions, the information of the nonlinearity and parameter uncertainties embedded in the footprint of uncertainty are brought to the stability analysis. Based on the Lyapunov stability theory, LMI-based stability conditions are obtained to guarantee the stability of the IT2 FMB control system and synthesize the IT2 fuzzy controller. Simulation examples have been given to demonstrate the merits of the proposed approach.

ACKNOWLEDGMENT

The work described in this paper was supported by King’s College London.

REFERENCES

- [1] G. Feng, “A survey on analysis and design of model-based fuzzy control systems,” *IEEE Trans. Fuzzy Systems*, vol. 14, no. 5, pp. 676–697, Oct. 2006.
- [2] T. Takagi and M. Sugeno, “Fuzzy identification of systems and its applications to modelling and control,” *IEEE Trans. Syst., Man., Cybern.*, vol. smc-15, no. 1, pp. 116–132, Jan. 1985.
- [3] M. Sugeno and G. T. Kang, “Structure identification of fuzzy model,” *Fuzzy sets and systems*, vol. 28, no. 1, pp. 15–33, Oct. 1988.
- [4] H. O. Wang, K. Tanaka, and M. F. Griffin, “An approach to fuzzy control of nonlinear systems: Stability and design issues,” *IEEE Trans. Fuzzy Systems*, vol. 4, no. 1, pp. 14–23, Feb. 1996.
- [5] K. Tanaka, T. Ikeda, and H. O. Wang, “Fuzzy regulators and fuzzy observers: Relaxed stability conditions and LMI-based designs,” *IEEE Trans. Fuzzy Systems*, vol. 6, no. 2, pp. 250–265, May 1998.
- [6] S. P. Boyd, *Linear matrix inequalities in system and control theory*. Society for Industrial and Applied Mathematics (SIAM), 1994.
- [7] E. Kim and H. Lee, “New approaches to relaxed quadratic stability condition of fuzzy control systems,” *IEEE Trans. Fuzzy Systems*, vol. 8, no. 5, pp. 523–534, Oct. 2000.
- [8] M. C. M. Teixeira, E. Assuncao, and R. G. Avellar, “On relaxed LMI-based designs for fuzzy regulators and fuzzy observers,” *IEEE Trans. Fuzzy Systems*, vol. 11, no. 5, pp. 613–623, Oct. 2003.
- [9] X. Liu and Q. Zhang, “New approaches to H_∞ controller designs based on fuzzy observers for Takagi-Sugeno fuzzy systems via LMI,” *Automatica*, vol. 39, no. 9, pp. 1571–1582, Sep. 2003.
- [10] —, “Approaches to quadratic stability conditions and H_∞ control designs for Takagi-Sugeno fuzzy systems,” *IEEE Trans. Fuzzy Systems*, vol. 11, no. 6, pp. 830–839, Dec. 2003.
- [11] C. H. Fang, Y. S. Liu, S. W. Kau, L. Hong, and C. H. Lee, “A new LMI-based approach to relaxed quadratic stabilization of Takagi-Sugeno fuzzy control systems,” *IEEE Trans. Fuzzy Systems*, vol. 14, no. 3, pp. 386–397, Jun. 2006.
- [12] A. Sala and C. Ariño, “Asymptotically necessary and sufficient conditions for stability and performance in fuzzy control: Applications of Polya’s theorem,” *Fuzzy Sets Syst.*, vol. 158, no. 24, pp. 2671–2686, Jul. 2007.
- [13] —, “Relaxed stability and performance conditions for Takagi-Sugeno fuzzy systems with knowledge on membership function overlap,” *IEEE Trans. Syst., Man and Cybern., Part B: Cybernetics*, vol. 37, no. 3, pp. 727–732, Jun. 2007.
- [14] —, “Relaxed stability and performance LMI conditions for Takagi-Sugeno fuzzy systems with polynomial constraints on membership function shapes,” *IEEE Trans. Fuzzy Systems*, vol. 16, no. 5, pp. 1328–1336, Oct. 2008.
- [15] A. Kruszcwski, A. Sala, T. Guerra, and C. Arino, “A triangulation approach to asymptotically exact conditions for fuzzy summations,” *IEEE Trans. on Fuzzy Systems*, vol. 17, no. 5, pp. 985–994, Oct. 2009.
- [16] H. K. Lam and F. H. F. Leung, “Stability analysis of fuzzy control systems subject to uncertain grades of membership,” *IEEE Trans. Syst., Man and Cybern., Part B: Cybernetics*, vol. 35, no. 6, pp. 1322–1325, Dec. 2005.
- [17] C. Ariño and A. Sala, “Extensions to “stability analysis of fuzzy control systems subject to uncertain grades of membership,”,” *IEEE Trans. Syst., Man and Cybern., Part B: Cybernetics*, vol. 38, no. 2, pp. 558–563, Apr. 2008.
- [18] H. K. Lam and M. Narimani, “Stability analysis and performance design for fuzzy-model-based control system under imperfect premise matching,” *IEEE Trans. Fuzzy Systems*, vol. 17, no. 4, pp. 949–961, Aug. 2009.
- [19] M. Narimani and H. K. Lam, “Relaxed LMI-based stability conditions for Takagi-Sugeno fuzzy control systems using regional-membership-function-shape-dependent analysis approach,” *IEEE Trans. Fuzzy Systems*, vol. 17, no. 5, pp. 1221–1228, Oct. 2009.
- [20] H. K. Lam and M. H. Narimani, “Quadratic stability analysis of fuzzy-model-based control systems using staircase membership functions,” *IEEE Trans. Fuzzy Systems*, vol. 18, no. 1, pp. 125–137, Feb. 2010.
- [21] J. M. Mendel, R. I. John, and F. Liu, “Interval type-2 fuzzy logic systems made simple,” *IEEE Trans. Fuzzy Systems*, vol. 14, no. 6, pp. 808–821, Dec. 2006.
- [22] J. M. Mendel and F. Liu, “Super-exponential convergence of the Karnik–Mendel algorithms for computing the centroid of an interval type-2 fuzzy set,” *IEEE Trans. Fuzzy Systems*, vol. 15, no. 2, pp. 309–320, Apr. 2007.
- [23] S. Coupland and R. John, “Geometric type-1 and type-2 fuzzy logic systems,” *IEEE Trans. Fuzzy Systems*, vol. 15, no. 1, pp. 3–15, Feb. 2007.
- [24] E. A. Jammeh, M. Fleury, C. Wagner, H. Hagrass, and M. Ghanbari, “Interval type-2 fuzzy logic congestion control for video streaming across IP networks,” *IEEE Trans. on Fuzzy Systems*, vol. 17, no. 5, pp. 1123–1142, Oct. 2009.
- [25] Q. Liang and J. M. Mendel, “Equalization of nonlinear time-varying channels using type-2 fuzzy adaptive filters,” *IEEE Trans. Fuzzy Systems*, vol. 8, no. 5, pp. 551–563, Oct. 2000.
- [26] H. K. Lam and L. D. Seneviratne, “Stability analysis of interval type-2 fuzzy-model-based control systems,” *IEEE Trans. Syst., Man and Cybern., Part B: Cybernetics*, vol. 38, no. 3, pp. 617–628, Jun. 2008.