

Output Regulation Using Integral Fuzzy Predictive Control with Piecewise Lyapunov Functions

Chien-Hung Liu

Department of Avionics
China University of Science and Technology
Hsinchu 31241, Taiwan
Telephone: 886-3-5935707-305
Fax: 886-3-5936591
E-mail: lch@cc.cust.edu.tw

Kuang-Yow Lian

Department of Electrical Engineering
National Taipei University of Technology
Taipei 10608, Taiwan
Telephone: 886-2-27712171-217
Fax: 886-2-27317187
E-mail: kylian@ntut.edu.tw

Abstract—This study proposes an LMI-based integral fuzzy model predictive control (MPC) for output regulation via piecewise Lyapunov function. In order to eliminate the system's bias and guarantee the zero-offset output regulation performance, firstly we take coordinate translation on equilibrium point and introduce an added integral state of output regulation error for the nonlinear plant, and then convert the resulting augmented system into a Takagi-Sugeno fuzzy model. Under the input constraint, an integrator based state feedback control law is computed by solving a convex optimization problem based on the infinite horizon predictive control scheme. Moreover, the proposed control design has following merits: i) reducing the conservatism by using piecewise Lyapunov function; ii) improving the transient performance by adding a decay rate condition; and iii) ensuring asymptotic stability for output regulation in the presence of time varying parameter. Finally, a numerical example is presented to demonstrate the effectiveness of the proposed integral fuzzy predictive controller.

Keywords- *Integral fuzzy model predictive control; linear matrix inequalities (LMIs); output regulation; Takagi-Sugeno fuzzy model; piecewise Lyapunov function*

I. INTRODUCTION

Model predictive control (MPC) has developed considerably over the last two decades [1]. The strategy of MPC is to use the knowledge of the plant to predict how the system behaves and take an appropriate control action. Compared with other control techniques, the key advantage of MPC is that its control law is obtained from an optimization procedure at each sampling time, whose objective function is related to control efforts and the prediction of future states. In addition, it can handle input/output constraints directly. Recently, MPC has been applied widely to many systems, such as nonlinear complex systems, hybrid systems and piecewise affine systems [1]. In practice, most physical plants are nonlinear and contain uncertainties. For a highly nonlinear system, a linear MPC algorithm may not satisfy dynamic performance. Recently, several researchers have developed nonlinear model predictive control (NMPC) algorithms to deal with highly nonlinear industrial processes. In fact, NMPC generally requires an accurate model and then computes an optimal or suboptimal

solution for a nonlinear nonconvex optimization problem at each sampling time. The increase of the prediction horizon further intensifies the time-consuming online computation load [2]. On the other hand, in order to handle the control of uncertain systems and reduce the computational burden, Kothare *et al.* [3] proposed an efficient procedure for robust constrained MPC by using linear matrix inequalities, which is suitable for online predictive controller's implementation. Since then, many LMI-based MPC approaches emerge, such as [4],[5]. More recently, the T-S fuzzy model approach has been incorporated into LMI-based MPC strategy to overcome previous drawbacks of NMPC because of its ability to accurately approximate complex nonlinear systems by a set of linear subsystems with associated membership functions [6],[7]. In stability analysis, it has been known that a common Lyapunov function might not exist for many fuzzy systems, especially for highly nonlinear complex systems. Instead, many authors [8]-[10] have presented a stability result of fuzzy systems using a piecewise Lyapunov function because of which can able to deal with a larger class of fuzzy dynamic systems in controller design.

The study of output regulation problems for nonlinear systems keeps attracting considerable attention due to demands from practical dynamical processes in mechanics, economics and biology. Based on this observation, many LMI-based fuzzy controller designs have been developed to solve output regulation control problems for nonlinear systems [11],[12]. In recently year, several well-developed approaches are proposed to design LMI-based NMPC algorithms for the purpose of regulating the system output at constant values [13],[14]. In this work, we combine coordinate translation, integral control technique, and fuzzy MPC scheme; meanwhile, a piecewise Lyapunov function is defined and utilized to synthesize the controller, to deal with the output regulation problem for nonlinear systems. As the result, at each time instant k , the fuzzy MPC controller with the input constraint is computed by solving a convex optimization problem over the infinite horizon objective function. Sufficient conditions are derived for asymptotic stabilizing the closed-loop predictive control system in linear matrix inequalities (LMIs). In addition, a decay rate conditions are also derived to improve the transient

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control performance. Moreover, the piecewise Lyapunov function has been used to reduce the conservatism of the existing predictive controller. In order to reveal the validity of the proposed controller design, a nonlinear chaotic system with time varying parameter is considered to demonstrate the output regulation control task.

The contents of this paper are organized as follows. In Section II, problem formulation: coordinate translation and integral control are introduced. In Section III, Output regulation using integral fuzzy MPC based on piecewise Lyapunov functions is presented. The theorem result also includes the input constraint and decay rate conditions. Numerical simulation is carried out for the chaotic system to verify the proposed controller in Section IV. Finally, some conclusions are given in Section V.

II. PROBLEM FORMULATION

A. Coordinate Translation and Integral Control

Consider a general discrete-time nonlinear system as follows:

$$\begin{aligned} x_p(k+1) &= f(x_p(k)) + g(x_p(k))u(k) + \eta \\ y(k) &= h(x_p(k)) \end{aligned} \quad (1)$$

where $x(k) \in R^{n_x}$ denotes the state variables; $y(k) \in R^{n_y}$ is the output vector; $u(k) \in R^{n_u}$ is input vector; $f(x)$, $g(x)$ and $h(x)$ are nonlinear function vectors with appropriate dimensions and η is system bias term. Note that if the system is controllable and subject to a constant bias term, it may have $f(x_p(k)) = 0$ at $x_p(k) = 0$, $g(x_p(k)) \neq 0 \forall x_p(k)$ and output function may have an offset such that $h(x_p(k)) \neq 0$ at $x_p(k) = 0$. Let $y_d \in R^{n_y}$ be a constant reference. In order to ensure zero offset output regulation performance in the presence of plant parameter variations, we want to design an integrator based controller such that $y(k) \rightarrow y_d$ as $k \rightarrow \infty$. Its principle is based on the procedure of introducing an integral action in the forward channel of tracking system to guarantee tracking property. To this end, we introduce a new state variable to account the resulting integrated tracking error, denoted by $x_e(k)$, is computed as

$$x_e(k+1) = x_e(k) + y_d - y(k) \quad (2)$$

Combining (1) with (2), we obtain the augmented dynamics as follows

$$\begin{aligned} x_p(k+1) &= f(x_p(k)) + g(x_p(k))u(k) + \eta \\ x_e(k+1) &= x_e(k) + y_d - h(x_p(k)) \end{aligned} \quad (3)$$

Let us define the regulation error as $e(k) = y_d - y(k)$. Without loss of generality, there always exists some equilibrium point for constant output reference y_d in practical industrial process control, denoted by the steady state $\bar{x}_p \in R^{n_x}$ and the corresponding input $\bar{u} \in R^{n_u}$. Then, the objective of output regulation for biased nonlinear system will be achieved by stabilizing the system at an equilibrium state which produces $e(k) = 0$. For

this purpose, it follows that

$$\begin{aligned} \bar{x}_p - f(\bar{x}_p) - g(\bar{x}_p)\bar{u} - \eta &= 0 \\ y_d - h(\bar{x}_p) &= 0. \end{aligned} \quad (4)$$

Here (4) is assumed to have a unique solution $(\bar{x}_p, \bar{u})$. To eliminate the constant bias term η from the dynamics (3), we consider the coordinate translation: let $\tilde{x}_p(k) = x_p(k) - \bar{x}_p$ and $\tilde{u}(k) = u(k) - \bar{u}$. By using the property of (4), the dynamics of the system (3) can be written as follows:

$$\begin{aligned} \tilde{x}_p(k+1) &= f_s(\tilde{x}_p(k)) + g_s(\tilde{x}_p(k))\tilde{u}(k) \\ x_e(k+1) &= x_e(k) + h_s(\tilde{x}_p(k)) \end{aligned} \quad (5)$$

where $f_s(\tilde{x}_p(k)) = f(\tilde{x}_p(k) + \bar{x}_p) - f(\bar{x}_p) + [g(\tilde{x}_p(k) + \bar{x}_p) - g(\bar{x}_p)]\bar{u}$; $g_s(\tilde{x}_p(k)) = g(\tilde{x}_p(k) + \bar{x}_p)$; and $h_s(\tilde{x}_p(k)) = h(\tilde{x}_p(k) + \bar{x}_p) - h(\bar{x}_p)$. Clearly, $f_s(\tilde{x}_p(k)) = 0$, $h_s(\tilde{x}_p(k)) = 0$, $g_s(\tilde{x}_p(k)) \neq 0$, once $\tilde{x}_p(k) = 0$.

B. T-S Fuzzy Modeling

From (5), the transformed augmented system can be expressed as a state-space form with no bias terms and free-offset error signal $e(k)$:

$$\begin{aligned} \tilde{x}_p(k+1) &= A(\tilde{x}_p(k))\tilde{x}_p(k) + B(\tilde{x}_p(k))\tilde{u}(k) \\ x_e(k+1) &= x_e(k) + C(\tilde{x}_p(k))\tilde{x}_p(k) \end{aligned} \quad (6)$$

where A , B and C are the system matrix, the input matrix and the output matrix, respectively. Next, we employ the T-S fuzzy dynamic model proposed in [15] to represent the system (6) with fuzzy inference rules and local analytic linear models as follows:

Model Rule s :

IF $\tilde{z}_1(k)$ is F_{1s} and $\dots$ and $\tilde{z}_g(k)$ is F_{gs}

THEN $\tilde{x}_p(k+1) = A_s\tilde{x}_p(k) + B_s\tilde{u}(k)$

$$x_e(k+1) = x_e(k) + C_s\tilde{x}_p(k), \quad 1 \leq s \leq r$$

where $\tilde{z}(k) = [\tilde{z}_1(k) \tilde{z}_2(k) \dots \tilde{z}_g(k)]$ are known premise variables that may be functions of the state variables; $F_{js} (j = 1, 2, \dots, g)$ is the fuzzy set and r is the number of model rules; A_s , B_s and C_s are system matrices of appropriate dimensions. Using singleton fuzzifier, product inference and weighted average defuzzifier, the fuzzy system is inferred as follows:

$$\begin{aligned} \tilde{x}_p(k+1) &= \sum_{s=1}^r \mu_s(\tilde{z}(k)) \{A_s\tilde{x}_p(k) + B_s\tilde{u}(k)\} \\ x_e(k+1) &= x_e(k) + \sum_{s=1}^r \mu_s(\tilde{z}(k)) C_s\tilde{x}_p(k) \end{aligned} \quad (7)$$

where $\mu_s(\tilde{z}(k)) = w_s(\tilde{z}(k)) / \sum_{s=1}^r w_s(\tilde{z}(k))$ with $w_s(\tilde{z}(k)) = \prod_{j=1}^g F_{js}(\tilde{z}_j(k))$ for all k and $F_{js}(\tilde{z}_j(k))$ is the grade of membership of $\tilde{z}_j(k)$ in F_{js} . Note that $\sum_{s=1}^r \mu_s(\tilde{z}(k)) = 1$ for all k and $\mu_s(\tilde{z}(k)) \geq 0$, for $s = 1, 2, \dots, r$, are regarded as grade function. Denoting the augmented state vector as $x_a(k) = [\tilde{x}_p(k)^T \ x_e(k)^T]^T$, the augmented system (7) becomes as

$$x_a(k+1) = \sum_{s=1}^r \mu_i(\tilde{z}(k)) \{\tilde{A}_s x_a(k) + \tilde{B}_s u_a(k)\} \quad (8)$$

where

$$\tilde{A}_s = \begin{bmatrix} A_s & 0 \\ C_s & I \end{bmatrix}, \quad \tilde{B}_s = \begin{bmatrix} B_s \\ 0 \end{bmatrix}.$$

Then the resulting integral state feedback controller via the PDC

$$\begin{aligned} u_a(k) &= - \sum_{j=1}^r \mu_j(\tilde{z}(k)) K_j x_a(k) \\ &= - \sum_{j=1}^r \mu_j(\tilde{z}(k)) [K_{pt} \ K_{et}] \cdot \begin{bmatrix} \tilde{x}_p(k) \\ x_e(k) \end{bmatrix} \end{aligned} \quad (9)$$

where K_{pt} denotes the feedback gain for $\tilde{x}_p(k)$ and K_{et} is the gain associated with $x_e(k)$.

III. OUTPUT REGULATION USING IFMPC BASED ON PIECEWISE LYAPUNOV FUNCTIONS

A. Stability for Piecewise Lyapunov Functions

According to [9], the state space of the premise variable can be partitioned into a number of closed polyhedral regions, denoted as $\{\mathfrak{S}_m\}_{m \in N} \subseteq R^{n_x}$, where N is the index set of regions. For subsequent use, a set Γ that represents all possible transitions from one region to any region at the instant k is defined as

$$\Gamma := \{n, m | x_a(k) \in \mathfrak{S}_n, x_a(k+1) \in \mathfrak{S}_m\}. \quad (10)$$

A piecewise Lyapunov function for current region is defined as follows:

$$V(k) = x_a(k)^T P_n x_a(k), \quad x_a(k) \in \mathfrak{S}_n \quad (11)$$

Then we have

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= x_a(k+1)^T P_m x_a(k+1) - x_a(k)^T P_n x_a(k) \end{aligned} \quad (12)$$

If the Lyapunov function (12) satisfies that $\Delta V < 0$ for all time, the system can be guaranteed to be stable in the sense of Lyapunov.

B. Integral Fuzzy MPC with Input Constraint

In general, the MPC problem is formulated as solving optimal control problem subject to system dynamics and constraints involving states and controls. In this paper, we introduce a integral T-S fuzzy infinite horizon MPC scheme (IFMPC) to predict the future state of the plant (8) and state feedback control law (9) as follows:

$$\begin{aligned} x_a(k+i+1|k) &= \sum_{s=1}^r \mu_s(\tilde{z}(k+i|k)) \{ \tilde{A}_s x_a(k+i|k) \\ &\quad + \tilde{B}_s u_a(k+i|k) \} \end{aligned} \quad (13)$$

$$u_a(k+i|k) = - \sum_{j=1}^r \mu_j(\tilde{z}(k+i|k)) K_j(k) x_a(k+i|k). \quad (14)$$

where $x_a(k+i|k)$ denotes the system state at time $k+i$, predicted based on the measurements at time k and belongs to some region $x_a(k+i|k) \in \mathfrak{S}_n$; $u_a(k+i|k)$ denote the control input at time $k+i$, computed by the optimization problem at time k . The objective of the integral fuzzy predictive controller

is to compute an optimal control sequence with respect to the following infinite horizon objective function [3], [4] at each time instant k .

$$\min_{u_a(k+i|k)=K_j(k)x_a(k+i|k), \ k, i \geq 0, \ 1 \leq j \leq r} J_0^\infty(k) \quad (15)$$

subject to

$$\|u_a(k+i|k)\|_2 \leq u_{max}, \quad k, i \geq 0 \quad (16)$$

where

$$\begin{aligned} J_0^\infty(k) &= \sum_{i=0}^{\infty} (x_a(k+i|k)^T Q x_a(k+i|k) \\ &\quad + u_a(k+i|k)^T R u_a(k+i|k)) \\ &= x_a(k|k)^T Q x_a(k|k) + u_a(k|k)^T R u_a(k|k) \\ &\quad + J_1^\infty(k) \end{aligned} \quad (17)$$

with $Q \geq 0$ and $R > 0$ are weighted symmetric matrices. We introduce the following Lemmas which are needed for the proof of theorem.

Lemma 1 : Let μ_{1s} , μ_{2j} and h_r be the normalized membership functions. Then,

$$\sum_{s=1}^p \sum_{j=1}^q \mu_{1s} \mu_{2j} = \sum_{r=1}^{pq} h_r$$

with $h_r = \mu_{1.s(r)} \mu_{2.j(r)}$, where $s(r) = 1 + \lfloor \frac{r-1}{q} \rfloor$; $j(r) = r - \lfloor \frac{r-1}{q} \rfloor q$; and $\lfloor \cdot \rfloor$ denotes the floor function.

Lemma 2 : Let M_s be a matrix and h_s be the normalized membership function. Then,

$$\sum_{s=1}^r \sum_{j=1}^r h_s h_j M_s^T M_j \leq \sum_{s=1}^r h_s M_s^2$$

C. Output Regulation Using IFMPC Based on Piecewise Lyapunov Functions

Without loss of generality, suppose that the state will transit from one region $\mathfrak{S}_n$ to any possible region $\mathfrak{S}_m$ after instant k with $n, m \in \Gamma$. The piecewise Lyapunov function for predictive system at any time instant k can be defined by

$$\begin{aligned} V(i, k) &= x_a(k+i|k)^T P_n(k) x_a(k+i|k), \\ V(i+1, k) &= x_a(k+i+1|k)^T P_m(k) x_a(k+i+1|k), \\ n, m \in \Gamma, P_n(k) &> 0, P_m(k) > 0, \quad k, i \geq 0 \end{aligned} \quad (18)$$

where $P_n(k)$ denote the common positive definite matrix at time k and belong to region n . Let us suppose temporarily, that for any instant k and $i \geq 0$, the following inequality is satisfied:

$$\begin{aligned} \Delta V(i, k) &\leq -((x_a(k+i|k)^T Q x_a(k+i|k) \\ &\quad + u_a(k+i|k)^T R u_a(k+i|k)) \end{aligned} \quad (19)$$

Inspired by (19) and applying Lemma 1, Lemma 2 and rule reduction method in [15], we can lead to the following sufficient condition of stability:

$$\begin{aligned} G_{ss}^T P_m(k) G_{ss} - P_n(k) + Q + K_s(k)^T R K_s(k) &< 0, \\ 1 \leq s \leq r \end{aligned} \quad (20)$$

$$\begin{aligned} (\frac{G_{sj} + G_{js}}{2})^T P_m(k) (\frac{G_{sj} + G_{js}}{2}) - P_n(k) + Q &\leq 0, \\ 1 \leq s < j \leq r \end{aligned} \quad (21)$$

where $G_{sj} = \tilde{A}_s - \tilde{B}_s K_j(k)$. Further, summing (19) from $i = 1$ up to $i = \infty$ with $x_a(\infty|k) = 0$ and $V(\infty, k) = 0$, the following condition is obtained:

$$J_1^\infty(k) \leq V(1, k) = x_a(k+1|k)^T P_m(k) x_a(k+1|k) \quad (22)$$

Combining (17) and (22), we obtain the upper bound for $J_0^\infty(k)$ as follows

$$J_0^\infty(k) \leq x_a(k|k)^T Q x_a(k|k) + u_a(k|k)^T R u_a(k|k) + x_a(k+1|k)^T P_m(k) x_a(k+1|k) \quad (23)$$

In this paper, the output regulation problem under consideration is equivalent to design an integral fuzzy feedback controller (14) such that the resulting closed-loop fuzzy system (13) is asymptotically stable and the performance constraint (19) is fulfilled. Therefore the minimization problem (15) can be considered as $J_0^\infty(k) < \gamma < \infty$. Obviously, minimizing $J_0^\infty(k)$ with $P_i(k) > 0$ is equivalent to

$$\min_{\gamma, u_a(k|k), \tilde{P}_i(k), M_i(k), i \in N} \gamma \quad (24)$$

subject to

$$\begin{aligned} & x_a(k|k)^T Q x_a(k|k) + u_a(k|k)^T R u_a(k|k) \\ & + (\tilde{A} x_a(k|k) + \tilde{B} u_a(k|k))^T P_i(k) \\ & \times (\tilde{A} x_a(k|k) + \tilde{B} u_a(k|k)) \leq \gamma \end{aligned} \quad (25)$$

where $\tilde{A} = \sum_{s=1}^r \mu_i(\tilde{z}(k|k)) \tilde{A}_s$ and $\tilde{B} = \sum_{s=1}^r \mu_s(\tilde{z}(k|k)) \tilde{B}_s$.

Summarizing the above arguments, the output regulation control gain via integral fuzzy MPC scheme can be obtained under the following theorem.

Theorem 1: Given $x_a(k|k)$, u_{max} , and weighting matrices Q and R for the system (1), the state feedback control law (14) will satisfy the performance constraint (19) and the minimized objective function (15) with input constraint (16) at each time instant k if there exist $u_a(k|k)$, a positive scalar γ , symmetric positive definite matrices $\tilde{P}_n(k)$ and $\tilde{P}_m(k)$, matrices $M_{ns}(k)$ such that the following convex objective minimization problem is feasible:

$$\min_{\gamma, u_a(k|k), \tilde{P}_n(k), \tilde{P}_m(k), M_{ns}(k)} \gamma$$

subject to

$$\begin{bmatrix} \tilde{P}_n(k) & * \\ M_{ns}(k) & u_{max}^2 I \end{bmatrix} \geq 0, \quad 1 \leq s \leq r \quad (26)$$

$$\begin{bmatrix} 1 & * & * & * \\ (\tilde{A} x_a(k|k) + \tilde{B} u_a(k|k)) & \tilde{P}_m(k) & * & * \\ Q^{1/2} x_a(k|k) & 0 & \gamma I & * \\ R^{1/2} u_a(k|k) & 0 & 0 & \gamma I \end{bmatrix} \geq 0, \quad 1 \leq s \leq r \quad (27)$$

$$\begin{bmatrix} \tilde{P}_n(k) & * & * & * \\ \tilde{A}_s \tilde{P}_n(k) - \tilde{B}_s M_{ns}(k) & \tilde{P}_m(k) & * & * \\ Q^{1/2} \tilde{P}_n(k) & 0 & \gamma I & * \\ R^{1/2} M_{ns}(k) & 0 & 0 & \gamma I \end{bmatrix} > 0, \quad n, m \in \Gamma, 1 \leq s \leq r \quad (28)$$

$$\begin{bmatrix} 4\tilde{P}_n(k) & * & * \\ \Xi & \tilde{P}_m(k) & * \\ 2Q^{1/2}\tilde{P}_n(k) & 0 & \gamma I \end{bmatrix} \geq 0, \quad 1 \leq s < j \leq r \quad (29)$$

where the symbol $*$ induces a symmetric structure, $\Xi = \tilde{A}_s \tilde{P}_n(k) - \tilde{B}_s M_{nj}(k) + \tilde{A}_j \tilde{P}_n(k) - \tilde{B}_j M_{ns}(k)$, $\tilde{P}_m(k) = \gamma P_m(k)^{-1}$, $\tilde{P}_n(k) = \gamma P_n(k)^{-1}$, $M_{ns}(k) = K_s(k) \tilde{P}_n(k)$. Moreover, the closed-loop regulation control system (13) is asymptotically stable.

In addition, the speed of the closed-loop system response can be influenced by specifying a desired decay rate on the piecewise Lyapunov function as follows [16]:

$$\Delta V(i, k) \leq (\alpha - 1)V(i, k) \quad (30)$$

The requirement (30) denotes the *Decay Rate* condition which will significantly improve the transient response of the closed-loop system by carefully choosing $0 < \alpha < 1$. Hence, it can be shown that (30) becomes the LMIs as follows:

$$\begin{bmatrix} \alpha \tilde{P}_n(k) & * \\ \tilde{A}_s \tilde{P}_n(k) - \tilde{B}_s M_{ns}(k) & \tilde{P}_m(k) \end{bmatrix} > 0, \quad 1 \leq s \leq r \quad (31)$$

$$\begin{bmatrix} 4\alpha \tilde{P}_n(k) & * \\ \Pi & \tilde{P}_m(k) \end{bmatrix} \geq 0, \quad 1 \leq s < j \leq r \quad (32)$$

where $\Pi = \tilde{A}_s \tilde{P}_n(k) - \tilde{B}_s M_{nj}(k) + \tilde{A}_j \tilde{P}_n(k) - \tilde{B}_j M_{ns}(k)$.

Remark 1: In above theorem 1, the controller design is called integral fuzzy MPC using piecewise Lyapunov function (IFMPC-PLF) which can directly extend to the integral fuzzy MPC with decay rate condition using piecewise Lyapunov function (IFMPC-PLFDR) by including LMIs (31) and (32).

Remark 2: In fact, the common Lyapunov function can be considered as the special case of the more general piecewise Lyapunov function [14]. So, the integral fuzzy MPC with decay rate using common Lyapunov function method (IFMPC-CLFDR) can be obtained by replacing $\tilde{P}_n(k)$ and $\tilde{P}_m(k)$ with $\bar{P}(k)$, from (26) to (29) and (31) to (32).

IV. NUMERICAL EXAMPLE

The aim of this numerical section is to verify the validity of the proposed integral fuzzy MPC regulation controller for the nonlinear chaotic system with time-varying parameter. For comparing control laws, three integral fuzzy MPC approaches based on common Lyapunov function with decay rate, piecewise Lyapunov function and piecewise Lyapunov function with decay rate are investigated respectively.

A. Plant Description

Consider the Hénon map chaotic system [15], [17] given as follows:

$$\begin{aligned} x_1(k+1) &= -x_1^2(k) + \beta \cdot x_2(k) + 1.4 + u(k) \\ x_2(k+1) &= x_1(k) \\ \beta &= 0.3 \end{aligned} \quad (33)$$

For the integral output regulation control purpose, we define output equation:

$$y(k) = x_2(k) + 1 \quad (34)$$

and assume that $y(k) \rightarrow y_d$ as $k \rightarrow \infty$.

From the model (33), let the desired output $y_d = \lambda$, we can obtain the equilibrium points of the state and control input $\bar{x}_1 = \lambda - 1$, $\bar{x}_2 = \lambda - 1$ and $\bar{u} = \lambda^2 - (1 + \beta)\lambda + \beta - 1.4$. Then the transformed augmented error dynamics (6) of the chaotic system is of the following form:

$$x_a(k+1) = \tilde{A}(x_p(k))x_a(k) + \tilde{B}u_a(k) \quad (35)$$

where $x_a(k) = [\tilde{x}_1(k) \ \tilde{x}_2(k) \ x_e(k)]^T$, $\tilde{B} = [1 \ 0 \ 0]^T$ and

$$\tilde{A}(x_p(k)) = \begin{bmatrix} -\tilde{x}_1(k) - 2\bar{x}_1(k) & \beta & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

In the simulation, we assume the parameter β is time-varying and has ± 0.1 deviation on its nominal value. i.e., $\beta(k) = 0.1 * \sin(2\pi \cdot k/5) + \beta_0$, and $\beta_0 = 0.3$ is the nominal value. From (35), two premise variables can be defined as $\tilde{z}_1(k) = \tilde{x}_1(k)$ and $\tilde{z}_2(k) = \beta(k)$. Since having two variables in $\tilde{A}$, only four fuzzy rules are needed. Then, the dynamical equation (35) can be exactly presented as the following four-rules T-S fuzzy model:

Model Rule r :

IF $\tilde{z}_1(k)$ is $F_{1s(r)}$ and $\tilde{z}_2(k)$ is $F_{2j(r)}$

Then $x_a(k+1) = \tilde{A}_r x_a(k) + \tilde{B}_r u_a(k)$
 $r = 1, 2, 3, 4$

Let $\mu_{1s(r)}$ and $\mu_{2j(r)}$ be the normalized membership functions for $F_{1s(r)}$ and $F_{2j(r)}$ respectively, i.e., $\sum_{s=1}^2 \mu_{1s} = \sum_{j=1}^2 \mu_{2j} = 1$ and $0 \leq \mu_{1s(r)} \cdot \mu_{2j(r)} \leq 1$ for each $s(r)$ and $j(r)$. The system is inferred as follows:

$$x_a(k+1) = \sum_{r=1}^4 h_r(\tilde{z}(k)) \{ \tilde{A}_r x_a(k) + \tilde{B}_r u_a(k) \} \quad (36)$$

where $h_r(\tilde{z}(k)) = \mu_{1s(r)}(\tilde{z}(k)) \cdot \mu_{2j(r)}(\tilde{z}(k))$ for $r = 1, 2, 3, 4$. Due to the $\beta(k)$ is time-varying, the regions of the nonlinear terms are assumed with $\tilde{x}_1(k) \in [-d, d]$ and $\beta(k) \in [d_1, D_1]$, where $d = 3$, $d_1 = \beta_0 - 0.1$, $D_1 = \beta_0 + 0.1$, $\tilde{x}_1(k) = \mu_{11}d + \mu_{12}(-d)$ and $\beta(k) = \mu_{21}D_1 + \mu_{22}d_1$. According to [17], we can define the normalized membership functions as

$$\mu_{11} = \frac{1}{2} + \frac{\tilde{x}_1(k)}{2d}, \quad \mu_{12} = \frac{1}{2} - \frac{\tilde{x}_1(k)}{2d}$$

$$\mu_{21} = \frac{\beta(k) - d_1}{D_1 - d_1}, \quad \mu_{22} = \frac{D_1 - \beta(k)}{D_1 - d_1}$$

From Lemma 1, we obtain $s(r)$ and $j(r)$ and then $\tilde{A}_r$, $\tilde{B}_r$ and h_r , which are summarized in Table I. As a result, the system matrices in the consequent part are:

$$\tilde{A}_1 = \begin{bmatrix} d - 2\bar{x}_1 & d_1 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix},$$

$$\tilde{A}_2 = \begin{bmatrix} d - 2\bar{x}_1 & D_1 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix},$$

$$\tilde{A}_3 = \begin{bmatrix} -d - 2\bar{x}_1 & d_1 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix},$$

$$\tilde{A}_4 = \begin{bmatrix} -d - 2\bar{x}_1 & D_1 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix},$$

and $\tilde{B}_r = [1 \ 0 \ 0]^T$, $r = 1, 2, 3, 4$.

TABLE I
System Matrices and Membership Functions of The Model

r	$s(r)$	$j(r)$	$\tilde{A}_r$	$\tilde{B}_r$	h_r
1	1	1	$\tilde{A}_1$	$\tilde{B}_1$	$h_1 = \mu_{11}\mu_{21}$
2	1	2	$\tilde{A}_2$	$\tilde{B}_2$	$h_2 = \mu_{11}\mu_{22}$
3	2	1	$\tilde{A}_3$	$\tilde{B}_3$	$h_3 = \mu_{12}\mu_{21}$
4	2	2	$\tilde{A}_4$	$\tilde{B}_4$	$h_4 = \mu_{12}\mu_{22}$

Consequently, the resulting fuzzy model can exactly represent the chaotic system, while the time varying parameter is put into the membership functions.

B. Simulation with IFMPC-CLFDR, IFMPC-PLF and IFMPC-PLFDR

To proceed on the simulation, we apply IFMPC-CLFDR, IFMPC-PLF, and IFMPC-PLFDR approaches to output regulation control the chaotic system with the same initial values and parameters for comparison as follows: $x_a(0) = [0.5 \ -0.5 \ 0.1]^T$, $\tilde{u}(0) = 1$, $u_{max} = 5$ and the desired output reference $y_d = 2$. Moreover, we choose $Q = 0.001 \times \text{diag}\{10, 10, 5\}$, $R = 0.01$, $\gamma = 2$, and $\alpha = 0.5$ by try and error method. For the regulation control problem, there is feasible solution for IFMPC-CLFDR, and the other two methods not only lead to feasible solutions but also obtain good performance for regulation purpose even though exist the parameter variation. For IFMPC-CLFDR, the corresponding common positive definite matrix and integral fuzzy MPC state feedback gains are given as follows:

$$P = \begin{bmatrix} 5.8632 & 2.0437 & -2.2263 \\ 2.0437 & 1.3425 & -1.2982 \\ -2.2263 & -1.2982 & 1.3313 \end{bmatrix},$$

and

$$K_1 = [1.2522 \ 0.5664 \ -0.3620],$$

$$K_2 = [1.2516 \ 0.7654 \ -0.3618],$$

$$K_3 = [-4.2460 \ 0.6587 \ -0.4702],$$

$$K_4 = [-4.2619 \ 0.8382 \ -0.4646].$$

For both IFMPC-PLF and IFMPC-PLFDR, the state space of premise variable is partitioned into two regions. The corresponding two different positive definite matrices and the integral fuzzy MPC state feedback gains are given, respectively,

$$P1 = \begin{bmatrix} 1.0557 & -0.0347 & -0.0006 \\ -0.0347 & 0.4289 & -0.0966 \\ -0.0006 & -0.0966 & 0.4188 \end{bmatrix},$$

$$P2 = \begin{bmatrix} 0.1028 & 0.0180 & 0.0016 \\ 0.0180 & 0.1291 & 0.0010 \\ 0.0016 & 0.0010 & 0.0954 \end{bmatrix},$$

and

$$\begin{aligned} K_1 &= \begin{bmatrix} 0.5268 & 0.1651 & 0.0075 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} 0.5255 & 0.3054 & -0.0008 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} -2.7093 & 0.1090 & 0.0077 \end{bmatrix}, \\ K_4 &= \begin{bmatrix} -2.7068 & 0.2177 & -0.0056 \end{bmatrix}, \end{aligned}$$

for IFMPC-PLF. Similarly,

$$\begin{aligned} P1 &= \begin{bmatrix} 1.4149 & -0.0431 & 0.0001 \\ -0.0431 & 0.6706 & -0.2723 \\ 0.0001 & -0.2723 & 0.6641 \end{bmatrix}, \\ P2 &= \begin{bmatrix} 0.1927 & 0.0172 & 0.0011 \\ 0.0172 & 0.1690 & 0.0011 \\ 0.0011 & 0.0011 & 0.1225 \end{bmatrix}, \end{aligned}$$

and

$$\begin{aligned} K_1 &= \begin{bmatrix} 0.7880 & 0.2005 & 0.0054 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} 0.7866 & 0.3856 & 0.0028 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} -3.8631 & 0.1533 & 0.0023 \end{bmatrix}, \\ K_4 &= \begin{bmatrix} -3.8564 & 0.3041 & -0.0092 \end{bmatrix}, \end{aligned}$$

for IFMPC-PLFDR. The open-loop response for chaotic system is shown in Fig. 1. The closed-loop control results are shown in Figs. 2 to 5. It is shown that the proposed IFMPC-PLFDR obtains much better transient control performance than IFMPC-CLFDR and IFMPC-PLF under the same turning parameters and input constraints. Notice that three integral fuzzy MPC control methods have steered the chaotic system to regulate $y_d = 2$ trajectory successfully. From the simulation results, we conclude that the integral fuzzy MPC scheme based on piecewise Lyapunov function with a decay rate is a powerful design. It can regularly control Hénon map system, achieving asymptotically stable, shorter transient time, lower overshoot and rejecting the parameter variation.

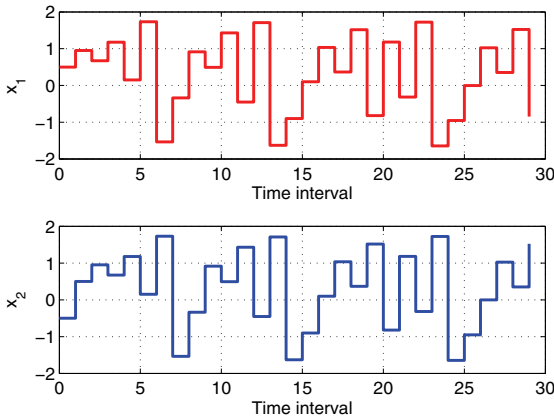


Fig. 1. Open-loop response for (a)state x_1 and (b)state x_2 .

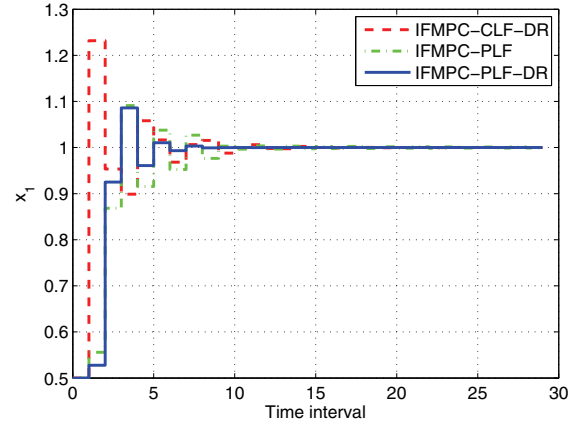


Fig. 2. Close-loop response for state x_1 controlled by three different IFMPC methods.

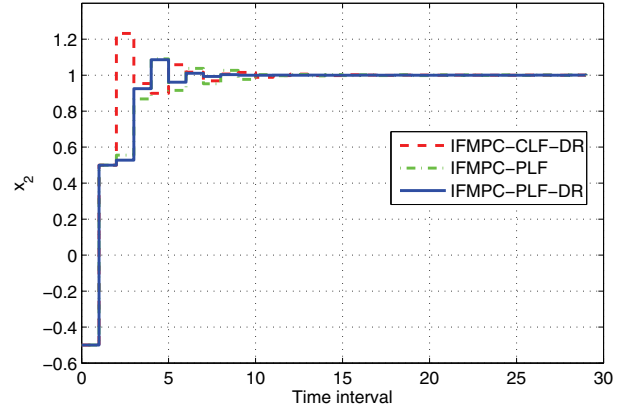


Fig. 3. Close-loop response for state x_2 controlled by three different IFMPC methods.

V. CONCLUSION

In this paper, an integral fuzzy MPC design for a nonlinear system using piecewise Lyapunov functions has been proposed to achieve output regulation. To this end, the nonlinear system is expressed by Takagi-Sugeno (T-S) fuzzy model via integral control and coordinate translation procedure. Then, the integral fuzzy regulation controller with input constraint is designed by infinite horizon MPC strategy and is formulated in terms of the LMIs. Finally, the numerical simulations for a chaotic system have shown the effectiveness of the proposed methods. It has been demonstrated that the IFMPC-PLFDR are less conservatism, but with short transient response, and zero-offset regulation performance when compared to IFMPC-CLF method.

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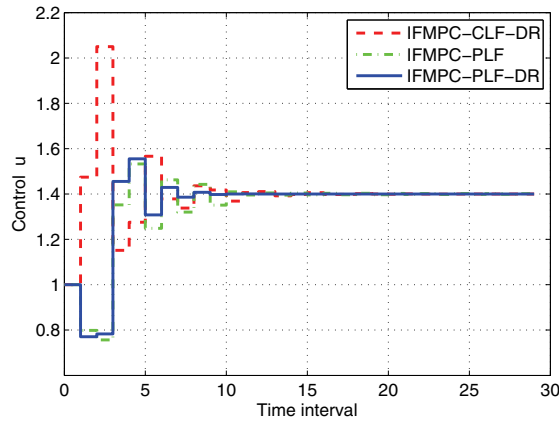


Fig. 4. Control input controlled by three different IFMPC methods.

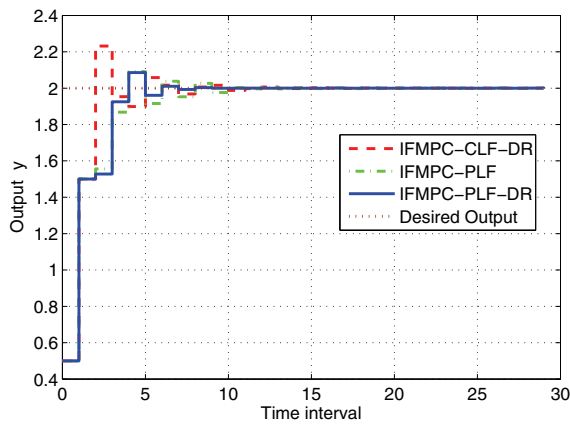


Fig. 5. Output regulation trajectory controlled by three different IFMPC methods.

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